1a) FALSE, $C_{sw} = \sqrt{gh}$

b) $E = \frac{1}{8}\rho g H^2 L$

c) Period

d) Refraction, Shoaling

e) True

f) Wave group

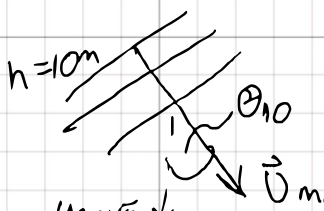
g) False

h) K_p, pressure reduction factor $\frac{\cosh(k(h+z))}{\cosh(kh)}$

i) Spilling, Plunging, Surging

j) True

2) In DW a T=12 s wave is 45° oblique to shore
At h=10 m, $u_{max} = 0.75$ m/s



$L_0 = \frac{gT^2}{2\pi} = 224.83$ m $\quad C_0 = \frac{224.83}{12}$

$C_{g_0} = \frac{18.74}{2} = 9.37$ m/s $\quad C_0 = 18.74$ m/s

From code: $L_{10} = 113.30$ m $\quad C_{g_{10}} = 8.60$ m/s

$C_{10} = 9.44$ m/s

$\theta_{10} = 20.88°$

a) What is $\vec{U}_{max}$ @ h=10m?

$$\vec{U}_{max} = \frac{u_{max}}{\cos\theta_0} = \frac{0.75}{\cos(20.88)} = \boxed{0.80 \text{ m/s} = \vec{U}_{max} @ h=10m}$$

b) $U = \frac{H}{2}\frac{gk}{\sigma}\frac{\cosh(k(h+z))}{\cosh kh}$ @ $z=-h$ $= \frac{H}{2}\frac{gk}{\sigma}\frac{1}{\cosh(kh)} = U$

$$H_{10} = \frac{2U_0 \cosh kh}{gk} = \frac{2(U_0)2\pi\cosh(kh)L}{Tg2\pi}$$

$k_{10} = \frac{2\pi}{113.3} = 0.055$

$(kh)_{10} = 0.055 \cdot 10 = 0.55$

$$= \frac{2(0.8)\cosh(0.55)113.30}{12[9.81]}$$

$\boxed{H_{10} = 1.78 \text{ m}}$ → Breaking: $H_b \geq 0.8 h_b = 0.8(10) = 8 \text{ m} \geq H_{10}$ ✓

c) What is H_0?

$$H_0 = H_{10}\sqrt{\frac{C_{g_{10}}}{C_{g_0}}}\sqrt{\frac{\cos\theta_{10}}{\cos\theta_0}} = 1.78\sqrt{\frac{8.60}{9.37}}\sqrt{\frac{\cos 20.88}{\cos 45}} = \boxed{1.54 \text{ m} = H_0}$$

Shoaling: $H_0 \sim < H_{10}$, $1.54 < 1.78$ ✓

d) What is h_0?

$$h_0 > \frac{L_0}{2} = \frac{224.83}{2} = \boxed{112.45 \text{ m} = h_0}$$

e) $(kh)_{sw} = \frac{\pi}{10}$ $\quad \sigma^2 = gk\tanh(kh) \Rightarrow k = \frac{\sigma^2}{g(kh)_{sw}} = \frac{4\pi^2(10)}{T^2 g \pi}$

$k = \frac{40\pi}{(12)^2(9.81)} = 0.0890$. $\quad h_{sw} = \frac{(kh)_{sw}}{k_{sw}} = \frac{\pi}{10(0.089)} = \boxed{3.53\,m = h_{sw}}$

$H_{sw} = H_0\sqrt{\frac{c_{g0}}{c_{gsw}}}\sqrt{\frac{\cos\theta_0}{\cos\theta_{sw}}}$

$= 1.59\sqrt{\frac{9.37}{5.6}}\sqrt{\frac{\cos(45)}{\cos(12.62)}} = \boxed{1.75\,m = H_{sw}}$

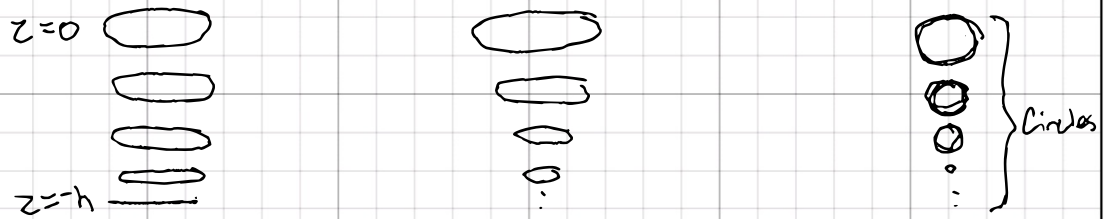
From code: $c_{g_{sw}} = 5.6$ m/s ✓, $\theta_{sw} = 12.62°$, $H_{sw} = 1.75$m, $L_{sw} = 69.45$ m

Breaking → $H_b \geq 0.8h = 0.8(3.53) = 2.82 \geq 1.75$ ∴ not breaking ✓

$L_{sw} = \frac{2\pi}{k_{sw}} = \frac{2\pi}{0.089} = 70.60\,m = T\sqrt{gh} = 12\sqrt{9.81(3.53)} = 70.62\,m$

$\boxed{L_{sw} = 70.62\,m} \neq 69.45$ margin of error? Discrepancy in code? Dispersion vs. shallow water approximation?

fi) SHALLOW | TRANSITION (10m) | DEEP



fii)

SURFACE	h_{sw}	h_{10}	h_0
S_x (m)	2.83	0.40	0.80
S_z (m)	0.88	0.45	0.80

fiii)

BOTTOM	h_{sw}	h_{10}	h_0
S_x (m)	2.69	0.78	0
S_z (m)	0	0	0

All values calculated using MATLAB code

SW: $h_{sw} = 3.53$, $T = 12$, $H_{sw} = 1.75$

TW: $h_{10} = 10$, $T_1 = 12$, $H_{10} = 1.78$

DW: $h_0 = 112.45$, $T = 12$, $H_0 = 1.59$

g) What is h_b and H_b. Derive the formula for h_b.

$$H_b = H_0\sqrt{\frac{C_{gB}}{C_{gb}}}\sqrt{\frac{\cos\theta_0}{\cos\theta_b}} = H_0\sqrt{\frac{C_0}{2\sqrt{gh}}}\sqrt{\cos\theta_0}, \text{ where i } \theta_b = 0,\ C_{g_0} = C_0/2,\ C_{gb} = \sqrt{gh} = C_b$$

$$H_b = Kh_b \Rightarrow h_b = \frac{H_0}{K}\sqrt{\frac{C_0}{2\sqrt{gh_b}}}\sqrt{\cos\theta_0} = \frac{H_0^2}{K^2}\sqrt{\frac{C_0^2}{4gh_b}}\cos\theta = h_b^2$$

$$h_b^4 = \frac{H_0^4}{K^4}\frac{C_0^2}{4gh_b}\cos^2\theta_0 \Rightarrow h_b = \sqrt[5]{\frac{H_0^4}{K^4}\frac{C_0^2}{4g}\cos^2\theta_0}$$

$$= \sqrt[5]{\frac{1.54^4}{0.84^4}\frac{18.74^2}{4(9.81)}\cos^2 45}$$

$$\boxed{h_b = 2.34\ m}$$