Braidan Duffy | Final Exam Q1 | 04/29/21

Two waves both have a height of 1m, However, $T_1 = 6s$, $T_2 = 12s$. Discuss and comparee the wave energy and power of each of therse waves.

$E_1 = 1/8 \rho g H_1^2$; $E_2 = 1/8 \rho g H_2^2$ since $H_1 = H_2$, $E_1 = E_2$

Both of the waves have the same energy per unit area

$\dot{E}_1 = E n_1 C_1$; $\dot{E}_2 = E n_2 C_2$ since $C \propto 1/T_1$, $\dot{E} \propto 1/T$ $\therefore$

$\dot{E}_1 \propto 1/T_1$; $\dot{E}_2 \propto 1/2T_1$ Assuming deepwater ($n_1 = n_2 = 1/2$) and $T_2 = 2T_1$

The second wave with a longer period has half of the power of the first wave period in deepwater. The specific number will vary with kh in intermediate water but, the second wave will always have less power per unit crest width than the first wave.

Braidan Duffy | Final Exam Q2 | 04/27/21

A 200-second record of wind waves was captured and can be seen in the attached document

a) State how to decompose the surface elevation into a wave-by-wave record and do it on the printout.

The surface data can be broken into individual waves via the zero-upcrossing method. At each index where the signal transitions from negative to positive, a delimiter is placed, indicating a discrete wave.

b) Record the wave heights and rank order them

c) Find H_1, H_{max}, H_{min}, H_{rms}, and H_s,

From MATLAB: $H_1 = 0.53\ m$
$H_{max} = 1.0\ m$
$H_{min} = 0.1\ m$
$H_{rms} = 0.57\ m$
$H_s = 0.78\ m$

d) Using $H_{rms} = 0.56\ m$ what is $P(H>2m)$?

$$P(H>2m) = e^{-(2/0.56)^2} = 2.89\times10^{-6} = \boxed{2.89\times10^{-4}\,\% = P(H>2m)}$$

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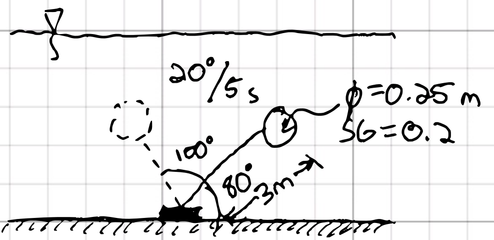
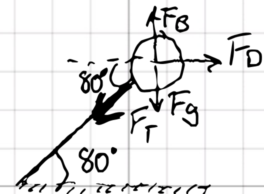
A buoy is mounted to a weighted instrument at the bottom of 10 m of water ($h = 10\,m$).



a) Show $U_{max} = 1.36$ m/s, balance forces



$$F_B = \rho_{sw} V_{buoy}\, g = \rho_{sw}\, \tfrac{4}{3}\pi r_b^3$$

$$= 1025(9.81)\, \tfrac{4}{3}\pi (0.125)^3 = 82.26\,N$$

$$\uparrow + \Sigma F_y = F_B - F_g - F_T \sin\theta = 0$$

$$F_T = \frac{F_B - F_g}{\sin\theta} = \frac{82.26 - 16.45}{\sin 80}$$

$$F_g = SG \cdot F_B = 0.2(82.26) = 16.45\,N$$

$$F_T = 66.83\,N$$

$$A_b = \pi r^2 = \pi (0.125)^2 = 0.049\,m^2$$

$$\rightarrow + \Sigma F_x = F_D - F_T \cos\theta = 0$$

$$F_D = F_T\cos\theta = 66.83\cos 80 = 11.60\,N = \tfrac{1}{2}\rho_{sw} C_D U_{max}^2 A_b$$

$$\sqrt{\frac{2F_D}{\rho_{sw} C_D A_b}} = U_{max} = \sqrt{\frac{2(11.60)}{1025(0.25)0.049}} = \boxed{1.36\,m/s = U_{max}}$$

b) What is H_{10} assuming $T = 10\,s$ and $z = -6.875\,m$?

$$U = \frac{H}{2}\frac{gk}{\sigma}\frac{\cosh(k(h+z))}{\cosh kh}\cancel{\cos\theta}^{\,1}$$

From MATLAB: $k = 0.068$
$kh = 0.68$
$\sigma = 0.6283$

$$= \frac{H}{2} \cdot \frac{gk}{\sigma} \cdot \frac{\cosh(k(h+z))}{\cosh kh}$$

$$\frac{2U\sigma\cosh(kh)}{gk\cosh(k(h+z))} = H = \frac{2(1.36)(0.6283)\cosh(0.68)}{9.81(0.068)\cosh(0.068(10-6.875))}$$

$$\boxed{H = 3.11\,m}$$

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c) What is h_0, H_0, L_0? From MATLAB: $C_{g_{10}} = 8.07\ m/s$

$$L_0 = \frac{gT^2}{2\pi} = \frac{9.81(10)^2}{2\pi} = \boxed{156.13\ m = L_0} \quad h_0 = \frac{L_0}{2} = \frac{156.13}{2} = \boxed{78.07\ m = h_0}$$

$$H_0 = H_{10}\sqrt{\frac{C_{g_{10}}}{C_{g_0}}} = H_{10}\sqrt{\frac{2C_{g_{10}}T}{L_0}} = 3.11\sqrt{\frac{2(8.07)10}{156.13}} = \boxed{3.16\ m = H_0}$$

Breaking: $\frac{H_0}{L_0} \geq \frac{1}{7}$, $\frac{3.16}{156.13} < \frac{1}{7}$ $\therefore$ Not breaking ✓

d) What is $h_{shallow}$, $H_{shallow}$, $L_{shallow}$?

