A DELAYED FLOW INTERSECTION MODEL FOR DYNAMIC TRAFFIC ASSIGNMENT

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Abstract. Day-to-Day and Within-Day dynamics are classically observed in dynamic traffic assignment, but smaller ones due to traffic lights phases also occur. These micro variations induce flow fluctuations defined at a cycle time scale. Their precise knowledge is irrelevant in a dynamic traffic assignment context. We propose to integrate these micro dynamics into a new intersection model without stages in which their average effects must be taken into account, especially delay and flow restriction generated by the presence of traffic lights.

1. Introduction

The objective of dynamic traffic assignment (DTA) models is to propose a representation of traffic evolutions for a road network when traffic conditions change. This includes demand variations but also supply variations, for example when an accident occurs or when phase plans at signalized intersections change. Those variations often happen at a large scale, i.e. with important characteristic times. That is why DTA models tend to work at a large time and spatial scale.

DTA models are composed of two interdependent parts:

- a traffic model that describes traffic evolutions for known demand and supply conditions and that provides travel times on the road net;
- an assignment model that determines from the calculated travel times the flow assignment between all possible paths for each origin-destination pair.

The determination of traffic assignment equilibrium needs an iterative process consisting in both travel times and associated flow assignment calculation. Because of the number of iterations and large work scales the traffic model used in the DTA model must be easily and rapidly calculated. However it needs to remain accurate enough if the final traffic equilibrium state is to be used for operational purposes.

The traffic models for DTA are classically derived either from traffic assignment models, such travel time models [1] or CONTRAM [6] or link-based formulation [9], or

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from traffic flow models, e.g. DYNASMART [4]. Traffic assignment models are usually efficient from the calculation time point of view but lack of precision in traffic phenomena description. Traffic flow models generally have reciprocal properties: long calculation time but accurate traffic description. We have chosen to develop the second approach consisting in simplifying a flow model for DTA use.

In that context we consider that the specific supply variations resulting from the existence of green and red stages are of lesser importance than the other supply and demand variations. They create small travel time fluctuations due to flow variations whose characteristic times are under the expected sensibility of users and then can't be directly taken into account for their route choice. Two different solutions are possible to remove those small fluctuations: smoothing a posteriori the results of a flow model in which green and red stages are explicitly considered ; or not representing them directly but integrating a priori the average effects of the stage alternation in the traffic model.

We propose here an intersection model corresponding to the second solution. We will first describe the arc traffic flow model used in parallel with the one for intersections. Then we will study the mean effects on flows of the stage alternation. The intersection model will be described in the part 4 and we will finish with an illustration on a small theoretical application.

2. Arc traffic flow model

Because of their obvious interdependency, both intersection and arc flow models must present consistent characteristics. Therefore the arc traffic flow model needs to precisely represent traffic flow phenomena such as congestion, but it must still be easily calculated for a use in DTA.

We choose the LWR model (Lighthill, Whitham [7] and Richards [11]) because it is a good compromise between our two opposite needs of calculation simplicity and accuracy in traffic phenomena description. It represents traffic as a homogenous and continuous flow described with three variables: the flow $Q(x,t)$, the density $K(x,t)$ and the flow speed $V(x,t)$. The LWR model is based on three equations:

- The conservation equation: $\frac{\partial K(x,t)}{\partial t}+\frac{\partial Q(x,t)}{\partial x}=0$
- The flow definition: $Q(x,t)=K(x,t).V(x,t)$
- The equilibrium fundamental relation: $Q(x,t)=Q_E(K(x,t))$

