

Assignment 1 (Due: Oct. 29, 2023)

1. (Math) In our lectures, we mentioned that matrices that can represent Euclidean transformations can form a group. Specifically, in 3D space, the set comprising matrices $\{M_i\}$ is actually a group, where

$$M_i = \begin{bmatrix} R_i & \mathbf{t}_i \\ \mathbf{0}^T & 1 \end{bmatrix} \in \mathbb{R}^{4 \times 4}, R_i \in \mathbb{R}^{3 \times 3} \text{ is an orthonormal matrix, } \det(R_i) = 1, \text{ and } \mathbf{t}_i \in \mathbb{R}^{3 \times 1} \text{ is a vector.}$$

Please prove that the set $\{M_i\}$ forms a group.

Hint: You need to prove that $\{M_i\}$ satisfies the four properties of a group, i.e., the closure, the associativity, the existence of an identity element, and the existence of an inverse element for each group element.

2. (Math) When deriving the Harris corner detector, we get the following matrix M composed of first-order partial derivatives in a local image patch w ,

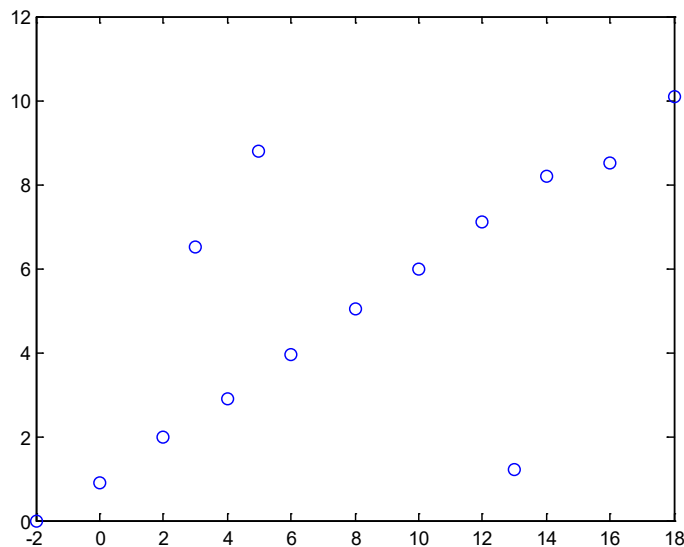
$$M = \begin{bmatrix} \sum_{(x_i, y_i) \in w} (I_x)^2 & \sum_{(x_i, y_i) \in w} (I_x I_y) \\ \sum_{(x_i, y_i) \in w} (I_x I_y) & \sum_{(x_i, y_i) \in w} (I_y)^2 \end{bmatrix}$$

- a) Please prove that M is positive semi-definite.
- b) In practice, M is usually positive definite. If M is positive definite, prove that in the Cartesian coordinate system, $[x, y] M \begin{bmatrix} x \\ y \end{bmatrix} = 1$ represents an ellipse.
- c) Suppose that M is positive definite and its two eigen-values are λ_1 and λ_2 and $\lambda_1 > \lambda_2 > 0$. For the ellipse defined by $[x, y] M \begin{bmatrix} x \\ y \end{bmatrix} = 1$, prove that the length of its semi-major axis is $\frac{1}{\sqrt{\lambda_2}}$ while the length of its semi-minor axis is $\frac{1}{\sqrt{\lambda_1}}$.

3. (Math) In the lecture, we talked about the least square method to solve an over-determined linear system $A\mathbf{x} = \mathbf{b}$, $A \in \mathbb{R}^{m \times n}$, $\mathbf{x} \in \mathbb{R}^{n \times 1}$, $m > n$, $\text{rank}(A) = n$. The closed form solution is $\mathbf{x} = (A^T A)^{-1} A^T \mathbf{b}$. Try to prove that $A^T A$ is non-singular (or in other words, it is invertible).

4. (Programming) RANSAC is widely used in fitting models from sample points with outliers. Please implement a program to fit a straight 2D line using RANSAC from the following sample points:

$(-2, 0)$, $(0, 0.9)$, $(2, 2.0)$, $(3, 6.5)$, $(4, 2.9)$, $(5, 8.8)$, $(6, 3.95)$, $(8, 5.03)$, $(10, 5.97)$, $(12, 7.1)$, $(13, 1.2)$, $(14, 8.2)$, $(16, 8.5)$, $(18, 10.1)$. Please show your result graphically.



5. (Programming) Get two images I_1 and I_2 of our campus and make sure that the major parts of I_1 and I_2 are from the same physical plane. Stitch I_1 and I_2 together to get a panorama view using scale-normalized LoG (or DoG) based interest point detector and SIFT descriptors. You can use OpenCV or Matlab. You are allowed to call the build-in functions provided by Matlab or OpenCV.
6. (Programming) ORB feature point detection and matching algorithms have been fully implemented in the OpenCV library. Please write a C++ program that invokes the OpenCV library's algorithm for ORB feature point detection and matching for two given images, and output feature point matching results similar to the following given example.

