FINAL Exam 2021

OCE 3521– Hydromechanics & Wave Theory
Term: Spring 2021
Exam Date: 4/27/2021

READ THE INSTRUCTIONS

This exam is to be worked on Tuesday April 27th. Turn the exam in by the end of the day before 5pm. A late submission will result in a zero for the exam.

- *Show all of your work!*
- *You are going to need to calculate critical variables along the way in order to reach the final answers, clearly identify these values by circling or boxing them.*
- *Write neatly so I can grade your work.*
- *Create a PDF of your solutions and upload your solutions in the same manner as done for the term exam 2 (do not ZIP your exam solutions).*
- *You can use your wave code, book, notes, etc.*
- *The last 2 problems are for those who want a chance to being up their test 2 score. ONLY work these if you want to have your test 1 score adjusted.*

- MAKE SURE TO READ THE PROBLEM AND PROCESS WHAT YOU ARE BEING ASKED TO FIND!

1. (**15 pts**) Two waves both have a wave height of 1 m; however, the first wave has a period of 6 seconds and the second has a period of 12 seconds. ***Discuss*** and ***compare***, **USING EQUATIONS**, the ***<u>wave energy and power</u>*** associated with each of these waves. Be sure to compute the values in order to support your statements.

2. **(30 pts)** Our field study of water surface elevation at Sebastian Park involved measuring wind waves. We collected a record of the sea surface outside the surf zone, using a wave gauge. A 200 second portion of the resulting time series of surface elevations is given in the figure on the next page:

First perform a **wave-by-wave analysis** on the time series given.

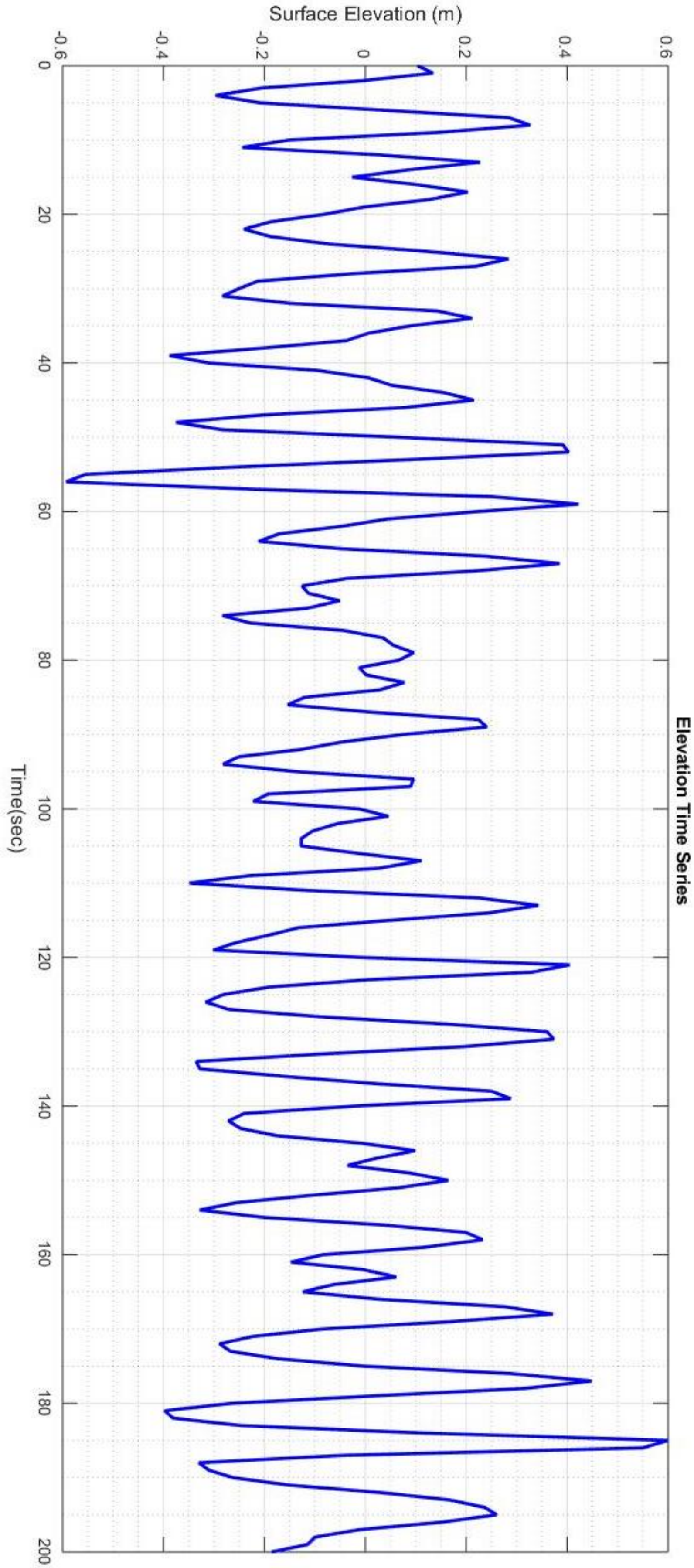
a) **(5pts)** ***State*** how you would go about decomposing the wave signal into a wave-by-wave record, AND ***Directly on the Figure*** identify and number each full "wave" in the signal.