These equations can be rewritten as a non-linear hyperbolic conservation equation:

$$\frac{\partial K(x,t)}{\partial t}+\frac{\partial Q_E(K(x,t))}{\partial x}=0$$

The LWR model is generally numerically solved by using the Godunov scheme introduced by Lebacque [5]. It is based on a finite difference scheme corresponding to a spatial and time discretization. Lebacque also proposes the two notions of supply Σ and the demand Δ (see figure 1 for their definitions) and shows that the flow Q is the minimum of Σ and Δ for each point at each moment:

$$Q(x,t)=\min(\Sigma(x,t),\Delta(x,t))$$

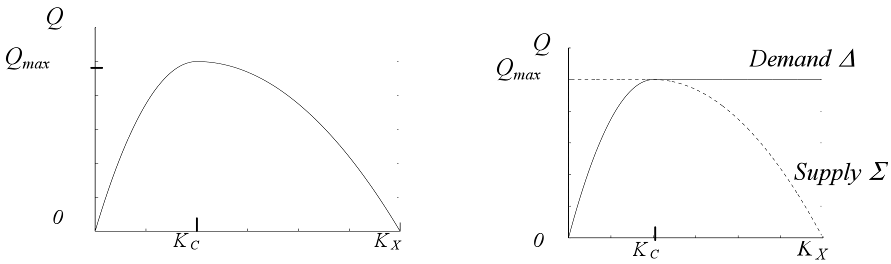

Figure 1. a. Example of fundamental relation $Q = Q_E(K)$: biparabolic diagram b. Corresponding demand $\Delta(K)$ and supply $\Sigma(K)$

Our large work scale in DTA imposes the use of a large discretization grid that is incompatible with a good intersection phenomena description [2]. So we prefer here another approach: the wave tracking (WT) resolution scheme. We will only briefly describe the WT scheme. Refer to [3] for a more detailed explanation.

This method consists in the explicit calculation of wave propagation and intersection in a space time diagram. These waves separate zones of different densities. This calculation is possible because the fundamental relationship is approximated by a piecewise linear function. Only linear shockwaves and linear characteristics are then created. Experienced travel times are easily calculated with the cumulative flow method [8]**.** The WT scheme gives the exact entropic solution of an approximated problem whereas the Godunov scheme calculates an approximated solution of the exact problem.

In DTA we can roughly approximate the demand variations by a set of successive states of constant demand. With the WT scheme only transitions between two states generate specific calculations, if no congestion occurs. Compared to the Godunov scheme which needs calculations for each instant of the discretization grid even in constant state, the WT scheme calculation time is then more acceptable for a use in DTA.

3. Mean stage alternation behaviour

The study of the effects on flow of phase alternation is the base of the building of a mean behaviour intersection model. Considering our large work scale phenomena whose characteristic times are lesser than the cycle time should not be described precisely but from an average point of view. Dynamics with greater characteristic times are supposed to be explicitly calculated and represented.

The alternation of green and red stages has three mainly correlated effects on traffic:

- A cyclic delay due to cyclic queues;
- A mean supply restriction induced by red phases;
- A specific shockwave propagation.

The first effect is the existence of cyclic queues that appear during red stages and resolve themselves during green stages. These queues induce a finite delay τ depending on the cycle duration c, the green time ratio λ, the maximal output flow Q_{max} and the flow through the intersection Q. Many authors propose delay formulations. We will consider here one of the most accepted concerning uniform delay given by the Highway Capacity Manual [11]:

$$\tau = \frac{c}{2} \cdot \frac{(1-\lambda)^2}{1-x} \qquad \text{with } x = \frac{Q}{Q_{\max}} \text{ saturation rate}$$

This HCM formulation corresponds to the first term of Webster's delay formulation. Use of other formulations is also possible. Some models, e.g. the travel-time based assignment model CONTRAM [6], also take into account a random delay in its travel time function. But this diverging delay is due to traffic dynamics whose characteristic times are greater than the cycle time (if the mean arrival rate is constant at each cycle time and lesser than $\lambda.Q_{max}$ then no user waits more than a cycle time). Therefore we do not integer this delay in our intersection model.

