```
> restart:

  assume(r>0);
  assume(M>0);

  libname:="H:\\Desktop\\grtensor-master\\lib\\",
  libname:grOptionMetricPath:="H:\\Desktop\\grtensor-
  master\\metrics\\":with(grtensor):
```

"GRTensor III v2.1.11 Apr 26, 2018"

"Copyright 2018, Peter Musgrave, Denis Pollney, Kayll Lake"

"Latest version is at http://github.com/grtensor/grtensor"

"For help ?grtensor"

"Support/contact grtensor3@gmail.com" **(1)**

```
> with(orthopoly):
  with(plots):
```

Start with definitions of the sphereical harmonics. First are the vectors, then tensors and Omega is the metric dependent component. Following Martel and Poisson Y=even and X=odd parity while we also drop the 'l' and 'm' with 2=theta and 3=phi

```
> Y__2:=diff(Y(theta,phi), theta); Y__3:=I*m*Y(theta,phi);
  X__2:=-I*m*Y(theta,phi)/sin(theta); X__3:=sin(theta)*(diff(Y
  (theta,phi), theta));
```

$$Y_2 := \frac{\partial}{\partial \theta} Y(\theta, \phi)$$

$$Y_3 := \mathrm{I}\, m\, Y(\theta, \phi)$$

$$X_2 := \frac{-\mathrm{I}\, m\, Y(\theta, \phi)}{\sin(\theta)}$$

$$X_3 := \sin(\theta) \left(\frac{\partial}{\partial \theta} Y(\theta, \phi) \right) \tag{2}$$

Next tensor spherical harmonics. This time there are three: even, odd and the one proportional the two-metric which I call Omega (again MP notation). Again Y=even and X=odd parity

```
> Y__22:=((1/2)*lambda+m^2*(1/sin(theta)^2))*Y(theta,phi)-cos
  (theta)*(diff(Y(theta,phi), theta))*(1/sin(theta));
  Y__23:=expand((I*m*(diff(Y(theta,phi), theta))*sin(theta)-I*cos
  (theta)*m*Y(theta,phi))*(1/sin(theta)));
  Y__33 := (-m^2+(1/2)*(-lambda*(sin(theta)^2)))*Y(theta,phi)+sin
  (theta)*cos(theta)*(diff(Y(theta,phi), theta));
```

$$Y_{22} := \left(\frac{\lambda}{2} + \frac{m^2}{\sin(\theta)^2} \right) Y(\theta, \phi) - \frac{\cos(\theta) \left(\frac{\partial}{\partial \theta} Y(\theta, \phi) \right)}{\sin(\theta)}$$

$$Y_{23} := \mathrm{I}\, m \left(\frac{\partial}{\partial \theta} Y(\theta, \phi) \right) - \frac{\mathrm{I} \cos(\theta)\, m\, Y(\theta, \phi)}{\sin(\theta)}$$

$$Y_{33} := \left(-m^2 - \frac{\lambda \sin(\theta)^2}{2}\right) Y(\theta, \phi) + \sin(\theta)\cos(\theta)\left(\frac{\partial}{\partial\theta} Y(\theta, \phi)\right) \tag{3}$$

Remember that $\lambda = -l(l+1)$. Here we add in the tensor components

```
> X__22:=-I*m*(diff(Y(theta,phi), theta))*(1/sin(theta))+I*cos
  (theta)*m*Y(theta,phi)*(1/sin(theta)^2);
  X__23 := (((1/2)*(sin(theta)*lambda))+m^2*(1/sin(theta)))*Y
  (theta,phi)-cos(theta)*(diff(Y(theta,phi), theta));
  X__33 := I*m*(diff(Y(theta,phi), theta))*sin(theta)-I*cos(theta)*
  m*Y(theta,phi);
```

$$X_{22} := -\frac{\mathrm{I}\, m \left(\frac{\partial}{\partial\theta} Y(\theta, \phi)\right)}{\sin(\theta)} + \frac{\mathrm{I}\cos(\theta)\, m\, Y(\theta, \phi)}{\sin(\theta)^2}$$

$$X_{23} := \left(\frac{\sin(\theta)\,\lambda}{2} + \frac{m^2}{\sin(\theta)}\right) Y(\theta, \phi) - \cos(\theta)\left(\frac{\partial}{\partial\theta} Y(\theta, \phi)\right)$$

$$X_{33} := \mathrm{I}\, m \left(\frac{\partial}{\partial\theta} Y(\theta, \phi)\right)\sin(\theta) - \mathrm{I}\cos(\theta)\, m\, Y(\theta, \phi) \tag{4}$$

Final tensor harmonics

```
> Omega__22:=Y(theta,phi);
  Omega__23:=0;
  Omega__33:=Y(theta,phi)*sin(theta)^2;
```

$$\Omega_{22} := Y(\theta, \phi)$$

$$\Omega_{23} := 0$$

$$\Omega_{33} := Y(\theta, \phi)\sin(\theta)^2 \tag{5}$$

'Linearize' linearizes in terms of epsilon to first order

```
> Linearize:=proc(T)
             local T1:
             T1:=convert(series(T, epsilon, 2), polynom);
             simplify(coeff(T1, epsilon, 0)+epsilon*subs(simplify
   (coeff(T1, epsilon, 1))));
  end proc:
```

Apply a fix to the cosine bias in the simplify command for maple

```
> FixSin:=proc(T)
             local T1;
             T1:=expand(subs(cos(theta)^2=1-sin(theta)^2, expand(T)));
  end proc:
```

Substitute in the Sphereical Harmonics into the metric perturbations.

This is expressed in Regge-Wheeler gauge. **********This needs to be done in general as well* *******eventually**

Gauge transformations are inputted in the last section. This section has to be revisited for the matter introductin. or does it? pretty sure it does lolz

```
> hExpand:=proc(T)
    local T1,T2,T3,T4,T5,T6;
    T1:=subs(H__scal(r,theta,phi)=h__rr(r)*Y(theta,phi),T):
    T2:=subs(H__V1(r,theta,phi)=h__Veven(r)*Y__2+h__Vodd(r)*X__2,
  T1):
    T3:=subs(H__V2(r,theta,phi)=h__Veven(r)*Y__3+h__Vodd(r)*X__3,
  T2):
    T4:=subs(H__T11(r,theta,phi)=h__Teven(r)*Y__22+h__Todd(r)*
  X__22+h__trace(r)*Omega__22,T3):
    T5:=subs(H__T12(r,theta,phi)=h__Teven(r)*Y__23+h__Todd(r)*
  X__23+h__trace(r)*Omega__23,T4):
    T6:=subs(H__T22(r,theta,phi)=h__Teven(r)*Y__33+h__Todd(r)*
  X__33+h__trace(r)*Omega__33,T5):
  expand(eval(T6));
  end proc:
```

Added a macro for my expression for Extrinsic Curvature. The grdef command would only accept these definitions k1 and not k[0] or k_[0]

```
> kExpand:=proc(T)
    local T1,T2,T3,T4,T5,T6;
    T1:=subs(k__scal(r,theta,phi)=k__rr(r)*Y(theta,phi),T):
    T2:=subs(k__V1(r,theta,phi)=k__Veven(r)*Y__2+k__Vodd(r)*X__2,
  T1):
    T3:=subs(k__V2(r,theta,phi)=k__Veven(r)*Y__3+k__Vodd(r)*X__3,
  T2):
    T4:=subs(k__T11(r,theta,phi)=k__Teven(r)*Y__22+k__Todd(r)*
  X__22+k__trace(r)*Omega__22,T3):
    T5:=subs(k__T12(r,theta,phi)=k__Teven(r)*Y__23+k__Todd(r)*
  X__23+k__trace(r)*Omega__23,T4):
    T6:=subs(k__T22(r,theta,phi)=k__Teven(r)*Y__33+k__Todd(r)*
  X__33+k__trace(r)*Omega__33,T5):
  expand(eval(T6));
  end proc:
```

ApplyId applies the scalar spherical harmonic eigenvalue equation to get rid of double theta derivatives

```
> ApplyId:=proc(T)
    local T1;
    T1:=expand(subs(diff(Y(theta,phi),theta,theta)=m^2*Y(theta,
  phi)/sin(theta)^2-cot(theta)*diff(Y(theta,phi),theta)+lambda*Y
  (theta,phi),T)):
  end proc:
```

RemPhi replaces phi derivatives with I*m*Y.

```
> RemPhi:=proc(T)
    local T1;
    T1:=expand(subs(diff(Y(theta,phi),phi)=I*m*Y(theta,phi),T)):
  end proc:
```

CollectY groups together theta derivatives and powers of m

```
> CollectY:=proc(T)
    local T1,T2,T3;
    T1:=simplify(coeff(T,diff(Y(theta,phi),theta)))*diff(Y(theta,
  phi),theta):
    T2:=factor(FixSin(simplify(T-T1))):
    T3:=T1+T2;
  end proc:
```

The next two functions take the theta and phi components of a two-vector and use them to identity the even/odd decomposition. They should be redundant but I include both so I can check for consistency.

```
> IdV2:=proc(V2)
        local Ycoeff,Xcoeff;
        Ycoeff:=expand(coeff(V2,diff(Y(theta,phi),theta)));
        Xcoeff:=expand(simplify(sin(theta)/m/Y(theta,phi)*I*(V2-
  Ycoeff*diff(Y(theta,phi),theta))));
        return(expand(Ycoeff/epsilon),expand(Xcoeff/epsilon));
      end proc:
> IdV3:=proc(V3)
        local Ycoeff,Xcoeff;
        Xcoeff:=expand(coeff(V3,diff(Y(theta,phi),theta)));
        Ycoeff:=expand(simplify(sin(theta)/m/Y(theta,phi)*I*(V3-
  Xcoeff*diff(Y(theta,phi),theta))));
        return(expand(-Ycoeff/epsilon/sin(theta)),expand
  (Xcoeff/epsilon/sin(theta)));
      end proc:
```

Next decomposing two-tensors. It's easiest to start with the theta-phi component to get the Y_AB and X_AB components and and get the Omega_AB component from the theta-theta. Finally we can check it all for consistency against phi-phi.

```
> IdT23:=proc(T23)
       local Ycoeff, Xcoeff;
         Ycoeff:=expand(simplify(coeff(T23,m,1)/Y__23));
         Xcoeff:=expand(simplify((T23-m*Ycoeff*Y__23)/X__23));
         return(expand(FixSin(expand(m*Ycoeff/epsilon))),expand
  (FixSin(expand(Xcoeff/epsilon))));
       end proc:
> IdT22:=proc(T22,Ycoeff,Xcoeff)
       expand(simplify((T22-epsilon*Ycoeff*Y__22-epsilon*Xcoeff*
  X__22)/Omega__22/epsilon));
       end proc:
> IdT33:=proc(T33,Ycoeff,Xcoeff)
       expand(simplify((T33-epsilon*Ycoeff*Y__33-epsilon*Xcoeff*
  X__33)/Omega__33/epsilon));
       end proc:
```

Load the Metric

```
> qload(lpschwNewFixed);
  grdisplay(g(dn, dn));
Error, (in H[T11]) wrong number of arguments
Error, (in H[T11]) wrong number of arguments
Calculated ds for lpschwNewFixed (0.000000 sec.)
```

Default spacetime = lpschwNewFixed

For the lpschwNewFixed spacetime:

Coordinates

x(up)

$$x^a = \begin{bmatrix} r & \theta & \phi \end{bmatrix}$$

For the lpschwNewFixed spacetime:

constraints

$$constraint = [\epsilon^2 = 0, \epsilon^3 = 0, \epsilon^4 = 0, \epsilon^5 = 0, \epsilon^6 = 0, \epsilon^7 = 0, \epsilon^8 = 0, \epsilon^9 = 0, \epsilon^{10} = 0]$$

For the lpschwNewFixed spacetime:

Covariant metric tensor

g(dn, dn)

$$g_{ab} = \begin{bmatrix} 1 + \epsilon H_{scal}(r, \theta, \phi) & \epsilon H_{V1}(r, \theta, \phi) & \epsilon H_{V2}(r, \theta, \phi) \\ \epsilon H_{V1}(r, \theta, \phi) & r^2 + \epsilon H_{T11}(r, \theta, \phi) & \epsilon H_{T12}(r, \theta, \phi) \\ \epsilon H_{V2}(r, \theta, \phi) & \epsilon H_{T12}(r, \theta, \phi) & r^2 \sin(\theta)^2 + \epsilon H_{T22}(r, \theta, \phi) \end{bmatrix} \tag{6}$$

Calculate the inverse metric

```
> grcalc(g(up, up));
  grmap(g(up, up), Linearize, 'x');
  grdisplay(_);
Calculated detg for lpschwNewFixed (0.000000 sec.)
Calculated g(up,up) for lpschwNewFixed (0.047000 sec.)
```

CPU Time $= 0.047$

```
Procedure name:LinearizeApplying routine Linearize to g(up,up)
```

For the lpschwNewFixed spacetime:

Contravariant metric tensor

g(up, up)

$$g^{ab} = \begin{bmatrix} 1 - \epsilon H_{scal}(r, \theta, \phi) & -\frac{H_{V1}(r, \theta, \phi)\, \epsilon}{r^2} & -\frac{H_{V2}(r, \theta, \phi)\, \epsilon}{r^2 \sin(\theta)^2} \\ -\frac{H_{V1}(r, \theta, \phi)\, \epsilon}{r^2} & \frac{1}{r^2} - \frac{\epsilon H_{T11}(r, \theta, \phi)}{r^4} & -\frac{H_{T12}(r, \theta, \phi)\, \epsilon}{\sin(\theta)^2 r^4} \\ -\frac{H_{V2}(r, \theta, \phi)\, \epsilon}{r^2 \sin(\theta)^2} & -\frac{H_{T12}(r, \theta, \phi)\, \epsilon}{\sin(\theta)^2 r^4} & \frac{1}{r^2 \sin(\theta)^2} - \frac{\epsilon H_{T22}(r, \theta, \phi)}{r^4 \sin(\theta)^4} \end{bmatrix} \tag{7}$$

Need to force the definition for the perturbed extrinsic curvature

```
> grdef(`er{a}:=[1, 0, 0]`);
  grdef(`et{a}:=[0, 1, 0]`);
  grdef(`ep{a}:=[0, 0, 1]`);
  grcalc(er(dn), et(dn), ep(dn));

  grdisplay(_);

  grdef(`KK{a b}:=(sqrt(2)/2*sqrt(M)/r^(3/2)+epsilon*k__scal(r,
  theta,phi))*er{a}*er{b}+(epsilon*k__V1(r,theta,phi))*er{a}*et{b}+
  (epsilon*k__V2(r,theta,phi))*er{a}*ep{b}+(epsilon*k__V1(r,theta,
  phi))*er{b}*et{a}+(epsilon*k__V2(r,theta,phi))*er{b}*ep{a}+(-sqrt
  (2)*sqrt(M)*sqrt(r)+epsilon*k__T11(r,theta,phi))*et{a}*et{b}+
  (epsilon*k__T12(r,theta,phi))*et{a}*ep{b}+(epsilon*k__T12(r,
  theta,phi))*et{b}*ep{a}+(-sqrt(2)*sqrt(M)*sqrt(r)*sin(theta)^2+
  epsilon*k__T22(r,theta,phi))*ep{a}*ep{b}`);
  grcalc(KK(dn, dn));
  grdisplay(KK(dn, dn));
```

```
Components assigned for metric: lpschwNewFixed
Created definition for er(dn)
Components assigned for metric: lpschwNewFixed
Created definition for et(dn)
Components assigned for metric: lpschwNewFixed
Created definition for ep(dn)
```

$$CPU\ Time\ = 0.$$

For the lpschwNewFixed spacetime:

er(dn)

$er(dn)$

$$er_a = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

ep(dn)

$ep(dn)$

$$ep_a = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$$

```
Created definition for KK(dn,dn)
Calculated KK(dn,dn) for lpschwNewFixed (0.000000 sec.)
```

$$CPU\ Time\ = 0.$$

For the lpschwNewFixed spacetime:

KK(dn,dn)

$KK(dn, dn)$

$$KK_{a\,b} = \left[\left[\frac{2\,\epsilon\,k_{scal}(r,\theta,\phi)\,r^{3/2} + \sqrt{2}\,\sqrt{M}}{2\,r^{3/2}}, \epsilon\,k_{V1}(r,\theta,\phi), \epsilon\,k_{V2}(r,\theta,\phi)\right],\right. \quad \textbf{(8)}$$
$$\left[\epsilon\,k_{V1}(r,\theta,\phi), -\sqrt{2}\,\sqrt{M}\,\sqrt{r} + \epsilon\,k_{T11}(r,\theta,\phi), \epsilon\,k_{T12}(r,\theta,\phi)\right],$$
$$\left.\left[\epsilon\,k_{V2}(r,\theta,\phi), \epsilon\,k_{T12}(r,\theta,\phi), -\sqrt{2}\,\sqrt{M}\,\sqrt{r}\,\sin(\theta)^2 + \epsilon\,k_{T22}(r,\theta,\phi)\right]\right]$$

Now to form the constraint equation.

First get the extrinsic curvature squared

```
> grdef(`K2:=KK{^a ^b}*KK{a b}`);
  grcalc(K2);
  grmap(K2, Linearize, 'x');
  grmap(K2, FixSin, 'x');
  grdisplay(K2);
Created definition for KK(up,up)
Created definition for K2
Calculated KK(up,up) for lpschwNewFixed (0.016000 sec.)
Calculated K2 for lpschwNewFixed (0.000000 sec.)
```

$$CPU\ Time\ = 0.016$$

```
Procedure name:LinearizeApplying routine Linearize to K2
Procedure name:FixSinApplying routine FixSin to K2
```

For the lpschwNewFixed spacetime:

K2

$$K2 = -\frac{M H_{scal}(r,\theta,\phi)\,\epsilon}{r^3} + \frac{9\,M}{2\,r^3} + \frac{\sqrt{2}\,k_{scal}(r,\theta,\phi)\,\epsilon\sqrt{M}}{r^{3/2}} - \frac{2\sqrt{2}\,k_{T11}(r,\theta,\phi)\,\epsilon\sqrt{M}}{r^{7/2}} - \frac{2\sqrt{2}\,k_{T22}(r,\theta,\phi)\,\epsilon\sqrt{M}}{r^{7/2}\sin(\theta)^2} - \frac{4\,M H_{T11}(r,\theta,\phi)\,\epsilon}{r^5} - \frac{4\,M H_{T22}(r,\theta,\phi)\,\epsilon}{r^5\sin(\theta)^2} \tag{9}$$

Next formulate the trace of the extrinsic curvature

```
> grdef(`KTr:=g{^a ^b}*KK{a b}`);
  grcalc(KTr);
  grmap(KTr, Linearize, 'x');
  grmap(KTr, FixSin, 'x');
  grdisplay(_);
Created definition for KTr
Calculated KTr for lpschwNewFixed (0.000000 sec.)
```

$$CPU\ Time\ = 0.$$

```
Procedure name:LinearizeApplying routine Linearize to KTr
Procedure name:FixSinApplying routine FixSin to KTr
```

For the lpschwNewFixed spacetime:

KTr

$$KTr = \frac{k_{T11}(r,\theta,\phi)\,\epsilon}{r^2} + \frac{k_{T22}(r,\theta,\phi)\,\epsilon}{r^2\sin(\theta)^2} + k_{scal}(r,\theta,\phi)\,\epsilon - \frac{H_{scal}(r,\theta,\phi)\sqrt{2}\sqrt{M}\,\epsilon}{2\,r^{3/2}} + \frac{H_{T11}(r,\theta,\phi)\sqrt{2}\sqrt{M}\,\epsilon}{r^{7/2}} - \frac{3\sqrt{2}\sqrt{M}}{2\,r^{3/2}} + \frac{\sqrt{2}\sqrt{M}\,H_{T22}(r,\theta,\phi)\,\epsilon}{r^{7/2}\sin(\theta)^2} \tag{10}$$

Start the Hamilton constraint equation

```
> grdef(`Cons:=KTr^2-K2`);
  grcalc(Cons);
```

```
grmap(Cons, Linearize, 'x');
grmap(Cons, FixSin, 'x');
Created definition for Cons
Calculated Cons for lpschwNewFixed (0.000000 sec.)
```

$$CPU\ Time\ = 0.$$

```
Procedure name:LinearizeApplying routine Linearize to Cons
Procedure name:FixSinApplying routine FixSin to Cons
```