b) **(10pts)** Record the **wave height, H**, in the provided table on the next page, and then rank order (a rank of 1 is the largest to rank of N is the smallest) in the adjacent column.

c) **(10pts)** Find: H_1 , H_{max}, H_{min}, H_{RMS} and $H_{1/3}$ From the time-series data by ***directly computing*** the values from the data.

d) **(5pts)** Using $H_{RMS} = 0.56\ m$, what is the probability that a wave will exceed 2 m?

Wave #	Wave Height H(m)	Rank Order
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		
13		
14		
15		
16		
17		
18		
19		
20		
21		
22		
23		
24		
25		
26		



(40 pts) A buoy is mounted onto a weighted instrument and dropped to the bottom at the **10 m** contour. The buoy is a **0.25 m** diameter foam float (**SG = 0.2**) attached by a **3 m** tether to the instrument on the sea floor (see Figure F1). The instrument attached to the buoy recorded the angle between the float tether and the bottom to be 80 and 100 degrees at buoy maximum excursion. And it takes **5 seconds** for the buoy to travel from the 80 degree extent to the 100 degree extent and then another **5 seconds** to travel back from 100 degrees to the 80 degree again.

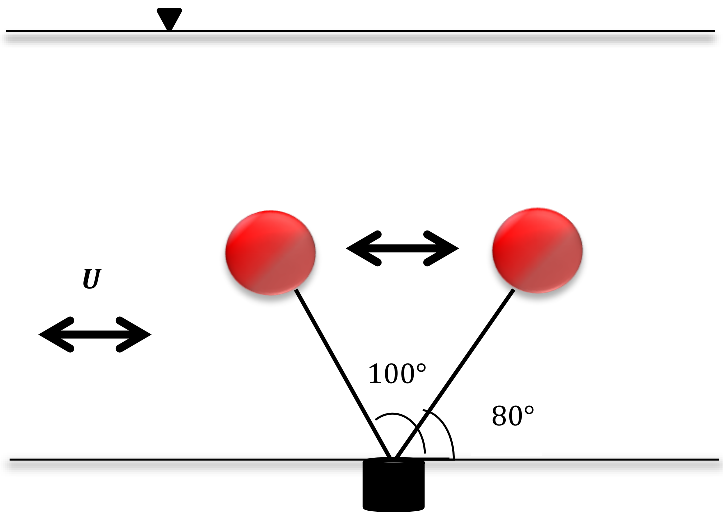


Figure F1

(Assume: $\boldsymbol{\rho_w = 1020\frac{kg}{m^3}, C_D = 0.25}$)

a) **(10 pts)** Show that the maximum horizontal wave velocity is $\underline{\boldsymbol{U = 1.36\ m/s}}$. Draw a free body diagram to help balance the forces in x and y. ***Hint: force balances: assume that*** $\sum F = ma = 0$ *when the tether is holding the buoy at the extreme excursions (e.g. theta = 80 degrees* ***)***

$$A = \pi r^2$$
$$V = \frac{4}{3}\pi r^3$$
$$F_B = \gamma V = \rho_w g V$$
$$F_D = \frac{1}{2}\rho_w C_D U|U|A$$
$$W = \rho_f g\, V = SG\, \rho_w g V$$

b) **(10pts)** What is the wave height at the location of the buoy, (**assume that $\boldsymbol{U = 1.36\ m/s}$** at the depth and location of the ***center of the buoy,*** and the wave period is **10 seconds**)?

