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Metastability in the formation of an experimental traffic jam

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Abstract. We show detailed data about the process of jam formation in a traffic experiment on a circuit without any bottlenecks. The experiment was carried out using a circular road on a flat ground. At the initial stage, vehicles are running homogeneously distributed on the circuit with the same velocity, but roughly 10 min later a traffic jam emerges spontaneously on the circuit. In the process of the jam formation, we found a homogeneous flow with large velocity is temporarily realized before a jam cluster appears. The instability of such a homogeneous flow is the key to understanding jam formation.

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1. Introduction

In recent years, many authors have studied traffic flow from the physical viewpoint [1]–[7]. They have proposed various models to investigate the properties of traffic flow, especially the physical mechanism of the formation of a traffic jam. In their models, the traffic jam emerges spontaneously if the vehicle density is larger than a certain critical value. The transition from a free flow to a jammed flow is understood as a kind of phase transition [8]–[11].

It is widely believed that the origin of a traffic jam is a bottleneck and the jam never appears without a bottleneck. In fact, jams on real highways are usually observed upstream of a bottleneck, and the traffic phenomena near the bottleneck have been investigated [9], [12]–[15]. However, it does not mean that the essential origin of traffic jams from the physical viewpoint is also the bottleneck. Even if a bottleneck exists, no jam appears when the vehicle density is low. In a previous paper, we have carried out a traffic jam experiment to demonstrate that a traffic jam can appear without bottlenecks [16, 17]. The experiment justifies the physical mechanism of jam formation: traffic flow at a sufficiently large density is unstable and transits to a jammed flow. The essential origin of a traffic jam is the instability of the flow and the transition from a free flow to a jammed flow occurs at a certain critical value of the vehicle density. The role of a bottleneck on real highways is to increase the density in order to realize such a traffic flow at a large density for the instability in a open system such as a straight road.

In this paper, we show detailed data about headway and velocity in the previous experiment to investigate the process of jam formation. We carried out a few runs by changing the number of vehicles [17]. We show the data for two cases where a traffic jam emerged. It is indicated that a metastable homogeneous flow is realized before a jam cluster appears. This result supports the physical mechanism of jam formation, the instability of a homogeneous flow.

In section 2, we give an overview of the experiment. Detailed data and the analysis of the process of jam formation are shown in section 3. We summarize the results in section 4.

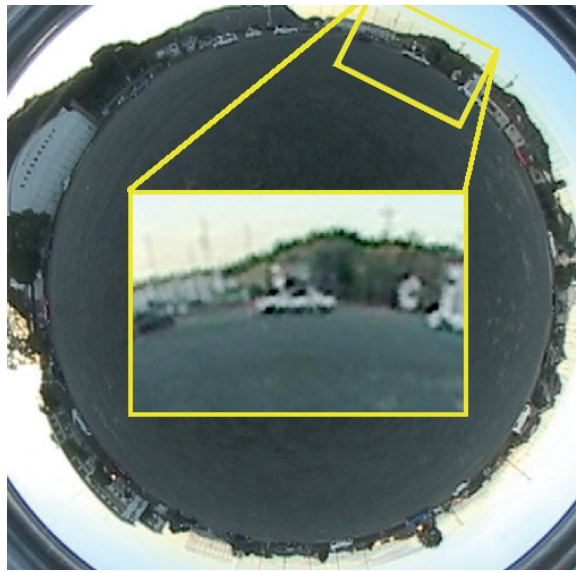


Figure 1. One of the frames of the movie taken by a 360° video camera located at the center of the circuit. Part of the frame is enlarged to show the size of a vehicle.

2. Experiment

2.1. Overview

The experiment was carried out on flat ground, where vehicles run along a circle with circumference 230 m (see figure 1). We have performed a few runs on this circuit. The drivers were requested to cruise at about 30 km h^{-1} . The number of vehicles and the cruising velocity were estimated by using the optimal velocity model in order to satisfy the unstable condition for a homogeneous flow [18]. At the beginning of each run, drivers were instructed so that they cruise along the circle at the same velocity ($\sim 30 \text{ km h}^{-1}$) with almost uniform spacing. We can identify the motion of the queue of vehicles with a free flow. After the start of run, the drivers were only instructed to follow the vehicle ahead in safety, in addition to trying to maintain their cruising velocity.

In the following, we examine the formation of traffic jams in detail, in two cases where the number of vehicles is 22 and 23 (see figure 2). The experimental data are extracted from the movie that was taken by a 360° video camera. We read positions of vehicles every $1/3 \text{ s}$ (10 frames). Velocities are calculated from these data^{11,12}. In the jam experiment, it typically takes 10 min until a jam appears. Then we show the data only for several minutes around the time when the jam appears.

2.2. Space-time diagrams

Figure 2 shows the space-time diagrams of vehicles in two runs: (I) the run of 22 vehicles and (II) the run of 23 vehicles. In figure 2(I), we show the data for a longer period than in the

¹¹ The accuracy of positions is estimated by enlarging a part of a frame. The error of position is roughly $\pm 0.5 \text{ m}$, which indicates that the error of velocity is $\pm 3 \text{ m s}^{-1}$.

¹² The raw data will be available from our web site <http://traffic.phys.cs.is.nagoya-u.ac.jp/>.