Insert the harmonics for the metric and the Extrinsic curvature

```
> grmap(Cons, hExpand, 'x');
  grmap(Cons, kExpand, 'x');
  grdisplay(_);
Procedure name:hExpandApplying routine hExpand to Cons
Procedure name:kExpandApplying routine kExpand to Cons
```

For the lpschwNewFixed spacetime:

Cons

$$Cons = -\frac{2\sqrt{2}\,\epsilon\sqrt{M}\,k_{trace}(r)\,Y(\theta,\phi)}{r^{7/2}} - \frac{4\sqrt{2}\,k_{rr}(r)\,Y(\theta,\phi)\,\epsilon\sqrt{M}}{r^{3/2}} + \frac{4\,M\,h_{rr}(r)\,Y(\theta,\phi)\,\epsilon}{r^{3}} - \frac{4\,M\,\epsilon\,h_{trace}(r)\,Y(\theta,\phi)}{r^{5}} \quad \textbf{(11)}$$

Next we need the Ricci Scalar

```
> grcalc(Ricciscalar); grdisplay(_);
Calculated g(dn,dn,pdn) for lpschwNewFixed (0.000000 sec.)
Calculated Chr(dn,dn,dn) for lpschwNewFixed (0.000000 sec.)
Calculated Chr(dn,dn,up) for lpschwNewFixed (0.000000 sec.)
Calculated R(dn,dn) for lpschwNewFixed (0.031000 sec.)
Calculated Ricciscalar for lpschwNewFixed (0.016000 sec.)
```

$$CPU\ Time\ = 0.047$$

For the lpschwNewFixed spacetime:

Ricci scalar

R = 149090 words. Exceeds grOptionDisplayLimit **(12)**

Run some macros on R before finishing the contraint equation

```
> grmap(Ricciscalar, Linearize, 'x');
  grmap(Ricciscalar, FixSin, 'x');
  grmap(Ricciscalar, hExpand, 'x');
  grmap(Ricciscalar, ApplyId, 'x');
  grmap(Ricciscalar, RemPhi, 'x');
  grmap(Ricciscalar, RemPhi, 'x');
  grmap(Ricciscalar, FixSin, 'x');
  grdisplay(Ricciscalar);
Procedure name:LinearizeApplying routine Linearize to
Ricciscalar
Procedure name:FixSinApplying routine FixSin to Ricciscalar
Procedure name:hExpandApplying routine hExpand to Ricciscalar
Procedure name:ApplyIdApplying routine ApplyId to Ricciscalar
```

```
Procedure name:RemPhiApplying routine RemPhi to Ricciscalar
Procedure name:RemPhiApplying routine RemPhi to Ricciscalar
Procedure name:FixSinApplying routine FixSin to Ricciscalar
```

For the lpschwNewFixed spacetime:

Ricci scalar

$$R = 15892 \text{ words. Exceeds grOptionDisplayLimit} \tag{13}$$

Now we can form final expression for the constraint equation for the Initial-Value Problem

```
> grdef(`Con:=Ricciscalar+Cons`);
  grcalc(Con);
  grmap(Con, hExpand, 'x');
  grmap(Con, ApplyId, 'x');
  gralter(Con, simplify);
  gralter(Con, expand);

  collect(collect(collect(grcomponent(Con), h__trace(r)),h__Teven
  (r)),h__rr(r)):
```

```
Created definition for Con
Calculated Con for lpschwNewFixed (0.000000 sec.)
```

$$\mathit{CPU\ Time} = 0.$$

```
Procedure name:hExpandApplying routine hExpand to Con
Procedure name:ApplyIdApplying routine ApplyId to Con
Component simplification of a GRTensorIII object:

Applying routine simplify to object Con
```

$$\mathit{CPU\ Time} = 0.032$$

```
Component simplification of a GRTensorIII object:

Applying routine expand to object Con
```

$$\mathit{CPU\ Time} = 0. \tag{14}$$

```
> grdef(`Con0:=Con/Y(theta,phi)/epsilon*r/2`);
  grcalc(Con0);
  gralter(Con, simplify);
  gralter(Con0, expand);
  H0X:=collect(collect(collect(grcomponent(Con0), h__trace(r)),
  h__Teven(r)),h__rr(r));
```

```
Created definition for Con0
Calculated Con0 for lpschwNewFixed (0.000000 sec.)
```

$$\mathit{CPU\ Time} = 0.$$

```
Component simplification of a GRTensorIII object:

Applying routine simplify to object Con
```

$$\mathit{CPU\ Time} = 0.016$$

```
Component simplification of a GRTensorIII object:

Applying routine expand to object Con0
```

$$\mathit{CPU\ Time} = 0.$$

$$H0X := \left(\frac{2\,M}{r^2} - \frac{\lambda}{2\,r} + \frac{1}{r}\right) h_{rr}(r) + \left(\frac{\lambda^2}{4\,r^3} + \frac{\lambda}{2\,r^3}\right) h_{Teven}(r) + \left(-\frac{2\,M}{r^4} - \frac{\lambda}{2\,r^3}\right. \tag{15}$$

$$- \frac{1}{r^3}\Bigg) h_{trace}(r) + \frac{h_{Veven}(r)\,\lambda}{r^2} + \frac{\left(\frac{d}{dr} h_{Veven}(r)\right)\lambda}{r} + \frac{d}{dr} h_{rr}(r) - \frac{\frac{d^2}{dr^2} h_{trace}(r)}{r}$$

$$+ \frac{\frac{d}{dr} h_{trace}(r)}{r^2} - \frac{\sqrt{2}\,\sqrt{M}\,k_{trace}(r)}{r^{5/2}} - \frac{2\sqrt{2}\,k_{rr}(r)\,\sqrt{M}}{\sqrt{r}}$$

Now we want to formulate the momentum constraint

```
> grdef(`JJ{^a ^b}:=KK{^a ^b}-g{^a ^b}*KTr`);
  grcalc(JJ(up, up));

  grdef(`J{b}:=JJ{^a b ;a}`);

  grcalc(J(dn));

  grmap(J(dn), Linearize, 'x');
  grmap(J(dn), FixSin, 'x');
  grmap(J(dn), hExpand, 'x');
  grmap(J(dn), kExpand, 'x');
  grmap(J(dn), RemPhi, 'x');
  grmap(J(dn), ApplyId, 'x');
  grmap(J(dn), CollectY, 'x');

  grcalc(J(dn));

  gralter(J(dn), simplify);
  gralter(J(dn), expand);
Created definition for JJ(up,up)
Calculated JJ(up,up) for lpschwNewFixed (0.000000 sec.)
```

$$CPU\ Time = 0.$$

```
Created definition for JJ(up,dn)
Created a definition for JJ(up,dn,cdn)
Created definition for J(dn)
Calculated JJ(up,dn) for lpschwNewFixed (0.000000 sec.)
Calculated JJ(up,dn,cdn) for lpschwNewFixed (0.672000 sec.)
Calculated J(dn) for lpschwNewFixed (0.188000 sec.)
```

$$CPU\ Time = 0.860$$

```
Procedure name:LinearizeApplying routine Linearize to J(dn)
Procedure name:FixSinApplying routine FixSin to J(dn)
Procedure name:hExpandApplying routine hExpand to J(dn)
Procedure name:kExpandApplying routine kExpand to J(dn)
Procedure name:RemPhiApplying routine RemPhi to J(dn)
Procedure name:ApplyIdApplying routine ApplyId to J(dn)
Procedure name:CollectYApplying routine CollectY to J(dn)
```

$$CPU\ Time = 0.$$

```
Component simplification of a GRTensorIII object:

Applying routine simplify to object J(dn)
```

$$CPU\ Time = 0.140$$

```
Component simplification of a GRTensorIII object:
```

```
Applying routine expand to object J(dn)
```
$$CPU\ Time\ = 0. \tag{16}$$

Below are the consistency checks. The r component is fine but the theta and phi are not matching Ivan's. trying many different things.

r component

```
> H1:=-epsilon*Y(theta, phi)*(-4*k__rr(r)*r^(13/2)-2*k__Veven(r)*
  lambda*r^(11/2)+4*(diff(k__trace(r), r))*r^(11/2)-4*k__trace(r)*
  r^(9/2)+sqrt(M)*sqrt(2)*h__Veven(r)*lambda*r^4+2*sqrt(M)*sqrt(2)*
  h__rr(r)*r^5+sqrt(M)*sqrt(2)*(diff(h__trace(r), r))*r^4-4*sqrt(M)
  *sqrt(2)*h__trace(r)*r^3)/(2*r^(15/2)):

  Test0:=simplify(H1-grcomponent(J(dn), [r]));
```
$$Test0 := 0 \tag{17}$$

Theta check

```
> H2:=-(((-(1/2)*lambda-1)*k__Teven(r)+k__trace(r))*(r^(3/2))+
  (k__rr(r)-(diff(k__Veven(r), r)))*(r^(7/2))-2*k__Veven(r)*(r^
  (5/2))+(1/4)*(sqrt(M)*(-4*(diff(h__Veven(r), r))*(r^2)+(-2*
  lambda-4)*h__Teven(r)+h__rr(r)*(r^2)-2*h__Veven(r)*r+4*h__trace
  (r))*sqrt(2)))*epsilon*(diff(Y(theta, phi), theta))*(1/r^(7/2))-
  I*epsilon*Y(theta, phi)*m*(2*(diff(k__Vodd(r), r))*(r^(11/2))+4*
  k__Vodd(r)*(r^(9/2))+k__Todd(r)*lambda*(r^(7/2))+2*k__Todd(r)*(r^
  (7/2))+2*sqrt(M)*sqrt(2)*(diff(h__Vodd(r), r))*(r^4)+sqrt(M)*sqrt
  (2)*h__Todd(r)*lambda*(r^2)+sqrt(M)*sqrt(2)*h__Vodd(r)*(r^3)+2*
  sqrt(M)*sqrt(2)*h__Todd(r)*(r^2))*(1/(2*r^(11/2)*sin(theta))):

  Test2:=simplify(expand(H2-grcomponent(J(dn), [theta])));
```
$$Test2 := 0 \tag{18}$$

Phi check

```
> H3:=2*sin(theta)*epsilon*((1/4)*(k__Todd(r)*(lambda+2)*(r^(3/2)))
  +k__Vodd(r)*(r^(5/2))+(1/2)*((diff(k__Vodd(r), r))*(r^(7/2)))+
  (1/4)*(sqrt(M)*(2*(diff(h__Vodd(r), r))*(r^2)+(lambda+2)*h__Todd
  (r)+h__Vodd(r)*r)*sqrt(2)))*(diff(Y(theta, phi), theta))*(1/r^
  (7/2))+I*epsilon*m*Y(theta, phi)*(4*(diff(k__Veven(r), r))*(r^
  (11/2))-4*k__rr(r)*(r^(11/2))+8*k__Veven(r)*(r^(9/2))+2*k__Teven
  (r)*lambda*(r^(7/2))+4*k__Teven(r)*(r^(7/2))-4*k__trace(r)*(r^
  (7/2))+4*sqrt(M)*sqrt(2)*(diff(h__Veven(r), r))*(r^4)-sqrt(2)*
  sqrt(M)*h__rr(r)*(r^4)+2*sqrt(M)*sqrt(2)*h__Veven(r)*(r^3)+2*sqrt
  (M)*sqrt(2)*h__Teven(r)*lambda*(r^2)+4*sqrt(M)*sqrt(2)*h__Teven
  (r)*(r^2)-4*sqrt(M)*sqrt(2)*h__trace(r)*(r^2))*(1/(4*r^(11/2))):

  Test3:=simplify(H3-grcomponent(J(dn), [phi]));
```
$$Test3 := 0 \tag{19}$$

Start with the decomposition of the vectors from the IDV2 commands

```
> H2__even, H2__odd:=IdV2(grcomponent(J(dn), [theta])):
  H3__even, H3__odd:=IdV3(grcomponent(J(dn), [phi])):
```

Check that the components are the same for each mode type

```
> simplify(FixSin(H2__even-H3__even));
  simplify(FixSin(H2__odd-H3__odd));
```

$$0$$

$$0 \tag{20}$$

Now we can setup the constraint equations in vaccuum: Following Baumgarte and Shapiro pg 124

Hamiltonian Constraint

```
> H0__vac:=collect(collect(collect(collect(expand(subs(H0X)),h__rr
  (r)),h__trace(r)),h__Teven(r)),m);
```

$$H0_{vac} := \left(\frac{2\,M}{r^2} - \frac{\lambda}{2\,r} + \frac{1}{r}\right) h_{rr}(r) + \left(\frac{\lambda^2}{4\,r^3} + \frac{\lambda}{2\,r^3}\right) h_{Teven}(r) + \left(-\frac{2\,M}{r^4} - \frac{\lambda}{2\,r^3} - \frac{1}{r^3}\right) h_{trace}(r) + \frac{h_{Veven}(r)\,\lambda}{r^2} + \frac{\left(\frac{d}{dr}\,h_{Veven}(r)\right)\lambda}{r} + \frac{d}{dr}\,h_{rr}(r) - \frac{\frac{d^2}{dr^2}\,h_{trace}(r)}{r} + \frac{\frac{d}{dr}\,h_{trace}(r)}{r^2} - \frac{\sqrt{2}\,\sqrt{M}\,k_{trace}(r)}{r^{5/2}} - \frac{2\,\sqrt{2}\,k_{rr}(r)\,\sqrt{M}}{\sqrt{r}} \tag{21}$$

```
> H0__vacX:=collect(collect(collect(expand(subs(h__rr(r)=H__rr(r)
  /r,h__Veven(r)=H__Veven(r)/r,r*H0__vac)),H__rr(r)),h__trace(r)),
  h__Teven(r));;
```

$$H0_{vacX} := \left(\frac{\lambda^2}{4\,r^2} + \frac{\lambda}{2\,r^2}\right) h_{Teven}(r) + \left(-\frac{2\,M}{r^3} - \frac{\lambda}{2\,r^2} - \frac{1}{r^2}\right) h_{trace}(r) + \left(\frac{2\,M}{r^2} - \frac{\lambda}{2\,r}\right) H_{rr}(r) + \frac{\lambda\left(\frac{d}{dr}\,H_{Veven}(r)\right)}{r} + \frac{d}{dr}\,H_{rr}(r) - \left(\frac{d^2}{dr^2}\,h_{trace}(r)\right) + \frac{\frac{d}{dr}\,h_{trace}(r)}{r} - \frac{\sqrt{2}\,\sqrt{M}\,k_{trace}(r)}{r^{3/2}} - 2\,\sqrt{r}\,\sqrt{2}\,k_{rr}(r)\,\sqrt{M} \tag{22}$$

Momentum Constraint

```
> H1__vac:=collect(expand(-subs(subs(k__trace(r)=r*K__trace(r),
  expand(r/2*grcomponent(J(dn), [r])/epsilon/Y(theta,phi))))), m);
```

$$H1_{vac} := -\frac{k_{Veven}(r)\,\lambda}{2\,r} + \frac{d}{dr}\,K_{trace}(r) - k_{rr}(r) + \frac{h_{rr}(r)\,\sqrt{2}\,\sqrt{M}}{2\,r^{3/2}} + \frac{\sqrt{2}\,\sqrt{M}\,h_{Veven}(r)\,\lambda}{4\,r^{5/2}} + \frac{\sqrt{2}\,\sqrt{M}\left(\frac{d}{dr}\,h_{trace}(r)\right)}{4\,r^{5/2}} - \frac{\sqrt{2}\,\sqrt{M}\,h_{trace}(r)}{r^{7/2}} \tag{23}$$

```
> expand(subs(h__trace(r)=r^4*H__trace(r),expand((sqrt(2)*h__Veven
  (r)*lambda*(1/(4*r^(5/2)))+sqrt(2)*h__rr(r)*(1/(2*r^(3/2)))+sqrt
```

```
(2)*(diff(h__trace(r), r))*(1/(4*r^(5/2)))-sqrt(2)*h__trace(r)*
(1/r^(7/2)))/sqrt(2)*r^(3/2)*2)));
```

$$\frac{h_{Veven}(r)\,\lambda}{2\,r} + h_{rr}(r) + \frac{r^3\left(\frac{d}{dr}\,H_{trace}(r)\right)}{2} \tag{24}$$

```
> Heven__vac:=collect(collect(collect(collect(expand(subs(k__Veven
  (r)=K__Veven(r)/r^2,h__Veven(r)=H2__Veven(r)/sqrt(r),expand(r^2*
  H2__even))),k__Teven(r)),k__trace(r)),h__Teven(r)),M);
```

$$Heven_{vac} := \left(\left(\frac{\sqrt{2}\,\lambda}{2\,r^{3/2}} + \frac{\sqrt{2}}{r^{3/2}}\right)h_{Teven}(r) + \sqrt{2}\left(\frac{d}{dr}\,H2_{Veven}(r)\right) - \frac{\sqrt{r}\,h_{rr}(r)\,\sqrt{2}}{4} - \frac{\sqrt{2}\,h_{trace}(r)}{r^{3/2}}\right)\sqrt{M} + \left(\frac{\lambda}{2}+1\right)k_{Teven}(r) - k_{trace}(r) - k_{rr}(r)\,r^2 + \frac{d}{dr}\,K_{Veven}(r) \tag{25}$$

```
> collect(expand(op(1,Heven__vac)/sqrt(2*M)*sqrt(r)),h__Teven);
```

$$\left(\frac{\lambda}{2\,r} + \frac{1}{r}\right)h_{Teven}(r) + \sqrt{r}\left(\frac{d}{dr}\,H2_{Veven}(r)\right) - \frac{r\,h_{rr}(r)}{4} - \frac{h_{trace}(r)}{r} \tag{26}$$

```
> Hodd__vac:=collect(collect(collect(expand(subs(h__Vodd(r)=H__Vodd
  (r)/sqrt(r),k__Vodd(r)=K__Vodd(r)/r^2,r^2*H2__odd)),k__Todd(r)),
  h__Todd(r)), M);
```

$$Hodd_{vac} := \left(\left(\frac{\sqrt{2}\,\lambda}{2\,r^{3/2}} + \frac{\sqrt{2}}{r^{3/2}}\right)h_{Todd}(r) + \sqrt{2}\left(\frac{d}{dr}\,H_{Vodd}(r)\right)\right)\sqrt{M} + \left(\frac{\lambda}{2}+1\right)k_{Todd}(r) + \frac{d}{dr}\,K_{Vodd}(r) \tag{27}$$

Initial Data Formulation

Time Evolution

Metric

The unperturbed values are from PG and the lapse and shift are left free to insert the gauge at a later time

```
> NX:=1+epsilon*n__L(r)*Y(theta, phi);
```

$$NX := 1 + \epsilon\, n_L(r)\, Y(\theta, \phi) \tag{28}$$

```
> grdef(`VX{a}:=[sqrt(2*M/r)+epsilon*v__scalar(r)*Y(theta,phi),
  epsilon*v__even(r)*Y__2+epsilon*v__odd(r)*X__2,epsilon*v__even(r)
  *Y__3+epsilon*v__odd(r)*X__3]`);
  grcalc(VX(dn));

  grmap(VX(dn), Linearize, 'x');
  grcalc(VX(up));
  grdisplay(VX(dn));
Components assigned for metric: lpschwNewFixed
Created definition for VX(dn)
```

$$CPU\ Time\ = 0.$$

```
Procedure name:LinearizeApplying routine Linearize to VX(dn)
```

```
Created definition for VX(up)
Calculated VX(up) for lpschwNewFixed (0.000000 sec.)
```

$$CPU\ Time\ = 0.$$

For the lpschwNewFixed spacetime:

VX(dn)

$$VX(dn)$$

$$VX_r = \frac{\epsilon\, v_{scalar}(r)\, Y(\theta, \phi)\, r + \sqrt{2}\,\sqrt{M}\,\sqrt{r}}{r}$$

$$VX_\theta = -\frac{\epsilon\left(\mathrm{I}\, v_{odd}(r)\, m\, Y(\theta, \phi) - v_{even}(r)\left(\frac{\partial}{\partial\theta} Y(\theta, \phi)\right)\sin(\theta)\right)}{\sin(\theta)}$$

$$VX_\phi = \left(\mathrm{I}\, v_{even}(r)\, m\, Y(\theta, \phi) + v_{odd}(r)\sin(\theta)\left(\frac{\partial}{\partial\theta} Y(\theta, \phi)\right)\right)\epsilon \tag{29}$$

```
> grdef(`DV{a b}:=VX{b ;a}`);
  grcalc(DV(dn, dn));
  grdisplay(DV(dn, dn));
```

```
Created a definition for VX(dn,cdn)
Created definition for DV(dn,dn)
Calculated VX(dn,cdn) for lpschwNewFixed (0.016000 sec.)
Calculated DV(dn,dn) for lpschwNewFixed (0.000000 sec.)
```

$$CPU\ Time\ = 0.016$$

For the lpschwNewFixed spacetime:

DV(dn,dn)

$$DV(dn, dn)$$

$DV_{r\,r} =$ *5642 words. Exceeds grOptionDisplayLimit*

$DV_{\theta\,r} =$ *6129 words. Exceeds grOptionDisplayLimit*

$DV_{\phi\,r} =$ *6143 words. Exceeds grOptionDisplayLimit*

$DV_{r\,\theta} =$ *6220 words. Exceeds grOptionDisplayLimit*

$DV_{\theta\,\theta} =$ *6349 words. Exceeds grOptionDisplayLimit*

$DV_{\phi\,\theta} =$ *6330 words. Exceeds grOptionDisplayLimit*

$DV_{r\,\phi} =$ *6236 words. Exceeds grOptionDisplayLimit*

$DV_{\theta\,\phi} =$ *6244 words. Exceeds grOptionDisplayLimit*

$DV_{\phi\,\phi} =$ *7136 words. Exceeds grOptionDisplayLimit* **(30)**

Coordinate time derivative of the metric

```
> grdef(`hdot{a b}:=2*NX*KK{a b}+DV{a b}+DV{b a}`);
  grcalc(hdot(dn, dn));

  grmap(hdot(dn, dn), Linearize, 'x');
  grmap(hdot(dn, dn), FixSin, 'x');
  grmap(hdot(dn, dn), hExpand, 'x');
  grmap(hdot(dn, dn), kExpand, 'x');
```

```
grmap(hdot(dn, dn), RemPhi, 'x');
grmap(hdot(dn, dn), ApplyId, 'x');
```

```
Created definition for hdot(dn,dn)
Calculated hdot(dn,dn) for lpschwNewFixed (0.016000 sec.)
```

$$CPU\ Time = 0.016$$

```
Procedure name:LinearizeApplying routine Linearize to hdot(dn,
dn)
Procedure name:FixSinApplying routine FixSin to hdot(dn,dn)
Procedure name:hExpandApplying routine hExpand to hdot(dn,dn)
Procedure name:kExpandApplying routine kExpand to hdot(dn,dn)
Procedure name:RemPhiApplying routine RemPhi to hdot(dn,dn)
Procedure name:ApplyIdApplying routine ApplyId to hdot(dn,dn)
```

Now we need to treat each of the components. First the rr component

```
> hdot__C11:=collect(expand(grcomponent(hdot(dn, dn), [1, 1])
  /epsilon/Y(theta,phi)),sqrt(M)):
  hdot__C11:=factor(op(1,hdot__C11))+op(2,hdot__C11)+op(3,
  hdot__C11);
```

$$hdot_{C11} := \frac{\sqrt{2}\left(-r\left(\frac{d}{dr} h_{rr}(r)\right) + n_L(r)\right)\sqrt{M}}{r^{3/2}} + 2\left(\frac{d}{dr} v_{scalar}(r)\right) + 2\,k_{rr}(r) \tag{31}$$

Check the hdot__C11 matches Ivan's

```
> hdot__C11Ivan:=sqrt(2)*(-r*(diff(h__rr(r), r))+n__L(r))*sqrt(M)
  /r^(3/2)+2*(diff(v__scalar(r), r))+2*k__rr(r);

  simplify(hdot__C11-hdot__C11Ivan);
```

$$hdot_{C11Ivan} := \frac{\sqrt{2}\left(-r\left(\frac{d}{dr} h_{rr}(r)\right) + n_L(r)\right)\sqrt{M}}{r^{3/2}} + 2\left(\frac{d}{dr} v_{scalar}(r)\right) + 2\,k_{rr}(r)$$

$$0 \tag{32}$$

Next we move to the vector components and check that they are the same and then look at the components

```
> IdV2(grcomponent(hdot(dn, dn), [1, 2]))-IdV3(grcomponent(hdot(dn,
  dn), [1, 3]));
```

$$0 \tag{33}$$

Seperate into the even and odd components

```
> hdot__CVeven, hdot__CVodd:=IdV2(grcomponent(hdot(dn, dn), [1, 2])
  ):
  hdot__CVeven;
  hdot__CVodd;
```

$$-\frac{\sqrt{2}\,h_{rr}(r)\sqrt{M}}{\sqrt{r}} + \frac{2\sqrt{2}\sqrt{M}\,h_{Veven}(r)}{r^{3/2}} + \frac{d}{dr} v_{even}(r) + v_{scalar}(r) - \frac{2\,v_{even}(r)}{r}$$

$$+2\,k_{Veven}(r)$$

$$2\,k_{Vodd}(r) + \frac{d}{dr}\,v_{odd}(r) + \frac{2\sqrt{2}\,h_{Vodd}(r)\sqrt{M}}{r^{3/2}} - \frac{2\,v_{odd}(r)}{r} \qquad \textbf{(34)}$$

Lastly, we attend to the tensor components

The [2, 3] component matches Ivan's

```
> hdot__Teven, hdot__Todd:=IdT23(grcomponent(hdot(dn, dn), [2, 3]))
  :

  hdot__Teven;
  hdot__Todd;
```

$$-\frac{2\sqrt{2}\,h_{Veven}(r)\sqrt{M}}{\sqrt{r}} + \frac{\sqrt{2}\sqrt{M}\left(\frac{d}{dr}\,h_{Teven}(r)\right)}{\sqrt{r}} + 2\,k_{Teven}(r) + 2\,v_{even}(r)$$

$$-\frac{2\sqrt{2}\,h_{Vodd}(r)\sqrt{M}}{\sqrt{r}} + \frac{\sqrt{2}\sqrt{M}\left(\frac{d}{dr}\,h_{Todd}(r)\right)}{\sqrt{r}} + 2\,k_{Todd}(r) + 2\,v_{odd}(r) \qquad \textbf{(35)}$$

```
> hdot__TOmega:=collect(IdT22(grcomponent(hdot(dn, dn), [2, 2]),
  hdot__Teven,hdot__Todd),sqrt(M));
  hdot__T33:=collect(IdT33(grcomponent(hdot(dn, dn), [3, 3]),
  hdot__Teven,hdot__Todd),sqrt(M));
```

$$hdot_{TOmega} := \left(\frac{\sqrt{2}\left(\frac{d}{dr}\,h_{trace}(r)\right)}{\sqrt{r}} - \frac{\sqrt{2}\,h_{Veven}(r)\,\lambda}{\sqrt{r}} - 2\sqrt{r}\,h_{rr}(r)\sqrt{2} - 2\sqrt{r}\,n_L(r)\sqrt{2}\right)\sqrt{M} + v_{even}(r)\,\lambda + 2\,v_{scalar}(r)\,r + 2\,k_{trace}(r)$$

$$hdot_{T33} := \left(\frac{\sqrt{2}\left(\frac{d}{dr}\,h_{trace}(r)\right)}{\sqrt{r}} - \frac{\sqrt{2}\,h_{Veven}(r)\,\lambda}{\sqrt{r}} - 2\sqrt{r}\,h_{rr}(r)\sqrt{2} - 2\sqrt{r}\,n_L(r)\sqrt{2}\right)\sqrt{M} + v_{even}(r)\,\lambda + 2\,v_{scalar}(r)\,r + 2\,k_{trace}(r) \qquad \textbf{(36)}$$

There to be an issue here where the check should be against the [3, 3] component but instead the code is the [2, 2] component against itself.
********** As can be seen above the [2, 2] and the [3, 3] components are indeeed the same as expected.

```
> hdot__TOmega__Check:=collect(expand(simplify(grcomponent(hdot(dn,
  dn), [2, 2])*sin(theta)^2+expand(epsilon*hdot__Todd*X__33+
  epsilon*hdot__Teven*Y__33))/sin(theta)^2/epsilon/Y(theta,phi)),
```

```
sqrt(M));
```

$$hdot_{TOmega_{Check}} := \left(\frac{\sqrt{2} \left(\frac{d}{dr} h_{trace}(r) \right)}{\sqrt{r}} - \frac{\sqrt{2}\, h_{Veven}(r)\, \lambda}{\sqrt{r}} - 2\sqrt{r}\, h_{rr}(r) \sqrt{2} - 2\sqrt{r}\, n_L(r) \sqrt{2} \right) \sqrt{M} + v_{even}(r)\, \lambda + 2\, v_{scalar}(r)\, r + 2\, k_{trace}(r) \quad (37)$$

```
> expand(hdot__TOmega-hdot__TOmega__Check);
```

$$0 \quad (38)$$

```
> expand(op(1,hdot__TOmega)/sqrt(M)/sqrt(2)/sqrt(r));
```

$$\frac{\frac{d}{dr} h_{trace}(r)}{r} - \frac{h_{Veven}(r)\, \lambda}{r} - 2\, h_{rr}(r) - 2\, n_L(r) \quad (39)$$

K__ij Evolution Equations

Start with the Ricci term - from Ivan's notes Ricci is trivial and N0 is 1 so we include the lapse N in case of changes later on

```
> grcalc(R(dn, dn));
  grmap(R(dn, dn),Linearize, 'x');
  grmap(R(dn, dn), FixSin, 'x');
  grmap(R(dn, dn), hExpand, 'x');
  grmap(R(dn, dn), kExpand, 'x');
  grmap(R(dn, dn), RemPhi, 'x');
  grmap(R(dn, dn), ApplyId, 'x');
  grmap(R(dn, dn), ApplyId, 'x');
  grmap(R(dn, dn), RemPhi, 'x');
  gralter(R(dn, dn), simplify);
  grmap(R(dn, dn), RemPhi, 'x');
  gralter(R(dn, dn), simplify);
```

$$CPU\ Time\ = 0.$$

```
Procedure name:LinearizeApplying routine Linearize to R(dn,dn)
Procedure name:FixSinApplying routine FixSin to R(dn,dn)
Procedure name:hExpandApplying routine hExpand to R(dn,dn)
Procedure name:kExpandApplying routine kExpand to R(dn,dn)
Procedure name:RemPhiApplying routine RemPhi to R(dn,dn)
Procedure name:ApplyIdApplying routine ApplyId to R(dn,dn)
Procedure name:ApplyIdApplying routine ApplyId to R(dn,dn)
Procedure name:RemPhiApplying routine RemPhi to R(dn,dn)
Component simplification of a GRTensorIII object:

Applying routine simplify to object R(dn,dn)
```

$$CPU\ Time\ = 0.297$$

```
Procedure name:RemPhiApplying routine RemPhi to R(dn,dn)
Component simplification of a GRTensorIII object:

Applying routine simplify to object R(dn,dn)
```

$$CPU\ Time\ = 0.203 \quad (40)$$

The scalar component is first

```
> NRc__rr:=Linearize(NX*grcomponent(R(dn, dn), [1, 1]));
```

$$NRc_{rr} := \frac{1}{r^4}\left(Y(\theta, \phi)\left(\left(\frac{d}{dr} h_{rr}(r)\right) r^3 + \left(\frac{d}{dr} h_{Veven}(r)\right)\lambda r^2 - \frac{h_{rr}(r)\,\lambda\, r^2}{2} - \left(\frac{d^2}{dr^2} h_{trace}(r)\right) r^2 + 2\left(\frac{d}{dr} h_{trace}(r)\right) r - 2\, h_{trace}(r)\right)\epsilon\right) \quad \textbf{(41)}$$

Next is the vector components

```
> NRc__12:=Linearize(NX*grcomponent(R(dn, dn), [1, 2])):
  NRc__13:=Linearize(NX*grcomponent(R(dn, dn), [1, 3])):
```

Check against Ivan's

```
> NRc__12Ivan:=(epsilon/(4*sin(theta)*r^3))*(-(2*(-(1/2)*r*
  (lambda+2)*(diff(h__Teven(r), r))+(diff(h__trace(r), r))*r+
  (lambda+2)*h__Teven(r)-h__rr(r)*r^2+2*h__Veven(r)*r-2*h__trace(r)
  ))*sin(theta)*(diff(Y(theta, phi), theta))-I*((diff(h__Todd(r),
  r))*r-2*h__Vodd(r)*r-2*h__Todd(r))*(lambda+2)*Y(theta, phi)*m):

  simplify(expand(NRc__12-NRc__12Ivan));
```

$$0 \quad \textbf{(42)}$$

```
> NRc__13Ivan:=(epsilon/(4*r^3))*(sin(theta)*(lambda+2)*((diff
  (h__Todd(r), r))*r-2*h__Vodd(r)*r-2*h__Todd(r))*(diff(Y(theta,
  phi), theta))-(2*I)*(-(1/2)*r*(lambda+2)*(diff(h__Teven(r), r))+
  (diff(h__trace(r), r))*r+(lambda+2)*h__Teven(r)-h__rr(r)*r^2+2*
  h__Veven(r)*r-2*h__trace(r))*Y(theta, phi)*m):

  simplify(NRc__13-NRc__13Ivan);
```

$$0 \quad \textbf{(43)}$$

Seperate the vector components into odd/even components

```
> NRc__V12even, NRc__V12odd:=IdV2(NRc__12):
  NRc__V12even;
  NRc__V12odd;
```

$$\frac{\left(\frac{d}{dr} h_{Teven}(r)\right)\lambda}{4\, r^2} + \frac{\frac{d}{dr} h_{Teven}(r)}{2\, r^2} - \frac{\frac{d}{dr} h_{trace}(r)}{2\, r^2} - \frac{h_{Teven}(r)\,\lambda}{2\, r^3} - \frac{h_{Teven}(r)}{r^3} + \frac{h_{rr}(r)}{2\, r} - \frac{h_{Veven}(r)}{r^2} + \frac{h_{trace}(r)}{r^3}$$

$$-\frac{\lambda\, h_{Vodd}(r)}{2\, r^2} + \frac{\lambda\left(\frac{d}{dr} h_{Todd}(r)\right)}{4\, r^2} - \frac{\lambda\, h_{Todd}(r)}{2\, r^3} - \frac{h_{Vodd}(r)}{r^2} + \frac{\frac{d}{dr} h_{Todd}(r)}{2\, r^2} - \frac{h_{Todd}(r)}{r^3} \quad \textbf{(44)}$$

```
> NRc__V13even, NRc__V13odd:=IdV3(NRc__13):
```

```
NRc__V13even;
NRc__V13odd;
```

$$\frac{\left(\frac{d}{dr}h_{Teven}(r)\right)\lambda}{4r^2}+\frac{\frac{d}{dr}h_{Teven}(r)}{2r^2}-\frac{\frac{d}{dr}h_{trace}(r)}{2r^2}-\frac{h_{Teven}(r)\lambda}{2r^3}-\frac{h_{Teven}(r)}{r^3}+\frac{h_{rr}(r)}{2r}-\frac{h_{Veven}(r)}{r^2}+\frac{h_{trace}(r)}{r^3}$$

$$-\frac{\lambda h_{Vodd}(r)}{2r^2}+\frac{\lambda\left(\frac{d}{dr}h_{Todd}(r)\right)}{4r^2}-\frac{\lambda h_{Todd}(r)}{2r^3}-\frac{h_{Vodd}(r)}{r^2}+\frac{\frac{d}{dr}h_{Todd}(r)}{2r^2}-\frac{h_{Todd}(r)}{r^3} \tag{45}$$

Finally the tensor components

Start oin the [2, 3] component

```
> NRc__23:=Linearize(NX*grcomponent(R(dn, dn), [2, 3])):
  NRc__T23even, NRc__T23odd:=IdT23(NRc__23):
  NRc__T23even;
  NRc__T23odd;
```

$$-\frac{h_{rr}(r)}{2}+\frac{d}{dr}h_{Veven}(r)-\frac{\frac{d^2}{dr^2}h_{Teven}(r)}{2}+\frac{\frac{d}{dr}h_{Teven}(r)}{r}-\frac{h_{Teven}(r)}{r^2}$$

$$\frac{d}{dr}h_{Vodd}(r)-\frac{\frac{d^2}{dr^2}h_{Todd}(r)}{2}+\frac{\frac{d}{dr}h_{Todd}(r)}{r}-\frac{h_{Todd}(r)}{r^2} \tag{46}$$

Check against Ivan

```
> NRc__T23Ivan:=-(1/2)*(diff(h__Teven(r), r, r))+diff(h__Veven(r),
  r)-(1/2)*h__rr(r)+(diff(h__Teven(r), r))/r-h__Teven(r)/r^2, diff
  (h__Vodd(r), r)-(1/2)*(diff(h__Todd(r), r, r))+(diff(h__Todd(r),
  r))/r-h__Todd(r)/r^2:

  simplify(NRc__T23Ivan-IdT23(NRc__23));
```

$$0 \tag{47}$$

Next the [2, 2] component which will be checked against the [3, 3] component

```
> NRc__T22:=Linearize(NX*grcomponent(R(dn, dn), [2, 2])):

  NRc__TOmega:=simplify(ApplyId(FixSin(IdT22(NRc__T22,
  NRc__T23even, NRc__T23odd))));
```

$$NRc_{TOmega} := \frac{1}{4r^2}\left(-2\left(\frac{d^2}{dr^2}h_{trace}(r)\right)r^2+2\left(\frac{d}{dr}h_{Veven}(r)\right)\lambda r^2+2\left(\frac{d}{dr}h_{rr}(r)\right)r^3+\left(\lambda^2+2\lambda\right)h_{Teven}(r)-r^2(\lambda-4)h_{rr}(r)+4\left(h_{Veven}(r)\,r-\frac{h_{trace}(r)}{2}\right)\lambda\right) \tag{48}$$

Now the check against the [3, 3] component

```
> NRc__T33:=Linearize(NX*grcomponent(R(dn, dn), [3, 3])):
  RcCheck:=simplify(ApplyId(expand(subs(NRc__T33-epsilon*
  NRc__T23even*Y__33-epsilon*NRc__T23odd*X__33-epsilon*NRc__TOmega*
  Omega__33))));
```

$$RcCheck := 0 \tag{49}$$

The Lapse term

We defie the lapse as foollows from Baumgarte and Shaprio (?????double check definintion) Seems like a standard defintion from literature regarding the lapse function

```
> grdef(`DN:=1+epsilon*n__L(r)*Y(theta, phi)`);
  grcalc(DN);
Created definition for DN
Calculated DN for lpschwNewFixed (0.000000 sec.)
```

$$CPU\ Time\ = 0. \tag{50}$$

```
> grdef(`DDN{i j}:=DN{;i ;j}`);
  grcalc(DDN(dn, dn));
  grmap(DDN(dn, dn), Linearize, 'x');
Created a definition for DN(cdn)
Created a definition for DN(cdn,cdn)
Created definition for DDN(dn,dn)
Calculated DN(cdn) for lpschwNewFixed (0.000000 sec.)
Calculated DN(cdn,cdn) for lpschwNewFixed (0.000000 sec.)
Calculated DDN(dn,dn) for lpschwNewFixed (0.000000 sec.)
```

$$CPU\ Time\ = 0.$$

```
Procedure name:LinearizeApplying routine Linearize to DDN(dn,dn)
```

With the second covariatn derivatives of the Lapse we can now seperate out all the components

```
> DDN__rr:=grcomponent(DDN(dn, dn), [1, 1]);
```

$$DDN_{rr} := \left(\frac{d^2}{dr^2}\, n_L(r)\right) Y(\theta, \phi)\, \epsilon \tag{51}$$

```
> DDN__12:=grcomponent(DDN(dn, dn), [1, 2]):
  DDN__13:=grcomponent(DDN(dn, dn), [1, 3]):

  DDN__Veven, DDN__Vodd:=IdV2(DDN__12);
```

$$DDN_{Veven}, DDN_{Vodd} := \frac{d}{dr}\, n_L(r) - \frac{n_L(r)}{r}, 0 \tag{52}$$

```
> DDN__22:=ApplyId(Linearize(grcomponent(DDN(dn, dn), [2,2]))):
  DDN__23:=RemPhi(Linearize(grcomponent(DDN(dn, dn), [2,3]))):
  DDN__33:=RemPhi(Linearize(grcomponent(DDN(dn, dn), [3,3]))):

  DDN__Teven,DDN__Todd:=IdT23(DDN__23);

  DDN__TOmega:=IdT22(DDN__22,DDN__Teven,DDN__Todd);
```

$$DDN_{Teven}, DDN_{Todd} := n_L(r), 0$$

$$DDN_{TOmega} := \frac{n_L(r)\,\lambda}{2} + \left(\frac{d}{dr}\, n_L(r)\right) r \qquad \mathbf{(53)}$$

