

## Homework 1

*Arindam Karmakar***1 Assignments:**

- a) A integer number  $n$  is divisible by three if the sum of it's digits is divisible by three. For example, 5432157 is divisible by 3 because  $5 + 4 + 3 + 2 + 1 + 5 + 7 = 27$  is divisible by 3. But there is a problem. We can only divide single digit numbers. Therefore, we cannot check whether 3 divides 27 or not. We add the digits again,  $2 + 7 = 9$ . Now we can verify that 9 is divisible by 3, Therefore 5432157 is divisible by 3. **Divisibility by 3** problem in this case required two iterations.

Given any arbitrarily large integer number  $N$  with  $n$  digits, determine the number of iterations required to solve **Divisibility by 3** problem as a function of  $n$ . At each iteration, we add up the digits of the number and find whether the summation is a single digit number or not. Continue the process until I get a single digit number.

**Hint:** Any integer number  $X$  can be represented by  $\log_{10} X$  digits.

- b) In the following algorithm, explain how many iterations (in terms of  $m$ ) will be required to obtain the final solution.

**Algorithm 1** Euclid's algorithm

---

1: <b>procedure</b> EUCLID( $m, n$ )	▷ The g.c.d. of $m$ and $n$
2: $r \leftarrow m - n$	
3: <b>while</b> $r \neq 0$ <b>do</b>	▷ We have the answer if $r$ is 0
4: $m \leftarrow n$	
5: $n \leftarrow r$	
6: $r \leftarrow m \bmod n$	
7: <b>end while</b>	
8: <b>return</b> $n$	▷ The gcd is $n$
9: <b>end procedure</b>	

---

- c) Find a closed formula for the following summation:

$$\sum_{i=0}^{n-1} \sum_{j=i}^{n-1} j - i + 1$$