Acoustic Intensity - Sound Power

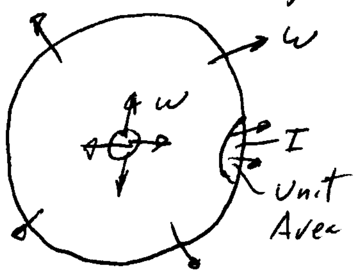
1) $\leftarrow O \rightarrow \quad E_i = \frac{1}{2} m U_i^2$

2) Rate of Transmission of Energy is Power

3) Intensity Power (Acoustic Intensity) flow per unit area at a point (I)

4) Sound Power: acoustic power Transmited to a fluid by the source

consider a spherical system



U_r (radial)
U_n (normal)

Power generated by a source must equal the power over the enclosing surfaces

$$I = \frac{Power}{AREA} = \frac{w}{4\pi r^2}$$

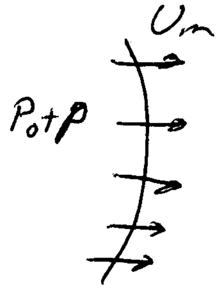
$$I \propto \frac{1}{r^2}$$

Power = Force X Velocity

Power = Pressure x Area x Velocity

Intensity = Pressure x Velocity

$$I_i = (P_0 + P) \times U_m$$



Small section of sphere

For a pulsating sphere:

$$P = RP(\tilde{P}e^{i\omega t}) \quad \text{and} \quad U_m = RP(\tilde{U}e^{i\omega t})$$

$$I_i = RP(P_0 \tilde{U}e^{i\omega t}) + RP(\tilde{P}e^{i\omega t})RP(\tilde{U}e^{i\omega t})$$

$$I_m = \frac{1}{T}\int_0^T I_i \, dt \quad \text{Total intensity}$$

but $RP(\tilde{P}e^{i\omega t}) = RP(|\tilde{P}| e^{i\omega t + \phi_P})$

$$= |\tilde{P}| \cos(\omega t + \phi_p)$$

$$U_r = \frac{\tilde{P}e^{i\omega t}}{\rho_0 C_0}$$

$$\tilde{U}e^{i\omega t} = \frac{\tilde{P}e^{i\omega t}}{\rho_0 C_0}$$

$$\boxed{I = \frac{W}{4\pi r^2}} \qquad W = \text{watt}$$

$$I_m = \frac{1}{T}\int_0^T I_i \, dt$$ Intensity with respect to the mass under investigation

$$I_i = RP(P_0 \tilde{U}e^{i\omega t}) + RP(\tilde{P}e^{i\omega t})RP(Ue^{i\omega t})$$

$$I_m = \frac{1}{T}\int_0^T \Big[\underbrace{P_0 |\tilde{U}| \cos(\omega t + \phi_U)}_{=0} + |\tilde{P}||\tilde{U}| \cos(\omega t + \phi_p)\cos(\omega t + \phi_U)\Big] dt$$

$$I_m = \frac{1}{T}\int_0^T |\tilde{P}||\tilde{U}| \cos(\omega t + \phi_p)\cos(\omega t + \phi_U) \, dt$$

$$I_m = \frac{|\tilde{P}||\tilde{U}|}{2T}\int_0^T \left[\cos(\underbrace{2\omega t + \phi_p + \phi_u}_{0}) + \cos(\phi_p - \phi_u)\right] dt$$

$$I_m = \frac{1}{2}|\tilde{P}||\tilde{U}|\cos(\phi_p - \phi_u)$$

$$I_m = \frac{1}{2} RP\left(|\tilde{P}||\tilde{U}| e^{i\phi_p - i\phi_u}\right)$$

$$I_m = \frac{1}{2} RP\left(\underbrace{|\tilde{P}| e^{i\phi_p}}_{\tilde{P}} \cdot |\tilde{U}| e^{-i\phi_u}\right.$$

Recall $z = x + iy$ $\quad z^* =$ complex conjugate

$z^* = x - ij$ $\quad |\tilde{U}| e^{-i\phi_u} = \tilde{U}^*$

$zz^* = x^2 + y^2$ $\quad (i = \sqrt{-1})$

$$I_m = \frac{1}{2} RP(\tilde{P}\tilde{U}^*)$$

Acoustic Momentum Equation gave:

$$\tilde{U} e^{i\omega t} = \frac{\tilde{P} e^{i\omega t}}{\rho_0 c_0}$$

$$\tilde{U} = \frac{\tilde{P}}{\rho_0 c_0}$$

or

$$U^* = \frac{\tilde{P}^*}{\rho_0 c_0}$$

$$I_m = \frac{1}{2} Re\left(\frac{\tilde{P}\tilde{P}^*}{\rho_0 c_0}\right)$$

Recall: $(x + iy)(x - iy) = x^2 + y^2 = zz^*$

$zz^* = |z|^2$

$$I_m = \frac{1}{2}\frac{RP\,|P|^2}{\rho_0 c_0}$$ $|P|$ = Peak Pressure

$$\boxed{I_m = \frac{1}{2}\frac{|P|^2}{\rho_0 c_0}}$$

also $I = \frac{1}{2}|P|u\cos\theta$ Pg 32 eqn 2.70

Pg 66 shows outline

mechanics of underwater noise

We have a relationship for W, I, P, U

Recall $P_{rms}^2 = \frac{|P|^2}{2}$

$$\boxed{I_m = \frac{P_{rms}^2}{\rho_0 c_0}}$$

$\rho_0 c_0 = 415$ Rayl (air)