The extrinsic curvature term is next for the evolution equation

```
> grdef(`NKK{i j}:=NX*(KTr*KK{i j}-2*g{^k ^l}*KK{i k}*KK{j l})`);
  grcalc(NKK(dn, dn));

  grmap(NKK(dn, dn), Linearize, 'x');
  grmap(NKK(dn, dn), kExpand, 'x');
  grmap(NKK(dn, dn), hExpand, 'x');
  gralter(NKK(dn, dn), simplify);
  grmap(NKK(dn, dn), FixSin, 'x');
  grmap(NKK(dn, dn), Linearize, 'x');
```

```
Created definition for NKK(dn,dn)
Calculated NKK(dn,dn) for lpschwNewFixed (0.015000 sec.)
```

$$CPU\ Time\ = 0.015$$

```
Procedure name:LinearizeApplying routine Linearize to NKK(dn,dn)
Procedure name:kExpandApplying routine kExpand to NKK(dn,dn)
Procedure name:hExpandApplying routine hExpand to NKK(dn,dn)
Component simplification of a GRTensorIII object:

Applying routine simplify to object NKK(dn,dn)
```

$$CPU\ Time\ = 0.203$$

```
Procedure name:FixSinApplying routine FixSin to NKK(dn,dn)
Procedure name:LinearizeApplying routine Linearize to NKK(dn,dn)
```

```
> NKK__rr:=expand(grcomponent(NKK(dn, dn), [1, 1]));
```

$$NKK_{rr} := \frac{\sqrt{2}\,\epsilon\,\sqrt{M}\,k_{trace}(r)\,Y(\theta,\phi)}{r^{7/2}} - \frac{3\sqrt{2}\,k_{rr}(r)\,Y(\theta,\phi)\,\epsilon\,\sqrt{M}}{r^{3/2}} + \frac{M\,h_{rr}(r)\,Y(\theta,\phi)\,\epsilon}{2\,r^3} + \frac{2\,M\,\epsilon\,h_{trace}(r)\,Y(\theta,\phi)}{r^5} - \frac{5\,M}{2\,r^3} - \frac{5\,M\,n_L(r)\,Y(\theta,\phi)\,\epsilon}{2\,r^3} \qquad \mathbf{(54)}$$

```
> NKK__even, NKK__odd:=IdV2(grcomponent(NKK(dn, dn), [1, 2]));
  IdV3(grcomponent(NKK(dn, dn), [1, 3])):
```

$$NKK_{even}, NKK_{odd} := -\frac{\sqrt{2}\,k_{Veven}(r)\,\sqrt{M}}{2\,r^{3/2}} - \frac{2\,M\,h_{Veven}(r)}{r^3}, -\frac{\sqrt{2}\,k_{Vodd}(r)\,\sqrt{M}}{2\,r^{3/2}} - \frac{2\,M\,h_{Vodd}(r)}{r^3} \qquad \mathbf{(55)}$$

```
> NKK__23:=FixSin(expand(grcomponent(NKK(dn, dn), [2, 3]))):
  NKK__Teven, NKK__Todd:=IdT23(NKK__23);
```

$$NKK_{Teven}, NKK_{Todd} := \frac{5\sqrt{2}\,k_{Teven}(r)\,\sqrt{M}}{2\,r^{3/2}} + \frac{4\,M\,h_{Teven}(r)}{r^3}, \frac{5\sqrt{2}\,\sqrt{M}\,k_{Todd}(r)}{2\,r^{3/2}} + \frac{4\,M\,h_{Todd}(r)}{r^3} \qquad \mathbf{(56)}$$

```
> NKK__TOmega:=IdT22(grcomponent(NKK(dn, dn), [2, 2])+M/r,
  NKK__Teven,NKK__Todd);
```

$$NKK_{TOmega} := -\sqrt{r}\,\sqrt{2}\,k_{rr}(r)\,\sqrt{M} + \frac{\sqrt{2}\,\sqrt{M}\,k_{trace}(r)}{2\,r^{3/2}} - \frac{M\,n_L(r)}{r} + \frac{M\,h_{rr}(r)}{r} \quad \textbf{(57)}$$

Lastly we need to construct the shift terms for the extrinsic curvature equaiton before we can start into the time derivates. We already have the shift vector defined as VX[dn]

```
> grdef(`LVK{i j}:=VX{^k}*KK{i j ,k}+KK{i k}*VX{^k ,j}+KK{k j}*VX
  {^k ,i}`);
  grcalc(LVK(dn, dn));

  grmap(LVK(dn, dn), Linearize, 'x');
  grmap(LVK(dn, dn), hExpand, 'x');
  grmap(LVK(dn, dn), kExpand, 'x');
  grmap(LVK(dn, dn), ApplyId, 'x');
  grmap(LVK(dn, dn), RemPhi, 'x');
  gralter(LVK(dn, dn), simplify);
  grmap(LVK(dn, dn), FixSin, 'x');
Created a definition for KK(dn,dn,pdn)
Created a definition for VX(up,pdn)
Created definition for LVK(dn,dn)
Calculated KK(dn,dn,pdn) for lpschwNewFixed (0.000000 sec.)
Calculated VX(up,pdn) for lpschwNewFixed (0.000000 sec.)
Calculated LVK(dn,dn) for lpschwNewFixed (0.016000 sec.)
```

$$CPU\ Time\ = 0.016$$

```
Procedure name:LinearizeApplying routine Linearize to LVK(dn,dn)
Procedure name:hExpandApplying routine hExpand to LVK(dn,dn)
Procedure name:kExpandApplying routine kExpand to LVK(dn,dn)
Procedure name:ApplyIdApplying routine ApplyId to LVK(dn,dn)
Procedure name:RemPhiApplying routine RemPhi to LVK(dn,dn)
Component simplification of a GRTensorIII object:

Applying routine simplify to object LVK(dn,dn)
```

$$CPU\ Time\ = 0.282$$

```
Procedure name:FixSinApplying routine FixSin to LVK(dn,dn)
> LVK__rr:=collect(grcomponent(LVK(dn, dn), [1, 1]), Y(theta, phi))
  ;
```

$$LVK_{rr} := \left(-\frac{2\,M\left(\frac{d}{dr}\,h_{rr}(r)\right)\epsilon}{r^2} + \frac{\sqrt{2}\left(\frac{d}{dr}\,k_{rr}(r)\right)\epsilon\sqrt{M}}{\sqrt{r}} + \frac{\sqrt{2}\left(\frac{d}{dr}\,v_{scalar}(r)\right)\epsilon\sqrt{M}}{r^{3/2}} - \frac{3\sqrt{2}\,v_{scalar}(r)\,\epsilon\sqrt{M}}{4\,r^{5/2}} - \frac{\sqrt{2}\,k_{rr}(r)\,\epsilon\sqrt{M}}{r^{3/2}} + \frac{5\,M\,h_{rr}(r)\,\epsilon}{2\,r^3} \right) Y(\theta,\phi) - \frac{5\,M}{2\,r^3} \quad \textbf{(58)}$$

```
> LVK__Veven, LVK__Vodd:=IdV2(grcomponent(LVK(dn, dn), [1, 2])):
  LVK__Veven3, LVK__Vodd3:=IdV3(grcomponent(LVK(dn, dn), [1, 3])):
```

```
simplify(LVK__Veven-LVK__Veven3);
simplify(LVK__Vodd-LVK__Vodd3);
```

$$0$$

$$0 \tag{59}$$

```
> LVK__Teven, LVK__Todd:=IdT23(grcomponent(LVK(dn, dn), [2, 3]));
```

$$LVK_{Teven}, LVK_{Todd} := \frac{\sqrt{2}\left(\frac{d}{dr} k_{Teven}(r)\right)\sqrt{M}}{\sqrt{r}} - \frac{2\sqrt{2}\sqrt{M}\, v_{even}(r)}{r^{3/2}} + \frac{4 M h_{Veven}(r)}{r^2}, \frac{\sqrt{2}\sqrt{M}\left(\frac{d}{dr} k_{Todd}(r)\right)}{\sqrt{r}} - \frac{2\sqrt{2}\sqrt{M}\, v_{odd}(r)}{r^{3/2}} + \frac{4 M h_{Vodd}(r)}{r^2} \tag{60}$$

```
> LVK__TOmega0:=coeff(grcomponent(LVK(dn, dn), [2, 2]), epsilon, 0)
;

LVK__TOmega:=(expand(IdT22(epsilon*coeff(grcomponent(LVK(dn, dn),
[2, 2]),epsilon,1),LVK__Teven,LVK__Todd)));
```

$$LVK_{TOmega0} := -\frac{M}{r}$$

$$LVK_{TOmega} := \frac{M h_{rr}(r)}{r} - \frac{\sqrt{2}\sqrt{M}\, v_{even}(r)\,\lambda}{r^{3/2}} - \frac{\sqrt{2}\sqrt{M}\, v_{scalar}(r)}{2\sqrt{r}} + \frac{\sqrt{2}\sqrt{M}\left(\frac{d}{dr} k_{trace}(r)\right)}{\sqrt{r}} + \frac{2 M h_{Veven}(r)\,\lambda}{r^2} \tag{61}$$

```
> LVK__TOmegaIvan:=M*h__rr(r)/r-sqrt(2)*v__even(r)*lambda*sqrt(M)
/r^(3/2)-sqrt(2)*v__scalar(r)*sqrt(M)/(2*sqrt(r))+sqrt(2)*(diff
(k__trace(r), r))*sqrt(M)/sqrt(r)+2*M*h__Veven(r)*lambda/r^2:

simplify(LVK__TOmega-LVK__TOmegaIvan);
```

$$0 \tag{62}$$

The time rate of changes are the final thing to do with this evolution equation

```
> Kdot__rr:=collect(collect(collect(expand((DDN__rr-NRc__rr-
NKK__rr+LVK__rr)/epsilon/Y(theta,phi)), h__trace(r)), h__rr(r)),
M):

Kdot__rrX:=collect(collect(collect(expand(subs(h__trace(r)=r*
H__trace(r), k__rr(r)=K__rr(r)/r^2, r*Kdot__rr)), H__trace(r)),
H__rr(r)), M);
```

$$Kdot_{rrX} := \left(\frac{2 h_{rr}(r)}{r^2} - \frac{2 H_{trace}(r)}{r^3} + \frac{5 n_L(r)}{2 r^2} - \frac{2\left(\frac{d}{dr} h_{rr}(r)\right)}{r}\right) M + \left(-\frac{\sqrt{2}\, k_{trace}(r)}{r^{5/2}} + \frac{\sqrt{2}\left(\frac{d}{dr} K_{rr}(r)\right)}{r^{3/2}} + \frac{\sqrt{2}\left(\frac{d}{dr} v_{scalar}(r)\right)}{\sqrt{r}} - \frac{3\sqrt{2}\, v_{scalar}(r)}{4 r^{3/2}}\right)\sqrt{M} + \frac{h_{rr}(r)\,\lambda}{2 r} \tag{63}$$

$$+ r\left(\frac{d^2}{dr^2} n_L(r)\right) - \left(\frac{d}{dr} h_{rr}(r)\right) - \frac{\left(\frac{d}{dr} h_{Veven}(r)\right)\lambda}{r} + \frac{d^2}{dr^2} H_{trace}(r)$$

```
> Kdot__Veven:=collect(collect(collect(expand(subs(DDN__Veven-
  NRc__V12even-NKK__even+LVK__Veven)), h__Veven(r)), h__Teven(r)),
  M);
```

$$Kdot_{Veven} := \left(-\frac{3\, h_{Veven}(r)}{r^3} + \frac{2\left(\frac{d}{dr} h_{Veven}(r)\right)}{r^2} - \frac{h_{rr}(r)}{r^2}\right) M + \left(\frac{\sqrt{2}\left(\frac{d}{dr} k_{Veven}(r)\right)}{\sqrt{r}} + \frac{\sqrt{2}\, v_{scalar}(r)}{2\, r^{3/2}} - \frac{\sqrt{2}\left(\frac{d}{dr} v_{even}(r)\right)}{r^{3/2}} + \frac{2\sqrt{2}\, v_{even}(r)}{r^{5/2}}\right)\sqrt{M} + \left(\frac{\lambda}{2\, r^3} + \frac{1}{r^3}\right) h_{Teven}(r) + \frac{h_{Veven}(r)}{r^2} + \frac{d}{dr} n_L(r) - \frac{n_L(r)}{r} - \frac{\left(\frac{d}{dr} h_{Teven}(r)\right)\lambda}{4\, r^2} - \frac{\frac{d}{dr} h_{Teven}(r)}{2\, r^2} + \frac{\frac{d}{dr} h_{trace}(r)}{2\, r^2} - \frac{h_{rr}(r)}{2\, r} - \frac{h_{trace}(r)}{r^3} \quad (64)$$

```
> Kdot__Vodd:=collect(collect(collect(expand(DDN__Vodd-NRc__V12odd-
  NKK__odd+LVK__Vodd), h__Vodd(r)), h__Todd(r)), M):

  Kdot__VoddIvan:=(-3*h__Vodd(r)/r^3+2*(diff(h__Vodd(r), r))/r^2)*
  M+((diff(k__Vodd(r), r))*sqrt(2)/sqrt(r)-sqrt(2)*(diff(v__odd(r),
  r))/r^(3/2)+2*sqrt(2)*v__odd(r)/r^(5/2))*sqrt(M)+(lambda/(2*r^3)
  +1/r^3)*h__Todd(r)+(lambda/(2*r^2)+1/r^2)*h__Vodd(r)-lambda*(diff
  (h__Todd(r), r))/(4*r^2)-(diff(h__Todd(r), r))/(2*r^2):

  simplify(Kdot__Vodd-Kdot__VoddIvan);
```

$$0 \quad (65)$$

```
> Kdot__Veven:=collect(collect(collect(collect(expand(subs(h__Veven
  (r)=r^(3/2)*H__Veven(r), v__even(r)=r^2*V__even(r), h__Teven(r)=
  r^2*H__Teven(r), h__trace(r)=r^2*H__trace(r), n__L(r)=r*N__L(r),
  DDN__Veven-NRc__V12even-NKK__even+LVK__Veven)), H__Veven(r)),
  H__Teven(r)), diff(H__Teven(r), r)), M):

  Kdot__VevenIvan:=(2*(diff(H__Veven(r), r))/sqrt(r)-h__rr(r)/r^2)*
  M+(sqrt(2)*(diff(k__Veven(r), r))/sqrt(r)+sqrt(2)*v__scalar(r)/
  (2*r^(3/2))-sqrt(r)*sqrt(2)*(diff(V__even(r), r)))*sqrt(M)+(-
  (1/4)*lambda-1/2)*(diff(H__Teven(r), r))+r*(diff(N__L(r), r))+
  (1/2)*(diff(H__trace(r), r))-h__rr(r)/(2*r)+H__Veven(r)/sqrt(r):

  simplify(Kdot__Veven-Kdot__VevenIvan);
```

$$0 \quad (66)$$

```
> simplify(Kdot__Veven-Kdot__Vodd);
```

$$-\frac{1}{r^{7/2}}\left(\left(\left(-\frac{\lambda}{4} - \frac{1}{2}\right)\left(\frac{d}{dr} h_{Todd}(r)\right) + 2\, M\left(\frac{d}{dr} h_{Vodd}(r)\right) + \left(\frac{\lambda}{2} + 1\right) h_{Vodd}(r) \quad (67)$$

$$+ h_{rr}(r)\, M\Big) r^{3/2} + \left(\left(\frac{\lambda}{4} + \frac{1}{2}\right)\left(\frac{d}{dr} H_{Teven}(r)\right) - \frac{\left(\frac{d}{dr} H_{trace}(r)\right)}{2}\right) r^{7/2}$$

$$+ \frac{h_{rr}(r)\, r^{5/2}}{2} - r^{9/2}\left(\frac{d}{dr} N_L(r)\right) + 2\left(\frac{r^3\left(\frac{d}{dr} V_{even}(r)\right)}{2} + \frac{\left(\frac{d}{dr} k_{Vodd}(r)\right) r^2}{2}\right.$$

$$\left. - \frac{\left(\frac{d}{dr} k_{Veven}(r)\right) r^2}{2} - \frac{v_{scalar}(r)\, r}{4} - \frac{\left(\frac{d}{dr} v_{odd}(r)\right) r}{2} + v_{odd}(r)\right) r\sqrt{2}\,\sqrt{M} + \Bigg($$

$$-3\, M\, h_{Vodd}(r) + \frac{(\lambda+2)\, h_{Todd}(r)}{2}\Bigg)\sqrt{r} - 2\, r^3\left(M\left(\frac{d}{dr} H_{Veven}(r)\right) + \frac{H_{Veven}(r)}{2}\right)\Bigg)$$

```
> expand(simplify((coeff(Kdot__Veven,sqrt(M)))/sqrt(2/r))) assuming
  r>0;
```

$$-r\left(\frac{d}{dr} V_{even}(r)\right) + \frac{d}{dr} k_{Veven}(r) + \frac{v_{scalar}(r)}{2\, r} \tag{68}$$

Mismatch here.......???

```
> Kdot__Vodd:=collect(collect(collect(collect(expand(subs(h__Vodd
  (r)=r^(3/2)*H__Vodd(r),v__odd(r)=r^2*V__odd(r),h__Todd(r)=r^2*
  H__Todd(r),DDN__Vodd-NRc__V12odd-NKK__odd+LVK__Vodd)),H__Vodd(r)
  ),H__Todd(r)),diff(H__Todd(r),r)),M);
```

$$Kdot_{Vodd} := \frac{2\, M\left(\frac{d}{dr} H_{Vodd}(r)\right)}{\sqrt{r}} + \left(\frac{\sqrt{2}\left(\frac{d}{dr} k_{Vodd}(r)\right)}{\sqrt{r}} - \sqrt{2}\,\sqrt{r}\left(\frac{d}{dr} V_{odd}(r)\right)\right)\sqrt{M} + \left(-\frac{\lambda}{4} - \frac{1}{2}\right)\left(\frac{d}{dr} H_{Todd}(r)\right) + \left(\frac{\lambda}{2\sqrt{r}} + \frac{1}{\sqrt{r}}\right) H_{Vodd}(r) \tag{69}$$

```
> factor(op(1,Kdot__Vodd)+op(4,Kdot__Vodd));
```

$$\frac{4\, M\left(\frac{d}{dr} H_{Vodd}(r)\right) + H_{Vodd}(r)\,\lambda + 2\, H_{Vodd}(r)}{2\sqrt{r}} \tag{70}$$

```
> simplify(op(3,Kdot__Vodd)/sqrt(2*M/r)) assuming M>0,r>0;
```

$$-\frac{(\lambda+2)\left(\frac{d}{dr} H_{Todd}(r)\right)\sqrt{2}\,\sqrt{r}}{8\sqrt{M}} \tag{71}$$

```
> Kdot__Teven:=collect(collect(expand(DDN__Teven-NRc__T23even-
  NKK__Teven+LVK__Teven),h__Teven(r)),M);
```

$$Kdot_{Teven} := \left(-\frac{4\, h_{Teven}(r)}{r^3} + \frac{4\, h_{Veven}(r)}{r^2}\right) M + \left(-\frac{5\sqrt{2}\, k_{Teven}(r)}{2\, r^{3/2}} + \frac{\sqrt{2}\left(\frac{d}{dr} k_{Teven}(r)\right)}{\sqrt{r}} - \frac{2\sqrt{2}\, v_{even}(r)}{r^{3/2}}\right)\sqrt{M} + \frac{h_{Teven}(r)}{r^2} + n_L(r) + \frac{h_{rr}(r)}{2} \tag{72}$$

$$-\left(\frac{d}{dr} h_{Veven}(r)\right) + \frac{\left(\frac{d^2}{dr^2} h_{Teven}(r)\right)}{2} - \frac{\frac{d}{dr} h_{Teven}(r)}{r}$$

```
> Kdot__TevenX:=collect(collect(expand(subs(h__Teven(r)=r*H__Teven(r),k__Teven(r)=K__Teven(r)*r^(5/2),Kdot__Teven)),h__Teven(r)),M);
```

$$Kdot_{TevenX} := \left(-\frac{4 H_{Teven}(r)}{r^2} + \frac{4 h_{Veven}(r)}{r^2}\right) M + \left(r^2 \sqrt{2} \left(\frac{d}{dr} K_{Teven}(r)\right) - \frac{2\sqrt{2}\, v_{even}(r)}{r^{3/2}}\right)\sqrt{M} + n_L(r) + \frac{h_{rr}(r)}{2} - \left(\frac{d}{dr} h_{Veven}(r)\right) + \frac{r\left(\frac{d^2}{dr^2} H_{Teven}(r)\right)}{2} \tag{73}$$

```
> factor(factor(op(1,Kdot__TevenX))/M*r^2);
simplify(expand(factor(op(2,Kdot__TevenX))/sqrt(2*M/r^3)))
assuming r>0,M>0;
```

