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Waves Homework 3

Problem 1 – DD 4.1

Part a)

$$L_0 = \frac{gT^2}{2\pi} \rightarrow T = \sqrt{\frac{L_0 2\pi}{g}} = 13.86 \text{ s}$$

Part b)

$$E = \frac{1}{8}\rho g H_{30} = 4.37 \frac{\text{kJ}}{\text{m}^2}$$

Part c)

Values below calculated using MATLAB code in APPENDIX

$$\theta = 60°,$$

$$L_{30,60°} = 212.76 \text{ m},$$

$$Cg_{30,60°} = 12.44 \frac{\text{m}}{\text{s}},$$

$$H_{30,60°} = 1.49 \text{ m}$$

Problem 2 – DD 4.3

Beginning, the distance between wave rays is defined by:

$$b = lcos(\theta) = l$$

If two wave rays are chosen at the edges of the flume at station A, then $b_A = 100$ m. Since the wave rays will hug the flume wall, the distance between rays at station B will give the slope of the walls. Starting with wave energy conservation, b_B can be determined by

$$H_B = H_A \sqrt{\frac{Cg_A}{Cg_B}} \sqrt{\frac{b_A}{b_B}}$$

Rearranging and using APPENDIX to solve for H_B, Cg_A, Cg_b,

$$b_B = \frac{b_A H_A^2 Cg_A}{H_B^2 Cg_B} = \frac{100(5)^2(14.05)}{2^2(9.304)} = 943.81 \text{ m}$$

To find the slope, we first must find the change in Y of the wall defined by

$$y = \frac{b_B - b_A}{2} = \frac{943.81 - 100}{2} = 421.91 \text{ m}$$

The final answer can be solved with

$$m = \frac{y}{x} = \frac{421.91}{6000} = 0.07$$

Problem 3 – DD 4.4

In elliptical particle trajectory, the semi-major and semi-minor axes of the ellipse are defined respectively as

$$A = \frac{H}{2}\frac{\cosh\big(k(h+z)\big)}{\sinh(kh)} = \frac{d\phi^2}{dxdt}$$

$$B = \frac{H}{2}\frac{\sinh\big(k(h+z)\big)}{\sinh(kh)} = \frac{d\phi^2}{dzdt}$$

If we take the semi-minor axis of the particle trajectory at the middle of the water column $\left(z = \frac{h}{2}\right)$ and assume shallow water, the definition reduces to

$$B = \frac{H}{2}\frac{\sinh\left(\frac{3}{2}kh\right)}{\sinh(kh)} = \frac{H}{2}\frac{3}{2}\frac{kh}{kh} = \frac{3H}{4}$$

Rearranging to solve for H yields

$$H = \frac{4B}{3} = \frac{4(0.05)}{3} = 0.06\text{ m}$$

If the same simplifications are made for the semi-major axis, its definition reduces to

$$A = \frac{H}{2}\frac{\cosh\left(\frac{3}{2}kh\right)}{\sinh(kh)} = \frac{H}{2}\frac{1}{kh}$$

Rearranging and substituting $k = \frac{2\pi}{L}$ allows the equation to transform and yield the wavelength

$$L = \frac{4\pi A h}{H} = \frac{4\pi(0.1)(1)}{0.06} = 20.94\text{ m}$$

The shallow-water approximation also gives an easy definition for T as a rearrangement of $\frac{L}{T} = \sqrt{gh}$

$$T = \frac{L}{\sqrt{gh}} = \frac{20.94}{\sqrt{9.81(1)}} = 6.69\text{ s}$$

Problem 4 – DD 4.8

The number of waves passing through the wave group, n can be defined as a frequency of waves through space. Therefore, the time a wave takes to travel through the wave group can be defined as the period of the group's period or

$$t = \frac{1}{n} = T_1$$

In deep water, a wave will encounter a wave group that is travelling at half of its celerity. The distance the wave group travels in the time it takes the wave passes through it can be expressed as

$$Cg = \frac{L_1}{2T_1} * T_1 = \frac{1}{2}L_1 = d$$

Problem 5 – DD 4.9

Dynamic pressure underneath a wave is defined by

$$P_{D_1} = \rho g \frac{H}{2}\cos(\Omega)\frac{1}{\cosh(kh)}$$

For simplicity, Ω is assumed to be 0 so the equation is reduced to

$$P_{D_1} = \rho g \frac{H}{2}\frac{1}{\cosh(kh)}$$

However, this equation has multiple unknown variables that must be solved for and cannot be otherwise, so it is unclear how to proceed from here

Problem 6 – DD 4.12

Part a)

Using the wave parameters code in APPENDIX, H and L at $h = 5\text{ m}$ are determined as

$$H_5 = 1.32\text{ m},$$

$$L_5 = 103.49\text{ m}$$

Part b)

The water depth at which a wave breaks can be determined by

$$h_b = \sqrt[5]{\frac{H_0^4}{\kappa^4}\frac{C_0^2}{4g}\cos(\theta)} = \sqrt[5]{\frac{1^4}{0.8^4}\frac{351.23^2}{4(15^2)(9.81)}} = 2.03\text{ m}$$

Problem 7 – DD 4.13

If a shallow water approximation is used, $\Omega = \frac{\pi}{2}$, $H = 0.08h$, the horizontal velocity at the seabed can be determined by

$$u = \frac{H}{2}\sqrt{\frac{g}{h}}\sin(\Omega) = \frac{0.08h}{2}\sqrt{\frac{g}{h}}$$

Rearranging yields an equation for h and a maximum depth where the sand depths can be moved

$$h = \frac{4u^2}{0.64g} = \frac{4(0.3)}{0.64(9.81)} = 0.191\text{ m}$$

Using the shallow water approximation definition $\left(h < \frac{L}{20}\right)$ and using APPENDIX to determine the wavelength $(L = 156.13\ \mathrm{m})$, the wave is known to be in shallow water and therefore the approximation is appropriate.

Problem 8 – DD 4.17

There are two reasons the pressure beneath a standing wave's node is purely hydrostatic. First, the standing wave does not propagate, therefore there is no dynamic pressure; second, the surface at the node does not move vertically or horizontally – meaning it is not a function of time and remains constantly hydrostatic.