Suggestions for further reading

Rather than attempt a complete bibliography, I will give a number of basic references. I will begin with historical references and textbooks. I will then give references for specific topics, more or less in the order in which topics appear in the text. Where material has been collected in one or another book, I have often referred to such books rather than to original articles. However, the importance and quality of exposition of some of the original sources often make them still to be preferred today. The subject in its earlier days was blessed with some of the finest expositors of mathematics, for example Steenrod, Serre, Milnor, and Adams. Some of the references are intended to give historical perspective, some are classical papers in the subject, some are follow-ups to material in the text, and some give an idea of the current state of the subject. In fact, many major parts of algebraic topology are nowhere mentioned in any of the existing textbooks, although several were well established by the mid-1970s. I will indicate particularly accessible references for some of them; the reader can find more of the original references in the sources given.

1. A classic book and historical references

The axioms for homology and cohomology theories were set out in the classic:

S. Eilenberg and N. Steenrod. Foundations of algebraic topology. Princeton University Press. 1952.

I believe the only historical monograph on the subject is:

J. Dieudonné. A history of algebraic and differential topology, 1900–1960. Birkhäuser. 1989.

A large collection of historical essays will appear soon:

I.M. James, editor. The history of topology. Elsevier Science. To appear.

Among the contributions, I will advertise one of my own, available on the web:

J.P. May. Stable algebraic topology, 1945–1966. http://hopf.math.purdue.edu

2. Textbooks in algebraic topology and homotopy theory

These are ordered roughly chronologically (although this is obscured by the fact that the most recent editions or versions are cited). I have included only those texts that I have looked at myself, that are at least at the level of the more elementary chapters here, and that offer significant individuality of treatment. There are many other textbooks in algebraic topology.

Two classic early textbooks:

P.J. Hilton and S. Wylie. Homology theory. Cambridge University Press. 1960.

E. Spanier. Algebraic topology. McGraw-Hill. 1966.

An idiosyncratic pre-homology level book giving much material about groupoids:

R. Brown. Topology. A geometric account of general topology, homotopy types, and the fundamental groupoid. Second edition. Ellis Horwood. 1988.

A homotopical introduction close to the spirit of this book:

B. Gray. Homotopy theory, an introduction to algebraic topology. Academic Press. 1975.

The standard current textbooks in basic algebraic topology:

M.J. Greenberg and J. R. Harper. Algebraic topology, a first course. Benjamin/Cummings. 1981.

W.S. Massey. A basic course in algebraic topology. Springer-Verlag. 1991.

A. Dold. Lectures on algebraic topology. Reprint of the 1972 edition. Springer-Verlag. 1995.

J.W. Vick. Homology theory; an introduction to algebraic topology. Second edition. Springer-Verlag. 1994.

J.R. Munkres. Elements of algebraic topology. Addison Wesley. 1984.

J.J. Rotman. An introduction to algebraic topology. Springer-Verlag. 1986.

G.E. Bredon. Topology and geometry. Springer-Verlag. 1993.

Sadly, the following are still the only more advanced textbooks in the subject:

R.M. Switzer. Algebraic topology. Homotopy and homology. Springer-Verlag. 1975.

G.W. Whitehead. Elements of homotopy theory. Springer-Verlag. 1978.

3. Books on CW complexes

Two books giving more detailed studies of CW complexes than are found in textbooks (the second giving a little of the theory of compactly generated spaces):

A.T. Lundell and S. Weingram The topology of CW complexes. Van Nostrand Reinhold. 1969.

R. Fritsch and R.A. Piccinini. Cellular structures in topology. Cambridge University Press. 1990.

4. Differential forms and Morse theory

Two introductions to algebraic topology starting from de Rham cohomology:

R. Bott and L.W. Tu. Differential forms in algebraic topology. Springer-Verlag. 1982.

I. Madsen and J. Tornehave. From calculus to cohomology. de Rham cohomology and characteristic classes. Cambridge University Press. 1997.

