Suppose $\mathbb{X} = U\Lambda V^\top$. Then a plug-in PCR-based estimate of β is $\tilde{\beta}_d = V_d \Lambda_d^{-1} U_d^\top Y$.

We can make a plug-in estimate (using the notation from Ding Mcdonald) $\hat{\beta}_d = U_d(F)\Lambda_d(F)^{-1}U_d(F)^\top \mathbb{X}^\top Y$. Also, write $U = [U_d, U_{p-d}]$ to be the decomposition of U into the first d columns and last $p-d$ columns and write $\tilde{U}_d$ and $\tilde{U}_{p-d}$ to indicate padding with zeros to give the same dimension as U. Now, $U_d(F) \approx V_d$, $\Lambda_d(F)^{-1} \approx \Lambda^{-2}$, and $U_d(F)^\top V \approx \begin{bmatrix} I_d & 0 \end{bmatrix}$. So, I'll proceed as usual to distill this expression down to these parts[1]

$$
\begin{aligned}
\|\hat{\beta}_d - \tilde{\beta}_d\|_2^2 &= \|U_d(F)\Lambda_d(F)^{-1}U_d(F)^\top \mathbb{X}^\top Y - V_d\Lambda_d^{-1}U_d^\top Y\|_2^2 && (1)\\
&= \|U_d(F)\Lambda_d(F)^{-1}U_d(F)^\top (V\Lambda[U_d, U_{p-d}]^\top)Y - V_d\Lambda_d^{-1}U_d^\top Y\|_2^2 && (2)\\
&= \|U_d(F)\Lambda_d(F)^{-1}U_d(F)^\top V\Lambda(\tilde{U}_d^\top Y + \tilde{U}_{p-d}^\top Y) - V_d\Lambda_d^{-1}U_d^\top Y\|_2^2 && (3)\\
&\le \|U_d(F)\Lambda_d(F)^{-1}U_d(F)^\top V\Lambda U_d^\top Y - V_d\Lambda_d^{-1}U_d^\top Y\|_2^2 + \|U_d(F)\Lambda_d(F)^{-1}U_d(F)^\top V\Lambda\tilde{U}_{p-d}^\top Y\|_2^2 && (4)\\
&\le \|U_d(F)\Lambda_d(F)^{-1}U_d(F)^\top V_d\Lambda_d - V_d\Lambda_d^{-1}U_d^\top Y\|_2^2\|U_d^\top Y\|_2^2 + \|U_d(F)\Lambda_d(F)^{-1}U_d(F)^\top V\Lambda\tilde{U}_{p-d}^\top Y\|_2^2 && (5)\\
&= \|U_d(F)\Lambda_d(F)^{-1}U_d(F)^\top V_d\Lambda_d - V_d\Lambda_d^{-1}U_d^\top Y\|_2^2 M_d + R_d && (6)\\
&= \|U_d(F)\Lambda_d(F)^{-1}U_d(F)^\top V_d\Lambda_d - (U_d(F)\Lambda_d(F)^{-1}\Lambda_d + U_d(F)\Lambda_d(F)^{-1}\Lambda_d) - V_d\Lambda_d^{-1}U_d^\top Y\|_2^2 M_d + R_d && (7)\\
&\le M_d\left(\|U_d(F)\Lambda_d(F)^{-1}\|_2^2\|U_d(F)^\top V_d - I\|_2^2\|\Lambda_d\|_2^2 - \Lambda_d + \|U_d(F)\Lambda_d(F)^{-1}\Lambda_d) - V_d\Lambda_d^{-1}U_d^\top Y\|_2^2\right) + R_d && (8)\\
&\text{and so on..} && (9)
\end{aligned}
$$

[1] Needing repeated constant 2 for $\|a+b\|_2^2 \le 2(\|a\|_2^2 + \|b\|_2^2)$