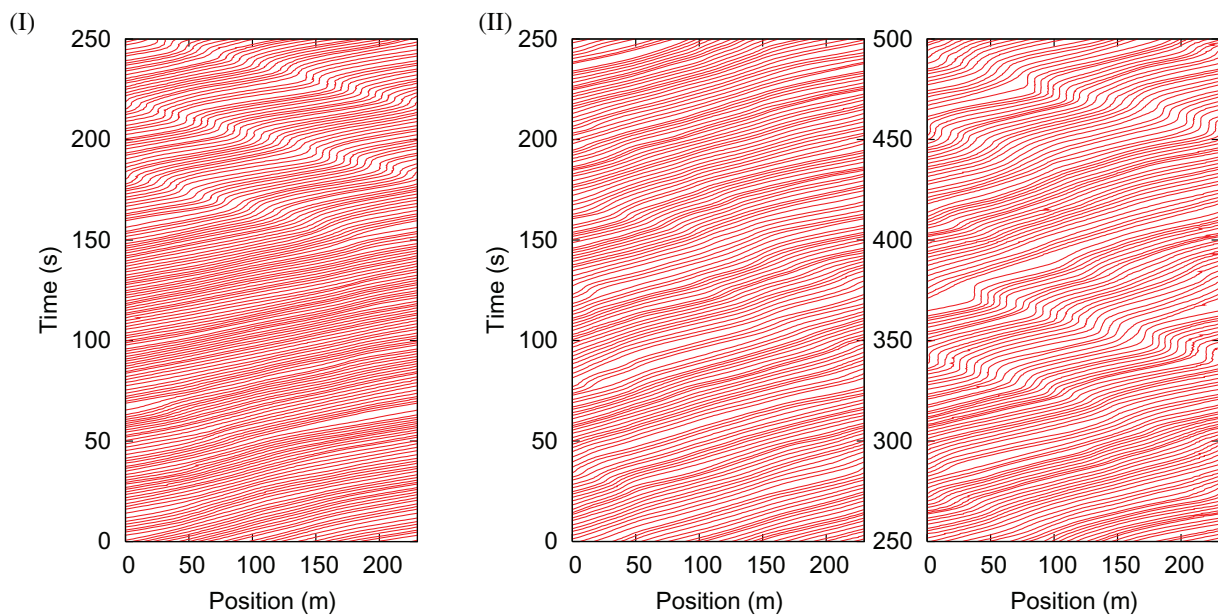


Figure 2. Space-time diagrams of vehicles: (I) a run of 22 vehicles. Data for 250 s around the time when a jam appears are shown. (II) A run of 23 vehicles. Data for 500 s are shown. In these graphs, the 0 m-point is identical to the 230 m-point.

previous paper [17]. A jam cluster emerges spontaneously in both cases. In figure 2(II), we can see a jam cluster is dissolved at $t \sim 370$ s, due to a driver who was aware of the jam ahead and decreased their velocity. Another jam soon emerged, however. This observation indicates that jam formation is really an inevitable nature of high-density traffic flow. As we already know, the jam formation originates in the instability of traffic flow and the vehicle density plays the essential role. Even if a driver dissolves the jam once, a jam appears again, because the total density cannot be changed in the circuit and the flow remains unstable.

2.3. Headway–velocity relation

We calculate headway and velocity from the data on the position for each vehicle. Here, we note that the position of each vehicle is defined by the position of its front wheel and the headway is defined by the distance between the positions of the vehicle and the one in front. A velocity is calculated from the positions of a vehicle in two successive frames, a mean velocity for 1/3 s (see footnote 13). Figure 3 shows headway–velocity relations for the two cases (I) and (II) in figure 2. The relations are qualitatively the same as those observed in real highways [19, 20]. We draw rough sketches of the headway–velocity relation in figure 3 with green curves, as references. Most of the drivers in run (II) participated in run (I) (run (II) was performed before run (I)). Therefore, the drivers got accustomed to the experiment. As a result, the average velocity in run (I) increased. We note that the difference of the number of vehicles between two cases is not essential to the maximum velocity as follows: vehicles move at the maximum velocity outside a jam cluster after the jam is formed. The density in the region is sufficiently

¹³ In this paper, we use the smoothed headway and velocity calculated from the moving average of position $x_{av}(t) \equiv [x(t - 1/3) + 2x(t) + x(t + 1/3)]/4$, in order to remove reading errors in the positions of vehicles.

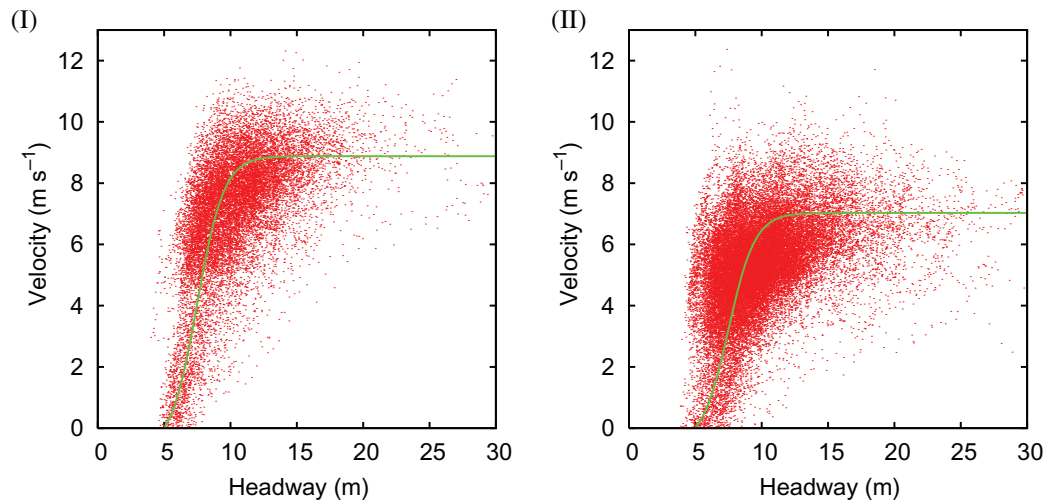


Figure 3. Red dots represent the headway and velocity data for all vehicles. (I) and (II) correspond to the cases of 22 and 23 vehicles, respectively. As references, we draw green curves representing the headway–velocity relations $V(x) = 4.8\{\tanh[0.5(x - 7.5)] + 0.85\}$ and $V(x) = 3.8\{\tanh[0.5(x - 7.5)] + 0.85\}$ for the cases (I) and (II), respectively.

small and is almost the same value in both cases. The number of vehicles only changes the length of the jam cluster.

2.4. Density–flow relation

Figure 4 shows the fundamental diagrams (density–flow relations). We note that the density and the flow rate in figure 4 are local quantities and are not the average ones measured on highways. The density in figure 4 is defined by the inverse of the headway of each vehicle. The spatial average of the density is obviously a constant in this experiment, and cannot be used for the fundamental diagram. The flow rate is calculated from the inverse of headway and the velocity for each vehicle. We discuss the average flow rate in the next section. Green lines in figure 4 are rough sketches of the density–flow relation, which is well known as the reverse λ shape. The shape of the diagram is quite similar to that observed on highways, except for the differences of absolute values of density and the flow rate.

3. Process of jam formation

3.1. Property of velocity and headway

In this section, we investigate the process of jam formation in this experiment. For this purpose, we focus on case (I).

