Doublet / Dipoles

Each basic physical mechanism that generates acoustic pressure fields corresponds mathematically to a dominant order of multipole.

thus, volume or mass fluctuations give rise to dominant simple sources. (zero-order poles → monopoles). Examples are pulsating bubbles, pistons in baffles and cavitation.

Fluctuating forces and vibratory motions of unbaffled rigid bodies are associated with dipoles and have a cosine directional pattern.

Monopoles and dipoles occur only at fluid boundaries.

<u>Types of Sound Sources</u> Pg 46 Mech of Underwater Noise

<u>Monopole</u>

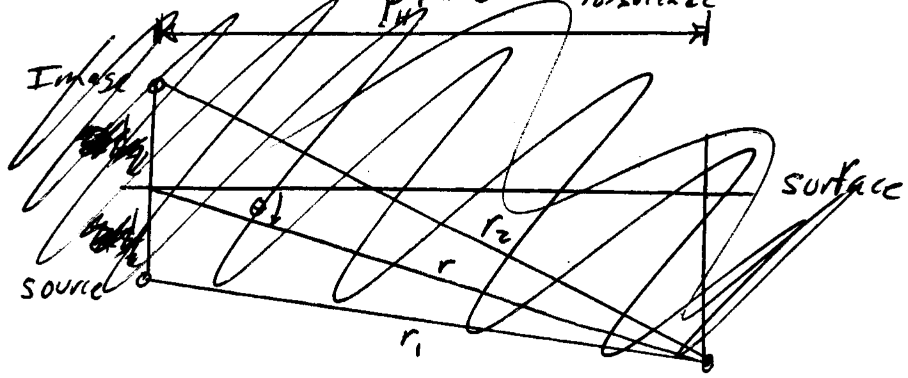
Mass addition, Heat addition (+) volume change, compression

<u>Dipole</u>

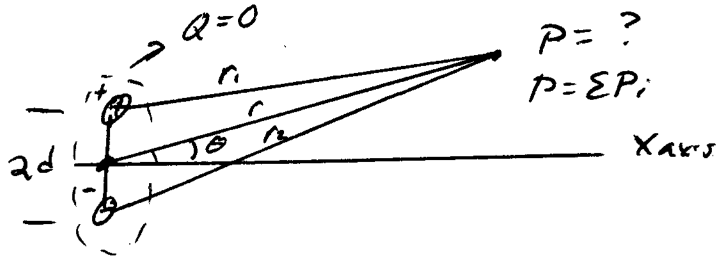
Force ⊕ → ⊖ Translation, Acceleration (sloshing)

<u>Quadrupole</u>

moment shear ⊕ → ⊖, ⊖ ← ⊕ Distortion, Rotation, Turbulence



Consider 2 equal monopoles radiating at exactly the same frequency and separated by distance 2d.



$P(r, \theta, t)$

Their mid point is taken as the origin.

The field point P at which the pressure is to be calculated is located at distance r.

θ is the angle of the field pt relative to the axis.

$$\tilde{P} = \tilde{P}_1 + \tilde{P}_2$$

$$\tilde{P}_1 e^{i\omega t} = \frac{i\omega\rho Q e^{i\omega t - ikr}}{4\pi r_1}$$

$$\tilde{P}_2 e^{i\omega t} = -\frac{i\omega\rho Q e^{i\omega t - ikr}}{4\pi r_2}$$

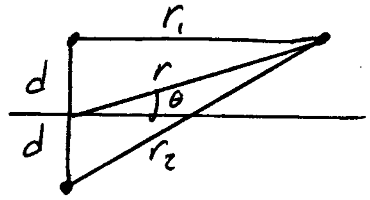
$$\tilde{P} e^{i\omega t} = \frac{i\omega\rho Q}{4\pi}\left[\frac{e^{-ikr_1}}{r_1} - \frac{e^{-ikr_2}}{r_2}\right] e^{i\omega t}$$

<u>IN FARFIELD</u>

$r >> d, \; kr >> 1$

$r_1 = r - d\sin\theta$

$r_2 = r + d\sin\theta$



$$\tilde{P}e^{i\omega t} = \frac{i\omega\rho Q}{4\pi} e^{i\omega t - ikr}\left[\frac{e^{ikd\sin\theta}}{r_1} - \frac{e^{-ikd\sin\theta}}{r_2}\right]$$

Numerator associated with phase
Denominator associated with amplitude

$$\frac{1}{r_1} \simeq \frac{1}{r_2} \simeq \frac{1}{r}$$

$$\tilde{P}e^{i\omega t} = \underbrace{\frac{i\omega\rho Q}{4\pi r} e^{i\omega t - ikr}}_{\text{Pressure field of one source at origin or midpoint}}\left[e^{ikd\sin\theta} - e^{-ikd\sin\theta}\right]$$

or $\quad \sin x = \dfrac{e^{ix} - e^{-ix}}{2i}$

$$\tilde{P}e^{i\omega t} = \tilde{P}_1 e^{i\omega t} \cdot \underbrace{2i\sin(kd\sin\theta)}_{\text{directionality}}$$

A dipole consist of two equal out-of-phase radiators whose separation is very small compared to both wavelength and distance to the field point.

