Assignment 2 (Due: Nov. 26, 2023)

1. (**Math**) In the augmented Euclidean plane, there is a line $x-3y+4=0$, what is the homogeneous coordinate of the infinity point of this line?

2. (**Math**) On the normalized retinal plane, suppose that $\mathbf{p}_n$ is an ideal point of projection without considering distortion. If distortion is considered, $\mathbf{p}_n=(x, y)^T$ is mapped to $\mathbf{p}_d=(x_d, y_d)^T$ which is also on the normalized retinal plane. Their relationship is,
$$\begin{cases} x_d = x\left(1+k_1r^2+k_2r^4\right)+2\rho_1xy+\rho_2\left(r^2+2x^2\right)+xk_3r^6 \\ y_d = y\left(1+k_1r^2+k_2r^4\right)+2\rho_2xy+\rho_1\left(r^2+2y^2\right)+yk_3r^6 \end{cases}$$
where $r^2=x^2+y^2$
For performing nonlinear optimization in the pipeline of camera calibration, we need to compute the Jacobian matrix of $\mathbf{p}_d$ w.r.t $\mathbf{p}_n$, i.e.,
$$\frac{d\mathbf{p}_d}{d\mathbf{p}_n^T}$$
It should be noted that in this question $\mathbf{p}_d$ is the function of $\mathbf{p}_n$ and all the other parameters can be regarded as constants.

3. (**Math**) In our lecture, we mentioned that for performing nonlinear optimization in the pipeline of camera calibration, we need to compute the Jacobian of the rotation matrix (represented in a vector) w.r.t its axis-angle representation. In this question, your task is to derive the concrete formula of this Jacobian matrix.
Suppose that
$\mathbf{d}=\theta\mathbf{n}\in\mathbb{R}^{3\times1}$, where $\mathbf{n}=\begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix}$ is a 3D unit vector and θ is a real number denoting the rotation angle.
With Rodrigues formula, $\mathbf{d}$ can be converted to its rotation matrix form,
$$\mathbf{R}=\cos\theta\mathbf{I}+(1-\cos\theta)\mathbf{n}\mathbf{n}^T+\sin\theta\mathbf{n}^\wedge$$
and obviously $\mathbf{R}\triangleq\begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$ is a 3×3 matrix.
Denote $\mathbf{r}$ by the vectorized form of $\mathbf{R}$, i.e.,
$$\mathbf{r}\triangleq\left(r_{11},r_{12},r_{13},r_{21},r_{22},r_{23},r_{31},r_{32},r_{33}\right)^T$$
Please give the concrete form of Jacobian matrix of $\mathbf{r}$ w.r.t $\mathbf{d}$, i.e., $\frac{d\mathbf{r}}{d\mathbf{d}^T}\in\mathbb{R}^{9\times3}$.
In order to make it easy to check your result, please follow the following notation requirements,
$$\alpha\triangleq\sin\theta,\beta\triangleq\cos\theta,\gamma\triangleq1-\cos\theta$$
In other words, the ingredients appearing in your formula are restricted to $\alpha,\beta,\gamma,\theta,n_1,n_2,n_3$.

4. (**Programming**) Bird's-eye-view generation. The geometric transform between the physical plane and its bird's-eye-view image can be simply described by a **similarity transformation** matrix. Bird's-eye-view is very useful in autonomous industrial inspection, ADAS, etc. In this question, your task is to create the bird's-eye-view image of a physical plane, e.g., the wall of your room. For this purpose, you may need to,
   1) make a calibration board with chessboard patterns;
   2) calibrate your camera (the camera mounted on your laptop or the camera of your mobile phone with fixed focal length) to get its intrinsics;
   3) attach regular patterns (e.g., chessboard patterns) to the wall, determine the 2D coordinate system C_W of the wall, and determine the coordinates $\left\{\mathbf{x}_{Wi}\right\}_{i=1}^N$ of the feature points of the regular patterns with

respect to C_W;

4) take the image I_d of the wall with regular patterns;
5) undistort image I_d with the camera's intrinsics to get the undistorted image I;
6) For each $\mathbf{x}_{Wi}$, determine its image $\mathbf{x}_{Ii}$ on I;
7) solve the homography matrix $P_{W \to I}$ between the wall and the image I wall using $\left\{ \mathbf{x}_{Wi} \leftrightarrow \mathbf{x}_{Ii} \right\}_{i=1}^{N}$;
8) generate the final bird's-eye-view image of the wall using the technique introduced in our lecture.

For submission, you **only** need to submit the following items to TA:

1) the intrinsic parameters of your camera;
2) the original image of the wall (or other physical planes) taken by your camera; make sure that your name is painted or attached on the wall (or the plane); (maybe similar to following image I provide to you)
3) the generated bird's-eye-view image of the wall (or other physical planes).