HW13 | Leidong Xu | Math 715

Let $\mathcal{J} = \langle e_n \rangle^{\perp} = \langle e_1, \dots, e_{n-1} \rangle$

For $x \in \mathcal{J} \Rightarrow x = \begin{bmatrix} x' \\ 0 \end{bmatrix}$ with $x' \in \mathbb{R}^{n-1}$

$$x^T A x = [x' \ \ 0] \left[\begin{array}{c|c} H & b \\ \hline b^T & u \end{array}\right] \begin{bmatrix} x' \\ 0 \end{bmatrix}$$

$$= (x')^T H (x')$$

$$x^T x = (x')^T x'$$

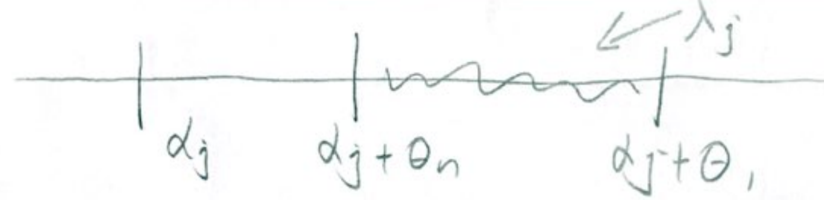
$$\Rightarrow \frac{x^T A x}{x^T x} = \frac{(x')^T H (x')}{(x')^T (x')}$$

$j \le n-1$ $$\alpha_j = \max_{\substack{U \subset \mathbb{R}^n \\ \dim U = j}} \min_{\substack{x \neq 0 \\ x \in U}} \frac{x^T A x}{x^T x}$$

$$\ge \max_{\substack{U < \mathcal{J} \\ \dim U = j}} \min_{\substack{x \neq 0 \\ x \in U}} \frac{x^T A x}{x^T x}$$

$$= \max_{\substack{U' \subset \mathbb{R}' \\ \dim U' j}} \min_{\substack{x' \neq 0 \\ x' \in U'}} \frac{(x')^T H x'}{(x')^T x'} = \theta_j$$

5.5



$$\lambda_j = \max_{\substack{U \subset \mathbb{R}^n \\ \dim U = j}} \min_{\substack{x \neq 0 \\ x \in U}} \underbrace{\frac{x^T(A+H)x}{x^T x}}_{\dfrac{x^TAx}{x^Tx} + \dfrac{x^THx}{x^Tx}}$$

$$\geq \max_{\substack{U \subset \mathbb{R}^n \\ \dim U = j}} \left[\min_{x \in U} \frac{x^TAx}{x^Tx} + \min_{\substack{x \neq 0 \\ x \in U}} \frac{x^THx}{x^Tx} \right]$$

$$\geq \max_{\substack{U \subset \mathbb{R}^n \\ \dim U = j}} \left[\min_{\substack{x \neq 0 \\ x \in U}} \frac{x^TAx}{x^Tx} + \min_{\substack{x \neq 0 \\ x \in \mathbb{R}^n}} \frac{x^THx}{x^Tx} \right] = \alpha_j + \theta_n$$

5.18

$$A = U\Sigma V^T$$
$$= UI\Sigma V^T$$
$$= (UV^T)(V\Sigma V^T)$$

$Q = UV^T$ where $Q^*Q = I$

and $\Sigma = \mathrm{diag}(\sigma_1, \ldots, \sigma_n)$

$P = V\Sigma V^T$ is positive-semi definite

$$\Leftrightarrow P = \sum_{j=1}^{n} \sigma_j v_j v_j^T$$
$$= \sigma_1 v_1 v_1^T + \sigma_2 v_2 v_2^T + \ldots$$
$$= \sigma_1 \mathrm{proj} \langle v_1 v_2 \rangle + \ldots$$

is unique

and Q is unitary

so the decomposotion is unique,