TMUA Homework 3

10 Questions

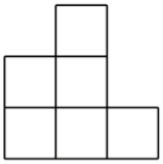
40 Minutes

请计时并不要使用计算器，完成后请填写线上表格提交作业

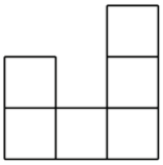
Question 1

Some identical unit cubes are used to construct a three-dimensional object by gluing them together face to face.

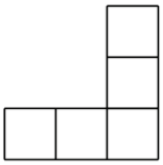
Sketches of this object are made by looking at it from the right-hand side, from the front and from above. These sketches are called the side elevation, the front elevation, and the plan view respectively.



This is the side elevation of the object.



This is the front elevation of the object.



This is the plan view of the object.

How many cubes were used to construct the object?

A exactly 6

B either 6 or 7

C exactly 7

D either 7 or 8

E exactly 8

F either 8 or 9

G exactly 9

Question 2

Each interior angle of a regular polygon with n sides is $\frac{3}{4}$ of each interior angle of a second regular polygon with m sides.

How many pairs of positive integers n and m are there for which this statement is true?

A none

B 1

C 2

D 3

E 4

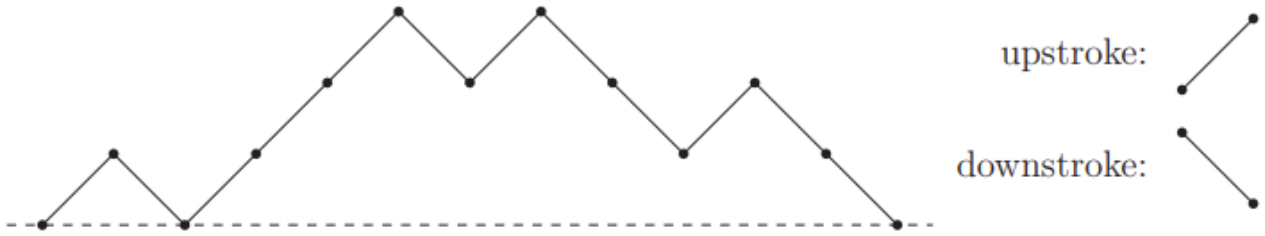
F 5

G 6

H infinitely many

Question 3

The diagram shows an example of a *mountain profile*.



This consists of *upstrokes* which go upwards from left to right, and *downstrokes* which go downwards from left to right. The example shown has six upstrokes and six downstrokes. The horizontal line at the bottom is known as *sea level*.

A *mountain profile of order n* consists of n upstrokes and n downstrokes, with the condition that the profile begins and ends at sea level and **never** goes **below** sea level (although it might reach sea level at any point). So the example shown is a mountain profile of order 6.

Mountain profiles can be coded by using U to indicate an upstroke and D to indicate a downstroke. The example shown has the code UDUUUDUDDUDD. A sequence of U's and D's obtained from a mountain profile in this way is known as a *valid code*.

Which of the following statements is/are true?

I If a valid code is written in reverse order, the result is always a valid code.

II If each U in a valid code is replaced by D and each D by U, the result is always a valid code.

III If U is added at the beginning of a valid code and D is added at the end of the code, the result is always a valid code.

A none of them

B I only

C II only

D III only

E I and II only

F I and III only

G II and III only

H I, II and III

Question 4

Triangles ABC and XYZ have the **same area**.

Which of these extra conditions, taken independently, would **imply** that they are congruent?

(1) $AB = XY$ **and** $BC = YZ$

(2) $AB = XY$ **and** $\angle ABC = \angle XYZ$

(3) $\angle ABC = \angle XYZ$ **and** $\angle BCA = \angle YZX$

	Condition (1)	Condition (2)	Condition (3)
A	Does not imply congruent	Does not imply congruent	Does not imply congruent
B	Does not imply congruent	Does not imply congruent	Implies congruent
C	Does not imply congruent	Implies congruent	Does not imply congruent
D	Does not imply congruent	Implies congruent	Implies congruent
E	Implies congruent	Does not imply congruent	Does not imply congruent
F	Implies congruent	Does not imply congruent	Implies congruent
G	Implies congruent	Implies congruent	Does not imply congruent
H	Implies congruent	Implies congruent	Implies congruent

Question 5

Consider the quadratic $f(x) = x^2 - 2px + q$ and the statement:

(*) $f(x) = 0$ has two real roots whose difference is greater than 2 and less than 4.

Which one of the following statements is true **if and only if** (*) is true?

A $q < p^2 < q + 4$

B $\sqrt{q+1} < p < \sqrt{q+4}$

C $q - 3 \leq p^2 - 4 \leq q$

D $q < p^2 - 1 < q + 3$

E $q - 2 < p^2 - 3 < q + 2$

Question 6

Consider the equation $2^x = mx + c$, where m and c are real constants.

Which of the following statements is/are true?

I The equation has a negative real solution **only if** $c > 1$.

II The equation has two distinct real solutions **if** $c > 1$.

III The equation has two distinct positive real solutions **if and only if** $c \leq 1$.

A none of them

B I only

C II only

D III only

E I and II only

F I and III only

G II and III only

H I, II and III

Question 7

A positive integer is called a *squaresum* **if and only if** it can be written as the sum of the squares of two integers. For example, 61 and 9 are both squaresums since $61 = 5^2 + 6^2$ and $9 = 3^2 + 0^2$.

A prime number is called *awkward* **if and only if** it has a remainder of 3 when divided by 4. For example, 23 is awkward since $23 = 5 \times 4 + 3$.

A (true) theorem due to Fermat states that:

> A positive integer is a squaresum **if and only if** each of its awkward prime factors occurs to an even power in its prime factorisation.

It follows that 5×23^2 is a squaresum, since 23 occurs to the power 2, but 5×23^3 is not, since 23 occurs to the power 3.

Which one of the following statements is **not** true?

A Every square number is a squaresum.

B If N and M are squaresums, then so is NM.

C If NM is a squaresum, then N and M are squaresums.

D If N is not a squaresum, then kN is a squaresum for some number k which is a product of awkward primes.

Question 8

Consider the following statement about the positive integer n:

Statement (*): *The sum of the four consecutive integers, the smallest of which is n, is a multiple of 6.*

Which one of the following is true?

A Statement (*) is true for all values of n.

B Statement (*) is true for all values of n which are odd, but not for any other values of n.

C Statement (*) is true for all values of n which are multiples of 3, but not for any other values of n.

D Statement (*) is true for all values of n which are multiples of 6, but not for any other values of n.

E Statement (*) is not true for any value of n.

Question 9

Five sealed urns, labelled P, Q, R, S, and T, each contain the **same** (non-zero) number of balls. The following statements are attached to the urns.

Urn P	This urn contains one or four balls.
Urn Q	This urn contains two or four balls.
Urn R	This urn contains more than two balls and fewer than five balls.
Urn S	This urn contains one or two balls.
Urn T	This urn contains fewer than three balls.

Exactly one of the urns has a true statement attached to it.

Which urn is it?

A Urn P

B Urn Q

C Urn R

D Urn S

E Urn T

Question 10

Five logicians each make a statement, as follows:

Mr P: Of these five statements, an odd number are true.

Ms Q: Both statements made by women are true.

Mr R: My first name is Robert and Mr P's statement is true.

Ms S: Exactly one statement made by a man is true.

Mr T: Neither statement made by a woman is true.

How many of the five statements can be simultaneously true?

A none

B 1 only

C 2 only

D 3 only

E 4 only

F none or 1 only

G 1 or 2 only

H 2 or 3 only