4.10 $A = H + S$ HW 12 Leidong Xu

(1) $H = H^*$

$S = -S^*$

$$A = A + A^* - A^*$$

$$= \underset{①}{\frac{A + A^*}{2}} + \underset{②}{\frac{A - A^*}{2}}$$

$①^* = \frac{A^* + A}{2} = ①$ so this is Hermitian

$②^* = \frac{A^* - A}{2} = -②$ so this is skew-Hermition

(4) $\|A\|_F = (\sqrt{trace(A^*A)})^2$

$$= Tr((P\Lambda P^*)^*(P\Lambda P^*))$$

$$= Tr(P^*\Lambda^*\Lambda P)$$

$$= Tr(\Lambda^*\Lambda)$$

$$= \sum_{i=1}^{n} (\lambda_i^*\lambda_i)$$

$$= \sum_{i=1}^{n} |\lambda_i|^2$$

Problem 5.1

Show that $A = B + iC$ is Hermitian if and only

$$M = \begin{bmatrix} B & -C \\ C & B \end{bmatrix}$$ is symmetric

(2) Express the eigenvalue and eigenvectors of A

We assume both B and C are real square matrix

If M is symmetric $\overset{\text{if and only if}}{\Longleftrightarrow}$ $\boxed{\begin{matrix} B = B^T \\ \text{and} \\ C = -C^T \end{matrix}}$ ①

For A is Heromitian, $A_{ij} = \overline{A_{ji}}$

$$\Leftrightarrow B_{ij} + C_{ij} = B_{ji} - C_{ji}$$

$B_{ij} = B_{ji}$
$C_{ij} = -C_{ji}$ $\Leftrightarrow$ $\boxed{\begin{matrix} B = B^T \\ C = -C^T \end{matrix}}$ ②

(2) ~~$\det(M - \lambda I)$~~

~~$= \det(B - I_n)\det((B - I_n) - C(B - I_n)(-C))$~~

$Mx = \lambda x$ $\quad \begin{bmatrix} B & -C \\ C & B \end{bmatrix} \Big[$

5.16 $A = D + \rho u u^T$

$D = diag(d_1 \cdots d_n)$

$u = [u_1 \cdots u_n]^T$

c) show that if $d_i = d_{i+1}$ or $u_i = 0$, then d_i is an eigenvalue of A.

$Ae_i = d_i e_i + \rho u (u^T e_i)$

$= d_i e_i + (\rho u_i) u$ ①

then if $\boxed{u_i = 0 \quad d_i \text{ is an eigenvalue of } A}$

if $d_i = d_{i+1}$

$d = diag(d_1, d_1 \cdots d_1)$

(b) same as (a)

$$Ae_i = d_i e_i + \rho u (u^T e_i)$$

$$= d_i e_i + (\rho u_i) u$$

If $u_i = 0$

$Ae_i = d_i e_i$ where e_i is the corresponsed eigenvector

(c) $A(ae_i + be_{i+1})$

$a d_i e_i + a(\rho u_i)u + b d_{i+1} e_i + b(\rho u_{i+1})u$

$$au_i + bu_{i+1} = 0$$

$$a = -b_i \frac{u_{i+1}}{u_i}$$

$$\Rightarrow \begin{cases} b = u_i \\ a = -u_{i+1} \end{cases}$$

$\Rightarrow -u_{i+1} e_i + u_i e_{i+1}$ is one eigenvecto for d_i