$$L_{sw} = \frac{gT^2}{2\pi}\left(\frac{\pi}{10}\right) = \frac{gT^2}{20} = \frac{9.81(10)^2}{20} = \boxed{49.05\ m = L_{sw}}$$

$$\frac{L_{sw}}{T} = \sqrt{gh} = \frac{L_{sw}^2}{T^2} = gh \quad \frac{L_{sw}^2}{gT^2} = h = \frac{49.05^2}{9.81(10)^2} = \boxed{2.45\ m = h_{sw}}$$

$$H_{sw} = H_0\sqrt{\frac{C_{g_0}}{C_{g_{sw}}}} = H_0\sqrt{\frac{L_0 \cancel{T}}{2\cancel{T} L_{sw}}} = H_0\sqrt{\frac{L_0}{2L_{sw}}} = 3.16\sqrt{\frac{156.13}{2(49.05)}} = \boxed{3.99\ m = H_{sw}}$$

The wave breaks well before the shallow water boundary

3. There are no impermeable surfaces. Assume:

$$\frac{\partial \phi}{\partial z} = Qe^{i(kx-\sigma t)} \text{ @ } z=-h; \quad \phi = [A\cosh(k(h+z)) + B\sinh(k(h+z))]e^{i\Omega}$$

$\gamma = k(h+z)$, $\Omega = kx - \sigma t$

$\partial\gamma/\partial z = k \qquad \partial\Omega/\partial t = -\sigma$

a) Solve for A or B

$$\frac{\partial}{\partial z}[A\cosh(\gamma(z)) + B\sinh(\gamma(z))]e^{i\Omega}$$

$$\frac{d\phi_I}{dz} = \partial/\partial z\, Ae^{i\Omega}\cosh(\gamma(z)) = Ae^{i\Omega}\sinh(\gamma(z)) \cdot \partial\gamma/\partial z$$

$$= Ae^{i\Omega}\sinh(\gamma(z))k$$

$$\frac{\partial \phi_{II}}{\partial z} = \partial/\partial z\, B e^{i\Omega}\sinh(\gamma(z)) = Be^{i\Omega}\cosh(\gamma(z)) \cdot \partial\gamma/\partial z$$

$$= Bke^{i\Omega}\cosh(\gamma(z))$$

$$\frac{d\phi}{dz} = [A\sinh(\gamma(z)) + B\cosh(\gamma(z))]\cancel{e^{i\Omega}}k = Q\cancel{e^{i\Omega}}$$

$$A\sinh(\gamma) = \frac{Q}{k} - B\cosh\gamma$$

$$A = \frac{Q - Bk\cosh\gamma}{k\sinh(\gamma)} = \boxed{\frac{Q - Bk\cosh(k(h+z))}{k\sinh(k(h+z))} = A}$$

b) Solve for other constant (B)

$$\eta = \frac{1}{g}\frac{\partial \phi}{\partial t} \text{ @ } z=0, \quad \eta = ae^{i(kx-\sigma t)} = ae^{i\Omega}$$

$$\frac{\partial \phi}{\partial t} = \frac{\partial}{\partial t}[A\cosh\gamma + B\sinh\gamma]e^{i\Omega(t)} = -\sigma e^{i\Omega}[A\cos\gamma + B\sinh\gamma]$$

$$1/g[-\sigma\cancel{e^{i\Omega}}(A\cosh\gamma + B\sinh\gamma)] = a\cancel{e^{i\Omega}} = -\sigma/g(A\cosh\gamma + B\sinh\gamma) = a$$

$$B\sinh\gamma = \frac{-ag}{\sigma} - A\cosh\gamma \Rightarrow \boxed{B = \frac{-ag - A\sigma\cosh(k(h+z))}{\sigma\sinh(k(h+z))}}$$

c) Apply KFSBC and determine linear dispersion relation. Show it reduces to the impermeable BBC when $Q=0$

$$\phi = \left[\left[\frac{-Q}{k}\tanh kh - \left(\frac{ag}{i\sigma\cosh kh}\right)\right]\cosh(k(h+z)) + \frac{Q}{k}\sinh(k(h+z))\right]e^{i\Omega}$$

$$KFSBC = -\frac{d\phi}{dz} = \frac{d\eta}{dt} \text{ @ } z=0$$

$$\frac{d\phi_I}{dz} = \left[\frac{-Q}{k}\tanh kh - \left(\frac{ag}{i\sigma\cosh kh}\right)\right]e^{i\Omega}k\sinh\gamma$$

$$\frac{d\phi_{II}}{dz} = \frac{Qe^{i\Omega}}{\cancel{k}}\cancel{k}\cosh\gamma = Qe^{i\Omega}\cosh\gamma$$

$$\frac{d\phi}{dz} = \left\{\left[\left(\frac{-Q\tanh kh}{k}\right) - \left(\frac{ag}{i\sigma\cosh kh}\right)\right]k\sinh\gamma + Q\cosh\gamma\right\}e^{i\Omega}$$

$$\frac{d\eta}{dt} = -ai\sigma e^{i\Omega}$$

$$-\left\{\left[\cancel{\left(\frac{-Q\tanh kh}{k}\right)}^{0} - \left(\frac{ag}{i\sigma\cosh kh}\right)\right]k\sinh \overset{kh}{\cancel{\gamma}} + \cancel{Q\cosh\gamma}^{0}\right\}\cancel{e^{i\Omega}} = -ai\sigma\cancel{e^{i\Omega}}$$

$$-\frac{-agk\sinh}{i\sigma\cosh kh} = -ai\sigma \iff -\overset{-1}{\cancel{i^2}}\sigma^2 = \frac{gk\overset{\tanh kh}{\cancel{\sinh kh}}}{\cancel{\cosh kh}} \Rightarrow \boxed{\sigma^2 = gk\tanh kh}$$