First, we show the time series of the average and standard deviation of velocities of all vehicles in figure 5(a). Figure 5(b) shows the average and standard deviation of headway. From figure 5, we can observe three different features of the average velocity. In order to specify each of them, we define three stages divided by blue lines in figure 5. The early stage is characterized as the state where the fluctuations of velocity and headway are relatively large. This state continues for roughly 10 min from the start of run. The data for the last 85 s of this

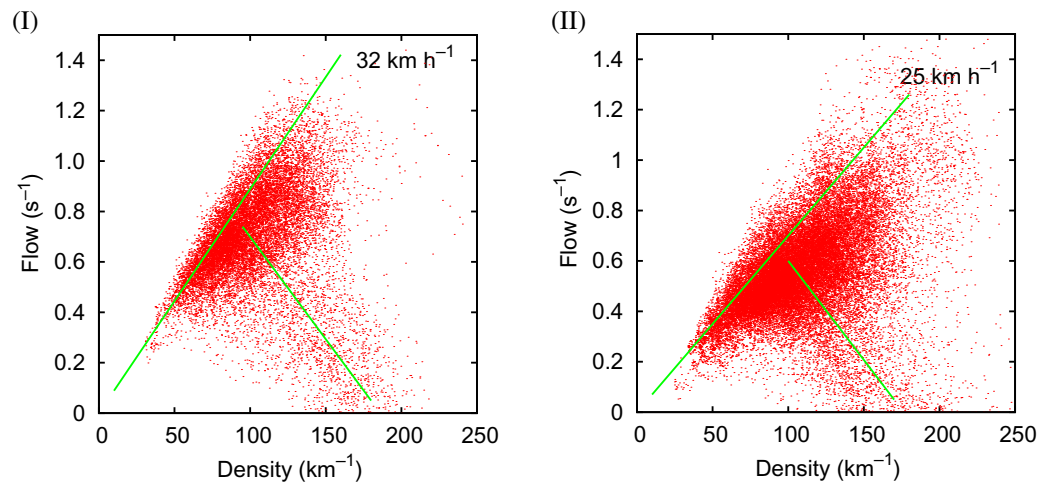


Figure 4. Fundamental diagrams for cases (I) and (II). The density and the flow rate are shown in units of km^{-1} and s^{-1} , respectively. Red dots represent the data for local density and flow rate for all vehicles through the whole period. Green lines in each figure represent a rough sketch of the density–flow relation. The left segments of the green lines correspond to the relation calculated from the maximum velocity 32 km h^{-1} (8.9 m s^{-1}) or 25 km h^{-1} (7.0 m s^{-1}) shown in figure 3.

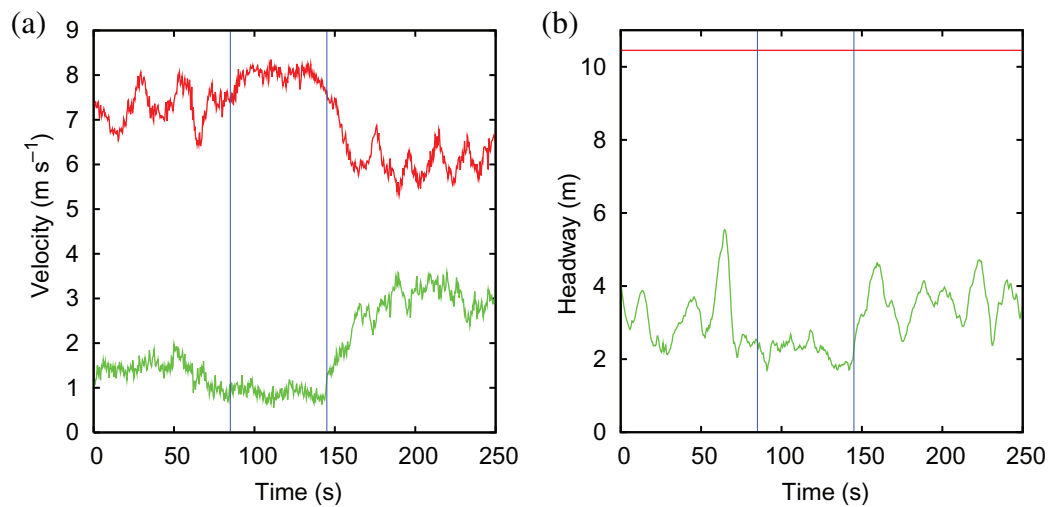


Figure 5. Data for (a) velocity and (b) headway are shown. Red and green represent the average and standard deviation of the data, respectively. Blue lines divide the whole period into three stages. The lines are drawn at $t = 85$ and 145 . Note that the average headway is constant.

stage are shown in figure 5. In the middle stage, the average velocity increases and the average headway is constant. The standard deviations of velocity and headway decrease. This state can be considered to be a homogeneous flow with large velocity. The flow quickly transits to the jammed flow. Therefore this process indicates that the state of the homogeneous flow with large velocity is metastable. It seems that the appearance of such a state is necessary for the

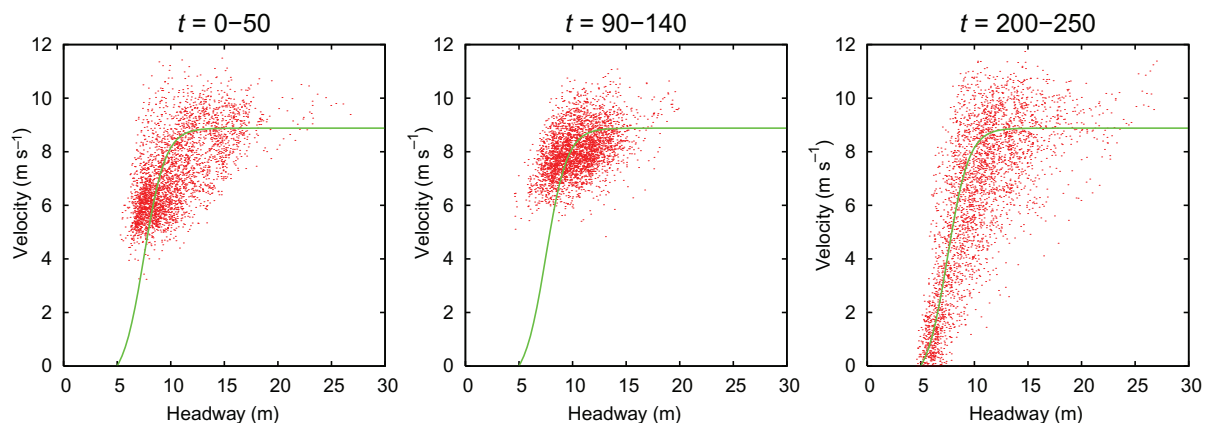


Figure 6. Headway and velocity for three stages in case (I). The periods $t = 0\text{--}50$, $90\text{--}140$ and $200\text{--}250$ correspond to the three stages mentioned in figure 5. Red dots represent data for headway and velocity, and green curves represent the same headway–velocity relation as that in figure 3(I).

transition to the jammed flow. We can observe a temporal appearance of the state before the second formation of a jam ($t \sim 420$) in case (II), figure 2(II) (see appendix A). In the final stage, the state of the jammed flow is characterized by a small velocity and large standard deviation. We can find similar features in the change of headway (see figure 5(b)).