The classic reference on Morse theory, with an exposition of the Bott periodicity theorem:

J. Milnor. Morse theory. Annals of Math. Studies No. 51. Princeton University Press. 1963.

A modern use of Morse theory for the analytic construction of homology:

M. Schwarz. Morse homology. Progress in Math. Vol. 111. Birkhäuser. 1993.

5. Equivariant algebraic topology

Two good basic references on equivariant algebraic topology, classically called the theory of transformation groups (see also §§16, 21 below):

G. Bredon. Introduction to compact transformation groups. Academic Press. 1972.

T. tom Dieck. Transformation groups. Walter de Gruyter. 1987.

A more advanced book, a precursor to much recent work in the area:

T. tom Dieck. Transformation groups and representation theory. Lecture Notes in Mathematics Vol. 766. Springer-Verlag. 1979.

6. Category theory and homological algebra

A revision of the following classic on basic category theory is in preparation:

S. Mac Lane. Categories for the working mathematician. Springer-Verlag. 1971.

Two classical treatments and a good modern treatment of homological algebra:

H. Cartan and S. Eilenberg. Homological algebra. Princeton University Press. 1956.

S. MacLane. Homology. Springer-Verlag. 1963.

C.A. Weibel. An introduction to homological algebra. Cambridge University Press. 1994.

7. Simplicial sets in algebraic topology

Two older treatments and a comprehensive modern treatment:

P. Gabriel and M. Zisman. Calculus of fractions and homotopy theory. Springer-Verlag. 1967.

J.P. May. Simplicial objects in algebraic topology. D. Van Nostrand 1967; reprinted by the University of Chicago Press 1982 and 1992.

P.G. Goerss and J.F. Jardine. Simplicial homotopy theory. Birkhäuser. To appear.

8. The Serre spectral sequence and Serre class theory

Two classic papers of Serre:

J.-P. Serre. Homologie singuliére des espaces fibrés. Applications. Annals of Math. (2)54(1951), 425–505.

J.-P. Serre. Groupes d'homotopie et classes de groupes abéliens. Annals of Math. (2)58(1953), 198–232.

A nice exposition of some basic homotopy theory and of Serre's work:

S.-T. Hu. Homotopy theory. Academic Press. 1959.

Many of the textbooks cited in §2 also treat the Serre spectral sequence.

9. The Eilenberg-Moore spectral sequence

There are other important spectral sequences in the context of fibrations, mainly due to Eilenberg and Moore. Three references:

S. Eilenberg and J.C. Moore. Homology and fibrations, I. Comm. Math. Helv. 40(1966), 199–236.

L. Smith. Homological algebra and the Eilenberg-Moore spectral sequences. Trans. Amer. Math. Soc. 129(1967), 58–93.

V.K.A.M. Gugenheim and J.P. May. On the theory and applications of differential torsion products. Memoirs Amer. Math. Soc. No. 142. 1974.

There is a useful guidebook to spectral sequences:

J. McCleary. User's guide to spectral sequences. Publish or Perish. 1985.

10. Cohomology operations

A compendium of the work of Steenrod and others on the construction and analysis of the Steenrod operations:

N.E. Steenrod and D.B.A. Epstein. Cohomology operations. Annals of Math. Studies No. 50. Princeton University Press. 1962.

A classic paper that first formalized cohomology operations, among other things:

J.-P. Serre. Cohomologie modulo 2 *des complexes d'Eilenberg-Mac Lane.* Comm. Math. Helv. 27(1953), 198–232.

A general treatment of Steenrod-like operations:

J.P. May. A general algebraic approach to Steenrod operations. In Lecture Notes in Mathematics Vol. 168, 153–231. Springer-Verlag. 1970.