$$-4 H_{Teven}(r) + 4 h_{Veven}(r)$$

$$\left(\frac{d}{dr} K_{Teven}(r)\right) r^{7/2} - 2 v_{even}(r) \tag{74}$$

```
> Kdot__Todd:=collect(expand((DDN__Todd-NRc__T23odd-NKK__Todd+LVK__Todd)),M);
```

$$Kdot_{Todd} := \left(-\frac{4 h_{Todd}(r)}{r^3} + \frac{4 h_{Vodd}(r)}{r^2}\right) M + \left(-\frac{5\sqrt{2}\, k_{Todd}(r)}{2 r^{3/2}} + \frac{\sqrt{2}\left(\frac{d}{dr} k_{Todd}(r)\right)}{\sqrt{r}} - \frac{2\sqrt{2}\, v_{odd}(r)}{r^{3/2}}\right)\sqrt{M} - \left(\frac{d}{dr} h_{Vodd}(r)\right) + \frac{\left(\frac{d^2}{dr^2} h_{Todd}(r)\right)}{2} - \frac{\frac{d}{dr} h_{Todd}(r)}{r} + \frac{h_{Todd}(r)}{r^2} \tag{75}$$

```
> Kdot__ToddX:=collect(collect(expand(subs(h__Todd(r)=r*H__Todd(r),
k__Todd(r)=K__Todd(r)*r^(5/2),Kdot__Todd)),h__Todd(r)),M);
```

$$Kdot_{ToddX} := \left(-\frac{4 H_{Todd}(r)}{r^2} + \frac{4 h_{Vodd}(r)}{r^2}\right) M + \left(r^2 \sqrt{2}\left(\frac{d}{dr} K_{Todd}(r)\right) - \frac{2\sqrt{2}\, v_{odd}(r)}{r^{3/2}}\right)\sqrt{M} - \left(\frac{d}{dr} h_{Vodd}(r)\right) + \frac{r\left(\frac{d^2}{dr^2} H_{Todd}(r)\right)}{2} \tag{76}$$

```
> Kdot__TOmega:=collect(collect(collect(collect(expand(
(DDN__TOmega-NRc__TOmega-NKK__TOmega+LVK__TOmega)),h__Teven(r)),
h__rr(r)),h__trace(r)),M);
```

$$Kdot_{TOmega} := \left(\frac{n_L(r)}{r} + \frac{2 h_{Veven}(r)\,\lambda}{r^2}\right) M + \left(\sqrt{r}\,\sqrt{2}\, k_{rr}(r) - \frac{\sqrt{2}\, k_{trace}(r)}{2 r^{3/2}}\right. \tag{77}$$

$$- \frac{\sqrt{2}\, v_{even}(r)\, \lambda}{r^{3/2}} - \frac{\sqrt{2}\, v_{scalar}(r)}{2\sqrt{r}} + \frac{\sqrt{2}\left(\frac{d}{dr} k_{trace}(r)\right)}{\sqrt{r}}\Bigg)\sqrt{M} + \left(\frac{\lambda}{4} - 1\right) h_{rr}(r)$$

$$+ \left(-\frac{\lambda^2}{4 r^2} - \frac{\lambda}{2 r^2}\right) h_{Teven}(r) + \frac{n_L(r)\, \lambda}{2} + \left(\frac{d}{dr} n_L(r)\right) r - \frac{r\left(\frac{d}{dr} h_{rr}(r)\right)}{2}$$

$$- \frac{\left(\frac{d}{dr} h_{Veven}(r)\right)\lambda}{2} + \frac{\left(\frac{d^2}{dr^2} h_{trace}(r)\right)}{2} - \frac{h_{Veven}(r)\, \lambda}{r} + \frac{\lambda\, h_{trace}(r)}{2 r^2}$$

```
> Kdot__TOmegaX:=collect(collect(collect(collect(expand(subs(h__rr
  (r)=H__rr(r)/r^2,k__trace(r)=sqrt(r)*K__trace(r),h__Veven(r)=
  H__Veven(r)/r^2,2*Kdot__TOmega)),h__Teven(r)),h__rr(r)),h__trace
  (r)),M);
```

$$Kdot_{TOmegaX} := \left(\frac{2\, n_L(r)}{r} + \frac{4\, H_{Veven}(r)\, \lambda}{r^4}\right) M + \Bigg(2\sqrt{r}\,\sqrt{2}\, k_{rr}(r) - \frac{2\sqrt{2}\, v_{even}(r)\, \lambda}{r^{3/2}}$$

$$- \frac{\sqrt{2}\, v_{scalar}(r)}{\sqrt{r}} + 2\sqrt{2}\left(\frac{d}{dr} K_{trace}(r)\right)\Bigg)\sqrt{M} + \left(-\frac{\lambda^2}{2 r^2} - \frac{\lambda}{r^2}\right) h_{Teven}(r)$$

$$+ \frac{H_{rr}(r)\, \lambda}{2 r^2} + n_L(r)\, \lambda + 2\left(\frac{d}{dr} n_L(r)\right) r - \frac{\frac{d}{dr} H_{rr}(r)}{r} - \frac{\lambda\left(\frac{d}{dr} H_{Veven}(r)\right)}{r^2} + \frac{d^2}{dr^2}$$

$$h_{trace}(r) + \frac{\lambda\, h_{trace}(r)}{r^2} \tag{78}$$

```
> expand(coeff(Kdot__TOmegaX,sqrt(M))/sqrt(2/r)) assuming r>0;
```

$$2\, r\, k_{rr}(r) - \frac{2\, v_{even}(r)\, \lambda}{r} - v_{scalar}(r) + 2\sqrt{r}\left(\frac{d}{dr} K_{trace}(r)\right) \tag{79}$$

Gauge Tranformations

Spatial

****************I really need to make more notes on all this stuff***************************

```
> grdef(`xi{a}:=[epsilon*xi__r(r)*Y(theta, phi), epsilon*xi__even
  (r)*Y__2+epsilon*xi__odd(r)*X__2, epsilon*xi__even(r)*Y__3+
  epsilon*xi__odd(r)*X__3]`);
  grcalc(xi(dn));

  grmap(xi(dn), Linearize, 'x');
  grmap(xi(dn), hExpand, 'x');
  grmap(xi(dn), RemPhi, 'x');
  grmap(xi(dn), ApplyId, 'x');
  grmap(xi(dn), FixSin, 'x');
  grmap(xi(dn), RemPhi, 'x');
  grmap(xi(dn), RemPhi, 'x');
```

```
grdisplay(_);
```

```
Components assigned for metric: lpschwNewFixed
Created definition for xi(dn)
```

$$CPU\ Time\ = 0.$$

```
Procedure name:LinearizeApplying routine Linearize to xi(dn)
Procedure name:hExpandApplying routine hExpand to xi(dn)
Procedure name:RemPhiApplying routine RemPhi to xi(dn)
Procedure name:ApplyIdApplying routine ApplyId to xi(dn)
Procedure name:FixSinApplying routine FixSin to xi(dn)
Procedure name:RemPhiApplying routine RemPhi to xi(dn)
Procedure name:RemPhiApplying routine RemPhi to xi(dn)
```

For the lpschwNewFixed spacetime:

xi(dn)

$$\xi(dn)$$

$$\xi_r = \epsilon\, \xi_r(r)\, Y(\theta, \phi)$$

$$\xi_\theta = \epsilon\, \xi_{even}(r) \left(\frac{\partial}{\partial\theta} Y(\theta, \phi)\right) - \frac{I\, \epsilon\, \xi_{odd}(r)\, m\, Y(\theta, \phi)}{\sin(\theta)}$$

$$\xi_\phi = I\, \epsilon\, \xi_{even}(r)\, m\, Y(\theta, \phi) + \epsilon\, \xi_{odd}(r) \sin(\theta) \left(\frac{\partial}{\partial\theta} Y(\theta, \phi)\right) \tag{80}$$

```
> grdef(`Lg__xi{a b}:=xi{a ;b}+xi{b ;a}`);
  grcalc(Lg__xi(dn, dn));


  grmap(Lg__xi(dn, dn), Linearize, 'x');
  grmap(Lg__xi(dn, dn), hExpand, 'x');
  grmap(Lg__xi(dn, dn), RemPhi, 'x');
  grmap(Lg__xi(dn, dn), ApplyId, 'x');
  grmap(Lg__xi(dn, dn), FixSin, 'x');
  grmap(Lg__xi(dn, dn), RemPhi, 'x');
  grmap(Lg__xi(dn, dn), RemPhi, 'x');

  grdisplay(_);
```

```
Created a definition for xi(dn,cdn)
Created definition for Lg__xi(dn,dn)
Calculated xi(dn,cdn) for lpschwNewFixed (0.000000 sec.)
Calculated Lg__xi(dn,dn) for lpschwNewFixed (0.000000 sec.)
```

$$CPU\ Time\ = 0.$$

```
Procedure name:LinearizeApplying routine Linearize to Lg__xi(dn,
dn)
Procedure name:hExpandApplying routine hExpand to Lg__xi(dn,dn)
Procedure name:RemPhiApplying routine RemPhi to Lg__xi(dn,dn)
Procedure name:ApplyIdApplying routine ApplyId to Lg__xi(dn,dn)
Procedure name:FixSinApplying routine FixSin to Lg__xi(dn,dn)
Procedure name:RemPhiApplying routine RemPhi to Lg__xi(dn,dn)
Procedure name:RemPhiApplying routine RemPhi to Lg__xi(dn,dn)
```

For the lpschwNewFixed spacetime:

$$Lg_{xi(dn,dn}$$

$$Lg_{\xi}(dn, dn)$$

$$Lg_{\xi_r\, r} = 2 \left(\frac{d}{dr}\, \xi_r(r)\right) Y(\theta, \phi)\, \epsilon$$

$$Lg_{\xi_\theta\, r} = -\frac{I \epsilon \left(\frac{d}{dr}\, \xi_{odd}(r)\right) Y(\theta, \phi)\, m}{\sin(\theta)} + \frac{2\, I \epsilon\, \xi_{odd}(r)\, Y(\theta, \phi)\, m}{r \sin(\theta)} + \epsilon \left(\frac{d}{dr}\, \xi_{even}(r)\right) \left(\frac{\partial}{\partial\theta}\, Y(\theta, \phi)\right) + \epsilon\, \xi_r(r) \left(\frac{\partial}{\partial\theta}\, Y(\theta, \phi)\right) - \frac{2\, \epsilon\, \xi_{even}(r) \left(\frac{\partial}{\partial\theta}\, Y(\theta, \phi)\right)}{r}$$

$$Lg_{\xi_\phi\, r} = \epsilon \left(\frac{d}{dr}\, \xi_{odd}(r)\right) \left(\frac{\partial}{\partial\theta}\, Y(\theta, \phi)\right) \sin(\theta) - \frac{2\, \epsilon\, \xi_{odd}(r) \sin(\theta) \left(\frac{\partial}{\partial\theta}\, Y(\theta, \phi)\right)}{r} + I \epsilon\, \xi_r(r)\, m\, Y(\theta, \phi) + I \epsilon \left(\frac{d}{dr}\, \xi_{even}(r)\right) Y(\theta, \phi)\, m - \frac{2\, I \epsilon\, \xi_{even}(r)\, m\, Y(\theta, \phi)}{r}$$

$$Lg_{\xi_r\, \theta} = -\frac{I \epsilon \left(\frac{d}{dr}\, \xi_{odd}(r)\right) Y(\theta, \phi)\, m}{\sin(\theta)} + \frac{2\, I \epsilon\, \xi_{odd}(r)\, Y(\theta, \phi)\, m}{r \sin(\theta)} + \epsilon \left(\frac{d}{dr}\, \xi_{even}(r)\right) \left(\frac{\partial}{\partial\theta}\, Y(\theta, \phi)\right) + \epsilon\, \xi_r(r) \left(\frac{\partial}{\partial\theta}\, Y(\theta, \phi)\right) - \frac{2\, \epsilon\, \xi_{even}(r) \left(\frac{\partial}{\partial\theta}\, Y(\theta, \phi)\right)}{r}$$

$$Lg_{\xi_\theta\, \theta} = -\frac{2\, I \epsilon\, \xi_{odd}(r) \left(\frac{\partial}{\partial\theta}\, Y(\theta, \phi)\right) m}{\sin(\theta)} + \frac{2\, I \epsilon\, \xi_{odd}(r)\, Y(\theta, \phi) \cos(\theta)\, m}{\sin(\theta)^2} + 2\, \epsilon\, \xi_r(r)\, Y(\theta, \phi)\, r + \frac{2\, \epsilon\, \xi_{even}(r)\, m^2\, Y(\theta, \phi)}{\sin(\theta)^2} - 2\, \epsilon\, \xi_{even}(r) \cot(\theta) \left(\frac{\partial}{\partial\theta}\, Y(\theta, \phi)\right) + 2\, \epsilon\, \xi_{even}(r)\, \lambda\, Y(\theta, \phi)$$

$$Lg_{\xi_\phi\, \theta} = 2\, I \epsilon\, \xi_{even}(r) \left(\frac{\partial}{\partial\theta}\, Y(\theta, \phi)\right) m - \frac{2\, I \epsilon\, \xi_{even}(r)\, Y(\theta, \phi) \cos(\theta)\, m}{\sin(\theta)} + \frac{2\, \epsilon\, \xi_{odd}(r)\, m^2\, Y(\theta, \phi)}{\sin(\theta)} - \epsilon\, \xi_{odd}(r) \left(\frac{\partial}{\partial\theta}\, Y(\theta, \phi)\right) \cos(\theta) - \epsilon \sin(\theta)\, \xi_{odd}(r) \cot(\theta) \left(\frac{\partial}{\partial\theta}\, Y(\theta, \phi)\right) + \epsilon \sin(\theta)\, \xi_{odd}(r)\, \lambda\, Y(\theta, \phi)$$

$$Lg_{\xi_r\, \phi} = \epsilon \left(\frac{d}{dr}\, \xi_{odd}(r)\right) \left(\frac{\partial}{\partial\theta}\, Y(\theta, \phi)\right) \sin(\theta) - \frac{2\, \epsilon\, \xi_{odd}(r) \sin(\theta) \left(\frac{\partial}{\partial\theta}\, Y(\theta, \phi)\right)}{r} + I \epsilon\, \xi_r(r)\, m\, Y(\theta, \phi) + I \epsilon \left(\frac{d}{dr}\, \xi_{even}(r)\right) Y(\theta, \phi)\, m - \frac{2\, I \epsilon\, \xi_{even}(r)\, m\, Y(\theta, \phi)}{r}$$

$$Lg_{\xi_{\theta}\,\phi} = 2\,\mathrm{I}\,\epsilon\,\xi_{even}(r)\left(\frac{\partial}{\partial\theta}\,Y(\theta,\phi)\right) m - \frac{2\,\mathrm{I}\,\epsilon\,\xi_{even}(r)\,Y(\theta,\phi)\,\cos(\theta)\,m}{\sin(\theta)} + \frac{2\,\epsilon\,\xi_{odd}(r)\,m^2\,Y(\theta,\phi)}{\sin(\theta)} - \epsilon\,\xi_{odd}(r)\left(\frac{\partial}{\partial\theta}\,Y(\theta,\phi)\right)\cos(\theta) - \epsilon\sin(\theta)\,\xi_{odd}(r)\cot(\theta)\left(\frac{\partial}{\partial\theta}\,Y(\theta,\phi)\right) + \epsilon\sin(\theta)\,\xi_{odd}(r)\,\lambda\,Y(\theta,\phi)$$

$$Lg_{\xi_{\phi}\,\phi} = -2\,\mathrm{I}\,\epsilon\,\xi_{odd}(r)\,Y(\theta,\phi)\cos(\theta)\,m + 2\,\epsilon\,\xi_r(r)\,Y(\theta,\phi)\,r\sin(\theta)^2 - 2\,\epsilon\,\xi_{even}(r)\,m^2\,Y(\theta,\phi) + 2\,\epsilon\,\xi_{even}(r)\left(\frac{\partial}{\partial\theta}\,Y(\theta,\phi)\right)\cos(\theta)\sin(\theta) + 2\,\mathrm{I}\,\epsilon\,\xi_{odd}(r)\sin(\theta)\,m\left(\frac{\partial}{\partial\theta}\,Y(\theta,\phi)\right) \tag{81}$$

```
> Lg__xi__r:=grcomponent(Lg__xi(dn, dn), [r, r])/epsilon/Y(theta,
  phi);
```

$$Lg_{\xi_r} := 2\left(\frac{d}{dr}\,\xi_r(r)\right) \tag{82}$$

```
> Lg__xi__Veven, Lg__xi__Vodd:=IdV2(grcomponent(Lg__xi(dn, dn), [r,
  theta]));
```

$$Lg_{\xi_{Veven}}, Lg_{\xi_{Vodd}} := \frac{d}{dr}\,\xi_{even}(r) + \xi_r(r) - \frac{2\,\xi_{even}(r)}{r}, \frac{d}{dr}\,\xi_{odd}(r) - \frac{2\,\xi_{odd}(r)}{r} \tag{83}$$

```
> Lg__xi__Teven, Lg__xi__Todd:=IdT23(grcomponent(Lg__xi(dn, dn),
  [theta, phi]));
```

$$Lg_{\xi_{Teven}}, Lg_{\xi_{Todd}} := 2\,\xi_{even}(r), 2\,\xi_{odd}(r) \tag{84}$$

```
> grcalc(xi(up));
  grmap(xi(up), Linearize, 'x');
  grdisplay(_);
Created definition for xi(up)
Calculated xi(up) for lpschwNewFixed (0.000000 sec.)
```

CPU Time $= 0.$

Procedure name:LinearizeApplying routine Linearize to xi(up)

For the lpschwNewFixed spacetime:

xi(up)

$\xi(up)$

$$\xi^r = \epsilon\,\xi_r(r)\,Y(\theta,\phi)$$

$$\xi^{\theta} = \frac{\epsilon\left(-\mathrm{I}\,\xi_{odd}(r)\,m\,Y(\theta,\phi) + \xi_{even}(r)\left(\frac{\partial}{\partial\theta}\,Y(\theta,\phi)\right)\sin(\theta)\right)}{r^2\sin(\theta)}$$

$$\xi^{\phi} = \frac{\epsilon\left(\mathrm{I}\,\xi_{even}(r)\,m\,Y(\theta,\phi) + \xi_{odd}(r)\sin(\theta)\left(\frac{\partial}{\partial\theta}\,Y(\theta,\phi)\right)\right)}{r^2\sin(\theta)^2} \tag{85}$$

```
> Lg__xi__Trace:=IdT22(grcomponent(Lg__xi(dn, dn), [theta, theta]),
  Lg__xi__Teven, Lg__xi__Todd);
```

$$Lg_{\xi_{Trace}} := \xi_{even}(r)\,\lambda + 2\,r\,\xi_r(r) \tag{86}$$

```
> grdef(`LK__xi{a b}:=xi{^l}*KK{a b ,l}+ KK{a p}*xi{^p ,b}+KK{b q}*
  xi{^q ,a}`);
  grcalc(LK__xi(dn, dn));

  grmap(LK__xi(dn, dn), Linearize, 'x');
  grmap(LK__xi(dn, dn), FixSin, 'x');
  grmap(LK__xi(dn, dn), RemPhi, 'x');
  grmap(LK__xi(dn, dn), ApplyId, 'x');
  grmap(LK__xi(dn, dn), FixSin, 'x');
  grmap(LK__xi(dn, dn), RemPhi, 'x');
Created a definition for xi(up,pdn)
Created definition for LK__xi(dn,dn)
Calculated xi(up,pdn) for lpschwNewFixed (0.000000 sec.)
Calculated LK__xi(dn,dn) for lpschwNewFixed (0.000000 sec.)
```

$$CPU\ Time = 0.$$

```
Procedure name:LinearizeApplying routine Linearize to LK__xi(dn,
dn)
Procedure name:FixSinApplying routine FixSin to LK__xi(dn,dn)
Procedure name:RemPhiApplying routine RemPhi to LK__xi(dn,dn)
Procedure name:ApplyIdApplying routine ApplyId to LK__xi(dn,dn)
Procedure name:FixSinApplying routine FixSin to LK__xi(dn,dn)
Procedure name:RemPhiApplying routine RemPhi to LK__xi(dn,dn)
```

```
> LK__xi__r:=simplify(grcomponent(LK__xi(dn, dn), [r, r]));
  LK__xi__12:=simplify(grcomponent(LK__xi(dn, dn), [r, theta]));
  LK__xi__13:=simplify(grcomponent(LK__xi(dn, dn), [r, phi]));
```

$$LK_{\xi_r} := \frac{\epsilon \sqrt{2}\,\sqrt{M}\,Y(\theta,\phi)\left(4\left(\frac{d}{dr}\,\xi_r(r)\right)r - 3\,\xi_r(r)\right)}{4\,r^{5/2}}$$

$$LK_{\xi_{12}} := -\frac{1}{r^{5/2}\sin(\theta)}\Bigg(2\sqrt{2}\,\epsilon\sqrt{M}\Bigg(-\frac{\sin(\theta)\left(r\,\xi_r(r) - 2\left(\frac{d}{dr}\,\xi_{even}(r)\right)r + 4\,\xi_{even}(r)\right)\left(\frac{\partial}{\partial\theta}\,Y(\theta,\phi)\right)}{4} + \mathrm{I}\,Y(\theta,\phi)\left(-\frac{\left(\frac{d}{dr}\,\xi_{odd}(r)\right)r}{2} + \xi_{odd}(r)\right)m\Bigg)\Bigg)$$

$$LK_{\xi_{13}} := \frac{1}{2\,r^{5/2}}\Bigg(\sqrt{2}\Bigg(4\sin(\theta)\left(-\frac{\left(\frac{d}{dr}\,\xi_{odd}(r)\right)r}{2} + \xi_{odd}(r)\right)\left(\frac{\partial}{\partial\theta}\,Y(\theta,\phi)\right) + \mathrm{I}\left(r\,\xi_r(r) - 2\left(\frac{d}{dr}\,\xi_{even}(r)\right)r + 4\,\xi_{even}(r)\right)Y(\theta,\phi)\,m\Bigg)\epsilon\sqrt{M}\Bigg) \quad \textbf{(87)}$$