$\rho_0 c_0 = 1.54 \times 10^6$ Rayl (sea water)

For a pulsating sphere, I constant on a spherical surface

$$I_m = \frac{W}{4\pi r^2}$$ e.g. spherical Transducer/Pinger

$$W = 4\pi r^2 I_m$$

$$\boxed{W = \frac{4\pi r^2 P_{rms}^2}{\rho_0 c_0}}$$ for a sphere

example

Recall: $SPL = 10 \log\left[\frac{P_{rms}^2}{P_0^2}\right]$ relationship between W, db level.

10 w

Perfect source

10 m (air)

SPL ?

$$I_{@10m} = \frac{W_{(10)}}{4\pi r^2_{(10^2)}} = 0.008\ \frac{W}{m^2}$$

$$|P|^2 = 2\rho_0 c_0 I = 2(415)(0.008) = 6.64\ Pa^2$$

go To Pg 36

$$\frac{|p|^2}{2} = P_{rms}^2 = 3.32\ Pa^2$$

so $P_o = 20\ \mu Pa = 20\ (10)^{-6}\ Pa$

and $SPL = 10 \log \left(\frac{P_{rms}^2}{P_o^2} \right)$

$$= 10 \log \left(83\ (10)^8 \right)$$

$SPL = 99\ db$ ref $20\ \mu Pa$

This assumes perfect sources - (This is very high)

example: A simple sound source radiates harmonic diverging spherical waves into free space with 10 watts of acoustic power at a frequency of 500 cycle/sec.
Find the a) intensity b) acoustic pressure c) particle velocity d) Particle displacement e) condensation f) sound pressure level

a) $I = \frac{W}{4\pi r^2} = \frac{10}{(4)(\pi)1^2} = \underline{0.8 \frac{watt}{m^2}}$

b) $I = \frac{1}{2}\frac{p^2}{\rho c}$ Acoustic pressure $p = \sqrt{2\rho c I} = \sqrt{2(1.21)(343)(0.8)} = \underline{25.8 \frac{nT}{m^2}}$

c) particle velocity $u = \frac{P}{\rho c}\cos\theta$

$p = 25.8 \frac{nT}{m^2}$ $c = 343 \frac{m}{sec}$

$\rho = 1.21 \frac{kg}{m^3}$ $\cos\theta = \frac{kr}{\sqrt{1+k^2r^2}}$

θ = phase ∡ between the acoustic pressure and particle velocity

where $kr = \frac{wr}{c}$ $k = \frac{w}{c}$

$w = 2\pi f$

$kr = \frac{(2\pi f)(r)}{c} = \frac{(2)\pi(500)(1)}{343}$

$kr = 9.18$ so $\cos\theta = 0.99$

$u = \frac{25.8 \frac{nT}{m^2}}{(1.21 \frac{kg}{m^3})(343 \frac{m}{sec})} 0.99 = \underline{0.062 \frac{m}{sec}}$

d) particle displacement $\xi = \frac{u}{w} = \frac{0.062 \frac{m}{sec}}{(2)(\pi)(500)} = \underline{1.97(10)^{-5} m}$

e) condensation $s = \frac{P}{\rho c^2} = \frac{25.8}{(1.21)(343)^2} = 1.8 \times 10^{-4}$

f) sound Pressure Level $\underline{SPL = 20 \log P + 94} = 20 \log 25.8 + 94$

For std sound Pressure reference $P_0 = 2(10)^{-5} \frac{nT}{m^2}$ or 0.0002 microbar

$= \underline{122.3 \text{ db}}$

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example

A diverging spherical wave has a peak acoustic pressure of $2\,\frac{nT}{m^2}$ at a distance of 1m from the source at standard atmospheric pressure and temperature. What is its intensity at a distance of 10m from the source?

1st assume the source is emitting a constant amount of energy to the sound waves.

The intensity of the wave diminishes with distance of propagation

@ 1m from the source

$$I = \frac{p^2}{2\rho c} = \frac{2^2}{2(1.21)(343)} = 0.0048\ \frac{\text{watt}}{m^2}$$

$\rho = 1.21\ \frac{kg}{m^3}$ for air

$c = 343\ \frac{m}{s}$ speed of sound in air at std atm pressure & temp

@ 10m from the source the effective sound pressure will change but the power radiated will remain the same.

$$W = 4\pi r^2 I = 4(3.14)(1)^2(0.0048) = 0.062\ \text{watt}$$

$$I = \frac{W}{4\pi r^2} = \frac{0.062}{4(3.14)(10)^2} = 0.000048\ \frac{\text{watt}}{m^2}$$