This scenario of jam formation can be clearly seen also in the headway–velocity space. Figure 6 shows the data for all vehicles for the periods $t = 0\text{--}50$, $90\text{--}140$ and $200\text{--}250$, respectively, which are taken from each stage in case (I). In the second period, $t = 90\text{--}140$, the variances of velocity and headway become small compared with those in the first period, $t = 0\text{--}50$. All vehicles move with large velocity and almost equal headway. The state of the second period is a metastable homogeneous flow. It is necessary for the appearance of a jam.

3.2. Properties of the flow

Next, we investigate the change of flow rate in the process of jam formation. Figure 7 shows the time series of the flow rate in case (I). Here, we analyze the flow rate by two definitions. One is the number of vehicles passing through the observational point during 40 s. This duration corresponds to the period in which a jam cluster moves around the circuit once. The other is the average of the product of velocity and the inverse of headway over all vehicles. The flow rates by both definitions become large in the middle stage where the metastable homogeneous flow is realized. The flow rates decrease in the final stage where the jammed flow appears. The existence of the metastable homogeneous flow is understood also in the change of the flow rate. The increase of the flow rate means that the homogeneous flow in the middle stage has a large average velocity. The fact that the state with high flow rate soon transits to the jammed flow means the state is metastable.

The process of jam formation is clearly seen in the fundamental diagram (figure 8) by taking suitable averages of density and flow rate for small spacetime areas. In this experiment, the typical size of a jam cluster is several tens of meters, and it takes several seconds to go through the cluster. Therefore we choose an average over 23 m for space and 10 s for time.

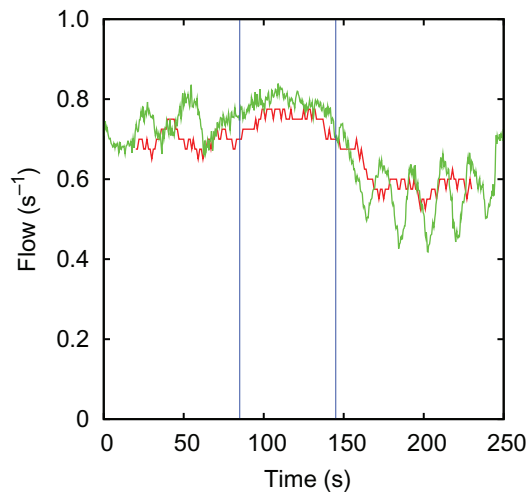


Figure 7. Red represents the section flow rate during 40 s. For example, the data point at $t = 20$ shows the flow rate for $t = 0$ –40. Green represents the average of flow rates calculated from the velocity and headway of each vehicle. Blue lines represent the borders of the three stages shown in figure 5.

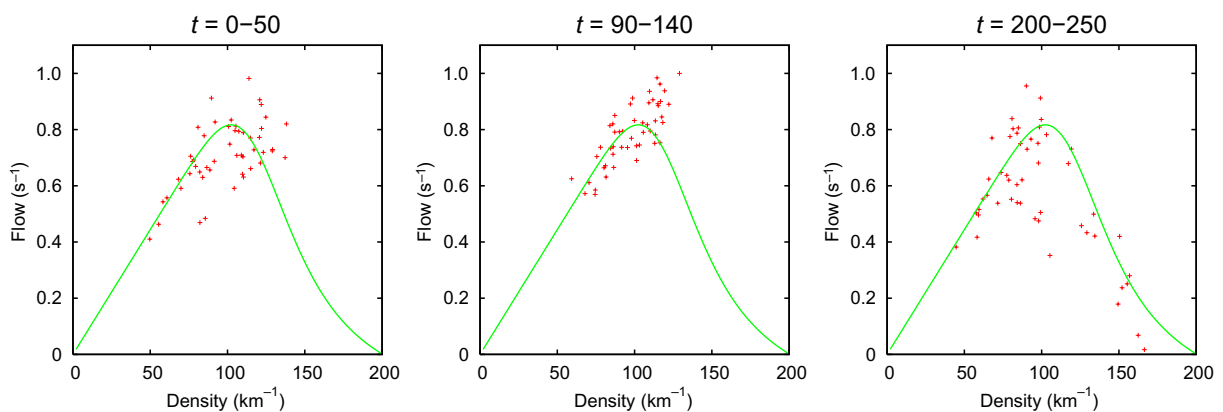


Figure 8. The periods $t = 0$ –50, 90–140 and 200–250 correspond to the three stages mentioned in figure 5. Red cross marks represent the density and the flow rate averaged over 23 m for space and 10 s for time. Green curves represent the density–flow relation calculated from the headway–velocity relation shown in figure 3(I).

Figure 8 shows the result of this calculation in case (I). Green curves in the figure represent the density–flow relation obtained from the headway–velocity relation $V(x)$ defined in figure 3. Fluctuations of density and flow rate are relatively large in the first period, $t = 0$ –50, but become small in the second period, $t = 90$ –140. The data points in the second period exist in the overhanging area above the green curve (see figure 4 also). This indicates that the flow is in a metastable state. The density–flow relation in the third period, $t = 200$ –250, shows that the jam is formed.

From this result, we can summarize the process of jam formation. In the early stage, vehicles run with various values of velocity and headway. The average velocity is relatively

small. In the middle stage, the average velocity is large and the variance of velocity is small. The flow is similar to the homogeneous flow but the density is high. The state of the flow is metastable. In this stage, a jam emerges due to a trigger like a fluctuation.