A nice book on mod 2 Steenrod operations and the Adams spectral sequence:

R. Mosher and M. Tangora. Cohomology operations and applications in homotopy theory. Harper and Row. 1968.

11. Vector bundles

A classic and a more recent standard treatment that includes K-theory:

N.E. Steenrod. Topology of fibre bundles. Princeton University Press. 1951. Fifth printing, 1965.

D. Husemoller. Fibre bundles. Springer-Verlag. 1966. Third edition, 1994.

A general treatment of classification theorems for bundles and fibrations:

J.P. May. Classifying spaces and fibrations. Memoirs Amer. Math. Soc. No. 155. 1975.

12. Characteristic classes

The classic introduction to characteristic classes:

J. Milnor and J.D. Stasheff. Characteristic classes. Annals of Math. Studies No. 76. Princeton University Press. 1974.

A good reference for the basic calculations of characteristic classes:

A. Borel. Topology of Lie groups and characteristic classes. Bull. Amer. Math. Soc. 61(1955), 297–432.

Two proofs of the Bott periodicity theorem that only use standard techniques of algebraic topology, starting from characteristic class calculations:

H. Cartan et al. Périodicité des groupes d'homotopie stables des groupes classiques, d'après Bott. Séminaire Henri Cartan, 1959/60. Ecole Normale Supérieure. Paris.

E. Dyer and R.K. Lashof. A topological proof of the Bott periodicity theorems. Ann. Mat. Pure Appl. (4)54(1961), 231–254.

13. K-theory

Two classical lecture notes on K-theory:

R. Bott. Lectures on $K(X)$. W.A. Benjamin. 1969.

This includes a reprint of perhaps the most accessible proof of the complex case of the Bott periodicity theorem, namely:

M.F. Atiyah and R. Bott. On the periodicity theorem for complex vector bundles. Acta Math. 112(1994), 229–247.

M.F. Atiyah. K-theory. Notes by D.W. Anderson. Second Edition. Addison-Wesley. 1967.

This includes reprints of two classic papers of Atiyah, one that relates Adams operations in K-theory to Steenrod operations in cohomology and another that sheds insight on the relationship between real and complex K-theory:

M.F. Atiyah. Power operations in K-theory. Quart. J. Math. (Oxford) (2)17(1966), 165–193.

M.F. Atiyah. K-theory and reality. Quart. J. Math. (Oxford) (2)17(1966), 367–386.

Another classic paper that greatly illuminates real K-theory:

M.F. Atiyah, R. Bott, and A. Shapiro. Clifford algebras. Topology 3(1964), suppl. 1, 3–38.

A more recent book on K-theory:

M. Karoubi. K-theory. Springer-Verlag. 1978.

Some basic papers of Adams and Adams and Atiyah giving applications of K-theory:

J.F. Adams. Vector fields on spheres. Annals of Math. 75(1962), 603–632.

J.F. Adams. On the groups $J(X)$ I, II, III, and IV. Topology 2(1963), 181–195; 3(1965), 137-171 and 193–222; 5(1966), 21–71.

J.F. Adams and M.F. Atiyah. K-theory and the Hopf invariant. Quart. J. Math. (Oxford) (2)17(1966), 31–38.

14. Hopf algebras; the Steenrod algebra, Adams spectral sequence

The basic source for the structure theory of (connected) Hopf algebras:

J. Milnor and J.C. Moore. On the structure of Hopf algebras. Annals of Math. 81(1965), 211–264.

The classic analysis of the structure of the Steenrod algebra as a Hopf algebra:

J. Milnor. The Steenrod algebra and its dual. Annals of Math. 67(1958), 150–171.

Two classic papers of Adams; the first constructs the Adams spectral sequence relating the Steenrod algebra to stable homotopy groups and the second uses secondary cohomology operations to solve the Hopf invariant one problem:

J.F. Adams. On the structure and applications of the Steenrod algebra. Comm. Math. Helv. 32(1958), 180–214.

