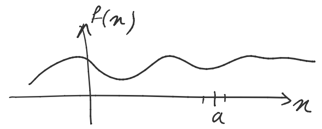
Oct 19, 2020

Friday, October 16, 2020 12:13 PM

$$\lim_{x\to a} f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)(x-a)^2}{2!} + \frac{f'''(a)(x-a)^3}{3!} + \dots$$



e.g. $x - a = 0.1$

$(x-a)^2 = 0.01$

0.001

if x is close to a

$$f(x) \approx f(a) + f'(a)(x-a)$$

$x - a = y$

$a = \Delta y$

$$f(y+\Delta y) \approx f(y) + f'(y)\Delta y$$

$f(x)$

$f(x,y)$ $\quad I(x,y)$

$$\frac{df(x,y)}{dx} \qquad \frac{d\left(\frac{df}{dx}\right)}{dx} = \frac{d^2f}{dx^2} \qquad \frac{df^2}{dx\,dy} \qquad \frac{\partial f}{\partial x}$$

Leibniz notation

Lagrange's Notation

$$f' = \frac{df}{dx} \qquad f'(x) = \frac{df(x)}{dx}$$

Newton's Notation

$$y(t) \rightarrow \dot{y}(t) = \frac{dy}{dt}$$

$$\ddot{y} = \frac{d^2y}{dt^2}$$

$f(x,y)$

$f_x = \frac{df}{dx}$

$f_y = \frac{df}{dy}$

$f_{xy} = \frac{df^2}{dx\,dy}$

f_{xx}

This should be d^2f

$I(x,y)$

$$I(x+u, y+v) \simeq I(x,y) + u \cdot \frac{\partial I(x,y)}{\partial x} + v \cdot \frac{\partial I(x,y)}{\partial y}$$

if u & v small

$$= I + u \cdot I_x + v\, I_y$$

$f(x)$ continuous $\quad \Delta x \quad 0.01$

a couple of things I didn't get the time to talk about during the lecture

1) Matrix A (e.g. 2×2) is "positive definite" if all its eigenvalues are positive (e.g. $\lambda_1 > 0$ & $\lambda_2 > 0$ if $A_{2\times2}$)

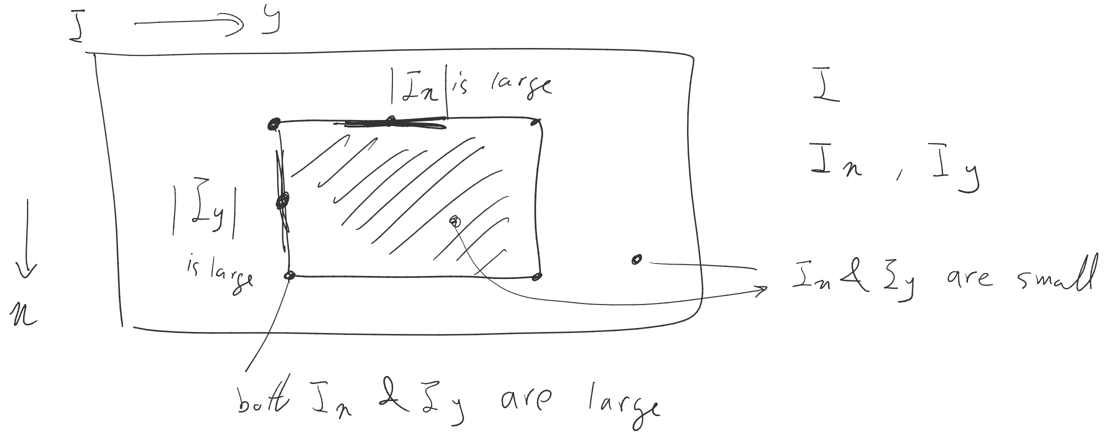
Equivalently, $\vec{v}^T A \vec{v} > 0$ for any vector $\vec{v}$ that is not $\vec{0}$ (i.e. $\begin{bmatrix}0\\0\end{bmatrix}$)