For the remaining questions, assume that at the 10 meter location you get the following wave parameters: $\boldsymbol{L_{10} = 92.37, H_{10} = 3.10\ m,}$

c) **(10pts)** At what depth is this wave considered to be in "deep" water and what is the deep water wave height and wavelength **(check for steepness limited breaking)**?

d) **(10pts)** At what depth is this wave "shallow" water and what is the associated wave height and wavelength (hint: ***think shallow water dispersion and when checking for breaking*** **assume kappa = 0.80**)?

3. **(15 pts total)** There are no impermeable surfaces on my home planet. As a result, our boundary value problems are more complicated. Everything else is the same as what you learned in Chapter 3, except the bottom boundary condition is:

$$\frac{\partial \phi}{\partial z} = Qe^{i(kx-\sigma t)} \quad at \;\; z = -h, \qquad (1)$$

where Q is the amplitude of vertical velocity through the bottom. Zxyorg the Great, our Master Mathematician, determined that the following form of the velocity potential, Φ, will work in this situation:

$$\phi = \left[A\cosh[k(h+z)] + B\ sinh[k(h+z)]\right]e^{i(kx-\sigma t)} \qquad (2)$$

where A and B are constants of integration whose values are based on the boundary conditions. Unfortunately, Zxyorg suffered a fatal rupture of his zyyzyx before he was able to complete the solution. It is your job to finish his derivation.

Determine the final form of ϕ in terms of the wave amplitude a(= H/2), the wavenumber k, the radian frequency σ, the water depth h, and Q.

a) **(5 pts)** Using the new bottom boundary condition given above (Eq. 1), solve for one of the constants of integration (A or B) by plugging ϕ from Eq. 2 into Eq. 1.

b) **(5 pts)** Using our same dynamic free surface boundary condition:

$$\eta = \frac{1}{g}\frac{\partial \phi}{\partial t} \qquad at \;\; z = 0, \qquad (3)$$

with

$$\eta = ae^{i(kx-\sigma t)}, \qquad (4)$$

solve Eq. 3 for the other constant of integration, using Eq. 4 and your results from part a).

c) **(5 pts)** Assuming you were able to solve for the final form of ϕ and you got:

$$\phi = \left[\left[\left(\frac{-Q}{k}\right)\tanh[kh] - \left(\frac{ag}{i\sigma\cosh[kh]}\right)\right]\cosh[k(h+z)] + \frac{Q}{k}\sinh[k(h+z)]\right]e^{i(kx-\sigma t)} \quad (5)$$

Apply the Kinematic Free Surface Boundary Condition:

$$-\frac{\partial\phi}{\partial z} = \frac{\partial\eta}{\partial t} \qquad at\;\; z = 0, \qquad (6)$$

Determine the linear dispersion relation. You may use η from part (b) above Eq 4. and ϕ from Eq. 5. After you have a linear dispersion relation, **SHOW** that it reduces to the impermeable bottom linear dispersion relation when $Q = 0$.

*EXAM 1 Section: only work these problems if you would like to have the score count towards your Test 1 grade

*(**15 pts total**) A 2-D velocity field is given by:

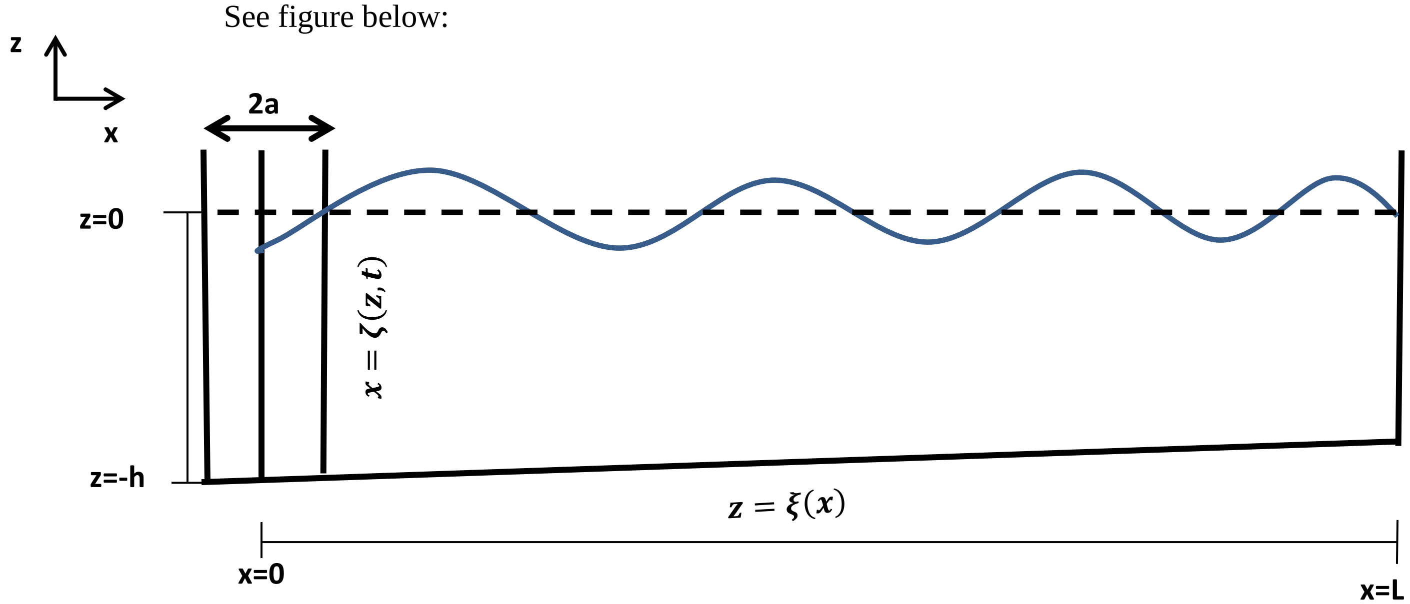
$$\phi = x^2 - y^2 + 5\,xy - 3$$

a. **(5 pts)** Find the components of velocity (u, w) and write them below:

b. **(10 pts)** Show that this flow is both irrotational and non-divergent?

***(10 pts)** In a wave tank, the waves are generated by a piston wave maker. Where the piston moves back and forth with a period of T. The maximum horizontal **excursion of the piston at the surface (z=0) is 2a**. The wave tank has a length of, L, with a **sloping bottom (m=0.05)**, and a depth of h at the wave flap.
Derive the boundary condition at the wave generating piston on the left hand side, by first developing an equation for $\zeta(z,t)$ that describes the position of the boundary and then use the appropriate BC formula to solve for the velocity BC at the piston. .

See figure below:



L=wavelength; k=wave number; h= water depth; σ=Angular wave frequency; T=period; C=wave celerity;

Cg=group velocity

Relative Depth	Transitional Water 1/20< h/L <1/2
Wave Profile	$\eta = \frac{H}{2}\cos[kx - \sigma t] = \frac{H}{2}\cos[\Omega]$
Wave Length	$L = \frac{gT^2}{2\pi}\tanh(kh)$
Wave Celerity	$C = \frac{\sigma}{k} = \frac{L}{T} = \frac{gT}{2\pi}\tanh(kh)$
Group Velocity	$C_g = nC = \frac{1}{2}\left[1 + \frac{2kh}{\sinh(2kh)}\right]C$
a) Horizontal particle Velocity	$u = \frac{H}{2}\frac{gk}{\sigma}\frac{\cosh[k(h+z)]}{\cosh(kh)}\cos\Omega$
b) Vertical particle Velocity	$w = \frac{H}{2}\frac{gk}{\sigma}\frac{\sinh[k(h+z)]}{\cosh(kh)}\sin\Omega$
a) Horizontal Particle acceleration	$a_x = \frac{H}{2}gk\frac{\cosh k(h+z)}{\cosh(kh)}\sin(\Omega)$
b) Vertical Particle acceleration	$a_z = -\frac{H}{2}gk\frac{\sinh k(h+z)}{\cosh(kh)}\cos(\Omega)$
a) Horizontal particle displacement	$\xi = -\frac{H}{2}\frac{\cosh k(h+z)}{\sinh(kh)}\sin(\Omega)$
b) Vertical particle displacement	$\zeta = \frac{H}{2}\frac{\sinh k(h+z)}{\sinh(kh)}\cos(\Omega)$
Sub-surface Pressure	$p = \rho g\frac{H}{2}\frac{\cosh k(h+z)}{\cosh(kh)}\cos(\Omega) - \rho gz$