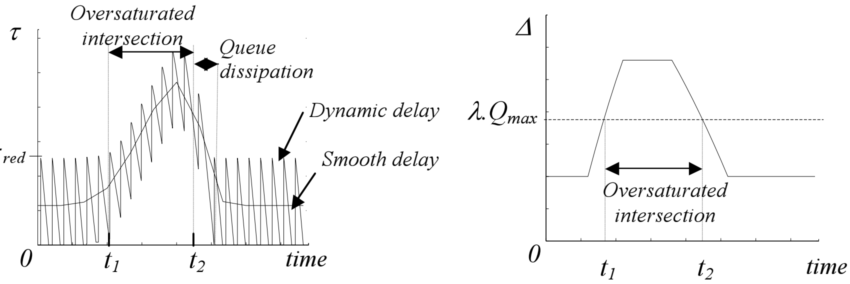

Figure 2. a. Dynamic and smooth delay in the intersection
b. Corresponding demand at the entry of the intersection

The stage alternation also creates a supply effect, i.e. a bottleneck effect: the maximum mean outflow of the intersection is $Q_{max}{}^{out} = \lambda.Q_{max}$ and the supply reduced to $\lambda.\Sigma$ (Σ supply at the entry of the downstream arc). For demands Δ exceeding Σ a diverging queue appears upwards the intersection and creates a diverging delay. This queue will resolve itself only if demand decreases under supply or if supply increases to the demand (if the stages plan changes for example) whereas the cyclic queue always disappears in the end of the green phase. Figure 2 shows these two different but simultaneous phenomena. Initial and final states show only cyclic queues (and then cyclic delays). When the intersection is oversaturated those two sorts of queues coexist but keep their own characteristics.

The last important impact of the stage alternation on flow is not so direct but it is still important. It concerns the shockwave propagation. Downwards propagating shock waves incur the same delay as vehicles but upwards propagating shock waves behave differently. If they reach the intersection when a cyclic queue is still present their propagation speed increases so that the shockwave is observed upward from the intersection earlier than if there were no cyclic queue. For that reason upwards propagating shockwaves don't induce delay but gain advances. Note that in that case the behaviour of the intersection highly depends on the precise instant when the shockwave arrives. In a DTA context this level of precision exceeds our ability to describe accurately traffic phenomena, especially demand variations. Therefore the use of stage intersection models may lead to falsely accurate results.

4. Delayed flow intersection model

We propose a new flow intersection model. It does not directly show the stage alternation but it must be able to represent its three mean effects on flow: the bottleneck effect, cyclic delay and the shockwave propagation. Phenomena whose characteristic times are lesser than the cycle time are not represented but are integrated in the intersection model.

The bottleneck effect is modelized by the use of λ in the supply-demand relation in the entry of the intersection: $Q = min(\lambda.\Sigma,\Delta)$. Diverging queues will appear upward from the intersection in same conditions as with the phase model (when $\Delta > \lambda.\Sigma$) and then create a diverging delay.

Creating a given delay is not self evident in the LWR model because travel time is not a basic variable of the model. The idea proposed here to introduce the cyclic delay is the use of a fictive arc. It is a supplementary artificial arc between the end of the upstream arc and the beginning of the intersection that increases the travel length and then creates a certain delay. The length of the fictive arc varies: if the inflow varies its length can vary to adjust the delay to the delay given by the HCM formulation.

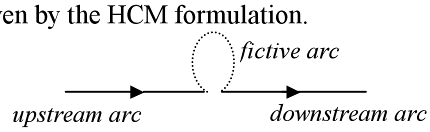

Figure 3. The fictive arc

The fictive arc has its own triangular fundamental relationship based on the one of the upstream arc:

$$Q_{max}^{fictive} = Q_{max}\ ,\ K_c^{fictive} = K_c \text{ and } K_x^{fictive} = 2\ .\ K_c^{fictive}$$

The triangular shape allows creating a pure delay for downstream propagation because it induces no platoon dispersion for non congested states (all users have the same speed). Two bottleneck restrictions are used at the entry and the exit of the fictive arc:

$$Q_{in}^{\ 1} = min\ (\Delta,\lambda.\Sigma_{fictive}) \text{ and } Q_{out} = min\ (\Delta_{fictive},\lambda.\Sigma).$$

with $\Sigma_{fictive}$ supply at the entry of the fictive arc and $\Delta_{fictive}$ demand at its exit.