Consistency check against Ivan's

```
> LK__xi__rIvan:=epsilon*sqrt(M)*sqrt(2)*Y(theta, phi)*(4*r*(diff
  (xi__r(r), r))-3*xi__r(r))/(4*r^(5/2)):
```

```
LK__xi__12Ivan:=-2*epsilon*(-sin(theta)*(-(1/2)*(diff(xi__even
(r), r))*r+(1/4)*r*xi__r(r)+xi__even(r))*(diff(Y(theta, phi),
theta))+I*m*(-(1/2)*(diff(xi__odd(r), r))*r+xi__odd(r))*Y(theta,
phi))*sqrt(M)*sqrt(2)/(r^(5/2)*sin(theta)):

LK__xi__13Ivan:=-epsilon*sqrt(M)*sqrt(2)*sin(theta)*(diff(Y
(theta, phi), theta))*(diff(xi__odd(r), r))/r^(3/2)+2*epsilon*
sqrt(M)*sqrt(2)*xi__odd(r)*sin(theta)*(diff(Y(theta, phi), theta)
)/r^(5/2)+I*epsilon*sqrt(M)*sqrt(2)*m*Y(theta, phi)*xi__r(r)/(2*
r^(3/2))-I*epsilon*sqrt(M)*sqrt(2)*(diff(xi__even(r), r))*Y
(theta, phi)*m/r^(3/2)+(2*I)*epsilon*sqrt(M)*sqrt(2)*xi__even(r)*
Y(theta, phi)*m/r^(5/2):

simplify(LK__xi__r-LK__xi__rIvan);
simplify(LK__xi__12-LK__xi__12Ivan);
simplify(LK__xi__13-LK__xi__13Ivan);
```

$$0$$

$$0$$

$$0 \qquad (88)$$

```
> LK__xi__Veven, LK__xi__Vodd:=IdV2(LK__xi__12);
IdV3(LK__xi__13);
```

$$LK_{\xi_{Veven}}, LK_{\xi_{Vodd}} := \frac{\sqrt{M}\,\sqrt{2}\,\xi_r(r)}{2\,r^{3/2}} - \frac{\sqrt{M}\,\sqrt{2}\left(\frac{d}{dr}\,\xi_{even}(r)\right)}{r^{3/2}} + \frac{2\,\sqrt{M}\,\sqrt{2}\,\xi_{even}(r)}{r^{5/2}},$$
$$-\frac{\sqrt{M}\,\sqrt{2}\left(\frac{d}{dr}\,\xi_{odd}(r)\right)}{r^{3/2}} + \frac{2\,\sqrt{M}\,\sqrt{2}\,\xi_{odd}(r)}{r^{5/2}}$$

$$\frac{\sqrt{M}\,\sqrt{2}\,\xi_r(r)}{2\,r^{3/2}} - \frac{\sqrt{M}\,\sqrt{2}\left(\frac{d}{dr}\,\xi_{even}(r)\right)}{r^{3/2}} + \frac{2\,\sqrt{M}\,\sqrt{2}\,\xi_{even}(r)}{r^{5/2}}, \qquad (89)$$
$$-\frac{\sqrt{M}\,\sqrt{2}\left(\frac{d}{dr}\,\xi_{odd}(r)\right)}{r^{3/2}} + \frac{2\,\sqrt{M}\,\sqrt{2}\,\xi_{odd}(r)}{r^{5/2}}$$

```
> LK__xi__22:=grcomponent(LK__xi(dn, dn), [theta, theta]);
LK__xi__23:=grcomponent(LK__xi(dn, dn), [theta, phi]);
LK__xi__33:=grcomponent(LK__xi(dn, dn), [phi, phi]);
```

$$LK_{\xi_{22}} := -\frac{2\,\epsilon\,\sqrt{M}\,\sqrt{2}\,\xi_{even}(r)\,m^2\,Y(\theta,\phi)}{r^{3/2}\sin(\theta)^2} + \frac{2\,\epsilon\,\sqrt{M}\,\sqrt{2}\,\xi_{even}(r)\cot(\theta)\left(\frac{\partial}{\partial\theta}\,Y(\theta,\phi)\right)}{r^{3/2}}$$
$$-\frac{2\,\epsilon\,\sqrt{M}\,\sqrt{2}\,\xi_{even}(r)\,\lambda\,Y(\theta,\phi)}{r^{3/2}} + \frac{2\,\mathrm{I}\,\epsilon\,\sqrt{M}\,\sqrt{2}\,\xi_{odd}(r)\left(\frac{\partial}{\partial\theta}\,Y(\theta,\phi)\right)m}{r^{3/2}\sin(\theta)}$$
$$-\frac{2\,\mathrm{I}\,\epsilon\,\sqrt{M}\,\sqrt{2}\,\xi_{odd}(r)\,Y(\theta,\phi)\cos(\theta)\,m}{r^{3/2}\sin(\theta)^2} - \frac{\epsilon\,\sqrt{M}\,\sqrt{2}\,\xi_r(r)\,Y(\theta,\phi)}{2\,\sqrt{r}}$$

$$LK_{\xi_{23}} := -\frac{2\,\mathrm{I}\,\epsilon\sqrt{M}\sqrt{2}\,\xi_{even}(r)\left(\frac{\partial}{\partial\theta}Y(\theta,\phi)\right)m}{r^{3/2}} + \frac{2\,\mathrm{I}\,\epsilon\sqrt{M}\sqrt{2}\,\xi_{even}(r)\,Y(\theta,\phi)\cos(\theta)\,m}{r^{3/2}\sin(\theta)} - \frac{2\,\epsilon\sqrt{M}\sqrt{2}\,\xi_{odd}(r)\,m^2\,Y(\theta,\phi)}{r^{3/2}\sin(\theta)} + \frac{\epsilon\sqrt{M}\sqrt{2}\,\xi_{odd}(r)\left(\frac{\partial}{\partial\theta}Y(\theta,\phi)\right)\cos(\theta)}{r^{3/2}} + \frac{\epsilon\sqrt{M}\sqrt{2}\,\sin(\theta)\,\xi_{odd}(r)\cot(\theta)\left(\frac{\partial}{\partial\theta}Y(\theta,\phi)\right)}{r^{3/2}} - \frac{\epsilon\sqrt{M}\sqrt{2}\,\sin(\theta)\,\xi_{odd}(r)\,\lambda\,Y(\theta,\phi)}{r^{3/2}}$$

$$LK_{\xi_{33}} := \frac{2\,\mathrm{I}\,\epsilon\sqrt{M}\sqrt{2}\,\xi_{odd}(r)\,Y(\theta,\phi)\cos(\theta)\,m}{r^{3/2}} - \frac{\epsilon\sqrt{M}\sqrt{2}\,\xi_{r}(r)\,Y(\theta,\phi)\sin(\theta)^2}{2\sqrt{r}} + \frac{2\,\epsilon\sqrt{M}\sqrt{2}\,\xi_{even}(r)\,m^2\,Y(\theta,\phi)}{r^{3/2}} - \frac{2\,\epsilon\sqrt{M}\sqrt{2}\,\xi_{even}(r)\left(\frac{\partial}{\partial\theta}Y(\theta,\phi)\right)\cos(\theta)\sin(\theta)}{r^{3/2}} - \frac{2\,\mathrm{I}\,\epsilon\sqrt{M}\sqrt{2}\,\xi_{odd}(r)\sin(\theta)\,m\left(\frac{\partial}{\partial\theta}Y(\theta,\phi)\right)}{r^{3/2}} \tag{90}$$

Consistency check against Ivan's

```
> LK__xi__22Ivan:=(2*I)*epsilon*sqrt(M)*sqrt(2)*xi__odd(r)*(diff(Y
(theta, phi), theta))*m/(r^(3/2)*sin(theta))+2*sqrt(2)*epsilon*
sqrt(M)*cos(theta)*xi__even(r)*(diff(Y(theta, phi), theta))/(r^
(3/2)*sin(theta))-epsilon*sqrt(M)*sqrt(2)*Y(theta, phi)*xi__r(r)/
(2*sqrt(r))-2*epsilon*sqrt(M)*sqrt(2)*xi__even(r)*lambda*Y(theta,
phi)/r^(3/2)-(2*I)*epsilon*sqrt(M)*sqrt(2)*cos(theta)*Y(theta,
phi)*xi__odd(r)*m/(r^(3/2)*sin(theta)^2)-2*sqrt(2)*epsilon*sqrt
(M)*xi__even(r)*m^2*Y(theta, phi)/(r^(3/2)*sin(theta)^2):
LK__xi__23Ivan:=-(2*I)*epsilon*sqrt(2)*sqrt(M*r)*xi__even(r)*
(diff(Y(theta, phi), theta))*m/r^2+2*epsilon*sqrt(2)*sqrt(M*r)*
cos(theta)*xi__odd(r)*(diff(Y(theta, phi), theta))/r^2-epsilon*
sqrt(2)*sqrt(M*r)*sin(theta)*xi__odd(r)*lambda*Y(theta, phi)/r^2
-2*epsilon*sqrt(2)*sqrt(M*r)*xi__odd(r)*m^2*Y(theta, phi)/(sin
(theta)*r^2)+(2*I)*epsilon*sqrt(2)*sqrt(M*r)*cos(theta)*xi__even
(r)*Y(theta, phi)*m/(sin(theta)*r^2):
LK__xi__33Ivan:=-(2*I)*epsilon*sqrt(M)*sqrt(2)*xi__odd(r)*sin
(theta)*(diff(Y(theta, phi), theta))*m/r^(3/2)-2*epsilon*sqrt(M)*
```

```
sqrt(2)*cos(theta)*xi__even(r)*sin(theta)*(diff(Y(theta, phi),
theta))/r^(3/2)+2*epsilon*sqrt(M)*sqrt(2)*m^2*xi__even(r)*Y
(theta, phi)/r^(3/2)+(2*I)*epsilon*sqrt(M)*sqrt(2)*cos(theta)*Y
(theta, phi)*xi__odd(r)*m/r^(3/2)+epsilon*sqrt(M)*sqrt(2)*cos
(theta)^2*Y(theta, phi)*xi__r(r)/(2*sqrt(r))-epsilon*sqrt(M)*sqrt
(2)*Y(theta, phi)*xi__r(r)/(2*sqrt(r)):

simplify(LK__xi__22-LK__xi__22Ivan);
simplify(LK__xi__23-LK__xi__23Ivan);
simplify(LK__xi__33-LK__xi__33Ivan);
```

$$0$$

$$0$$

$$0 \tag{91}$$

```
> LK__xi__Teven,LK__xi__Todd:=IdT23(LK__xi__23);
```

$$LK_{\xi_{Teven}}, LK_{\xi_{Todd}} := -\frac{2\sqrt{2}\sqrt{M}\,\xi_{even}(r)}{r^{3/2}}, -\frac{2\sqrt{2}\sqrt{M}\,\xi_{odd}(r)}{r^{3/2}} \tag{92}$$

```
> LK__xi__Trace:=factor(IdT22(LK__xi__22,LK__xi__Teven,
  LK__xi__Todd));
```

$$LK_{\xi_{Trace}} := -\frac{\sqrt{2}\sqrt{M}\left(2\,\xi_{even}(r)\,\lambda + r\,\xi_r(r)\right)}{2\,r^{3/2}} \tag{93}$$

Time

(Metric part is easy.....according to Ivan lol)

```
> xi__N:=epsilon*xi__0(r)*Y(theta, phi);
```

$$\xi_N := \epsilon\,\xi_0(r)\;Y(\theta,\phi) \tag{94}$$

```
> xidotg__11:=Linearize(xi__N*grcomponent(KK(dn, dn), [r, r]));
```

$$xidotg_{11} := \frac{\xi_0(r)\;Y(\theta,\phi)\,\sqrt{2}\,\sqrt{M}\,\epsilon}{2\,r^{3/2}} \tag{95}$$

```
> xidotg__12:=Linearize(xi__N*grcomponent(KK(dn, dn), [r, theta]));
  xidotg__13:=Linearize(xi__N*grcomponent(KK(dn, dn), [r, phi]));
```

$$xidotg_{12} := 0$$

$$xidotg_{13} := 0 \tag{96}$$

```
> xidotg__22:=Linearize(xi__N*grcomponent(KK(dn, dn), [theta,
  theta]));
  xidotg__23:=Linearize(xi__N*grcomponent(KK(dn, dn), [theta, phi])
  );
  xidotg__33:=Linearize(xi__N*grcomponent(KK(dn, dn), [phi, phi]));
```

$$xidotg_{22} := -\xi_0(r)\;Y(\theta,\phi)\,\sqrt{2}\,\sqrt{M}\,\sqrt{r}\,\epsilon$$

$$xidotg_{23} := 0$$

$$xidotg_{33} := -\xi_0(r)\;Y(\theta,\phi)\,\sqrt{2}\,\sqrt{M}\,\sqrt{r}\,\sin(\theta)^2\,\epsilon \tag{97}$$

```
> xidotg__r:=xidotg__11/epsilon/Y(theta, phi);
```

$$xidotg_r := \frac{\xi_0(r)\sqrt{2}\sqrt{M}}{2\, r^{3/2}} \quad \textbf{(98)}$$

```
> xidotg__Veven, xidotg__Vodd:=IdV2(xidotg__12);
```

$$xidotg_{Veven}, xidotg_{Vodd} := 0, 0 \quad \textbf{(99)}$$

```
> xidotg__Teven, xidotg__Todd:=IdT23(xidotg__23);
```

$$xidotg_{Teven}, xidotg_{Todd} := 0, 0 \quad \textbf{(100)}$$

```
> xidotg__Trace:=IdT22(xidotg__22, xidotg__Teven, xidotg__Todd);
```

$$xidotg_{Trace} := -\xi_0(r)\sqrt{2}\sqrt{M}\sqrt{r} \quad \textbf{(101)}$$

Extrinsic Curvature Deformation

There needed to be some additional setup for the first term in the deformation which is the double covariant derivative of the Lapse function

```
> grdef(`xi__NTest:=epsilon*xi__0(r)*Y(theta, phi)`);
  grcalc(xi__NTest);
  grcalc(xi__NTest(cdn, cdn));

  grmap(xi__NTest(cdn, cdn), Linearize, 'x');

  grdef(`xi__NLast{a b}:=xi__NTest{;a ;b}`);
```

```
Created definition for xi__NTest
Calculated xi__NTest for lpschwNewFixed (0.000000 sec.)
```

$$CPU\ Time\ = 0.$$

```
Created a definition for xi__NTest(cdn)
Created a definition for xi__NTest(cdn,cdn)
Calculated xi__NTest(cdn) for lpschwNewFixed (0.000000 sec.)
Calculated xi__NTest(cdn,cdn) for lpschwNewFixed (0.000000 sec.)
```

$$CPU\ Time\ = 0.031$$

```
Procedure name:LinearizeApplying routine Linearize to xi__NTest
(cdn,cdn)
Created definition for xi__NLast(dn,dn)
```

```
> grdef(`DDxi__N{a b}:=xi__NLast{a b}`);
  grcalc(DDxi__N(dn, dn));

  grdisplay(_);
```

```
Created definition for DDxi__N(dn,dn)
Calculated xi__NLast(dn,dn) for lpschwNewFixed (0.000000 sec.)
Calculated DDxi__N(dn,dn) for lpschwNewFixed (0.000000 sec.)
```

$$CPU\ Time\ = 0.$$

For the lpschwNewFixed spacetime:

$$DDxi_{N(dn,dn}$$

$$DDxi_N(dn, dn)$$

$$DDxi_{N_r\,r} = \left(\frac{d^2}{dr^2}\,\xi_0(r)\right) Y(\theta,\phi)\,\epsilon$$

$$DDxi_{N_\theta\,r} = \frac{\epsilon\left(\frac{\partial}{\partial\theta}\,Y(\theta,\phi)\right)\left(\left(\frac{d}{dr}\,\xi_0(r)\right) r - \xi_0(r)\right)}{r}$$

$$DDxi_{N_\phi\,r} = \frac{\epsilon\left(\frac{\partial}{\partial\phi}\,Y(\theta,\phi)\right)\left(\left(\frac{d}{dr}\,\xi_0(r)\right) r - \xi_0(r)\right)}{r}$$

$$DDxi_{N_r\,\theta} = \frac{\epsilon\left(\frac{\partial}{\partial\theta}\,Y(\theta,\phi)\right)\left(\left(\frac{d}{dr}\,\xi_0(r)\right) r - \xi_0(r)\right)}{r}$$

$$DDxi_{N_\theta\,\theta} = \epsilon\left(\left(\frac{d}{dr}\,\xi_0(r)\right) Y(\theta,\phi)\,r + \xi_0(r)\left(\frac{\partial^2}{\partial\theta^2}\,Y(\theta,\phi)\right)\right)$$

$$DDxi_{N_\phi\,\theta} = -\frac{\epsilon\,\xi_0(r)\left(\cos(\theta)\left(\frac{\partial}{\partial\phi}\,Y(\theta,\phi)\right) - \sin(\theta)\left(\frac{\partial^2}{\partial\phi\partial\theta}\,Y(\theta,\phi)\right)\right)}{\sin(\theta)}$$

$$DDxi_{N_r\,\phi} = \frac{\epsilon\left(\frac{\partial}{\partial\phi}\,Y(\theta,\phi)\right)\left(\left(\frac{d}{dr}\,\xi_0(r)\right) r - \xi_0(r)\right)}{r}$$

$$DDxi_{N_\theta\,\phi} = -\frac{\epsilon\,\xi_0(r)\left(\cos(\theta)\left(\frac{\partial}{\partial\phi}\,Y(\theta,\phi)\right) - \sin(\theta)\left(\frac{\partial^2}{\partial\phi\partial\theta}\,Y(\theta,\phi)\right)\right)}{\sin(\theta)}$$

$$DDxi_{N_\phi\,\phi} = -\epsilon\left(\left(\frac{d}{dr}\,\xi_0(r)\right) Y(\theta,\phi)\cos(\theta)^2\,r - \xi_0(r)\left(\frac{\partial}{\partial\theta}\,Y(\theta,\phi)\right)\cos(\theta)\sin(\theta) - \left(\frac{d}{dr}\,\xi_0(r)\right) Y(\theta,\phi)\,r - \xi_0(r)\left(\frac{\partial^2}{\partial\phi^2}\,Y(\theta,\phi)\right)\right) \tag{102}$$

```
> grdef(`xiKK{a b}:=xi__N*(KTr*KK{a b}-2*g{^k ^l}*KK{a k}*KK{b l})
  `);
  grcalc(xiKK(dn, dn));

  grmap(xiKK(dn, dn), Linearize, 'x');
  grmap(xiKK(dn, dn), RemPhi, 'x');
  grmap(xiKK(dn, dn), FixSin, 'x');
  grmap(xiKK(dn, dn), ApplyId, 'x');
  grmap(xiKK(dn, dn), RemPhi, 'x');
  grmap(xiKK(dn, dn), FixSin, 'x');

  grdisplay(_);
Created definition for xiKK(dn,dn)
Calculated xiKK(dn,dn) for lpschwNewFixed (0.000000 sec.)
```

$$CPU\ Time\ = 0.$$

```
Procedure name:LinearizeApplying routine Linearize to xiKK(dn,
dn)
Procedure name:RemPhiApplying routine RemPhi to xiKK(dn,dn)
```

```
Procedure name:FixSinApplying routine FixSin to xiKK(dn,dn)
Procedure name:ApplyIdApplying routine ApplyId to xiKK(dn,dn)
Procedure name:RemPhiApplying routine RemPhi to xiKK(dn,dn)
Procedure name:FixSinApplying routine FixSin to xiKK(dn,dn)
```

