PROGRAMMING ASSIGNMENT 3

Problem 1. Here we will write a short program in Python, based on Problem 2 of Homework 4 and on the example `cond_number_demo.ipynb` that can be found on Canvas.

The goal is to create a number of matrices A with random condition number, and then plot the condition number of A against the relative error in solving a linear system of the form

$$Ax = b.$$

Most of the structure of `cond_number_demo.ipynb` remains the same (with small variations). The main difference is in the # create A part of the code (aside from some minor changes, namely we don't need the variable `delta` anymore, but we will need to set a variable $N = 30$ for the size of the N-by-N we want to construct). To build the random matrices A, we will use the ideas discussed in Problem 2 of HW4.

Recall that given a basis $\{u_1, \ldots, u_n\}$ for $\mathbb{R}^n$ and positive numbers $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n > 0$, we may construct an n-by-n matrix A with the u's as eigenvectors and the λ's as eigenvalues, by letting

$$A = S^{-1}DS,$$

where $S = \begin{bmatrix} u_1 & \cdots & u_n \end{bmatrix}$ is the invertible matrix whose columns are the eigenvectors and $D = \mathrm{Diag}(\lambda_1, \ldots, \lambda_n)$ is the diagonal matrix of eigenvalues.

In the code, you will first create a random N-by-N matrix S with normally distributed entries. Then, you will create an array of N random numbers between 1 and 2, and scale the smallest of these numbers by 10^{-16u} where u is uniformly distributed in $[0, 1]$.

Finally, you will plot the condition number against the relative error as before.