4. Summary and discussion

We have shown the detailed data from a traffic experiment on a circuit without any bottlenecks. A traffic jam emerges spontaneously when the vehicle density is large. We have found a homogeneous flow is temporarily realized before a jam cluster appears. This homogeneous flow with high density and large velocity is metastable. It is necessary for the spontaneous creation of a jam. We have found the same conclusion also in the change of the flow rate and in the fundamental diagram.

Here, we mention the variation among drivers. The drivers are students of a college and are not professional drivers prepared for this experiment. They use their own vehicles. The lengths and performance of the vehicles are not the same [16, 17]. These variations are reflected in the motions of vehicles, and can be clearly observed in headway–velocity space. In appendix B, we show how the headway–velocity relations of all drivers vary in the process of jam formation. In the early and final stages, the difference in the motion of vehicles, due to the difference of the characters of the drivers, is relatively large. In the middle stage, however, all the vehicles are moving at an almost equal velocity with an almost equal headway. Variations among the drivers are lost in this state.

The formation of a traffic jam is understood as a physical phenomenon of a non-equilibrium system of interacting particles (vehicles). We are convinced that our experiment provides evidence for this understanding. From this physical viewpoint, we think that the variation of drivers is not so important. In practical situations, automatic driving systems can quantitatively reduce several kinds of practical problems of jam formation. However, it is important to recognize that a physical constraint exists in the fundamental basis of such a problem.

In order to justify the physical interpretation of traffic flow, it is necessary to carry out several experiments for various values of density in a large circuit. We are planning such experiments to investigate the detail of jam formation and to confirm the metastability.

Acknowledgments

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Appendix A. Jam formation in case (II)

We show the average and standard deviation of velocity in case (II). From the time series of the average velocity, we cannot clearly identify the homogeneity of the flow in the middle stage in figure A.1(a). However, it is found that the homogeneous flow appears temporarily before the

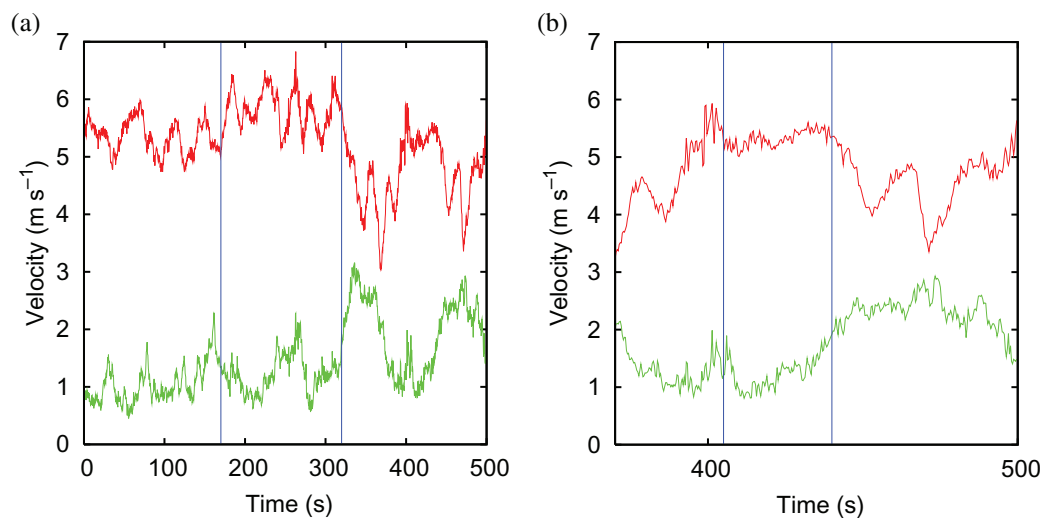


Figure A.1. Two graphs (a) and (b) show the data in the period $t = 0$ –500 and the period $t = 370$ –500, respectively. Red and green represent the average and standard deviation of velocity, respectively. Blue lines divide the period into three stages. In the cases (a) and (b), the lines are drawn at $t = 170$ and 320 and at $t = 405$ and 440 , respectively.

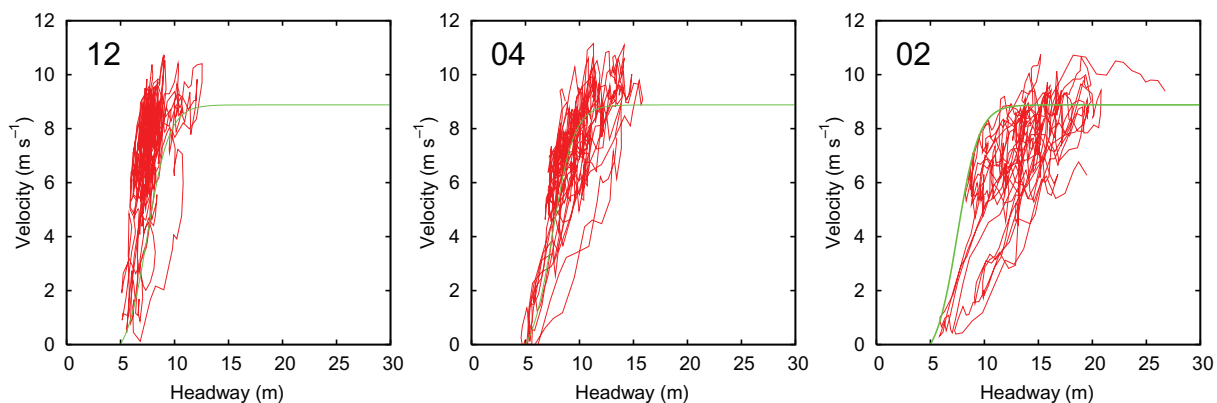


Figure B.1. Headway–velocity relations of three typical drivers in case (I). Numbers 12 (left), 04 (center) and 02 (right) are aggressive, average and careful drivers, respectively. Red is the plot of all headway and velocity data for the driver. Green curves represent the headway–velocity relation shown in figure 3(I), which is almost equal to that of average drivers.

second transition to the jammed flow in figure A.1(b). In order to see this clearly, we plotted the data in the period $t = 370$ –500, where a jam cluster appears again.

Appendix B. Variation among drivers

Here, we show the variation of characters among drivers. Figure B.1 shows the motions of three typical vehicles in case (I). Numbers 12, 04 and 02 show the motion of vehicles driven by

Table B.1. Parameter α for all drivers in the two cases (I) and (II). The numbering of drivers is not common for the two cases. An extremely large value of the No. 20 driver is an artifact, because we require that $V(5) = 0$ irrelevantly to the size of vehicle.