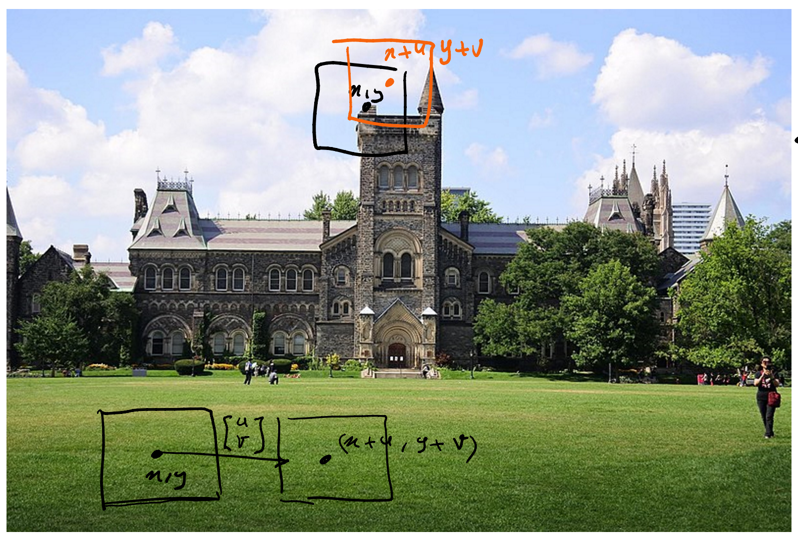
Similarly, matrix A is "positive semi-definite" if its eigenvalues are non-negative ≥ 0 or $\vec{v}^T A \vec{v} \geq 0$

2) if A is symmetric <u>and</u> if A can be written as $A = B^TB$ (Cholesky Decomposition) then A is positive definite.

$$\vec{v}^T A \vec{v} = \vec{v}^T (B^TB)\vec{v} = (\vec{v}^T B^T)(B\vec{v})$$

$f(x)$ continuous Δx 0.01
image smallest u & $v = 1$

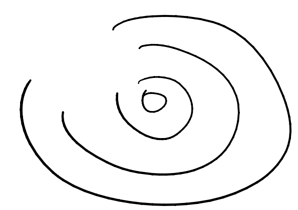


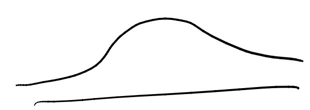


$E_{WSSD}(u,v) =$

$$\sum_x \sum_y w(x,y)\Big(\underbrace{I(x,y)}_{\text{intensity}} - \underbrace{I(x+u,\,y+v)}_{\text{shifted intensity}}\Big)^2$$

$G(x,y)$





$\sum w = 1$

W: window function
(weight)

we want $E_{WSSD}(u,v)$ to be large for any $\overset{\text{small}}{}$ u & v

$$\sum_x \sum_y W(x,y)\Big(I(x,y) - \underline{I(x+u,\,y+v)}\Big)^2$$

$v^T A v = v^T (B^T B) v =$

$= (B\vec v)^T (B\vec v) = \vec u^T \vec u > 0$

define $\vec u = B\vec v$ if $\vec u \neq \vec 0$

3) Re: Corner Detection

$$[u\ \ v]\, M \begin{bmatrix}u\\v\end{bmatrix} = [u\ \ v]\, V \Sigma V^{-1} \begin{bmatrix}u\\v\end{bmatrix}$$

$\vec e = \begin{bmatrix}u\\v\end{bmatrix}$, $V^{-1} = V^T$

$= (\vec e^{\,T} V)\, \Sigma\, (V^T \vec e)$

$= (V^T \vec e)^T\, \Sigma\, (V^T \vec e)$

$(\vec e\,' = V^T \vec e)$

$= \vec e\,'^T\, \Sigma\, \vec e\,'$ $\vec e\,' = \begin{bmatrix}a\\b\end{bmatrix}$

$= \lambda_1 a^2 + \lambda_2 b^2$

to make this large

$x \quad y$

$$\approx \sum_x \sum_y w(x,y) \left(\cancel{I(x,y)} - \cancel{I(x,y)} - \underbrace{u \frac{\partial I(x,y)}{\partial x}}_{u I_x} - \underbrace{v \frac{\partial I(x,y)}{\partial y}}_{v I_y} \right)^2$$

$$= \sum_x \sum_y w(x,y) \left(- u I_x - v I_y \right)^2$$

$$= \sum_x \sum_y w(x,y) \left(\underbrace{u^2 I_x^2 + 2 u v I_x I_y + v^2 I_y^2} \right)$$

$$= \sum_x \sum_y w(x,y) \left(\begin{bmatrix} u & v \end{bmatrix} \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} \right)$$

