Q IV 8.1 a) $\underline{sol^n}$

The Boolean expression implemented by the digital circuit is:

$$Y = (\neg A \uparrow \neg B) \uparrow ((A \uparrow B) \uparrow C)$$

We can also write the above exp. as:

$$\neg((\neg(\neg A \wedge \neg B)) \wedge \neg(\neg(A \wedge B) \wedge C))$$

$$= \neg((\neg\neg A \vee \neg\neg B) \wedge (\neg\neg(A \wedge B) \vee \neg C))$$

[Demorgan's law]

$$= (\neg(A \vee B) \vee \neg((A \wedge B) \vee \neg C)$$

[double negation]

$$= ((\neg A \wedge \neg B) \vee (\neg(A \wedge B) \wedge \neg\neg C))$$

[De morgan's law]

$$= ((\neg A \wedge \neg B) \vee ((\neg A \vee \neg B) \wedge C)$$

[double negation]

$$= ((\neg A \wedge \neg B) \vee ((\neg A \wedge C) \vee (\neg B \wedge C)))$$

[Distributivity]

$$= ((\neg A \wedge \neg B) \vee (\neg A \wedge C) \vee (\neg B \wedge C))$$

The above eqn. can also be written as,

$$((\neg A \wedge \neg B \wedge C) \vee (\neg A \wedge \neg B \wedge \neg C) \vee$$
$$(\neg A \wedge B \wedge C) \vee (\neg A \wedge \neg B \wedge C) \vee$$
$$(A \wedge \neg B \wedge C) \vee (\neg A \wedge \neg B \wedge C))$$

So, in minterm form we can write,

$$= m_1 + m_0 + m_3 + m_1 + m_5 + m_1$$
$$= m_0 + m_1 + m_3 + m_5 \quad \text{[idempotency]}$$

Q.N 8.(2) Soln

$$S = (A \dot{\vee} B) \dot{\vee} C_{in}$$
$$C_{out} = (A \wedge B) \vee (C_{in} \wedge (A \dot{\vee} B))$$

for S:

The S can also be written as,

$$(\neg((\neg A \wedge B) \vee (A \wedge \neg B)) \wedge C_{in})$$
$$\vee (((\neg A \wedge B) \vee (A \wedge \neg B)) \wedge \neg C_{in}))$$
$$= (\neg(((\neg A \wedge B) \wedge C_{in}) \vee ((A \wedge \neg B) \wedge C_{in})) \vee (\neg A \wedge \neg B \wedge \neg C_{in}))$$
$$((\neg A \wedge B \wedge \neg C_{in}) \vee$$

[Distributivity]

Q.N 8.2) soln

$$S = (A \dot{\vee} B) \dot{\vee} C_{in}$$

$$C_{out} = (A \wedge B) \vee (C_{in} \wedge (A \dot{\vee} B))$$

for S and Cout
As the truth table is easier to follow than the algebraic computation so,

A	B	Cin	S	Cout
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

So, sum for S is

$(\neg A \wedge \neg B \wedge C_{in}) \vee (\neg A \wedge B \wedge \neg C_{in}) \vee$
$(A \wedge \neg B \wedge \neg C_{in}) \vee (A \wedge B \wedge C_{in})$

The product for the S is:

$(\neg A \lor \neg B \lor \neg C_{in}) \land (\neg A \lor B \lor C_{in}) \land$

$(A \lor \neg B \lor C_{in}) \land (A \lor B \lor \neg C_{in})$

The sum for the C_{out} is:

$\cancel{(\neg A \lor \neg B \lor C_{in}) \land (}$

$(A \land B \land \neg C_{in}) \lor (\neg A \land B \land C_{in}) \lor$

$(A \land \neg B \land C_{in}) \lor (A \land B \land C_{in})$

The product for C_{out} is:

$(\neg A \lor \neg B \lor \neg C_{in}) \land (\neg A \lor \neg B \lor C_{in}) \land$