(I)											
No. 01	02	03	04	05	06	07	08	09	10	11	
1.56	0.92	2.61	1.85	2.34	2.02	1.61	2.47	1.68	1.97	1.22	
No. 12	13	14	15	16	17	18	19	20	21	22	
2.96	1.20	1.20	1.14	1.25	1.71	1.06	1.43	1.97	2.69	2.11	

(II)											
No. 01	02	03	04	05	06	07	08	09	10	11	12
1.12	1.90	1.90	1.17	0.93	0.96	0.70	1.28	1.17	1.19	0.90	1.90
No. 13	14	15	16	17	18	19	20	21	22	23	
0.72	3.20	2.93	1.80	1.17	1.10	2.42	70.30	0.99	1.41	1.02	

aggressive, average and careful persons, respectively (see also table B.1). We consider that the difference among drivers is characterized by the headway–velocity relation from the theoretical viewpoint. In addition, the aggressive and careful drivers can be distinguished by the magnitudes of fluctuations of headway and velocity. The aggressive driver seems to control the vehicle more sensitively than the careful driver does. However, the fluctuation includes the effect of dynamics, and it is difficult to connect the fluctuation to the difference in driving manner directly. Therefore the headway–velocity relation is the only quantity to distinguish drivers, at present.

In order to extract a parameter to identify each driver, we fit the headway–velocity relation by a piecewise linear function

$$V(x) = \begin{cases} 0, & x < 5, \\ \alpha(x - 5), & 5 \leq x \leq 5 + \frac{V_{\max}}{\alpha}, \\ V_{\max}, & 5 + \frac{V_{\max}}{\alpha} < x, \end{cases} \quad (\text{B.1})$$

where $V_{\max} = 4.8 \times 1.85$ for the case (I) and $V_{\max} = 3.8 \times 1.85$ for the case (II). The parameter α can be used as an easy index for drivers. Table B.1 shows the parameter α for each driver for the cases (I) and (II). We can see the parameters of drivers take almost random values. This guarantees that the drivers are not prepared and are not trained specially for this experiment. The unit of α is s^{-1} , but we do not assign any physical meaning to this parameter, for example, the inverse of the reaction time of the driver.

Next, we show how the driving manners of all the drivers change in the process of jam formation. Figures B.2 and B.3 show the headway and velocity data in the three stages of the process of jam formation in the cases (I) and (II), respectively. We can find that the drivers drive differently to each other, but maintain their own driving manner during the jam formation. The change of driving manner can be clearly seen in figure B.3. In the early and final stages, the difference among the driving manners of drivers is relatively large. In the middle stage, all

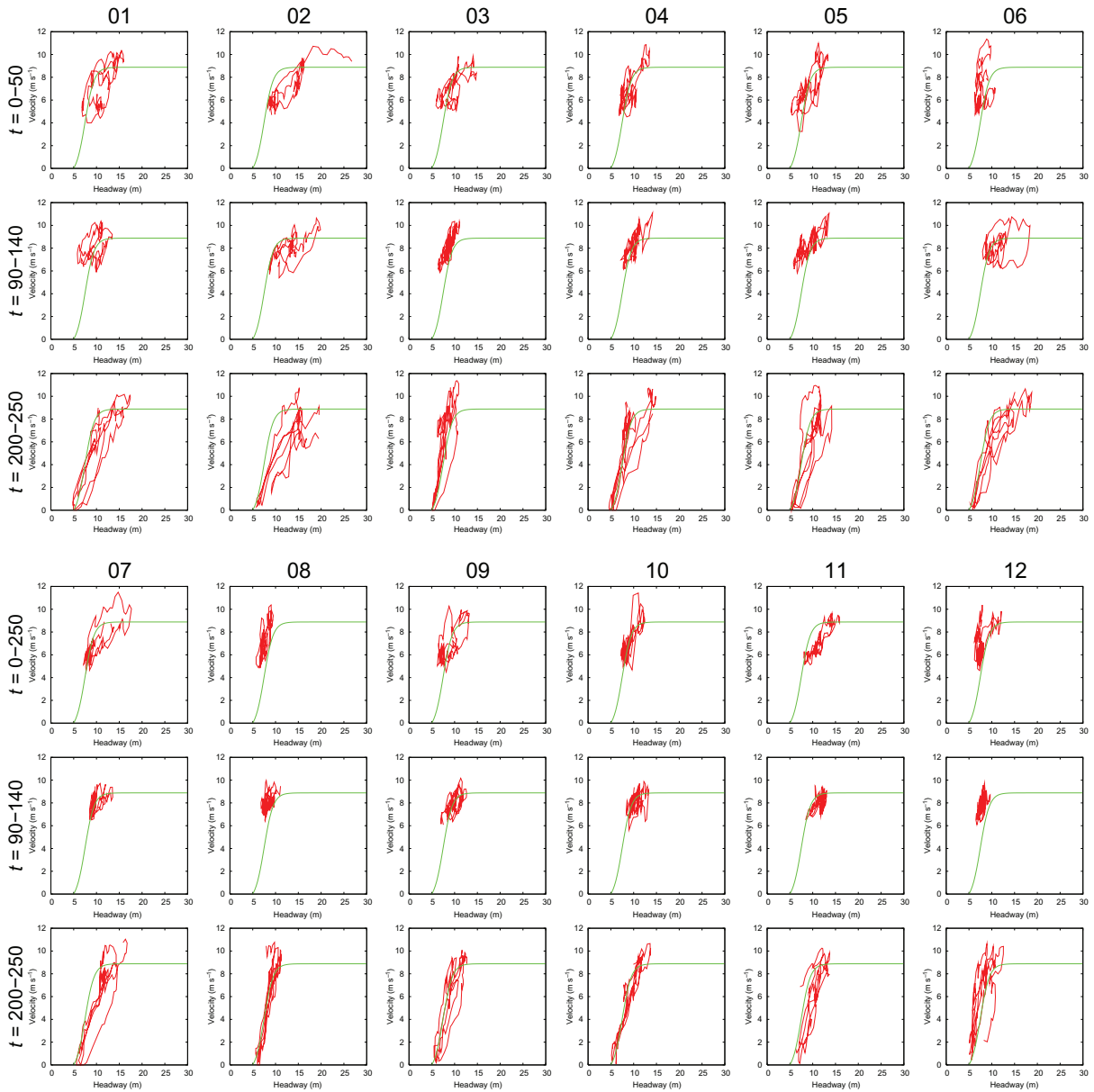


Figure B.2. Plots of the headway and velocity for the periods $t = 0-50$, $90-140$ and $200-250$ in case (I). The number above each figure shows the index of the drivers, but the numbering is different from that in figure B.3. Red in each figure is the plot of headway and velocity. Green curves represent the headway–velocity relation shown in figure 3(I).

the vehicles are moving at an almost equal velocity with an almost equal headway. Variation among the drivers is lost in this state. The flow is in a steady state with high density and large velocity, which can be identified as a homogeneous flow. Such a high density flow is known to be unstable and transits to the jammed flow. This result confirms the physical mechanism of the jam formation.