$$= \begin{bmatrix} u & v \end{bmatrix}_{1\times 2} \underbrace{\left(\sum_x \sum_y w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix} \right)_{2\times 2}}_{M} \begin{bmatrix} u \\ v \end{bmatrix}_{2\times 1}$$

$$\begin{bmatrix} u & v \end{bmatrix} M_{2\times 2} \begin{bmatrix} u \\ v \end{bmatrix} \qquad \vec{e} = \begin{bmatrix} u \\ v \end{bmatrix}$$

$$\boxed{E_{WSSD}(\vec{e}) = \vec{e}^{T} M \vec{e}}$$

for any small u & v
↳ we want large $E(u,v)$

λ_1 & λ_2 should be large

+ WSSD

↳ we want large $E(u,v)$

S.V.D $\quad A_{m\times n} = U \Sigma V^T$

$$\begin{bmatrix} & \\ & \end{bmatrix} = \begin{bmatrix} & & \\ & & \end{bmatrix}\begin{bmatrix} & \\ & \end{bmatrix}\begin{bmatrix} & \\ & \end{bmatrix}$$

if A is square : E.V.D.

$$A\vec{v} = \lambda \vec{v} \longrightarrow \underbrace{(A - \lambda I)}_{\times}\vec{v} = 0$$

$$A_{m\times m} = V \Sigma V^{-1} = V \Sigma V^T \qquad \sigma_i = |\lambda_i|$$

if A: real & symmetric $\longrightarrow \lambda$ real

$$V^T = V^{-1} \qquad V V^T = V^T V = I$$

$$M_{2\times 2} = \sum_x \sum_y w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

M: 2nd moment matrix (structure tensor)

$\sum_x \sum_y w \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$: symmetric

M: symmetric & real valued $\longrightarrow$ eigenvalues are real

$\det\left(\begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}\right) = 0$ $\qquad I_x(x,y)$

$$I_x^2 I_y^2 - I_x I_y I_x I_y = 0$$

$\det = 0 \longrightarrow \lambda_1 = 0$

$\lambda_2 \geq 0$

sum of positive semi-definite matrices $\longrightarrow$ +ive semi-definite

$M: \lambda_1, \lambda_2 \geq 0$

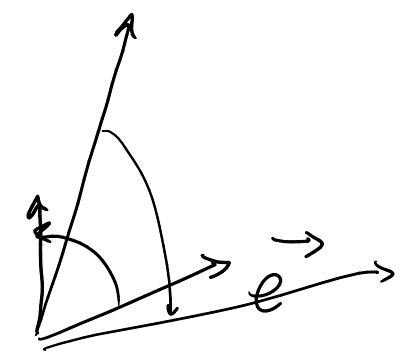
$M \vec{e} = (V \Sigma V^{-1}) \vec{e} = V \Sigma (V^{-1} \vec{e})$

