Math 715 | HW 9 | Leidong Xu

(a) $L_{jk}^2 = \| z_j' - z_k' \|_2^2$

$= \| (z_j - z_k) + (\delta z_j - \delta z_k) \|_2^2$

$= \| z_j - z_k \|_2^2 + \| \delta z_j - \delta z_k \|_2^2$

$+ 2 (z_j - z_k, \delta z_j - \delta z_k)$

discard the nonlinear term

$= \| z_j - z_k \|_2^2 + 2 (z_j - z_k, \delta z_j - \delta z_k)$

(b) set the system as $Ax = b$

$$\begin{cases} L_{01} = 1.1 \\ L_{02} = 0.9 \\ L_{12} = 1.5 \end{cases}$$

$$\Rightarrow \begin{cases} \| z_0 - z_1 \|_2^2 + 2(z_0 - z_1, \delta z_0 - \delta z_1) = 1.1^2 \\ \| z_0 - z_2 \|_2^2 + 2(z_0 - z_2, \delta z_0 - \delta z_2) = 0.9^2 \\ \| z_1 - z_2 \|_2^2 + 2(z_1 - z_2, \delta z_1 - \delta z_2) = 1.5^2 \end{cases}$$

$$\Rightarrow \begin{cases} 2(z_0 - z_1)(\delta z_0 - \delta z_1) = 1.1^2 - \| z_0 - z_1 \|_2^2 \\ \quad \sim \sim \sim \\ \quad \sim \sim \sim \end{cases}$$

$$= 2\begin{cases} z_0 \cdot \delta z_0 - z_1 \delta z_0 - z_0 \delta z_1 + z_1 \delta z_1 = 1.1^2 - \|z_0 - z_1\|_2^2 \\ \cdots \\ \cdots \end{cases}$$

$$\Rightarrow Ax = b$$

$$\Rightarrow 2\begin{bmatrix} (z_0 - z_1), & -z_0 + z_1, & 0 \\ (z_0 - z_2), & 0, & -z_0 + z_2 \\ 0, & (z_1 - z_2), & -z_1 + z_2 \end{bmatrix} \begin{bmatrix} \delta z_0 \\ \delta z_1 \\ \delta z_2 \end{bmatrix} = \begin{bmatrix} 1.1^2 - \|z_0 - z_1\|_2^2 \\ 0.9^2 - \|z_0 - z_2\|_2^2 \\ 1.5^2 - \|z_1 - z_2\|_2^2 \end{bmatrix}$$

(c) All the z_0, z_1, z_2 above are vectors

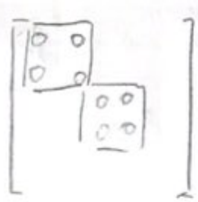
we can convert the $Ax = b$ system by

$$\begin{bmatrix} \quad & & \\ & & \\ & & \end{bmatrix}_{6\times 6} \begin{bmatrix} \delta z_{0x} \\ \delta z_{0y} \\ \delta z_{1x} \\ \delta z_{1y} \\ \delta z_{2x} \\ \delta z_{2y} \end{bmatrix}_{6\times 1} = \begin{bmatrix} 1.1^2 - \|z_{0x} - z_{1x}\|_2^2 \\ \vdots \\ \vdots \end{bmatrix}_{6\times 1}$$

since $z_0 = [0.0]$, $z_1 = [1.0]$, $z_2 = [0.1]$

the maxinum entry could be 1

(d)



flatten z will led to a multigroup system

Each group contain a 2x2 blockwise matrix, the first row correspond to $z(x, \ )$ the second row correspond to $z(\ , y)$

```
for k in rang(N):
    for i in rang(2):
        for j in rang(2):
            c += 1
            row[c] = 2*k + i
            col[c] = 2*k + j
            if i == 0 and j == 0:
                val[c] = 2(Z_x[k] - Z_x[j])
            elif i == 0, and j == 1:
                val[c] = 2(-Z_x[k] + Z_x[j])
            elif i == 1, and j == 0:
                val[c] = 2(Z_y[k] - Z_y[j])
            elif i == 0 and j == 1
                val[c] = 2(-Z_y[k] + Z_y[j])
```

e) this system is inverotble,

which should has a nontrival spae

or

Think about slide map.

if $Ax_1 = Ax_2$ and $x_1 \neq x_2$ ($\delta Z_1 \neq \delta Z_2$)

then $Ax = 0$

f) No idea.