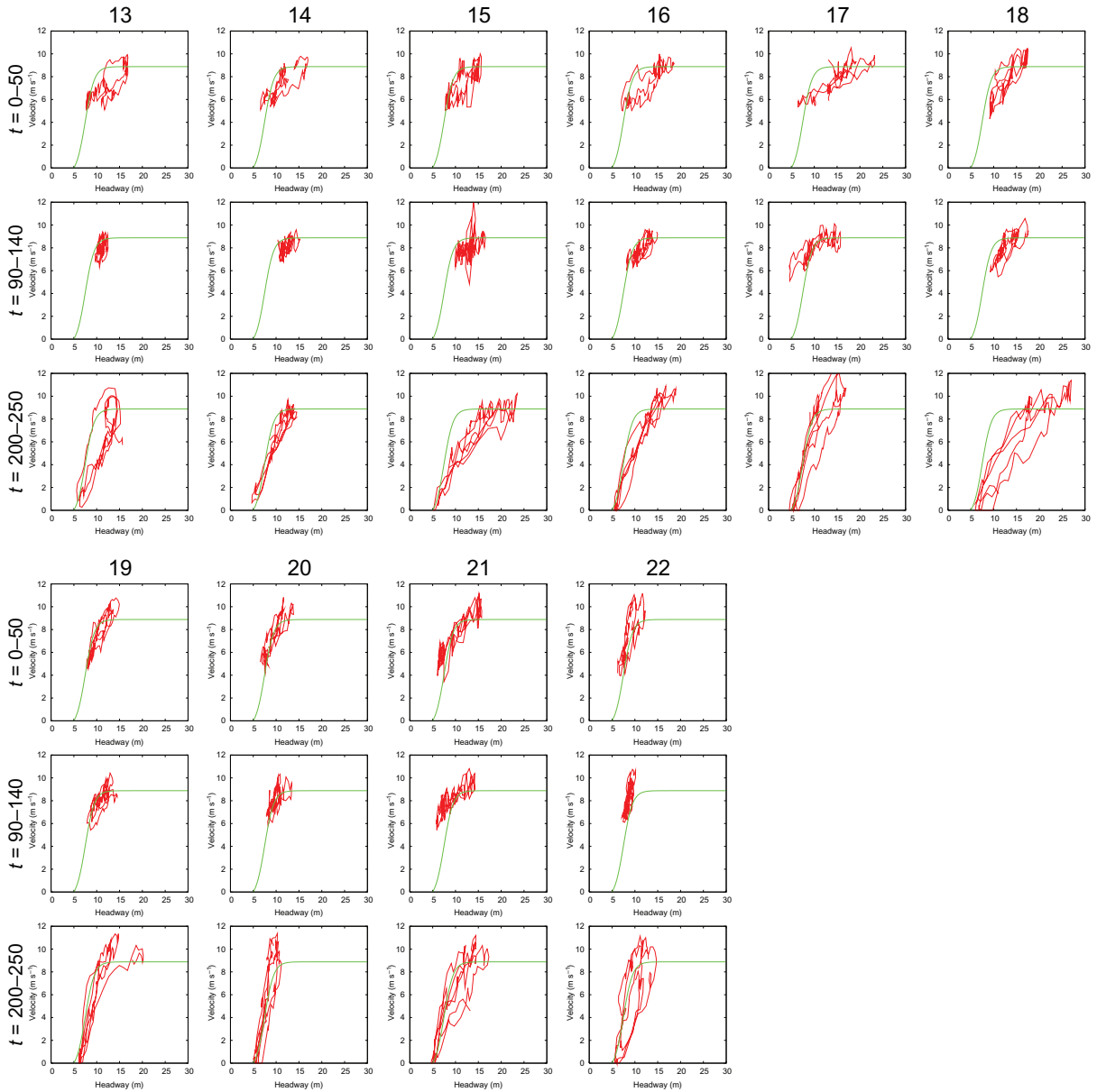


Figure B.2. Continued.