$$M\vec{e} = (V\Sigma V')\vec{e} = V\Sigma(V' e)$$

V: orthonormal

$VV^T = I$

$$R^T = R^{-1}$$



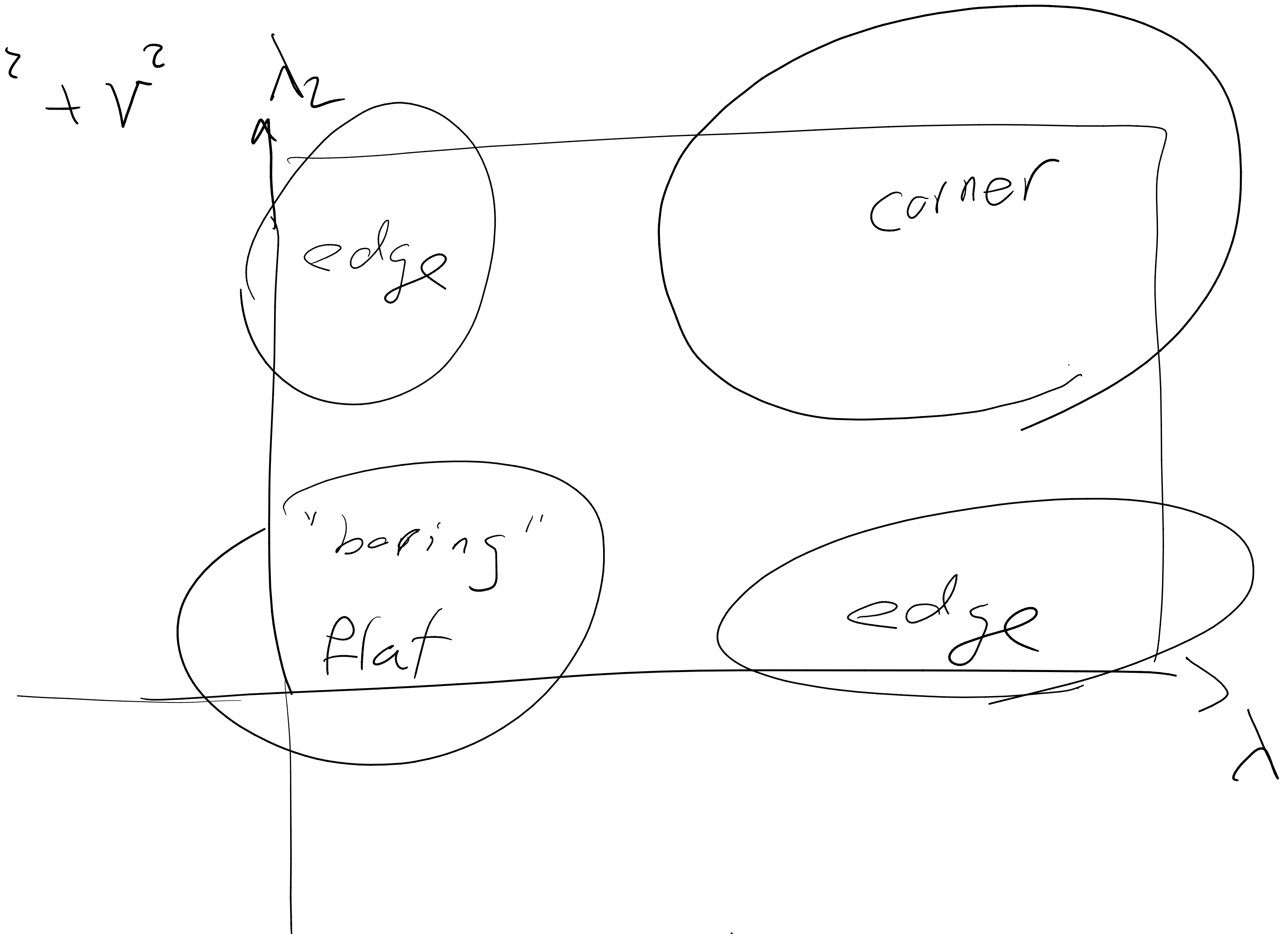
$$\begin{bmatrix}\lambda_1 & 0 \\ 0 & \lambda_2\end{bmatrix}\begin{bmatrix}a \\ b\end{bmatrix}$$

$$E_{wssD}(u, v) = [u, v] \; M \begin{bmatrix}u \\ v\end{bmatrix}$$

$$= [u \;\; v] \underbrace{V \begin{bmatrix}\lambda_1 & 0 \\ 0 & \lambda_2\end{bmatrix} V^{-1}}_{M} \begin{bmatrix}u \\ v\end{bmatrix}$$

$$\begin{bmatrix}1 & \\ & 1\end{bmatrix}$$

$$e^T e = [u \quad v]\begin{bmatrix} u \\ v \end{bmatrix} = \vec{e}\cdot\vec{e} = u^2 + v^2$$



λ_1 : large

λ_2 : large

$$[u \quad v]\, M \begin{bmatrix} u \\ v \end{bmatrix}$$

we want M to have large eigenvalues

M : real & symmetric

$$\det(AB) = \det(A)\det(B)$$

$M = \quad V \quad \Sigma \quad V^{-1}$

$\det(AB) = \det(A)\det(B)$

$M = V \Sigma V$

$\det(M) = \underline{\det}(V)\det(\Sigma)\underline{\det(V^{-1})}$

V: orthonormal $\quad V V^T = 1 \quad$ V: rotation

$$\det\left(\begin{bmatrix} \cos(\theta) & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}\right)$$

$\cos^2\theta + \sin^2\theta = 1$

$\det(V) = 1 \quad \det(V^{-1}) = 1$

$$\det(M) = \det\left(\begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}\right) = \lambda_1\lambda_2 - 0 = \underline{\lambda_1\lambda_2}$$

$\underline{\text{trace}}(M) = \lambda_1 + \lambda_2$