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<u>Assignment 2 Question 2</u>

Prove: rev_append l1 l2 = rev_append' l1 l2

Because *let rev_append' l1 l2 = append (rev l1) l2*

Final Proof: rev_append l1 l2 = append (rev l1) l2

Lemma: append l1 l2 = l1 @ l2

Base Case: l1 = [ ]

$\Rightarrow$ append [ ] l2

$\Rightarrow$ l2 by append

$\Rightarrow$ [ ] @ l2

$\Rightarrow$ l2 by @ operator

Step Case: l1 = h : : t

IH: append t l2 = t @ l2 for all l2

Prove that: append (h : : t) = (h : : t) @ l2

append (h : : t)

$\Rightarrow$ h : : append (t l2) by append

$\Rightarrow$ h : : (t @ l2) by I.H.

$\Leftarrow$ (h : : t) @ l2 by transitivity

Main Proof: **rev_append l1 l2 = append (rev l1) l2**

Base Case: l1 = [ ]

rev_append [ ] l2

$\Rightarrow$ l2 by rev_append

append (rev [ ]) l2
$\Rightarrow$ append [ ] l2 by rev
$\Rightarrow$ l2 by append

Step Case: l1 = h : : t

I.H. : rev_append t l2' = append (rev t) l2' for all l2

Prove that: rev_append (h : : t) l2 = append (rev (h::t)) l2

rev_append (h : : t) l2

$\Rightarrow$ rev_append t (h : : l2) by rev_append

$\Rightarrow$ rev_append t l2' by rev_append

$\Leftarrow$ append (rev t) l2' by I.H.

append (rev (h::t)) l2

$\Rightarrow$ append ((rev t) @ [h]) l2 by rev

$\Rightarrow$ ((rev t) @ [h]) @ l2 by lemma

$\Rightarrow$ (rev t) @ ([h] @ l2) by associativity of @

$\Rightarrow$ (rev t) @ (h : : l2) by unrolling def. of @

$\Rightarrow$ append (rev t) l2' by lemma

$\Leftarrow$ rev_append t l2' by I.H.