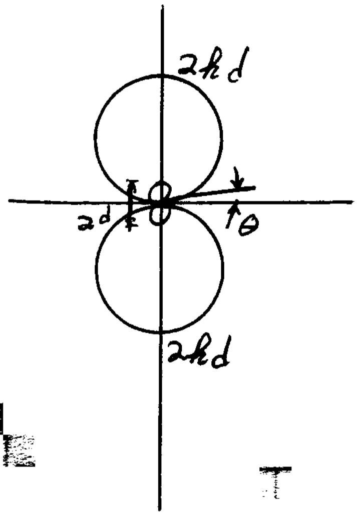
if we assume $kd \ll 1$, $x \ll 1$ $\sin(x) \to x$

$$\tilde{P} e^{i\omega t} = \tilde{P}_1 e^{i\omega t} \cdot 2 i d k \sin\theta$$

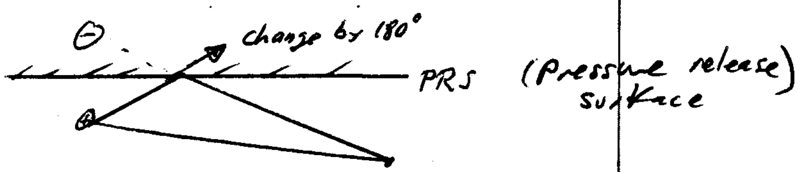
$$P(x,t) = |\tilde{P}| \cos(\omega t)$$

$$= |\tilde{P}_1| \, 2kd \sin\theta$$

As $kd \ll 1$, the pressure field is greatly reduced.

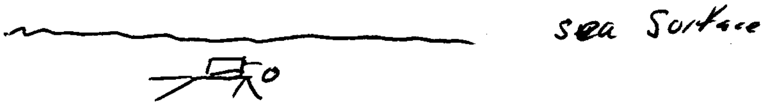


"Pressure field around a pressure release surface"



see
Pg 131 Urick
Fig 5,23

eg.



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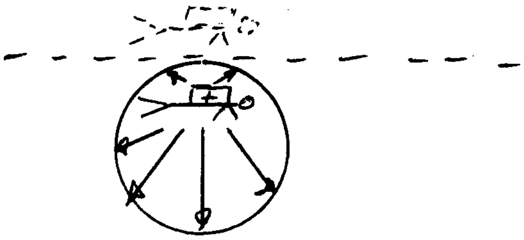
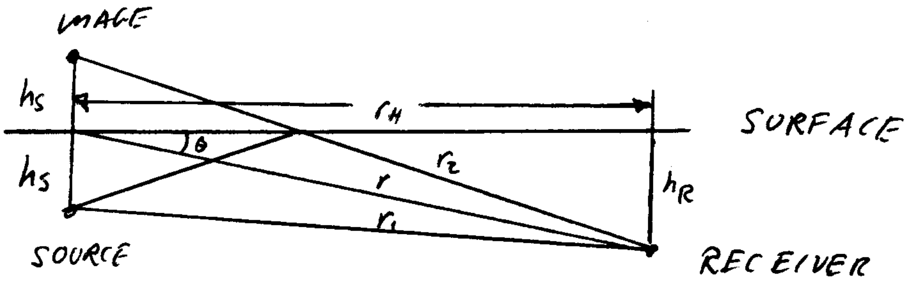


RECIPROCITY: Transmitters and Receivers are interchangeable



Near - Surface Sources

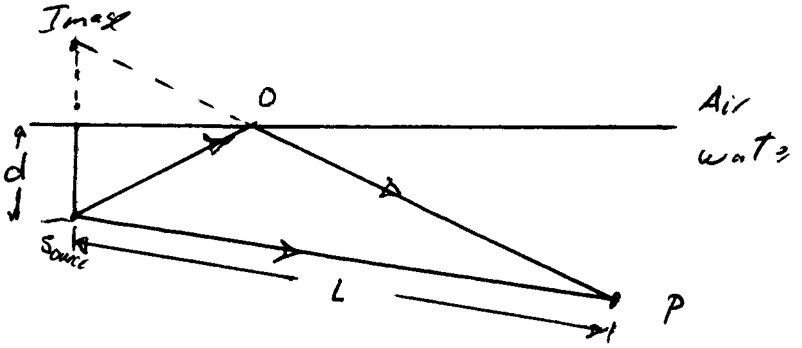
the surface of the ocean is a nearly perfect reflector of sound. Radiation from a source near a surface can be analyzed in terms of direct radiation from the source and from a negative image source located above the surface

the strength of the image source is proportional to the specular reflection coefficient α_r of the surface and so is a function of its roughness.

the rms distance $\bar{r} = \sqrt{r_H^2 + h_S^2 + h_R^2}$

Example 1.

