HW 9

Problem 1. In this problem we will do some of the leg work needed to tackle Question 3.19. Ignoring angle measurements, the problem is formulated as follows.

> You are given an original database of two-dimensional positions $\{z_j\}$, containing the approximate locations of various landmarks obtained with outdated measurement techniques. You are also given a set of new measurements $L_{jk} = \|z'_j - z'_k\|_2$, for some pairs of j and k, where $z'_j = z_j + \delta z_j$ is the "true" location of the j-th landmark.

Assuming that the magnitudes of the vectors δz_j are not too large, it is possible to update the database with the given measurements by solving a linear least squares problem.

(a) Begin by expanding the right-hand side of the equation

$$L_{jk}^2 = \|z'_j - z'_k\|_2^2 = \|(z_j - z_k) + (\delta z_j - \delta z_k)\|_2^2$$

and discarding the nonlinear terms in δz_j and δz_k. The resulting equation is linear in the unknowns δz_j and δz_k. You will have one such equation for each measurement. In general, you will use more measurements than you have unknowns, so you cannot hope to satisfy all equations exactly; you will need to perform a least squares approximation of the problem.

(b) To simplify, consider first a situation where there are 4 old data points and 3 new measurements (even though this is not overdetermined). To be explicit, the original positions are

$$[\ z_0 = [0,0], z_1 = [1,0], z_2 = [0,1], z_3 = [1,1]\ ]$$

and the measurements are

$$\begin{aligned} L_{01} &= 1.1 \\ L_{02} &= 0.9 \\ L_{12} &= 1.5 \end{aligned}$$

Set up a system of equations and turn it into an equation $Ax = b$ by specifying the matrix A and the vector b. You don't have to solve it.

(c) Continuing with the small example of part (b), what are the dimensions of the matrix A? the vector b? The vector x? What is the largest number of non-zero entries one can expect in A?

(d) Generalize parts (b) and (c). Suppose you are given a database of locations $z = [z_0, \ldots z_N]$, and a file of measurements where each row contains a triple of information (j, k, v) representing the fact that

$$L_{jk} = \|z'_j - z'_k\|_2 = v.$$

Describe an algorithm that will create the matrix A in sparse form. Namely, you will need three lists I, J, V of size n, where n is the number of possible non-zero entries in A. Also, you will need to flatten the array z so that it becomes a vector of size $(2N, 1)$.

(e) Prove that the system you derived has a nontrivial nullspace. (Hint: Think about how relative distances change as you slide a map around on a desk.)

(f) Suggest a method for resolving the non-uniqueness arising from the nontrivial nullspace of A.