$(\neg A \lor B \lor \neg C_{in}) \land (A \lor \neg B \lor \neg C_{in})$

c)

$\cancel{S = A \dot\lor B \dot\lor C_{in}}$

$\cancel{= (A \dot\lor B) \dot\lor C_{in}}$

$\cancel{= ((A \lor B) \land (\neg A \lor \neg B)) \dot\lor C_{in}}$

$\cancel{= (\neg(\neg A \land \neg B) \land (\neg(A \land B))) \dot\lor C_{in}}$

~~[De Morgan's law]~~

$$= \cancel{((\neg A \uparrow \neg B) \wedge (A \uparrow B))) \dot{\vee} C_{in}}$$

$$= \cancel{(\neg A \wedge A}$$

c) Lets consider for x, y

$x \dot{\vee} y = ?$ (In terms of $\neg$ & $\uparrow$)

x	y	$x \uparrow y$	$(\neg x \uparrow \neg y)$	$(x \uparrow y) \uparrow (\neg x \uparrow \neg y)$	$x \dot{\vee} y$
0	0	1	0	0	0
0	1	1	1	1	1
1	0	1	1	1	1
1	1	0	1	0	0

As we can see $x \dot{\vee} y$ is identical to

$(x \uparrow y) \uparrow (\neg x \uparrow \neg y)$

so,

$$x \dot{\vee} y = (x \uparrow y) \uparrow (\neg x \uparrow \neg y)$$

so $S = A \dot{\vee} B \dot{\vee} C_{in}$

$$= (A \dot{\vee} B) \dot{\vee} C_{in}$$

$$= ((A \uparrow B) \uparrow (\neg A \uparrow \neg B)) \dot{\vee} C_{in}$$

$$= (((A \uparrow B) \uparrow (\neg A \uparrow \neg B)) \uparrow C_{in}) \uparrow$$
$$(\neg((A \uparrow B) \uparrow (\neg A \uparrow \neg B)) \uparrow \neg C_{in})$$

which is the required form consisting only $\uparrow$ and $\neg$.

for Cout:

$$C_{out} = (A \wedge B) \vee (C_{in} \wedge (A \dot{\vee} B))$$

$$= \neg(\neg A \uparrow \neg B) \vee \neg(\neg C_{in} \uparrow \neg(A \dot{\vee} B))$$

[obvious]

$$= \neg((\neg A \uparrow \neg B) \wedge (\neg C_{in} \uparrow \neg(A \dot{\vee} B)))$$

[De Morgan's law]

$$= (\neg A \uparrow \neg B) \uparrow (\neg C_{in} \uparrow \neg(A \dot{\vee} B))$$

$$= (\neg A \uparrow \neg B) \uparrow (\neg C_{in} \uparrow \neg((A \uparrow B) \uparrow (\neg A \uparrow \neg B)))$$

which is the required form consisting only $\uparrow$ and $\neg$.

d)

for S:

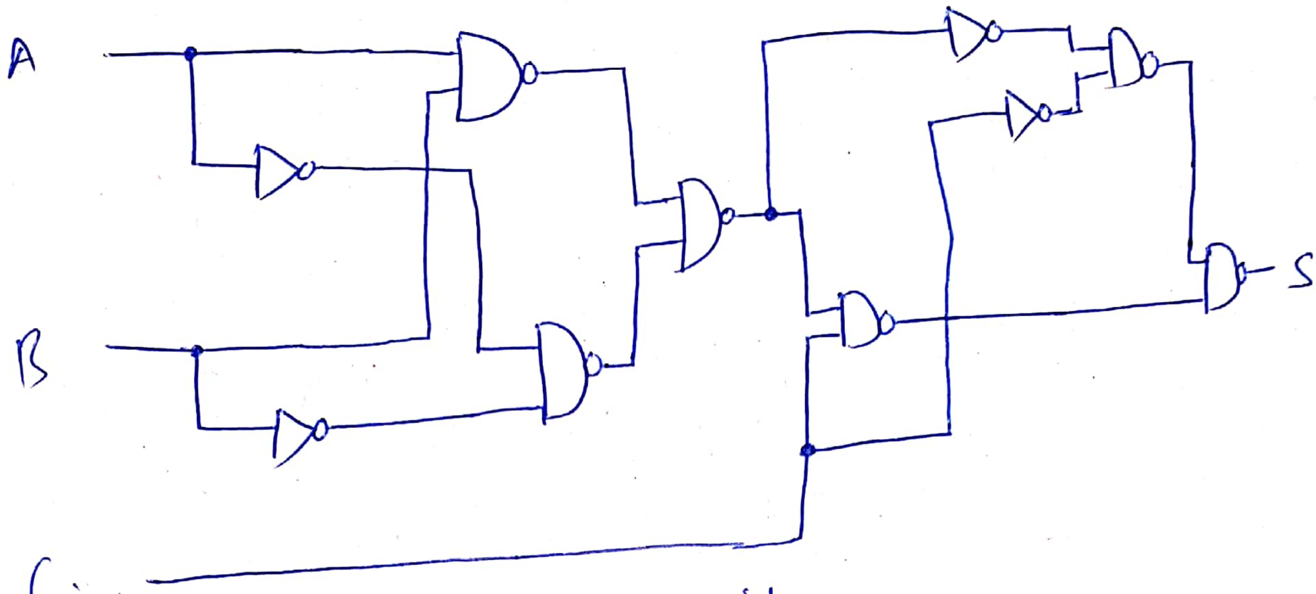

fig: S circuit

for Cout:

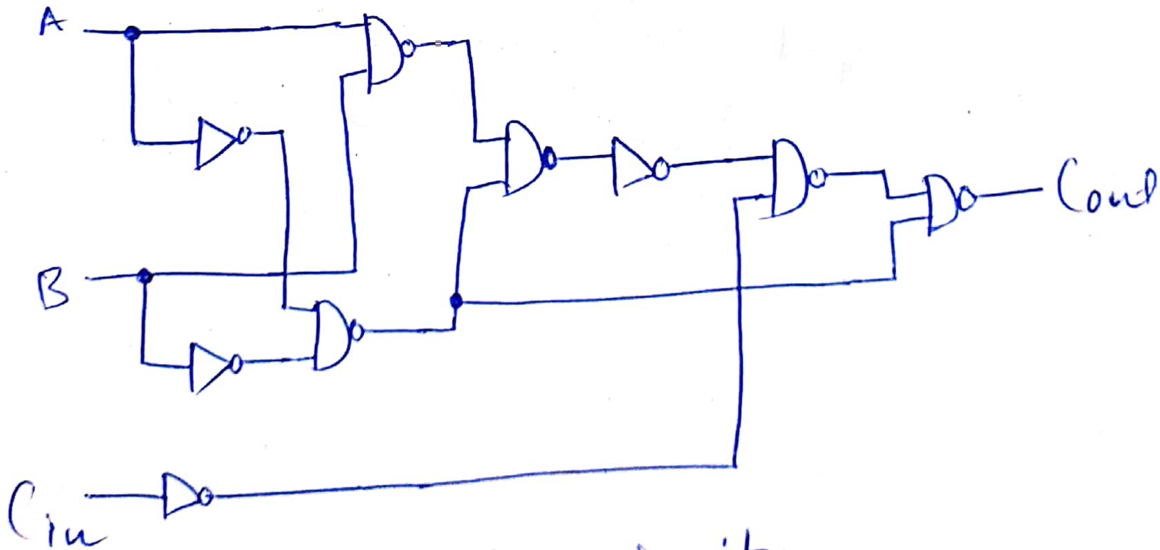

fig: Cout circuit