J.F. Adams. On the non-existence of elements of Hopf invariant one. Annals of Math. 72(1960), 20–104.

15. Cobordism

The beautiful classic paper of Thom is still highly recommended:

R. Thom. Quelques propriétés globals des variétés différentiables. Comm. Math. Helv. 28(1954), 17–86.

Thom computed unoriented cobordism. Oriented and complex cobordism came later. In simplest form, the calculations use the Adams spectral sequence:

J. Milnor. On the cobordism ring Ω^* and a complex analogue. Amer. J. Math. 82(1960), 505–521.

C.T.C. Wall. A characterization of simple modules over the Steenrod algebra mod 2. Topology 1(1962), 249–254.

A. Liulevicius. A proof of Thom's theorem. Comm. Math. Helv. 37(1962), 121–131.

A. Liulevicius. Notes on homotopy of Thom spectra. Amer. J. Math. 86(1964), 1–16.

A very useful compendium of calculations of cobordism groups:

R. Stong. Notes on cobordism theory. Princeton University Press. 1968.

16. Generalized homology theory and stable homotopy theory

Two classical references, the second of which also gives detailed information about complex cobordism that is of fundamental importance to the subject.

G.W. Whitehead. Generalized homology theories. Trans. Amer. Math. Soc. 102(1962), 227–283.

J.F. Adams. Stable homotopy and generalised homology. Chicago Lectures in Mathematics. University of Chicago Press. 1974. Reprinted in 1995.

An often overlooked but interesting book on the subject:

H.R. Margolis. Spectra and the Steenrod algebra. Modules over the Steenrod algebra and the stable homotopy category. North-Holland. 1983.

Foundations for equivariant stable homotopy theory are established in:

L.G. Lewis, Jr., J.P. May, and M.Steinberger (with contributions by J.E. McClure). Equivariant stable homotopy theory. Lecture Notes in Mathematics Vol. 1213. Springer-Verlag. 1986.

17. Quillen model categories

In the introduction, I alluded to axiomatic treatments of "homotopy theory." Here are the original and two more recent references:

D.G. Quillen. Homotopical algebra. Lecture Notes in Mathematics Vol. 43. Springer-Verlag. 1967.

W.G. Dwyer and J. Spalinski. Homotopy theories and model categories. In A handbook of algebraic topology, edited by I.M. James. North-Holland. 1995.

The cited "*Handbook*" (over 1300 pages) contains an uneven but very interesting collection of expository articles on a wide variety of topics in algebraic topology.

M. Hovey. Model categories. Amer. Math. Soc. Surveys and Monographs No. 63. 1998.

18. Localization and completion; rational homotopy theory

Since the early 1970s, it has been standard practice in algebraic topology to localize and complete topological spaces, and not just their algebraic invariants, at sets of primes and then to study the subject one prime at a time, or rationally. Two of the basic original references are:

D. Sullivan. The genetics of homotopy theory and the Adams conjecture. Annals of Math. 100(1974), 1–79.

A.K. Bousfield and D.M. Kan. Homotopy limits, completions, and localizations. Lecture Notes in Mathematics Vol. 304. Springer-Verlag. 1972.

A more accessible introduction to localization and a readable recent paper on completion are:

P. Hilton, G. Mislin, and J. Roitberg. Localization of nilpotent groups and spaces. North-Holland. 1975.

F. Morel. Quelques remarques sur la cohomologie modulo p continue des pro-p-espaces et les resultats de J. Lannes concernent les espaces fonctionnel Hom(BV, X). Ann. Sci. Ecole Norm. Sup. (4)26(1993), 309–360.

When spaces are rationalized, there is a completely algebraic description of the result. The main original reference and a more accessible source are:

D. Sullivan. Infinitesimal computations in topology. Publ. Math. IHES 47(1978), 269–332.

A.K. Bousfield and V.K.A.M. Gugenheim. On PL de Rham theory and rational homotopy type. Memoirs Amer. Math. Soc. No. 179. 1976.

