HW 11 | Leidong Xu | Math 715

4.6

1. similar system should have same solution.

$\boxed{\text{No idea}}$

4.8

Let A be m-by-n and B be n-by-m

Show the matrix

$$\begin{pmatrix} AB & 0 \\ B & 0 \end{pmatrix} \text{ and } \begin{pmatrix} 0 & 0 \\ B & BA \end{pmatrix}$$

are somiliar.

Conclude that the non-zero eigenvalues of AB are the same as those of BA

$$\begin{pmatrix} AB & 0 \\ B & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & B \\ 0 & AB \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & AB \\ 0 & B \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} AB & 0 \\ B & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 \\ B & BA \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} BA & B \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$\boxed{\text{No idea}}$

(2) we assume AB is invertible

$m \times m$ $\quad$ $\underset{m\times n}{A} \cdot \underset{n\times m}{B} = m$

$$ABv = \lambda v$$

$$\underset{n\times m}{B}\underset{m\times n}{A}\underset{n\times m}{B}v = \lambda Bv$$

$$\underset{n\times n}{(BA)}\underset{n\times m}{(Bv)} = \lambda (Bv)$$

since $Bv \neq 0$, so λ is a non-zero eigenvalue

$$BAu = \lambda u$$

$$\underset{m\times n}{A}\underset{n\times m}{B}\underset{m\times n}{A}u = \lambda Av$$

$$(AB)(Av) = \lambda (Av)$$

same here, so AB and BA has same

non-zero eigenvalues

4.12 Let $A = \begin{bmatrix} a & c \\ 0 & b \end{bmatrix}$ show that condition number of the eigenvalues of A are both equal to

$$\left(1 + \left(\frac{c}{a-b}\right)^2\right)^{\frac{1}{2}}$$

eigenvalues of $\begin{bmatrix} a & c \\ 0 & b \end{bmatrix}$ are a and b

$$A - \lambda I = \begin{bmatrix} a-\lambda & c \\ 0 & b-\lambda \end{bmatrix}.$$

since $\det(A - \lambda I) = 0$ and then we plug in $\lambda = a$ first

$$\cancel{S = \begin{bmatrix} 1 & \frac{c}{b-a} \\ 0 & 1 \end{bmatrix}} \Rightarrow \begin{bmatrix} 0 & c \\ 0 & b-a \end{bmatrix}$$

$$\cancel{S^{-1} = 2 \begin{bmatrix} \frac{1}{2} & -\frac{c/2}{b-a} \\ 0 & 1 \end{bmatrix} \text{ from lemma 4}}$$

for a, we can calculate the left and right eigenvector first

left: $\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}^T \begin{bmatrix} 0 & c \\ 0 & b-a \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$$\Rightarrow y_1 = -\frac{b-a}{c} y_2$$

right $\begin{bmatrix} 0 & c \\ 0 & b-a \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$$x = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

then $\operatorname{cond}(A) = \dfrac{\|x\| \, \|y\|}{|y^T x|}$

$$= \frac{1 \cdot \sqrt{1 + \left(\frac{b-a}{c}\right)^2}}{\left| -\frac{(b-a)}{c} \right|}$$

$$= \boxed{\sqrt{1 + \left(\frac{c}{a-b}\right)^2}}$$

if we plug in $\lambda = b$ and do the same process. The results are same