BE_PCLT_report

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Contents

Experiment setup

- K = 10, number of factors
- Q_U = 660, number of unreplicated arms, each with $N_q = 1$
- Q_R = 350, number of small replicated arms, each with $N_q = 2$
- Q_L = 14, number of large arms, each with $N_q = 30$.
- N = 1780, population

Generate data such that all the k-way ($k \geq 3$) interactions are zero. The effects are small: nonzero effects are randomly generated from uniform($[-0.5, -0.1] \cup [0.1, 0.5]$).

The nonzero factorial effects in the first two levels:

```
tau[abs(tau)>1e-4]
```

```
##          F2          F3          F5          F6          F8          F9       F2.F3
##  0.3175201   0.1738920   0.1297196   0.1167904   0.4029012   0.1007448   0.3550530
##       F2.F5       F2.F6       F2.F8       F2.F9       F3.F5       F3.F6       F3.F8
## -0.2621460  -0.1683530  -0.4384127   0.3199513  -0.3568972  -0.2766801   0.3098141
##       F5.F8       F5.F9       F6.F9
##  0.3514266   0.3307017   0.1565897
```

The true target effects and true variance for the WLS estimator:

```
# population level:
## tau
cat("target_tau\n")
```

```
## target_tau
```

```
target_tau
```

```
##               [,1]
## F2   3.175201e-01
## F4  -7.372575e-18
## F6   1.167904e-01
## F8   4.029012e-01
```

```
## F10  8.239937e-18
```

```
cat("\n")
```

```
## variance for the estimator
cat("true variance\n")
```

```
## true variance
```

```
true_cov_tauhat <- t(as.matrix(target_design)) %*% true_cov_Yhat %*% as.matrix(target_design) / 1024^2
true_cov_tauhat
```

```
##                F2            F4            F6            F8           F10
## F2   1.968737e-03  1.654976e-05 -4.117173e-05  1.778187e-05 -1.982575e-05
## F4   1.654976e-05  1.968551e-03 -6.562823e-05  4.423916e-05  3.853443e-05
## F6  -4.117173e-05 -6.562823e-05  1.968537e-03 -1.103128e-04  4.954102e-05
## F8   1.778187e-05  4.423916e-05 -1.103128e-04  1.968650e-03 -6.851526e-05
## F10 -1.982575e-05  3.853443e-05  4.954102e-05 -6.851526e-05  1.968592e-03
```

```
cat("\n")
```

```
## True standard deviation
cat("true sd\n")
```

```
## true sd
```

```
sqrt(diag(true_cov_tauhat))
```

```
##         F2         F4         F6         F8        F10
## 0.04437045 0.04436836 0.04436820 0.04436947 0.04436882
```

```
cat("\n")
```

Target effects we want to estimate: 'F2', 'F4', 'F6', 'F8', 'F10'

Numeric experiments

We run 1000 MC trials and report:

- histogram of the point estimates
- estimated standard deviation for 3 methods (see later parts on what these methods are)
- 95%-CI coverage

