

Extra Credit 3.3

(a) $T(n) = 2T(n/2) + n^3$
 homogeneous $T(n) = 2T(n/2)$
 $TP(n) = Cn$
 $TP(n) = Cn^3$
 $Cn^2 = \frac{1}{4}n^3 + n^3$
 $T(n) = T_h(n) + TP(n)$
 $T(1) = 1$ therefore $C = -1/3$
 $T(n) = 4/3(4n \leq 1)$

b) $T(n) = 2T(n/4) + \sqrt{n}$
 $2(2T(n/16) + \sqrt{n/4}) + \sqrt{n} = 4T(n/16) + 2\sqrt{n}$
 $2(4T(n/64) + 2\sqrt{n/16}) + \sqrt{n} = 8T(n/64) + 2\sqrt{n}$
 $2(8T(n/256) + 2\sqrt{n/64}) + \sqrt{n} = 16T(n/256) + 5/4\sqrt{n}$

c) $T(n) = T(\sqrt{n}) + 1$ (consider $n = 2^{2^k}$)
 $T(2^{2^k}) = T(2^{2^{k-1}}) + 1 \Rightarrow T(2^{2^{k-1}}) = 1$
 $T(2^{2^0}) + k = T(2^{2^k})$
 $T(2) = T(\sqrt{2}) + 1 \rightarrow T(k) = T(\sqrt{k}) + 1$

d) $T(n) = 4T(n/2) + n^2$
 $T(n) = n^2 + 4(4T(n/4) + n^2/4)$
 $= 2n^2 + 16T(n/4)$
 $k \cdot n^2 + 4^k T(n/2^k)$