For the lpschwNewFixed spacetime:

xiKK(dn,dn)

xiKK(dn, dn)

$$xiKK_{a\,b} = \begin{bmatrix} -\frac{5\,\xi_0(r)\,Y(\theta,\phi)\,M\,\epsilon}{2\,r^3} & 0 & 0 \\ 0 & -\frac{\xi_0(r)\,Y(\theta,\phi)\,M\,\epsilon}{r} & 0 \\ 0 & 0 & -\frac{\xi_0(r)\,Y(\theta,\phi)\,M\sin(\theta)^2\,\epsilon}{r} \end{bmatrix} \quad \textbf{(103)}$$

```
> xidotK__11:=simplify(grcomponent(DDxi__N(dn, dn), [r, r])-
  grcomponent(xiKK(dn, dn), [r, r]));
  xidotK__12:=simplify(grcomponent(DDxi__N(dn, dn), [r, theta])-
  grcomponent(xiKK(dn, dn), [r, theta]));
  xidotK__13:=simplify(grcomponent(DDxi__N(dn, dn), [r, phi])-
  grcomponent(xiKK(dn, dn), [r, phi]));
  xidotK__22:=ApplyId(RemPhi(simplify(grcomponent(DDxi__N(dn, dn),
  [theta, theta])-grcomponent(xiKK(dn, dn), [theta, theta]))));
  xidotK__23:=ApplyId(RemPhi(simplify(grcomponent(DDxi__N(dn, dn),
  [theta, phi])-grcomponent(xiKK(dn, dn), [theta, phi]))));
  xidotK__33:=ApplyId(RemPhi(simplify(grcomponent(DDxi__N(dn, dn),
  [phi, phi])-grcomponent(xiKK(dn, dn), [phi, phi]))));
```

$$xidotK_{11} := \frac{Y(\theta,\phi)\,\epsilon\left(2\left(\frac{d^2}{dr^2}\,\xi_0(r)\right)r^3 + 5\,\xi_0(r)\,M\right)}{2\,r^3}$$

$$xidotK_{12} := \frac{\epsilon\left(\frac{\partial}{\partial\theta}\,Y(\theta,\phi)\right)\left(\left(\frac{d}{dr}\,\xi_0(r)\right)r - \xi_0(r)\right)}{r}$$

$$xidotK_{13} := \frac{\epsilon\left(\frac{\partial}{\partial\phi}\,Y(\theta,\phi)\right)\left(\left(\frac{d}{dr}\,\xi_0(r)\right)r - \xi_0(r)\right)}{r}$$

$$xidotK_{22} := \epsilon\,r\left(\frac{d}{dr}\,\xi_0(r)\right)Y(\theta,\phi) + \frac{\xi_0(r)\,Y(\theta,\phi)\,M\,\epsilon}{r} + \frac{\epsilon\,\xi_0(r)\,m^2\,Y(\theta,\phi)}{\sin(\theta)^2} - \epsilon\,\xi_0(r)\cot(\theta)\left(\frac{\partial}{\partial\theta}\,Y(\theta,\phi)\right) + \epsilon\,\xi_0(r)\,\lambda\,Y(\theta,\phi)$$

$$xidotK_{23} := \mathrm{I}\,\epsilon\,\xi_0(r)\,m\left(\frac{\partial}{\partial\theta}\,Y(\theta,\phi)\right) - \frac{\mathrm{I}\,\epsilon\,\xi_0(r)\cos(\theta)\,m\,Y(\theta,\phi)}{\sin(\theta)}$$

$$xidotK_{33} := \mathrm{I}\,\epsilon\,\xi_0(r)\,m\left(\frac{\partial}{\partial\phi}\,Y(\theta,\phi)\right) + \epsilon\,\xi_0(r)\left(\frac{\partial}{\partial\theta}\,Y(\theta,\phi)\right)\cos(\theta)\sin(\theta) - \epsilon\,r\left(\frac{d}{dr}\right. \quad \textbf{(104)}$$

$$\xi_0(r)\Big)\,Y(\theta,\phi)\cos(\theta)^2 - \frac{\epsilon M \xi_0(r)\,Y(\theta,\phi)\cos(\theta)^2}{r} + \epsilon\, r\left(\frac{d}{dr}\,\xi_0(r)\right)Y(\theta,\phi)$$
$$+\frac{\xi_0(r)\,Y(\theta,\phi)\,M\epsilon}{r}$$

```
> xidotK__r:=xidotK__11;
```

$$xidotK_r := \frac{Y(\theta,\phi)\,\epsilon\left(2\left(\frac{d^2}{dr^2}\,\xi_0(r)\right)r^3 + 5\,\xi_0(r)\,M\right)}{2\,r^3} \tag{105}$$

```
> xidotK__Veven, xidotK__Vodd:=IdV2(xidotK__12);
```

$$xidotK_{Veven}, xidotK_{Vodd} := \frac{d}{dr}\,\xi_0(r) - \frac{\xi_0(r)}{r}, 0 \tag{106}$$

```
> xidotK__Teven, xidotK__Todd:=IdT23(xidotK__23);
```

$$xidotK_{Teven}, xidotK_{Todd} := \xi_0(r), 0 \tag{107}$$

```
> xidotK__Trace:=IdT22(xidotK__22, xidotK__Teven, xidotK__Todd);
```

$$xidotK_{Trace} := \left(\frac{d}{dr}\,\xi_0(r)\right)r + \frac{\xi_0(r)\,\lambda}{2} + \frac{\xi_0(r)\,M}{r} \tag{108}$$

Now the big combo

```
> delta__r:=Lg__xi__r + xidotg__r;
  deltag__Veven:=Lg__xi__Veven + xidotg__Veven;
  deltag__Vodd:=Lg__xi__Vodd + xidotg__Vodd;
  deltag__Teven:=Lg__xi__Teven + xidotg__Teven;
  deltag__Todd:=Lg__xi__Todd + xidotg__Todd;
  deltag__Trace:=Lg__xi__Trace + xidotg__Trace;
```

$$\delta_r := 2\left(\frac{d}{dr}\,\xi_r(r)\right) + \frac{\xi_0(r)\sqrt{2}\sqrt{M}}{2\,r^{3/2}}$$

$$deltag_{Veven} := \frac{d}{dr}\,\xi_{even}(r) + \xi_r(r) - \frac{2\,\xi_{even}(r)}{r}$$

$$deltag_{Vodd} := \frac{d}{dr}\,\xi_{odd}(r) - \frac{2\,\xi_{odd}(r)}{r}$$

$$deltag_{Teven} := 2\,\xi_{even}(r)$$

$$deltag_{Todd} := 2\,\xi_{odd}(r)$$

$$deltag_{Trace} := \xi_{even}(r)\,\lambda + 2\,r\,\xi_r(r) - \xi_0(r)\sqrt{2}\sqrt{M}\sqrt{r} \tag{109}$$

```
> deltaK__r:=collect((LK__xi__r + xidotK__r)/epsilon/Y(theta, phi),
  M);
  deltaK__Veven:=collect((LK__xi__Veven + xidotK__Veven), M);
  deltaK__Vodd:=collect((LK__xi__Vodd + xidotK__Vodd), M);
  deltaK_Teven:=LK__xi__Teven + xidotK__Teven;
  deltaK__Todd:=LK__xi__Todd + xidotK__Todd;
  deltaK__Trace:=collect(LK__xi__Trace + xidotK__Trace, M);
```

$$deltaK_r := \frac{5\,\xi_0(r)\,M}{2\,r^3} + \frac{\sqrt{2}\left(4\left(\frac{d}{dr}\,\xi_r(r)\right)r - 3\,\xi_r(r)\right)\sqrt{M}}{4\,r^{5/2}} + \frac{d^2}{dr^2}\,\xi_0(r)$$

$$deltaK_{Veven} := \left(\frac{\sqrt{2}\,\xi_r(r)}{2\,r^{3/2}} - \frac{\sqrt{2}\left(\frac{d}{dr}\,\xi_{even}(r)\right)}{r^{3/2}} + \frac{2\sqrt{2}\,\xi_{even}(r)}{r^{5/2}}\right)\sqrt{M} + \frac{d}{dr}\,\xi_0(r) - \frac{\xi_0(r)}{r}$$

$$deltaK_{Vodd} := \left(-\frac{\sqrt{2}\left(\frac{d}{dr}\,\xi_{odd}(r)\right)}{r^{3/2}} + \frac{2\sqrt{2}\,\xi_{odd}(r)}{r^{5/2}}\right)\sqrt{M}$$

$$deltaK\_Teven := -\frac{2\sqrt{2}\,\sqrt{M}\,\xi_{even}(r)}{r^{3/2}} + \xi_0(r)$$

$$deltaK_{Todd} := -\frac{2\sqrt{2}\,\sqrt{M}\,\xi_{odd}(r)}{r^{3/2}}$$

$$deltaK_{Trace} := -\frac{\sqrt{2}\,\sqrt{M}\left(2\,\xi_{even}(r)\,\lambda + r\,\xi_r(r)\right)}{2\,r^{3/2}} + \left(\frac{d}{dr}\,\xi_0(r)\right)r + \frac{\xi_0(r)\,\lambda}{2} + \frac{\xi_0(r)\,M}{r} \tag{110}$$

```
> expand(subs(xi__r(r)=Xi(r)*r^(3/4), op(2, deltaK__r)));
```

$$\frac{\sqrt{2}\,\sqrt{M}\left(\frac{d}{dr}\,\Xi(r)\right)}{r^{3/4}} \tag{111}$$

```
> factor(expand(subs(xi__even(r)=r^2*Xi(r), op(1, deltaK__Veven)))
  ;
```

$$\frac{\sqrt{M}\,\sqrt{2}\left(-2\,r^2\left(\frac{d}{dr}\,\Xi(r)\right) + \xi_r(r)\right)}{2\,r^{3/2}} \tag{112}$$

```
> deltaK__Trace;
```

$$-\frac{\sqrt{2}\,\sqrt{M}\left(2\,\xi_{even}(r)\,\lambda + r\,\xi_r(r)\right)}{2\,r^{3/2}} + \left(\frac{d}{dr}\,\xi_0(r)\right)r + \frac{\xi_0(r)\,\lambda}{2} + \frac{\xi_0(r)\,M}{r} \tag{113}$$

Now in the RW gauge.

Constraints

Hamiltonian Constraint

```
> HamilCon:=eval(subs(h__Teven(r)=0,h__Todd(r)=0,h__Veven(r)=0,
  H0__vac));
```

$$HamilCon := \left(\frac{2\,M}{r^2} - \frac{\lambda}{2\,r} + \frac{1}{r}\right)h_{rr}(r) + \left(-\frac{2\,M}{r^4} - \frac{\lambda}{2\,r^3} - \frac{1}{r^3}\right)h_{trace}(r) + \frac{d}{dr}\,h_{rr}(r) \tag{114}$$

$$- \frac{\frac{d^2}{dr^2} h_{trace}(r)}{r} + \frac{\frac{d}{dr} h_{trace}(r)}{r^2} - \frac{\sqrt{2}\,\sqrt{M}\,k_{trace}(r)}{r^{5/2}} - \frac{2\sqrt{2}\,k_{rr}(r)\,\sqrt{M}}{\sqrt{r}}$$

Momentum Constraint

```
> MomentumConR:=eval(subs(h__Teven(r)=0,h__Todd(r)=0,h__Veven(r)=0,
  H1__vac));
```

$$MomentumConR := -\frac{k_{Veven}(r)\,\lambda}{2\,r} + \frac{d}{dr} K_{trace}(r) - k_{rr}(r) + \frac{h_{rr}(r)\,\sqrt{2}\,\sqrt{M}}{2\,r^{3/2}} + \frac{\sqrt{2}\,\sqrt{M}\left(\frac{d}{dr} h_{trace}(r)\right)}{4\,r^{5/2}} - \frac{\sqrt{2}\,\sqrt{M}\,h_{trace}(r)}{r^{7/2}} \quad \textbf{(115)}$$

```
> MomentumConVEven:=eval(subs(h__Teven(r)=0,h__Todd(r)=0,h__Veven
  (r)=0,k__Teven(r)=0,Heven__vac));
  MomentumConVOdd:=eval(subs(h__Teven(r)=0,h__Todd(r)=0,h__Veven(r)
  =0,k__Teven(r)=0,Hodd__vac));
```

$$MomentumConVEven := \left(\sqrt{2}\left(\frac{d}{dr} H2_{Veven}(r)\right) - \frac{\sqrt{r}\,h_{rr}(r)\,\sqrt{2}}{4} - \frac{\sqrt{2}\,h_{trace}(r)}{r^{3/2}}\right)\sqrt{M} - k_{trace}(r) - k_{rr}(r)\,r^2 + \frac{d}{dr} K_{Veven}(r)$$

$$MomentumConVOdd := \sqrt{2}\left(\frac{d}{dr} H_{Vodd}(r)\right)\sqrt{M} + \left(\frac{\lambda}{2} + 1\right) k_{Todd}(r) + \frac{d}{dr} K_{Vodd}(r) \quad \textbf{(116)}$$

Time derivative of h

```
> HdotR:=expand(hdot__C11/epsilon/Y(theta,phi));
```

$$HdotR := -\frac{\sqrt{2}\,\sqrt{M}\left(\frac{d}{dr} h_{rr}(r)\right)}{Y(\theta,\phi)\,\epsilon\,\sqrt{r}} + \frac{\sqrt{2}\,\sqrt{M}\,n_L(r)}{Y(\theta,\phi)\,\epsilon\,r^{3/2}} + \frac{2\left(\frac{d}{dr} v_{scalar}(r)\right)}{Y(\theta,\phi)\,\epsilon} + \frac{2\,k_{rr}(r)}{Y(\theta,\phi)\,\epsilon} \quad \textbf{(117)}$$

```
> HdotVEven:=eval(subs(h__Teven(r)=0,h__Todd(r)=0,h__Veven(r)=0,
  k__Teven(r)=0,v__even(r)=0,v__odd(r)=0,hdot__CVeven));
  HdotVOdd:=eval(subs(h__Teven(r)=0,h__Todd(r)=0,h__Veven(r)=0,
  k__Teven(r)=0,v__even(r)=0,v__odd(r)=0,hdot__CVodd));
```

$$HdotVEven := -\frac{\sqrt{2}\,h_{rr}(r)\,\sqrt{M}}{\sqrt{r}} + v_{scalar}(r) + 2\,k_{Veven}(r)$$

$$HdotVOdd := 2\,k_{Vodd}(r) + \frac{2\sqrt{2}\,h_{Vodd}(r)\,\sqrt{M}}{r^{3/2}} \quad \textbf{(118)}$$

```
> HdotTEven:=eval(subs(h__Teven(r)=0,h__Todd(r)=0,h__Veven(r)=0,
  k__Teven(r)=0,v__even(r)=0,hdot__Teven));
  HdotTOdd:=eval(subs(h__Teven(r)=0,h__Todd(r)=0,h__Veven(r)=0,
  k__Teven(r)=0,v__even(r)=0,v__odd(r)=-k__Todd(r)+sqrt(M)*h__Vodd
  (r)*sqrt(2)*(1/sqrt(r)),hdot__Todd));
  HdotTTrace:=eval(subs(h__Teven(r)=0,h__Todd(r)=0,h__Veven(r)=0,
  k__Teven(r)=0,v__even(r)=0,hdot__TOmega));
```

$$HdotTEven := 0$$

$$HdotTOdd := 0$$

$$HdotTTrace := \left(\frac{\sqrt{2}\left(\frac{d}{dr}\,h_{trace}(r)\right)}{\sqrt{r}} - 2\sqrt{r}\,h_{rr}(r)\sqrt{2} - 2\sqrt{r}\,n_L(r)\sqrt{2}\right)\sqrt{M} + 2\,v_{scalar}(r)\,r + 2\,k_{trace}(r) \tag{119}$$

Time derivative of K

```
> KdotR:=eval(subs(h__Teven(r)=0,h__Todd(r)=0,h__Veven(r)=0,
  k__Teven(r)=0,v__even(r)=0,v__odd(r)=-k__Todd(r)+sqrt(M)*h__Vodd
  (r)*sqrt(2)*(1/sqrt(r)),Kdot__rrX));
```

$$KdotR := \left(\frac{2\,h_{rr}(r)}{r^2} - \frac{2\,H_{trace}(r)}{r^3} + \frac{5\,n_L(r)}{2\,r^2} - \frac{2\left(\frac{d}{dr}\,h_{rr}(r)\right)}{r}\right)M + \left(-\frac{\sqrt{2}\,k_{trace}(r)}{r^{5/2}} + \frac{\sqrt{2}\left(\frac{d}{dr}\,K_{rr}(r)\right)}{r^{3/2}} + \frac{\sqrt{2}\left(\frac{d}{dr}\,v_{scalar}(r)\right)}{\sqrt{r}} - \frac{3\sqrt{2}\,v_{scalar}(r)}{4\,r^{3/2}}\right)\sqrt{M} + \frac{h_{rr}(r)\,\lambda}{2\,r} + r\left(\frac{d^2}{dr^2}\,n_L(r)\right) - \left(\frac{d}{dr}\,h_{rr}(r)\right) + \frac{d^2}{dr^2}\,H_{trace}(r) \tag{120}$$

```
> KdotVEven:=eval(subs(h__Teven(r)=0,h__Todd(r)=0,h__Veven(r)=0,
  k__Teven(r)=0,v__even(r)=0,V__odd(r)=-k__Todd(r)+sqrt(M)*h__Vodd
  (r)*sqrt(2)*(1/sqrt(r)),Kdot__Veven));
```

$$KdotVEven := \left(\frac{2\left(\frac{d}{dr}\,H_{Veven}(r)\right)}{\sqrt{r}} - \frac{h_{rr}(r)}{r^2}\right)M + \left(\frac{\sqrt{2}\left(\frac{d}{dr}\,k_{Veven}(r)\right)}{\sqrt{r}} + \frac{\sqrt{2}\,v_{scalar}(r)}{2\,r^{3/2}} - \sqrt{2}\sqrt{r}\left(\frac{d}{dr}\,V_{even}(r)\right)\right)\sqrt{M} + \left(-\frac{\lambda}{4} - \frac{1}{2}\right)\left(\frac{d}{dr}\,H_{Teven}(r)\right) + r\left(\frac{d}{dr}\,N_L(r)\right) + \frac{\frac{d}{dr}\,H_{trace}(r)}{2} - \frac{h_{rr}(r)}{2\,r} + \frac{H_{Veven}(r)}{\sqrt{r}} \tag{121}$$

```
> KdotVOdd:=eval(subs(h__Teven(r)=0,h__Todd(r)=0,h__Veven(r)=0,
  k__Teven(r)=0,v__even(r)=0,v__odd(r)=-k__Todd(r)+sqrt(M)*h__Vodd
  (r)*sqrt(2)*(1/sqrt(r)),Kdot__Vodd));
```

$$KdotVOdd := \frac{2\,M\left(\frac{d}{dr}\,H_{Vodd}(r)\right)}{\sqrt{r}} + \left(\frac{\sqrt{2}\left(\frac{d}{dr}\,k_{Vodd}(r)\right)}{\sqrt{r}} - \sqrt{2}\sqrt{r}\left(\frac{d}{dr}\,V_{odd}(r)\right)\right)\sqrt{M} + \left(-\frac{\lambda}{4} - \frac{1}{2}\right)\left(\frac{d}{dr}\,H_{Todd}(r)\right) + \left(\frac{\lambda}{2\sqrt{r}} + \frac{1}{\sqrt{r}}\right)H_{Vodd}(r) \tag{122}$$

```
> KdotTEven:=eval(subs(h__Teven(r)=0,h__Todd(r)=0,h__Veven(r)=0,
  k__Teven(r)=0,v__even(r)=0,v__odd(r)=-k__Todd(r)+sqrt(M)*h__Vodd
  (r)*sqrt(2)*(1/sqrt(r)),Kdot__Teven));
```

$$KdotTEven := n_L(r) + \frac{h_{rr}(r)}{2} \tag{123}$$

```
> KdotTOdd:=expand(eval(subs(h__Teven(r)=0,h__Todd(r)=0,h__Veven(r)
```

```
=0,k__Teven(r)=0,v__even(r)=0,v__odd(r)=-k__Todd(r)+sqrt(M)*
h__Vodd(r)*sqrt(2)*(1/sqrt(r)),Kdot__Todd)));
```

$$KdotTOdd := -\frac{\sqrt{2}\,\sqrt{M}\,k_{Todd}(r)}{2\,r^{3/2}} + \frac{\sqrt{2}\,\sqrt{M}\left(\frac{d}{dr}\,k_{Todd}(r)\right)}{\sqrt{r}} - \left(\frac{d}{dr}\,h_{Vodd}(r)\right) \tag{124}$$

```
> KdotTTrace:=eval(subs(h__Teven(r)=0,h__Todd(r)=0,h__Veven(r)=0,
k__Teven(r)=0,v__even(r)=0,v__odd(r)=-k__Todd(r)+sqrt(M)*h__Vodd
(r)*sqrt(2)*(1/sqrt(r)),Kdot__TOmega));
```

$$KdotTTrace := \frac{M\,n_L(r)}{r} + \left(\sqrt{r}\,\sqrt{2}\,k_{rr}(r) - \frac{\sqrt{2}\,k_{trace}(r)}{2\,r^{3/2}} - \frac{\sqrt{2}\,v_{scalar}(r)}{2\sqrt{r}} + \frac{\sqrt{2}\left(\frac{d}{dr}\,k_{trace}(r)\right)}{\sqrt{r}}\right)\sqrt{M} + \left(\frac{\lambda}{4} - 1\right)h_{rr}(r) + \frac{n_L(r)\,\lambda}{2} + \left(\frac{d}{dr}\,n_L(r)\right)r - \frac{r\left(\frac{d}{dr}\,h_{rr}(r)\right)}{2} + \frac{\left(\frac{d^2}{dr^2}\,h_{trace}(r)\right)}{2} + \frac{\lambda\,h_{trace}(r)}{2\,r^2} \tag{125}$$

