HW6 Math 715 Leidong Xu

Problem 1. What does a left multiplication by a diagonal matrix D to a dense matrix A? How about right-multiplication?

$$D = \begin{bmatrix} a & & \\ & b & \\ & & c \end{bmatrix} \qquad A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$$

$$DA = \begin{bmatrix} aA_{11} & aA_{12} & aA_{13} \\ bA_{21} & bA_{22} & bA_{23} \\ cA_{13} & cA_{32} & cA_{33} \end{bmatrix}$$

$$AD = \begin{bmatrix} aA_{11} & bA_{12} & cA_{13} \\ aA_{21} & bA_{22} & cA_{23} \\ aA_{31} & cA_{32} & cA_{33} \end{bmatrix}$$

Consider $A = \begin{bmatrix} 3 & 4 & 0 \\ 1 & -1 & 1 \\ 0 & 9 & 2 \end{bmatrix}$

$\tilde{A} = D^{-1}AD$ is a symmetric matrix

$$D^{-1} = \begin{bmatrix} 1/a & & \\ & 1/b & \\ & & 1/c \end{bmatrix}$$

$$D^{-1}A = \begin{bmatrix} \frac{3}{a} & \frac{4}{a} & 0 \\ \frac{1}{b} & -\frac{1}{b} & \frac{1}{b} \\ 0 & \frac{9}{c} & \frac{2}{c} \end{bmatrix}$$

$$D^{-1}AD = \begin{bmatrix} \frac{3}{a} & \frac{4}{a} & 0 \\ \frac{1}{b} & -\frac{1}{b} & \frac{1}{b} \\ 0 & \frac{9}{c} & \frac{2}{c} \end{bmatrix} \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$$

$$= \begin{bmatrix} 3, & \frac{4b}{a}, & 0 \\ \frac{a}{b}, & -1, & \frac{c}{b} \\ 0, & \frac{9b}{c}, & 2 \end{bmatrix}$$

then $\begin{cases} \frac{a}{b} = \frac{4b}{a} \\ \frac{9b}{c} = \frac{c}{b} \end{cases} \Rightarrow \begin{cases} a^2 = 4b^2 \\ c^2 = 9b^2 \end{cases} \Rightarrow \begin{cases} |a| = 2|b| \\ |c| = 3|b| \end{cases}$

Problem 2,

$$L^T = \begin{bmatrix} x_1 & y_1 & & & \\ & x_2 & y_2 & & \\ & & x_3 & y_3 & \\ & & & \ddots & y_{n-1} \\ & & & & x_n \end{bmatrix}$$

$$LL^T = \begin{bmatrix} x_1 & & & \\ y_1 & x_2 & & \\ & y_2 & x_3 & \\ & & y_3 & x_4 \end{bmatrix} \begin{bmatrix} x_1 & y_1 & & \\ & x_2 & y_2 & \\ & & x_3 & y_3 \\ & & & x_4 \end{bmatrix}$$

$$= \begin{bmatrix} x_1^2, & x_1 y_1 & & \\ x_1 y_1, & y_1^2 + x_2^2, & x_2 y_2 & \\ , & x_2 y_2, & y_2^2 + x_3^2, & x_3 y_3 \\ & & x_3 y_3, & y_3^2 + x_4^2 \end{bmatrix}$$

Algorithm

operator_r = 0

operator_a = 0

x = [ ]

y = [ ]

at boundary

$x[0] = \sqrt{a[0]}$

operator_r += 1

Last element is $y_3^2 + x_4^2$ not x_4^2, spend two hour to figure it out

$x[n] = \sqrt{a[n]}$

operator_r ± 1

for i in [2 ..... n-1]:

$\quad y[i-1] = \dfrac{b[i-1]}{x[i-1]}$

$\quad$ operator_a += 1

$\quad$ temp = y[i-1] * y[i-1]

$\quad$ operator_a += 1

$\quad$ temp = a[i] - temp

$\quad$ operator_a += 1

$\quad x[i] = \sqrt{temp}$

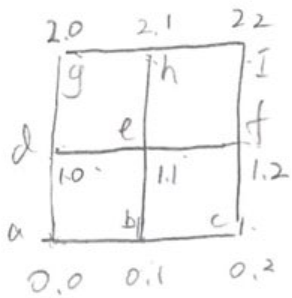
$\quad$ operator_r += 1

=> operator_r = 2 + n - 2

operator_a = 3(n-1)

= 3n - 3

problem 3



(a) $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2}$

$$= \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{\Delta X} + \frac{u_{i,j+1} - 2u_{i,j} + u_{i,j-1}}{\Delta X}$$

If the boundary condition is not clear, the whole system should be written as

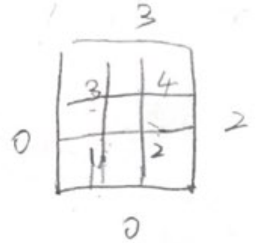
$$\frac{1}{h^2}\begin{array}{c} \begin{array}{ccccccccc} a & b & c & d & e & f & g & h & I \end{array} \\ \begin{bmatrix} -2 & 1 & & 1 & & & & & \\ 1 & -3 & 1 & & 1 & & & & \\ & 1 & -2 & & & 1 & & & \\ 1 & & & -2 & & & 1 & & \\ & 1 & & 1 & -4 & 1 & & 1 & \\ & & 1 & & & -2 & & & 1 \\ & & & 1 & & & -2 & 1 & \\ & & & & 1 & & 1 & -3 & 1 \\ & & & & & 1 & & 1 & -2 \end{bmatrix} \end{array}$$

(2) there are 9 equations, and only one node is not at boundary, $u_{1,1} = \frac{5}{4}$.

Does the ambiguous corn points matter?

For solving center points, No. But the conditions are against each other. In other word, this problem is not continue at corner

(3) since it's fixed and uniform boundary condition,



$$\begin{bmatrix} -4 & 1 & 1 & \\ 1 & -4 & & 1 \\ 1 & & -4 & 1 \\ & 1 & 1 & -4 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = - \begin{bmatrix} g_L + g_B \\ g_B + g_R \\ g_L + g_T \\ g_R + g_T \end{bmatrix} \quad \text{where} \quad \begin{matrix} g_L = 0 \\ g_B = 0 \\ g_R = 2 \\ g_T = 3 \end{matrix} \Rightarrow \begin{cases} u_1 = 0.625 \\ u_2 = 1.125 \\ u_3 = 1.375 \\ u_4 = 1.875 \end{cases}$$

We can still calculate the inside nodes values without using those ambiguous points