Hydroacoustics HW#1

Basic Properties of Fluids

Due Wednesday 21st September

An Incompressible fluid is defined by

$$\frac{\partial \varrho}{\partial t} + \bar{u}.\nabla\varrho = 0$$

A) Show for an incompressible fluid that the equation of continuity reduces to:

$$\nabla.\bar{u} = 0$$

B) What is Eulers Equation for an Incompressible fluid ?

C) What is c_o for an incompressible fluid? (use the definition for an adiabatic gas and the definition of c_o)

An incompressible fluid is defined by $\frac{\partial \rho}{\partial t} + \vec{u} \cdot \nabla \rho = 0$

Show for incompressible fluid that:

a) <u>Equation of continuity reduces to $\nabla \cdot \vec{u} = 0$</u>

$$\frac{\partial \rho}{\partial t} + \vec{u} \nabla \rho = 0 \longrightarrow \frac{\partial \rho}{\partial t} + \rho \nabla \vec{u} = 0$$

$$\frac{\partial \rho}{\partial t} + \rho \frac{\partial u_x}{\partial x} + u_x \frac{\partial \rho}{\partial x} + \rho \frac{\partial u_y}{\partial y} + u_y \frac{\partial \rho}{\partial y} + \rho \frac{\partial u_z}{\partial z} + u_z \frac{\partial \rho}{\partial z} = 0$$

incompressibillity $\longrightarrow$ constant density

$$\rho \frac{\partial u_x}{\partial x} + \rho \frac{\partial u_y}{\partial y} + \rho \frac{\partial u_z}{\partial z} = 0$$

$$\rho \left[\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z} \right] = 0$$

$$\rho \left[\nabla \cdot \vec{u} \right] = 0 \Longrightarrow \nabla \cdot \vec{u} = 0$$

b) Derive momentum equation for incompressible fluid

$$\vec{F} = m \vec{a}$$

$$\vec{F} = (\rho \, dv) \vec{a}$$

$$-\nabla P \, dv = \rho \, dv \, \vec{a}$$

$$\vec{a} = \frac{\partial \vec{u}}{\partial t} + (\vec{u} \nabla) \vec{u}$$

$$-\nabla P\, dv = (\rho\, dV)\, a$$

$$\therefore \quad -\nabla P \cancel{dv} = \left[\frac{\partial \vec{u}}{\partial t} + (\bar{u}\,\nabla)\,\vec{u} \right] \rho \cancel{dv}$$

Therefore $\quad -\nabla P = \rho \left[\frac{\partial \vec{u}}{\partial t} + (\bar{u} \cdot \nabla)\,\vec{u} \right]$

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c) What is c for an incompressible fluid

c is the speed of sound in a fluid, for incompressible fluids c would tend to infinity. WHY?