Assignment3

Course: Computer Vision

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Q1. Nonlinear least-squares. Suppose that $f(\mathbf{x}) = (f_1(\mathbf{x}), f_2(\mathbf{x}), \dots, f_m(\mathbf{x})) : \mathbb{R}^n \rightarrow \mathbb{R}^m$, $\mathbf{x} \in \mathbb{R}^n$, $f \in \mathbb{R}^m$ and some $f_i : \mathbb{R}^n \rightarrow \mathbb{R}$ is a (are) non-linear function(s). Then, the problem,

$$\mathbf{x}^* = \arg\min_{\mathbf{x}} \frac{1}{2}\|f(\mathbf{x})\|^2 = \arg\min_{\mathbf{x}} \frac{1}{2}(f(\mathbf{x}))^{\mathrm{T}} f(\mathbf{x})$$

is a nonlinear least-squares problem. In our lecture, we mentioned that Levenberg-Marquardt algorithm is a typical method to solve this problem. In L-M algorithm, for each updating step, at the current $\mathbf{x}$, a local approximation model is constructed as,

$$\begin{aligned} L(\mathbf{h}) &= \frac{1}{2}(f(\mathbf{x}+\mathbf{h}))^{\mathrm{T}}(f(\mathbf{x}+\mathbf{h})) + \frac{1}{2}\mu\mathbf{h}^{\mathrm{T}}\mathbf{h} \\ &= \frac{1}{2}(f(\mathbf{x}))^{\mathrm{T}} f(\mathbf{x}) + \mathbf{h}^{\mathrm{T}}(J(\mathbf{x}))^{\mathrm{T}} f(\mathbf{x}) + \frac{1}{2}\mathbf{h}^{\mathrm{T}}(J(\mathbf{x}))^{\mathrm{T}} J(\mathbf{x})\mathbf{h} + \frac{1}{2}\mu\mathbf{h}^{\mathrm{T}}\mathbf{h} \end{aligned}$$

where $J(\mathbf{x})$ is $f(\mathbf{x})$'s Jacobian matrix, and $\mu > 0$ is the damped coefficient. Please prove that $L(\mathbf{h})$ is a strictly convex function. (Hint: If a function $L(\mathbf{h})$ is differentiable up to at least second order, L is strictly convex if its Hessian matrix is positive definite.)

Ans. To prove that $L(\mathbf{h})$ is a strictly convex function, we need to prove that its Hessian matrix is positive definite.

$$\begin{aligned} \nabla L(\mathbf{h}) &= \frac{\partial L(\mathbf{h})}{\partial \mathbf{h}} = (J(\mathbf{x}))^{\mathrm{T}} f(\mathbf{x}) + (J(\mathbf{x}))^{\mathrm{T}} J(\mathbf{x})\mathbf{h} + \mu\mathbf{h} \\ \nabla^2 L(\mathbf{h}) &= \frac{\partial^2 L(\mathbf{h})}{\partial \mathbf{h}^2} = (J(\mathbf{x}))^{\mathrm{T}} J(\mathbf{x}) + \mu I \end{aligned}$$

where I is the identity matrix.

Hence, we need to prove that $(J(\mathbf{x}))^{\mathrm{T}} J(\mathbf{x}) + \mu I$ is positive definite.

For any $\mathbf{v} \in \mathbb{R}^n$, $\mathbf{v} \neq \mathbf{0}$

$$\begin{aligned} \mathbf{v}^{\mathrm{T}}((J(\mathbf{x}))^{\mathrm{T}} J(\mathbf{x}) + \mu I)\mathbf{v} &= \mathbf{v}^{\mathrm{T}}(J(\mathbf{x}))^{\mathrm{T}} J(\mathbf{x})\mathbf{v} + \mu\mathbf{v}^{\mathrm{T}}\mathbf{v} \\ &= \|J(\mathbf{x})\mathbf{v}\|_2^2 + \mu\|\mathbf{v}\|_2^2 \\ &> 0 \end{aligned}$$

Hence, $(J(\mathbf{x}))^{\mathrm{T}} J(\mathbf{x}) + \mu I$ is positive definite, and $L(\mathbf{h})$ is a strictly convex function.

Q2. I have established a dataset for training models for detecting speed-bumps and persons. This dataset can be downloaded from
https://github.com/csLinZhang/CVBook/tree/main/chapter-15-YOLO/For-yolov4.

Using this dataset, please train a speed-bump detection model and test your model on the provided test video (on the course website). For this question, you only need to hand in your video with detected bounding-boxes to the TA. A sample frame of our result video may like the following image.

Ans. The following is a frame with detected result in the video.