Brill-Lindquist

```
> spacetime(Br-Lind, coord=[r, theta, phi], ds=((2*M/r)/(1+M/(2*r))
*P__l)^4*(d[r]^2/(1-2*M/r)+r^2*(d[theta]^2+sin(theta)^2*d[phi]^2)
));
Calculated ds for Br-Lind (0.000000 sec.)
```

CPU Time $= 0.$

For the || (*Br* − *Lind*) || *spacetime:*

Line element

$$ds^2 = \frac{16\,M^4\,P_l^4\;d\,r^2}{r^4\left(1+\frac{M}{2\,r}\right)^4\left(1-\frac{2\,M}{r}\right)} + \frac{16\,M^4\,P_l^4\;d\,\theta^2}{r^2\left(1+\frac{M}{2\,r}\right)^4} + \frac{16\,M^4\,P_l^4\,\sin(\theta)^2\;d\,\phi^2}{r^2\left(1+\frac{M}{2\,r}\right)^4} \tag{126}$$

```
> BLH__rr:=grcomponent(g(dn,dn),[r,r]);
BLH__tt:=grcomponent(g(dn,dn),[theta, theta]);
BLH__pp:=grcomponent(g(dn,dn),[phi,phi]);
```

$$BLH_{rr} := \frac{16\,M^4\,P_l^4}{r^4\left(1+\frac{M}{2\,r}\right)^4\left(1-\frac{2\,M}{r}\right)}$$

$$BLH_{tt} := \frac{16\,M^4\,P_l^4}{r^2\left(1+\frac{M}{2\,r}\right)^4}$$

(127)

$$BLH_{pp} := \frac{16\,M^4\,P_l^4\,\sin(\theta)^2}{r^2\left(1+\frac{M}{2\,r}\right)^4} \tag{127}$$

Constraints

```
> BLMomentumConR:=eval(subs(h__rr(r)=BLH__rr,h__trace(r)=BLH__pp,
  MomentumConR));
```

$$BLMomentumConR := -\frac{k_{Veven}(r)\,\lambda}{2\,r} + \frac{d}{dr}\,K_{trace}(r) - k_{rr}(r) + \frac{8\,M^{9/2}\,P_l^4\,\sqrt{2}}{r^{11/2}\left(1+\frac{M}{2\,r}\right)^4\left(1-\frac{2\,M}{r}\right)} + \frac{\sqrt{2}\,\sqrt{M}\left(-\frac{32\,M^4\,P_l^4\,\sin(\theta)^2}{r^3\left(1+\frac{M}{2\,r}\right)^4} + \frac{32\,M^5\,P_l^4\,\sin(\theta)^2}{r^4\left(1+\frac{M}{2\,r}\right)^5}\right)}{4\,r^{5/2}} - \frac{16\,\sqrt{2}\,M^{9/2}\,P_l^4\,\sin(\theta)^2}{r^{11/2}\left(1+\frac{M}{2\,r}\right)^4} \tag{128}$$

```
> BLMomentumConVEven:=eval(subs(H2__Veven(r)=0,h__Todd(r)=0,h__rr
  (r)=BLH__rr,h__trace(r)=BLH__pp,MomentumConVEven));
  BLMomentumConVOdd:=eval(subs(H__Vodd(r)=0,MomentumConVOdd));
```

$$BLMomentumConVEven := \left(-\frac{4\,M^4\,P_l^4\,\sqrt{2}}{r^{7/2}\left(1+\frac{M}{2\,r}\right)^4\left(1-\frac{2\,M}{r}\right)} - \frac{16\,\sqrt{2}\,M^4\,P_l^4\,\sin(\theta)^2}{r^{7/2}\left(1+\frac{M}{2\,r}\right)^4}\right)\sqrt{M} - k_{trace}(r) - k_{rr}(r)\,r^2 + \frac{d}{dr}\,K_{Veven}(r)$$

$$BLMomentumConVOdd := \left(\frac{\lambda}{2}+1\right)k_{Todd}(r) + \frac{d}{dr}\,K_{Vodd}(r) \tag{129}$$

Time derivative

```
> hdot__r:=simplify(subs(h__rr(r)=BLH__rr,HdotR));
```

$$hdot_r := \tag{130}$$

$$\frac{1}{r^{3/2}\,Y(\theta,\phi)\,\epsilon\,(2\,r+M)^5\,(-r+2\,M)^2}\Bigg(8\Bigg(\frac{121\left(-\frac{3072\,P_l^4}{121}+n_L(r)\right)r^2\,\sqrt{2}\,M^{11/2}}{8} + \frac{9\left(\frac{128\,P_l^4}{9}+n_L(r)\right)r\,\sqrt{2}\,M^{13/2}}{2} + 10\,M^3\left(k_{rr}(r)+\frac{d}{dr}\,v_{scalar}(r)\right)r^{11/2}$$

$$-28\,M^2\left(k_{rr}(r)+\frac{d}{dr}\,v_{scalar}(r)\right)r^{13/2} - 12\,M\left(k_{rr}(r)+\frac{d}{dr}\,v_{scalar}(r)\right)r^{15/2}$$

$$+\left(8\,k_{rr}(r)+8\left(\frac{d}{dr}\,v_{scalar}(r)\right)\right)r^{17/2}+\frac{85\left(\frac{1024\,P_l^4}{85}+n_L(r)\right)r^3\sqrt{2}\,M^{9/2}}{4}$$

$$+M^7\left(k_{rr}(r)+\frac{d}{dr}\,v_{scalar}(r)\right)r^{3/2}+9\,M^6\left(k_{rr}(r)+\frac{d}{dr}\,v_{scalar}(r)\right)r^{5/2}$$

$$+\frac{121\,M^5\left(k_{rr}(r)+\frac{d}{dr}\,v_{scalar}(r)\right)r^{7/2}}{4}+\frac{85\,M^4\left(k_{rr}(r)+\frac{d}{dr}\,v_{scalar}(r)\right)r^{9/2}}{2}$$

$$-14\left(-\frac{M^{15/2}}{28}+r^4\left(-\frac{2\sqrt{M}\,r^3}{7}+\frac{3\,M^{3/2}\,r^2}{7}+M^{5/2}\,r-\frac{5\,M^{7/2}}{14}\right)\right)\sqrt{2}\,n_L(r)\Bigg)\Bigg)$$

```
> hdotVEven:=subs(h__rr(r)=BLH__rr,HdotVEven);
  hdotVOdd:=subs(h__Vodd(r)=0, HdotVOdd);
```

$$hdotVEven := -\frac{16\sqrt{2}\,M^{9/2}\,P_l^4}{r^{9/2}\left(1+\frac{M}{2\,r}\right)^4\left(1-\frac{2\,M}{r}\right)}+v_{scalar}(r)+2\,k_{Veven}(r)$$

$$hdotVOdd := 2\,k_{Vodd}(r) \tag{131}$$

```
> HdotTEven:=eval(subs(h__Teven(r)=0,h__Todd(r)=0,h__Veven(r)=0,
  k__Teven(r)=0,v__even(r)=0,hdot__Teven));
  HdotTOdd:=eval(subs(h__Teven(r)=0,h__Todd(r)=0,h__Veven(r)=0,
  k__Teven(r)=0,v__even(r)=0,v__odd(r)=-k__Todd(r)+sqrt(M)*h__Vodd
  (r)*sqrt(2)*(1/sqrt(r)),hdot__Todd));
  hdotTTrace:=simplify(subs(h__rr(r)=BLH__rr, h__trace(r)=BLH__pp,
  HdotTTrace));
```

$$HdotTEven := 0$$

$$HdotTOdd := 0$$

$$hdotTTrace := \frac{1}{\sqrt{r}\,(2\,r+M)^5\,(-r+2\,M)}\Bigg(4\Bigg(\tag{132}$$

$$-\frac{19\,r^2\left(-\frac{1280\cos(\theta)^2\,P_l^4}{19}+\frac{1024\,P_l^4}{19}+n_L(r)\right)\sqrt{2}\,M^{11/2}}{2}-r\sqrt{2}\,\Big(256\cos(\theta)^2\,P_l^4$$

$$-256\,P_l^4+n_L(r)\Big)M^{13/2}-35\,r^3\left(\frac{256\cos(\theta)^2\,P_l^4}{35}-\frac{512\,P_l^4}{35}+n_L(r)\right)\sqrt{2}\,M^{9/2}$$

$$+\left(40\,M^2\,v_{scalar}(r)-8\,M\,k_{trace}(r)\right)r^{11/2}+\left(-8\,M\,v_{scalar}(r)-16\,k_{trace}(r)\right)r^{13/2}$$

$$+M^5\left(M\,v_{scalar}(r)+\frac{19\,k_{trace}(r)}{2}\right)r^{3/2}+\left(\frac{19\,M^5\,v_{scalar}(r)}{2}+35\,M^4\,k_{trace}(r)\right)r^{5/2}$$

$$+\left(35\,M^4\,v_{scalar}(r)+60\,M^3\,k_{trace}(r)\right)r^{7/2}+\left(60\,M^3\,v_{scalar}(r)+40\,M^2\,k_{trace}(r)\right)r^{9/2}$$

$$+16\,n_L(r)\,\sqrt{2}\,\sqrt{M}\,r^7+8\,M^{3/2}\,n_L(r)\,\sqrt{2}\,r^6+M^6\,\sqrt{r}\,k_{trace}(r)-40\,M^{5/2}\,n_L(r)\,\sqrt{2}\,r^5$$

$$\left.\left.-60\,M^{7/2}\,n_L(r)\,\sqrt{2}\,r^4-16\,r^{15/2}\,v_{scalar}(r)\right)\right)$$

Time derivative of K

```
> BLKdotR:=eval(subs(h__rr(r)=BLH__rr, h__trace(r)=BLH__pp,
  H__trace(r)=BLH__pp,KdotR));
```

$$BLKdotR:=\left(\frac{32\,M^4\,P_l^4}{r^6\left(1+\frac{M}{2\,r}\right)^4\left(1-\frac{2\,M}{r}\right)}-\frac{32\,M^4\,P_l^4\sin(\theta)^2}{r^5\left(1+\frac{M}{2\,r}\right)^4}+\frac{5\,n_L(r)}{2\,r^2}-\frac{1}{r}\left(2\left(\right.\right.\right.$$

$$-\frac{64\,M^4\,P_l^4}{r^5\left(1+\frac{M}{2\,r}\right)^4\left(1-\frac{2\,M}{r}\right)}+\frac{32\,M^5\,P_l^4}{r^6\left(1+\frac{M}{2\,r}\right)^5\left(1-\frac{2\,M}{r}\right)}$$

$$\left.\left.\left.-\frac{32\,M^5\,P_l^4}{r^6\left(1+\frac{M}{2\,r}\right)^4\left(1-\frac{2\,M}{r}\right)^2}\right)\right)\right)M+\left(-\frac{\sqrt{2}\,k_{trace}(r)}{r^{5/2}}+\frac{\sqrt{2}\left(\frac{d}{dr}K_{rr}(r)\right)}{r^{3/2}}\right.$$

$$\left.+\frac{\sqrt{2}\left(\frac{d}{dr}v_{scalar}(r)\right)}{\sqrt{r}}-\frac{3\,\sqrt{2}\,v_{scalar}(r)}{4\,r^{3/2}}\right)\sqrt{M}+\frac{8\,M^4\,P_l^4\,\lambda}{r^5\left(1+\frac{M}{2\,r}\right)^4\left(1-\frac{2\,M}{r}\right)}\qquad\textbf{(133)}$$

$$+r\left(\frac{d^2}{dr^2}n_L(r)\right)+\frac{64\,M^4\,P_l^4}{r^5\left(1+\frac{M}{2\,r}\right)^4\left(1-\frac{2\,M}{r}\right)}-\frac{32\,M^5\,P_l^4}{r^6\left(1+\frac{M}{2\,r}\right)^5\left(1-\frac{2\,M}{r}\right)}$$

$$+\frac{32\,M^5\,P_l^4}{r^6\left(1+\frac{M}{2\,r}\right)^4\left(1-\frac{2\,M}{r}\right)^2}+\frac{96\,M^4\,P_l^4\sin(\theta)^2}{r^4\left(1+\frac{M}{2\,r}\right)^4}-\frac{192\,M^5\,P_l^4\sin(\theta)^2}{r^5\left(1+\frac{M}{2\,r}\right)^5}$$

$$+\frac{80\,M^6\,P_l^4\sin(\theta)^2}{r^6\left(1+\frac{M}{2\,r}\right)^6}$$

```
> BLKdotVEven:=eval(subs(h__rr(r)=BLH__rr, h__trace(r)=BLH__pp,
  H__trace(r)=BLH__pp, H__Teven=0, H__Veven=0,KdotVEven));
```

$$BLKdotVEven := -\frac{16\,M^5\,P_l^4}{r^6\left(1+\frac{M}{2\,r}\right)^4\left(1-\frac{2\,M}{r}\right)} + \left(\frac{\sqrt{2}\,\left(\frac{\mathrm{d}}{\mathrm{d}r}\,k_{Veven}(r)\right)}{\sqrt{r}} + \frac{\sqrt{2}\,v_{scalar}(r)}{2\,r^{3/2}} - \sqrt{2}\,\sqrt{r}\,\left(\frac{\mathrm{d}}{\mathrm{d}r}\,V_{even}(r)\right)\right)\sqrt{M} + r\left(\frac{\mathrm{d}}{\mathrm{d}r}\,N_L(r)\right) - \frac{16\,M^4\,P_l^4\,\sin(\theta)^2}{r^3\left(1+\frac{M}{2\,r}\right)^4} + \frac{16\,M^5\,P_l^4\,\sin(\theta)^2}{r^4\left(1+\frac{M}{2\,r}\right)^5} - \frac{8\,M^4\,P_l^4}{r^5\left(1+\frac{M}{2\,r}\right)^4\left(1-\frac{2\,M}{r}\right)} \quad \textbf{(134)}$$

```
> BLKdotVOdd:=eval(subs(h__rr(r)=BLH__rr, h__trace(r)=BLH__pp,
  H__trace(r)=BLH__pp, H__Teven=0,H__Todd=0, H__Veven=0, H__Vodd=0,
  KdotVOdd));
```

$$BLKdotVOdd := \left(\frac{\sqrt{2}\,\left(\frac{\mathrm{d}}{\mathrm{d}r}\,k_{Vodd}(r)\right)}{\sqrt{r}} - \sqrt{2}\,\sqrt{r}\,\left(\frac{\mathrm{d}}{\mathrm{d}r}\,V_{odd}(r)\right)\right)\sqrt{M} \quad \textbf{(135)}$$

```
> BLKdotTEven:=eval(subs(h__rr(r)=BLH__rr, h__trace(r)=BLH__pp,
  H__trace(r)=BLH__pp, H__Teven=0, H__Veven=0,KdotTEven));
```

$$BLKdotTEven := n_L(r) + \frac{8\,M^4\,P_l^4}{r^4\left(1+\frac{M}{2\,r}\right)^4\left(1-\frac{2\,M}{r}\right)} \quad \textbf{(136)}$$

```
> BLKdotTOdd:=expand(eval(subs(h__rr(r)=BLH__rr, h__trace(r)=
  BLH__pp, H__trace(r)=BLH__pp, H__Teven=0, H__Veven=0,KdotTOdd)));
```

$$BLKdotTOdd := -\frac{\sqrt{2}\,\sqrt{M}\,k_{Todd}(r)}{2\,r^{3/2}} + \frac{\sqrt{2}\,\sqrt{M}\,\left(\frac{\mathrm{d}}{\mathrm{d}r}\,k_{Todd}(r)\right)}{\sqrt{r}} - \left(\frac{\mathrm{d}}{\mathrm{d}r}\,h_{Vodd}(r)\right) \quad \textbf{(137)}$$

```
> BLKdotTTrace:=eval(subs(h__rr(r)=BLH__rr, h__trace(r)=BLH__pp,
  H__trace(r)=BLH__pp, H__Teven=0, H__Veven=0,KdotTTrace));
```

$$BLKdotTTrace := \frac{M\,n_L(r)}{r} + \left(\sqrt{r}\,\sqrt{2}\,k_{rr}(r) - \frac{\sqrt{2}\,k_{trace}(r)}{2\,r^{3/2}} - \frac{\sqrt{2}\,v_{scalar}(r)}{2\,\sqrt{r}} + \frac{\sqrt{2}\,\left(\frac{\mathrm{d}}{\mathrm{d}r}\,k_{trace}(r)\right)}{\sqrt{r}}\right)\sqrt{M} + \frac{16\left(\frac{\lambda}{4}-1\right)M^4\,P_l^4}{r^4\left(1+\frac{M}{2\,r}\right)^4\left(1-\frac{2\,M}{r}\right)} + \frac{n_L(r)\,\lambda}{2} + \left(\frac{\mathrm{d}}{\mathrm{d}r}\,n_L(r)\right)r - \frac{1}{2}\left(r\left(-\frac{64\,M^4\,P_l^4}{r^5\left(1+\frac{M}{2\,r}\right)^4\left(1-\frac{2\,M}{r}\right)} + \frac{32\,M^5\,P_l^4}{r^6\left(1+\frac{M}{2\,r}\right)^5\left(1-\frac{2\,M}{r}\right)}\right.\right.$$

$$
- \frac{32\, M^5\, P_l^4}{r^6 \left(1 + \frac{M}{2\, r}\right)^4 \left(1 - \frac{2\, M}{r}\right)^2} \Bigg)\Bigg) + \frac{48\, M^4\, P_l^4 \sin(\theta)^2}{r^4 \left(1 + \frac{M}{2\, r}\right)^4} - \frac{96\, M^5\, P_l^4 \sin(\theta)^2}{r^5 \left(1 + \frac{M}{2\, r}\right)^5}
$$

$$
+ \frac{40\, M^6\, P_l^4 \sin(\theta)^2}{r^6 \left(1 + \frac{M}{2\, r}\right)^6} + \frac{8\, \lambda\, M^4\, P_l^4 \sin(\theta)^2}{r^4 \left(1 + \frac{M}{2\, r}\right)^4}
$$