Q.N 6.1) Soln

As we know that NAND is the universal operator we are going to show that some combination of implication (→) and (¬) Negation gives us the ~~value~~ same value as the NAND ~~operators~~ operator.

Let X and Y be two ~~binary~~ boolean variables.

So, Creating the truth table for X and Y:

X	Y	(X → Y)	¬(X)	(X → Y) → (¬X)	X ↑ Y
1	1	1	0	0	0
0	1	1	1	1	1
1	0	0	0	1	1
0	0	1	1	1	1

As we can see from the truth table that (X → Y) → (¬ X) can produce same outcome as X ↑ Y so we can conclude that the combination of implication (→) and negation (¬) can be considered as universal operator.

Q.N 6.2) Soln

a) $\varphi(A, B) = (\neg A \vee \neg B) \wedge (\neg A \vee B) \wedge (A \vee \neg B)$

$= ((\neg A \vee \neg B) \wedge (\neg A \vee B)) \wedge (A \vee \neg B)$

$= (\neg A \vee (\neg B \wedge B)) \wedge (A \vee \neg B)$ [Distributivity]

$= (\neg A \vee 0) \wedge (A \vee \neg B)$

$= \neg A \wedge (A \vee \neg B)$ [identity]

$= (\neg A \wedge A) \vee (\neg A \vee \neg B)$ [Distributivity]

$= 0 \vee (\neg A \vee \neg B)$

$= (\neg A \vee \neg B$ [Identity]

$= \neg(A \wedge B)$ (de Morgan's laws)

b) $\cancel{\varphi(A, B, C) = (A \wedge \neg B) \vee (A \wedge \neg B \wedge C)}$

$\cancel{= ((A \wedge \neg B) \vee (A \wedge \neg B)) \wedge ((A \wedge \neg B) \vee C)}$ ~~[Distributivity]~~

$\cancel{= (A \wedge \neg B) \wedge ((A \wedge \neg B) \vee C}$

Q.N 6.2) soln

b) $\varphi(A,B,C) = (A\land\neg B)\lor(A\land\neg B\land C)$

$= ((A\land\neg B)\lor A)\land((A\land\neg B)\lor(\neg B\land C))$ [Distributivity]

$= A\land((A\land\neg B)\lor(\neg B\land C))$ [Absorption laws]

$= A\land(((A\land\neg B)\lor\neg B)\land((A\land\neg B)\lor C))$ [Distributivity]

$= A\land(\neg B\land((A\land\neg B)\lor C))$ [Absorption law]

$= A\land((\neg B\lor(A\land\neg B))\land(\neg B\lor C))$ [Distributivity]

$= A\land(\neg B\land(\neg B\lor C))$ [Absorption law]

$\cancel{= A\land((\neg B\land\neg B)\lor(\neg B\land C))}$ ~~[Distributivity]~~

$\cancel{= A\land[\neg B\lor(\neg B\land C)]}$ ~~[~~

$= A\land(\neg B)$ [Absorption law]

Q.N 6.2 c (soln)

$\varphi(A,B,C,D) = (A \vee \neg(B \wedge A)) \wedge (C \vee (D \vee C))$

$= (A \vee (\neg B \vee \neg A)) \wedge ((C \vee (D \vee C))$ (De Morgan's law)

$= (A \vee (\neg A \vee \neg B)) \wedge (C \vee (D \vee C))$ [~~Associativity~~ Commutalibity]

$= ((A \vee \neg A) \vee \neg B) \wedge ((C \vee (D \vee C))$ [Associativity]

$= (1 \vee \neg B) \wedge ((C \vee (D \vee C))$

$= 1 \wedge ((C \vee (D \vee C))$ [domination]

$= C \vee (D \vee C)$ [Identity]

$= C \vee (C \vee D)$ [Commutalivity]

$= (C \vee C) \vee D$ [Associativity]

$= C \vee D$ [Idemptoncy]

Q.N 6.2d soln)

$\varphi(A,B,C) = (\neg(A \wedge B) \vee \neg C) \wedge (\neg A \vee B \vee \neg C)$

$= ((\neg A \vee \neg B) \vee \neg C) \wedge (\neg A \vee B \vee \neg C)$ [de Morgan's law]

$= (\neg A \vee (\neg B \vee \neg C)) \wedge (\neg A \vee (B \vee \neg C))$ [Associativity]

$= (\neg A \vee (\neg C \vee \neg B)) \wedge (\neg A \vee (\neg C \vee B))$ [Commutativity]

$= ((\neg A \vee \neg C) \vee \neg B) \wedge ((\neg A \vee \neg C) \vee B)$ [Associativity]

$= (\neg A \vee \neg C) \vee (\neg B \wedge C)$ [Distributivity]

$= (\neg A \vee \neg C) \vee 0$

$= \neg A \vee \neg C$ [Identity]

$= \neg (A \wedge C)$ [de Morgan's laws]

Q.N 6.2 e) $\underline{\underline{\text{soln}}}$

$\varphi(A, B) = (A \vee B) \wedge (\neg A \vee B) \wedge (A \vee \neg B) \wedge (\neg A \vee \neg B)$

$= ((A \wedge \neg A) \vee B) \wedge ((A \wedge \neg A) \vee \neg B)$

[Distributivity twice]

$= (0 \vee B) \wedge (0 \vee \neg B)$

$= B \wedge \neg B$ [Identity twice]

$= 0$

Q . N 6.3 a) Sol^n

Creating the truth table for
$\varphi(P,Q,R,S) = (\neg P \vee Q) \wedge (\neg Q \vee R) \wedge (\neg R \vee S) \wedge (\neg S \vee P)$.

P	Q	R	S	$\neg P$	$\neg Q$	$\neg R$	$\neg S$	$\neg P \vee Q$	$\neg Q \vee R$	$\neg R \vee S$	$\neg S \vee P$	$(\neg P \vee Q) \wedge (\neg Q \vee R) \wedge (\neg R \vee S) \wedge (\neg S \vee P)$
0	0	0	0	1	1	1	1	1	1	1	1	1
0	0	0	1	1	1	1	0	1	1	1	0	0
0	0	1	0	1	1	0	1	1	1	0	1	0
0	0	1	1	1	1	0	0	1	1	1	0	0
0	1	0	0	1	0	1	1	1	0	1	1	0
0	1	0	1	1	0	1	0	1	0	1	0	0
0	1	1	0	1	0	0	1	1	1	0	1	0
0	1	1	1	1	0	0	0	1	1	1	0	0
1	0	0	0	0	1	1	1	0	1	1	1	0
1	0	0	1	0	1	1	0	0	1	1	1	0
1	0	1	0	0	1	0	1	0	1	0	1	0
1	0	1	1	0	1	0	0	0	1	1	1	0
1	1	0	0	0	0	1	1	1	0	1	1	0
1	1	0	1	0	0	1	0	1	0	1	1	0
1	1	1	0	0	0	0	1	1	1	0	1	0
1	1	1	1	0	0	0	0	1	1	1	1	1

As we can see that only two ~~value~~ interpretations of P and Q satisfy the condition.

i.e. $P = Q = R = S$

Q.N 6.3 b <u>soln</u>

from the truth table we did in question 6.3 a we can write the following DNF form.

$$(P \land Q \land R \land S) \lor (\neg P \land \neg Q \land \neg R \land \neg S)$$