19. Infinite loop space theory

Another area well established by the mid-1970s. The following book is a delightful read, with capsule introductions of many topics other than infinite loop space theory, a very pleasant starting place for learning modern algebraic topology:

J.F. Adams. Infinite loop spaces. Annals of Math. Studies No. 90. Princeton University Press. 1978.

The following survey article is less easy going, but gives an indication of the applications to high dimensional geometric topology and to algebraic K-theory:

J.P. May. Infinite loop space theory. Bull. Amer. Math. Soc. 83(1977), 456–494.

Five monographs, each containing a good deal of expository material, that give a variety of theoretical and calculational developments and applications in this area:

J.P. May. The geometry of iterated loop spaces. Lecture Notes in Mathematics Vol. 271. Springer-Verlag. 1972.

J.M. Boardman and R.M. Vogt. Homotopy invariant algebraic structures on topological spaces. Lecture Notes in Mathematics Vol. 347. Springer-Verlag. 1973.

F.R. Cohen, T.J. Lada, and J.P. May. The homology of iterated loop spaces. Lecture Notes in Mathematics Vol. 533. Springer-Verlag. 1976.

J.P. May (with contributions by F. Quinn, N. Ray, and J. Tornehave). E_∞ ring spaces and E_∞ ring spectra. Lecture Notes in Mathematics Vol. 577. Springer-Verlag. 1977.

R. Bruner, J.P. May, J.E. McClure, and M. Steinberger. H_∞ ring spectra and their applications. Lecture Notes in Mathematics Vol. 1176. Springer-Verlag. 1986.

20. Complex cobordism and stable homotopy theory

Adams' book cited in §16 gives a spectral sequence for the computation of stable homotopy groups in terms of generalized cohomology theories. Starting from complex cobordism and related theories, its use has been central to two waves of major developments in stable homotopy theory. A good exposition for the first wave:

D.C. Ravenel. Complex cobordism and stable homotopy groups of spheres. Academic Press. 1986.

The essential original paper and a very nice survey article on the second wave:

E. Devinatz, M.J. Hopkins, and J.H. Smith. Nilpotence and stable homotopy theory. Annals of Math. 128(1988), 207–242.

M.J. Hopkins. Global methods in homotopy theory. In Proceedings of the 1985 LMS Symposium on homotopy theory, edited by J.D.S. Jones and E. Rees. London Mathematical Society. 1987.

The cited *Proceedings* contain good introductory survey articles on several other topics in algebraic topology. A larger scale exposition of the second wave is:

D.C. Ravenel. Nilpotence and periodicity in stable homotopy theory. Annals of Math. Studies No. 128. Princeton University Press. 1992.

21. Follow-ups to this book

There is a leap from the level of this introductory book to that of the most recent work in the subject. One recent book that helps fill the gap is:

P. Selick. Introduction to homotopy theory. Fields Institute Monographs No. 9. American Mathematical Society. 1997.

There is a recent expository book for the reader who would like to jump right in and see the current state of algebraic topology; although it focuses on equivariant theory, it contains introductions and discussions of many non-equivariant topics:

J.P. May et al. Equivariant homotopy and cohomology theory. NSF-CBMS Regional Conference Monograph. 1996.

For the reader of the last section of this book whose appetite has been whetted for more stable homotopy theory, there is an expository article that motivates and explains the properties that a satisfactory category of spectra should have:

J.P. May. Stable algebraic topology and stable topological algebra. Bulletin London Math. Soc. 30(1998), 225–234.

The following monograph gives such a category, with many applications; more readable accounts appear in the *Handbook* cited in §17 and in the book just cited:

A. Elmendorf, I. Kriz, M.A. Mandell, and J.P. May, with an appendix by M. Cole. Rings, modules, and algebras in stable homotopy theory. Amer. Math. Soc. Surveys and Monographs No. 47. 1997.