The specific propagation of shockwave from downward to upward is introduced through the use of another bottleneck restriction. It ensures the instantaneous propagation of shockwaves through the intersection:

$$Q_{in}^{\ 2} = min\ (\Delta,\lambda.\Sigma).$$

It results that:

$$Q_{in} = min\ (Q_{in}^{\ 1},\ Q_{in}^{\ 2}) = min\ (\Delta\ ,\ \lambda.\Sigma\ ,\ \lambda.\Sigma_{fictive}).$$

Final flow restriction laws through the intersection are synthesized in the figure 4.

At constant inflow the delay and the length of the fictive arc are constant. Flow is just delayed in the intersection. When the inflow changes from Q_1 to Q_2 the delay through the intersection must also change from $\tau(Q_1)$ to $\tau(Q_2)$. Consequently the length of the fictive arc must vary from L_1 to L_2 that respectively corresponds to the delays $\tau(Q_1)$ and $\tau(Q_2)$. We are not interested in the precise description of the transition from state 1 to state 2 (its characteristic time is under the cycle times). We suppose then that the delay varies linearly

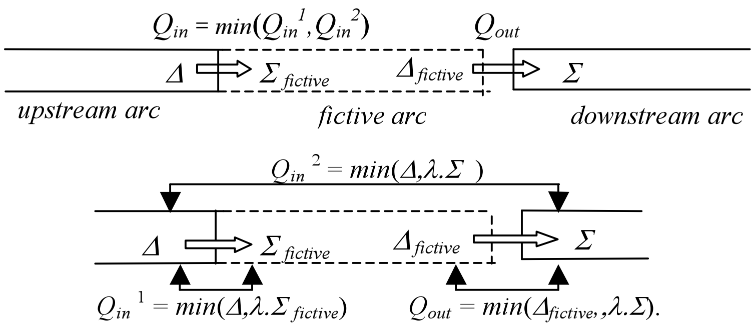

Figure 4. a. Flows through the intersection
b. Flow restriction laws

from $\tau(Q_1)$ to $\tau(Q_2)$ and that the length of the fictive arc also varies linearly from L_1 to L_2 at the speed $V_{frontier}$.

To vary the length of the fictive arc its downstream frontier moves. This also ensures a FIFO behaviour of the intersection. Note that abscissas of the exit of the upstream arc and of the entry downstream arc don't change because the fictive arc has no real physical extension just like vertical queues. When the downstream frontier of the fictive arc moves the flow Q_f leaving the fictive arc and the flow Q_{out} entering the downstream arc are linked by the relation:

$$Q_{out} = Q_f.(1 - \frac{V_{frontier}}{V_f})$$

where V_f is the speed of vehicles on the fictive arc. We make the assumption that Q_{out} doesn't change until the fictive arc reaches the length L_2. V_f is then given by relationship:

$$V_{frontier} = V_f.(1 - \frac{Q_{out}}{Q_f})$$

The intersection model creates no supplementary dynamics: when a shockwave reaches the intersection only one shockwave is generated and none is created in the stationary state. Therefore the calculation time is quite reduced compared to the stage model. Note that the use of the WT model as the arc flow model allows this event discretization (only shockwaves intersections are considered).

5. Illustration

We propose a simple theoretical application to illustrate our intersection model (see figure 5). We consider three successive arcs linked by two intersections whose cycle duration is $c=100\ s$ and green times respectively $60\ s$ and $50\ s$ ($\lambda_1=0.6$, $\lambda_2=0.5$), with $Q_{max} = 1\ veh/s$ and $K_x = 0.3\ veh/m$. Maximal outflow supplies are then $\Sigma_{max}^1 = 0.6$ and $\Sigma_{max}^2 = 0.5$. The demand variation at the entry of the first link is represented in figure 6 (the time step of the variation is one minute). The scenario lasts for four hours.

Note that fictive links have no real length even if vehicles must travel them. Their entries and their exits then have the same abscissa in the space-time diagram. For the sake of representation we have chosen to draw the entries of the downstream arcs at a constant distance from the respective entries of the fictive arcs. Therefore fictive arcs seem to have a constant length, but vehicles don't travel the blank spaces between their exits and the entries of the downstream arcs (they directly jump from the exit to the next entry).