Sound waves are produced at a depth d below the surface of the sea.
Derive an expression for the intensity at point P a distance L from the source S as shown in the figure.



For a homogeneous medium, sound waves reach P via 2 paths: SP directly from the source, and SOP after reflection at O on the boundary surface.

From the acoustic mirror phenomenon, the sound ray OP appears to come from the acoustic image I directly opposite the source S.

The total effect at P is therefore the sum of the direct and reflected waves.

Let p_1 be the acoustic pressure at P due to the direct wave alone

$$p_1 = P_1 \cos(\omega t - \theta_1) \ \frac{nT}{m^2}$$

P_1 = Pressure amplitude

$\theta_1 = \frac{\omega L}{c}$ phase difference between the pressure at S and at P

Similarly let p_2 be the acoustic pressure at P due to the reflected wave alone,

$$p_2 = P_2 \cos(\omega t - \theta_2 - 180^\circ) \ \frac{nT}{m^2}$$

$\theta_2 = \frac{\omega (IP)}{c}$ the phase difference between the pressure at the Image I and the receiver P and 180° is the phase change due to the reflection at the interface (from water to air).

The resultant pressure at P is therefore

$$p = p_1 + p_2 = P_1 \cos(\omega t - \theta_1) + P_2 \cos(\omega t - \theta_2 - 180^\circ)$$

$$P_1 \cos(\omega t - \theta_1) = P_1 \cos \omega t \cos\theta_1 + P_1 \sin \omega t \sin\theta_1$$

$$P_2 \cos(\omega t - \theta_2 - 180^\circ) = P_2 \cos \omega t \cos(\theta_2 + 180^\circ) + P_2 \sin \omega t \sin(\theta_2 + 180^\circ)$$

$$\cos(\theta_2 + 180^\circ) = -\cos\theta_2$$

$$\sin(\theta_2 + 180^\circ) = -\sin\theta_2$$

We have $p = \cos \omega t (P_1 \cos\theta_1 - P_2 \cos\theta_2) + \sin \omega t (P_1 \sin\theta_1 - P_2 \sin\theta_2)$

or $p = P \cos(\omega t - \phi)$

where $P = \sqrt{A^2 + B^2}$

$\phi = \tan^{-1}\left(\frac{B}{A}\right)$

$$A = P_1 \cos\theta_1 - P_2 \cos\theta_2$$

$$B = P_1 \sin\theta_1 - P_2 \sin\theta_2$$

The intensity of the resultant radiation at P becomes

$$I = \frac{p^2}{2\rho c} = (A^2 + B^2)/2\rho c \quad \frac{\text{watts}}{m^2}$$

$$A^2 = P_1^2 \cos^2\theta_1 + P_2^2 \cos^2\theta_2 - 2P_1P_2 \cos\theta_1 \cos\theta_2$$

$$B^2 = P_1^2 \sin^2\theta_1 + P_2^2 \sin^2\theta_2 - 2P_1P_2 \sin\theta_1 \sin\theta_2$$

$$A^2 + B^2 = P_1^2 + P_2^2 - 2P_1P_2 \cos(\theta_1 - \theta_2)$$

so $\boxed{I = \frac{P_1^2 + P_2^2 - 2P_1P_2\cos(\theta_1 - \theta_2)}{2\rho c}}$

It is convenient to express this intensity in terms of the intensity $I_0 = \frac{p_1^2}{2\rho c}$ produced at P by the direct radiation from source S.

Then $$I = \frac{p_1^2}{2\rho c}\left[\frac{P_1^2}{P_1^2} + \frac{P_2^2}{P_1^2} - \frac{2P_1P_2\cos(\theta_1 - \theta_2)}{P_1^2}\right]$$

$$= I_0\left[1 + R^2 - 2R\cos(\theta_1 - \theta_2)\right]$$

where $R = \frac{P_2}{P_1}$ (the ratio of the pressure amplitudes due to reflected and direct waves.)

Depending on the values of the phase ∢'s θ_1 and θ_2, $\cos(\theta_1 - \theta_2)$ will fluctuate between -1 and $+1$. the resultant intensity is seen to fluctuate between $I_0(1+R)^2$ and $I_0(1-R)^2$.

If the source and the receiver are close together near the surface, the phase angles are essentially zero and R approximates unity; then the resultant intensity will fluctuate between zero and $4I_0$