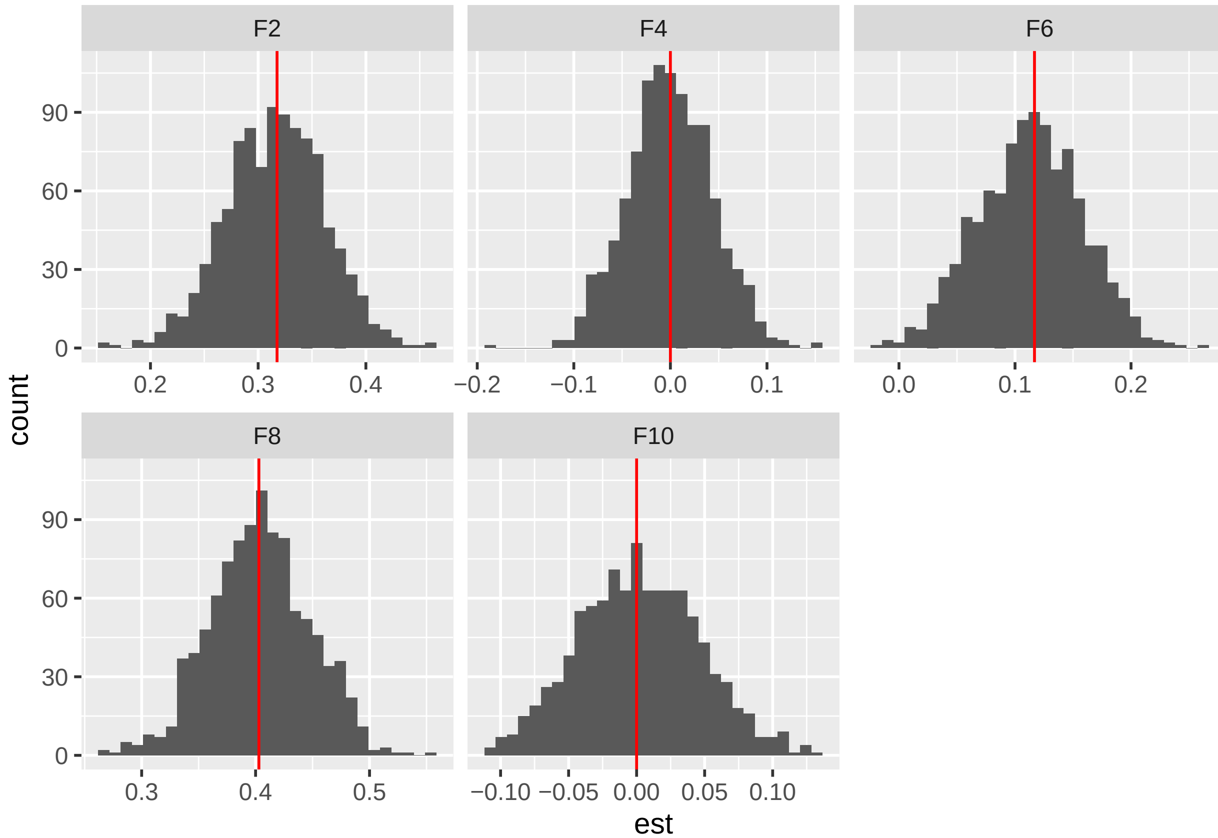
Distribution of point estimates

```
# report results
record <- readRDS("record_EFFECTS.RData")
# point estimates
hist_data <- data.frame(
  est = c(t(record$rec_point_est)),
  labs = factor(rep(target_effect, each = 1000), levels = c('F2', 'F4', 'F6', 'F8', 'F10'))
)

summary_data <- data.frame(
  target_tau = target_tau,
  labs = factor(target_effect, levels = c('F2', 'F4', 'F6', 'F8', 'F10'))
)
```

```r
ggplot(hist_data, aes(x = est)) +
  facet_wrap(~labs, scales = 'free_x') +
  geom_histogram() +
  geom_vline(data = summary_data, mapping = aes(xintercept = target_tau), col = 'red') # red lines are
```

```
## `stat_bin()` using `bins = 30`. Pick better value with `binwidth`.
```



Takeaways:

- CLT holds even the design is highly non-uniform.

Expectation of the sd estimators

We applied three methods for variance estimation:

- ehw_0: wls + hc2 var, with specification: $Y \sim F2 + F4 + F6 + F8 + F10$
- ehw_1: wls + hc2 var, with specification; $Y \sim (F2 + F4 + F6 + F8 + F10)^2$
- lex: lexicographical pairing

```r
# expectation of sd estimator
print(data.frame(
  true_sd    = sqrt(diag(true_cov_tauhat)),
  ehw_0_sd   = diag(apply(sqrt(record$rec_var_est_ehw_0), MARGIN = c(1,2), sum))/1000,
  ehw_1_sd   = diag(apply(sqrt(record$rec_var_est_ehw_1), MARGIN = c(1,2), sum))/1000,
  lex_sd   = diag(apply(sqrt(record$rec_var_est_lex), MARGIN = c(1,2), sum))/1000
))
```

```
## Warning in sqrt(record$rec_var_est_ehw_0): NaNs produced

## Warning in sqrt(record$rec_var_est_ehw_1): NaNs produced
```

```
## Warning in sqrt(record$rec_var_est_lex): NaNs produced

##        true_sd   ehw_0_sd   ehw_1_sd     lex_sd
## F2  0.04437045 0.05342504 0.05194368 0.04895025
## F4  0.04436836 0.05342504 0.05194368 0.04895025
## F6  0.04436820 0.05342504 0.05194368 0.04895025
## F8  0.04436947 0.05342504 0.05194368 0.04895025
## F10 0.04436882 0.05342504 0.05194368 0.04895025
```

Takeaways: in the small effect cases,

- wls + ehw: both `ehw_0` and `ehw_1` are robust. Adding two-way interactions gives less conservative variance estimation.
- lex: `lex` pairing is robust. It works better than `ehw` since there is smaller between group variation.

CI coverage

```
# CI coverage
print(data.frame(
  target_effect = target_effect,
  ehw_0_coverage  = rowSums(record$rec_coverage_ehw_0)/1000,
  ehw_1_coverage  = rowSums(record$rec_coverage_ehw_1)/1000,
  lex_coverage  = rowSums(record$rec_coverage_lex)/1000
))
```

```
##   target_effect ehw_0_coverage ehw_1_coverage lex_coverage
## 1            F2          0.977          0.973        0.963
## 2            F4          0.990          0.989        0.977
## 3            F6          0.981          0.978        0.974
## 4            F8          0.985          0.980        0.973
## 5           F10          0.983          0.978        0.971
```