When demand increases, the length of both fictive links also increases to fit with the used delay formula but at different speeds because λ_1 and λ_2 are different (see figure 5). When demand exceeds Σ_{max}^2 intersection 2 becomes saturated: a queue appears that grows upwards. When it reaches intersection 1 its exit supply immediately falls and congestion propagates upward from the intersection. When demand falls under Σ_{max}^2 the queue disappears progressively. As we use the WT scheme the shockwaves are piecewise linear. Consequently output flows and travel times are also piecewise linear.

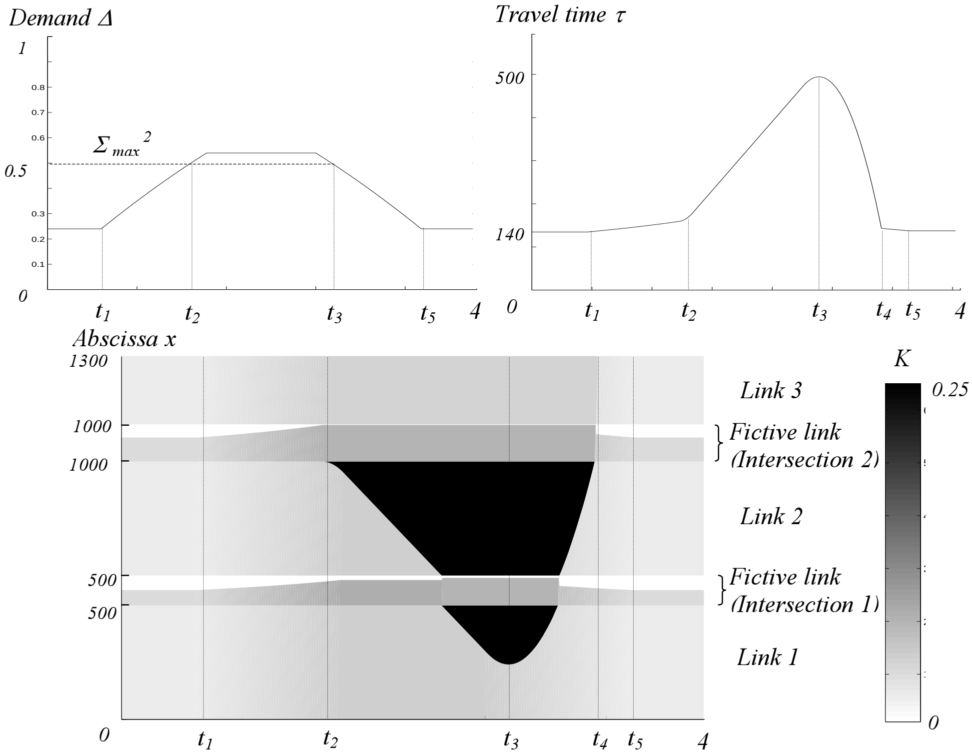

Figure 5: a. Demand $\Delta(t)$ in *veh/s*, *t* in *h* b. Experienced travel time $\tau(t)$ in s, *t* in *h* c. Corresponding space-time diagram (*x* in m, t in *h* and *K* in *veh/m*)

The scenario lasts for 4 hours but the calculation time remains acceptable for DTA purposes because the traffic state on the net varies only when the demand changes at its entry or when a shockwave propagates. With stage intersection models a concentration fan is created for each green stage so that calculation time is highly increased.

The different goals we fixed for the models are reached: the travel time needs no smoothing for DTA use, traffic is described precisely enough to locate the mean congestion shock waves on the links, and the calculation time is acceptable for DTA purposes.

6. Conclusions

In this extended abstract we have developed a new intersection flow model for DTA. Our approach consists in simplifying a precise traffic flow model by suppressing dynamics due to stage alternation. Our intersection model takes into account delays created at the intersection, local flow restriction and shock wave propagation. It uses an adapted supply-demand relationship and a fictive arc whose varying length creates the delay corresponding to the specific flow entering the intersection. This model allows quite a precise description of traffic flow phenomena and still remains rapidly calculated because only transitions between constant states are considered.

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