HW5 | Leidong Xu | Math 715

problem 1

Show that $|a_{ij}| < (a_{ii} a_{jj})^{\frac{1}{2}}$

for a 2×2 S.P.D A

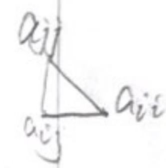
$\because$ all the eigenvalue of SPD > 0

$\therefore \det(A) = a_{ii} a_{jj} - a_{ij}^2 > 0$

$\Rightarrow |a_{ij}| < (a_{ii} a_{jj})^{\frac{1}{2}}$ ①

$\because$ from proposotion 2.2.2.

All the submatrix of SPD is SPD



$\therefore$ ① could be applied to any size spd.

Problem 2

First prove $1 + v^T A^{-1} u \neq 0$

$$1 + v^T A^{-1} u = u + u v^T A^{-1} u$$
$$= (A + u v^T) A^{-1} u \quad ①$$

since $A + uv^T$ and A^{-1} is non-singular (invertible)

and u is no-zero vector

$\Rightarrow ① \neq 0$

then $(A + uv^T)\left(A^{-1} - \dfrac{A^{-1} u v^T A^{-1}}{1 + v^T A^{-1} u}\right)$

$$= AA^{-1} + uv^T A^{-1} - \frac{AA^{-1} u v^T A^{-1}}{1 + v^T A^{-1} u} - \frac{u v^T A^{-1} u v^T A^{-1}}{1 + v^T A^{-1} u}$$

$$= I + uv^T A^{-1} - \left(\frac{uv^T A^{-1} + u v^T A^{-1} u v^T A^{-1}}{1 + v^T A^{-1} u}\right)$$

$$= I + uv^T A^{-1} - \frac{u(v^T A^{-1} + v^T A^{-1} u v^T A^{-1})}{1 + v^T A^{-1} u}$$

$$= I + uv^T A^{-1} - \frac{u(\cancel{I + v^T A^{-1} u}) v^T A^{-1}}{\cancel{1 + v^T A^{-1} u}}$$

$$= I$$

problem 3

solve $(A+uv^T)x = b$ in min operators and memory

$$x = (A+uv^T)^{-1} b$$
$$= \left(A^{-1} - \frac{A^{-1}uv^TA^{-1}}{1+v^TA^{-1}u}\right) b$$
$$= A^{-1}b - \frac{A^{-1}uv^TA^{-1}b}{1+v^TA^{-1}u}$$

Set $A^{-1}\cdot$vector as fastsolve(A, vector) → vector

Matrix dot a vector as A.dot(b) → vector

vector inner product as $u^T * v$ → scalar

① = fastsolve(A, b)

② = $v^T *$ ①

③ = ② u

④ = fast solve (A, ③)

⑤ = fastsolve(A, u)

⑥ = $v^T *$ ⑤

result = ① $- \frac{④}{1+⑥}$

I don't know if $A^{-1}b * \left(1 - \frac{v^T*(\text{fastsolve}(A,u))}{1+v^T*(\text{fastsolve}(A,u))}\right)$

will give the same solution)