Review of "Berry-Esseen bounds for design-based causal inference with possibly diverging treatment levels and varying group sizes"

This work studies the Berry-Esseen convergence of test statistics under completely randomized experiments with various design choices. This problem was originally investigated by Li and Ding [2017]; however, their work has several limitations. First, Li and Ding [2017] assumes that the number of arms, Q, is fixed as the total number of samples $N \to \infty$. Additionally, they require that the sample size in each treatment arm, N_q for $q = 1, \ldots, Q$, grows to infinity. Furthermore, while Li and Ding [2017] establishes a convergence result, they do not provide an explicit convergence rate.

This paper extends the results of Li and Ding [2017] in several key aspects. First, it considers varying asymptotic regimes regarding the divergence of Q and N_q, as well as mixtures of these regimes. Second, it broadens the applicability of the convergence result from Li and Ding [2017], thanks to this extension. Toward the end of the paper, the authors conduct numerical experiments to illustrate the theoretical findings from another perspective. The results are both practically significant and technically non-trivial. Overall, I am optimistic about its acceptance. I only have a few relatively minor comments.

1. The comments following the statement of Theorem 1 are very helpful, as they assist readers in easily understanding what is already established in the literature and what challenges remain. I recommend elaborating on the following statement in the revised manuscript: "To prove Theorem 1(ii), we need to further derive upper bounds in terms of I and II in (10) from two different perspectives, which is non-trivial to the best of our knowledge." You may skip this comment if a concise elaboration proves too difficult.

2. The literature review in Appendix A is illuminating. However, as far as I can see, it seems that the main text of the paper did not mention there is a literature review in Appendix A. I would recommend you to mention the existence of such a review in the introduction section, together with a short summary of the papers discussed in that section.

References

Xinran Li and Peng Ding. General forms of finite population central limit theorems with applications to causal inference. *Journal of the American Statistical Association*, 112(520):1759–1769, 2017.