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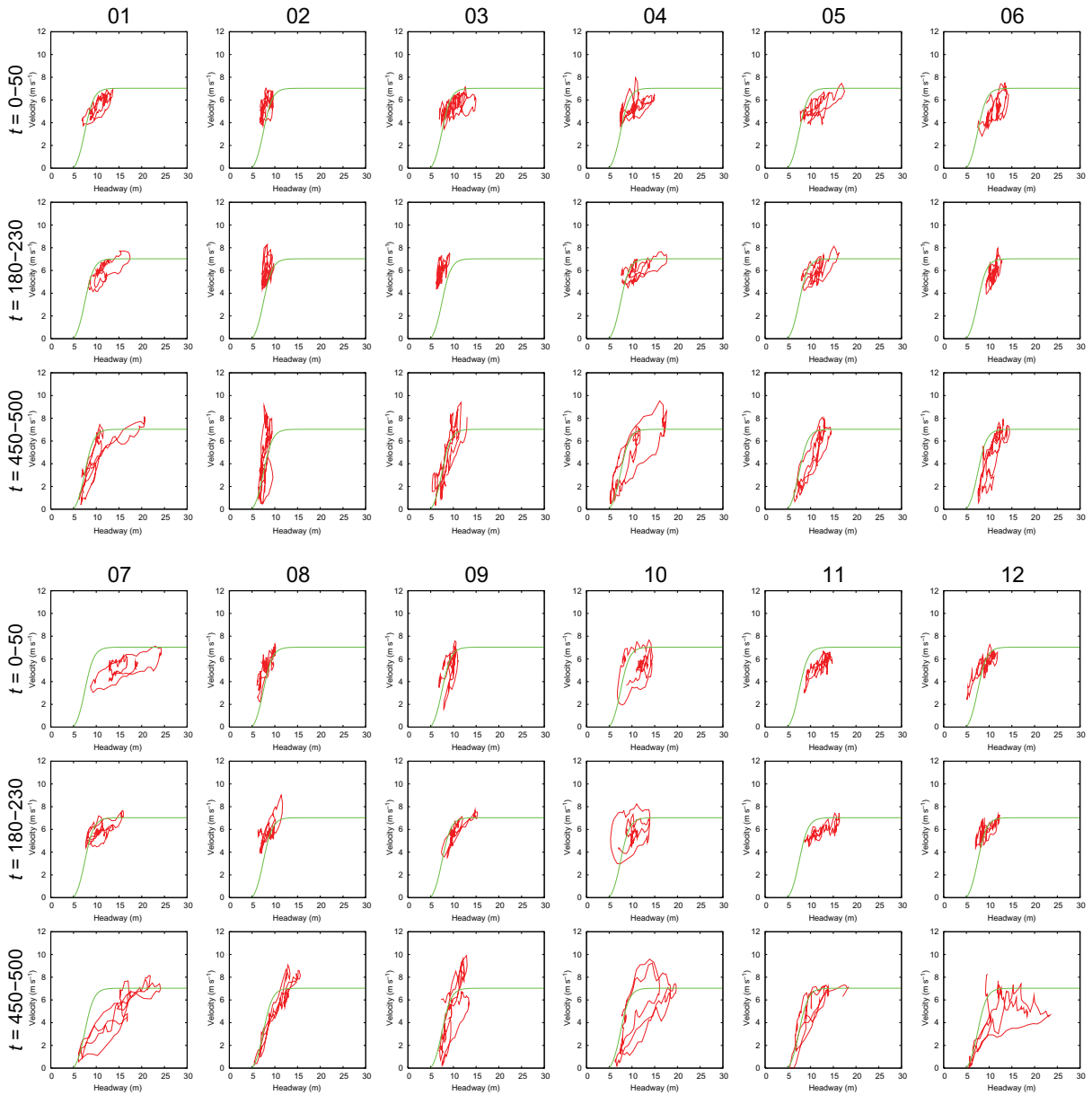


Figure B.3. Plots of the headway and velocity for the periods $t = 0-50$, $180-230$ and $450-500$ in the case (II). The number above each figure shows the index of the drivers. Red in each figure is the plot of headway and velocity. Green curves represent the headway-velocity function shown in figure 3(II).

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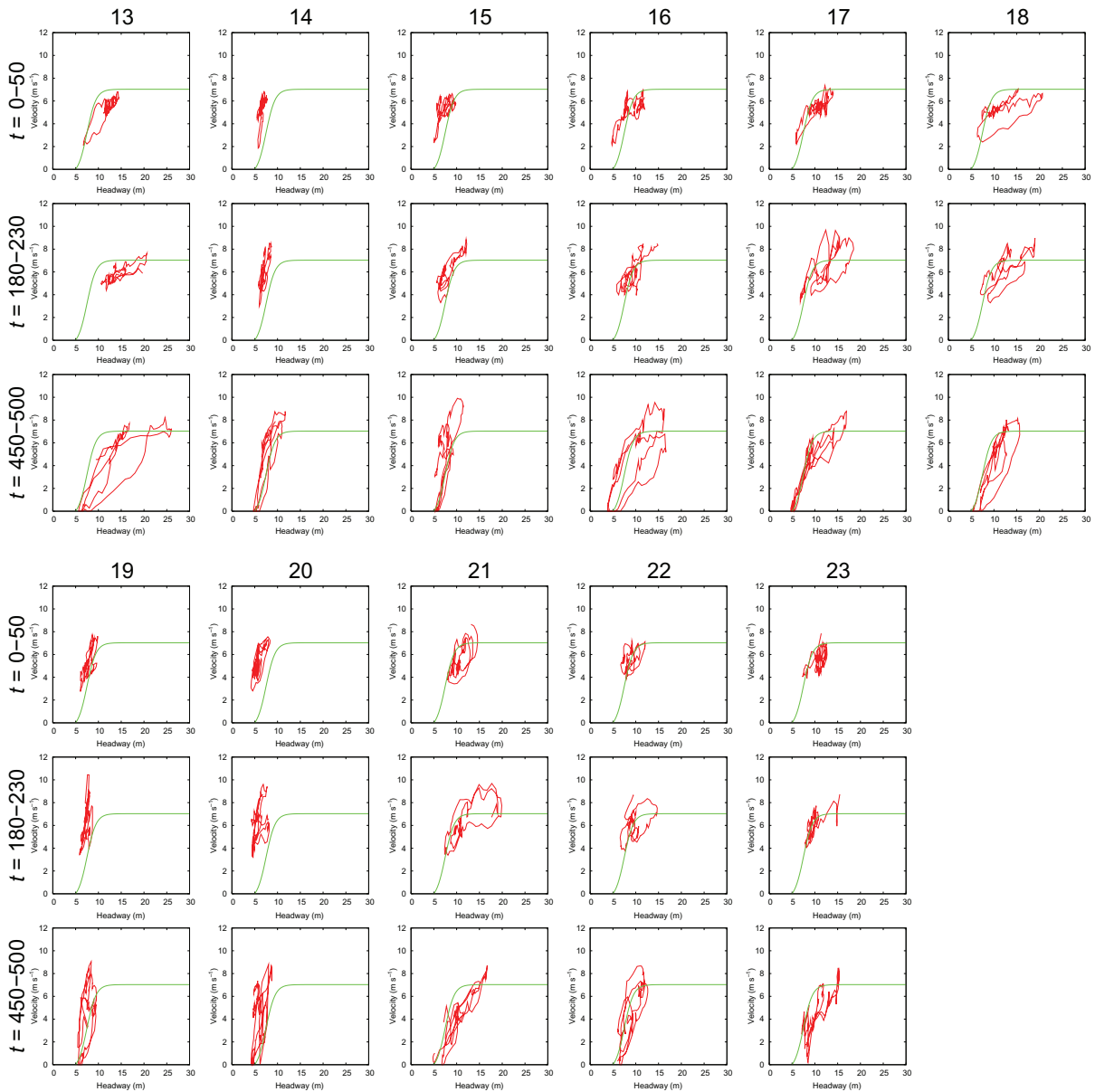


Figure B.3. Continued.

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