Proposal: Verification test for Euler traveling vortex in libCEED

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Overview

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2. Description of Technical Tasks
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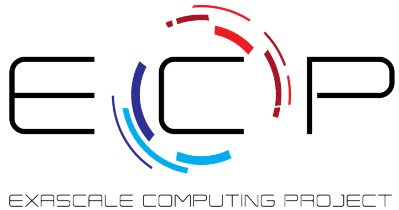
Introduction to libCEED

- The CEED API library
- The focus is High-order FEM
- Provides a portable algebraic interface, hence applicable for other numerical methods
- Includes optimized implementations for CPU and GPU
- Includes library examples
  - Fluid Mechanics (Navier-Stokes and Shallow Water)
  - Solid Mechanics (Elasticity)



Description of Technical Tasks

Euler Equations

$$\begin{aligned} \frac{\partial \rho}{\partial t} + \nabla \cdot \boldsymbol{U} &= 0 \\ \frac{\partial \boldsymbol{U}}{\partial t} + \nabla \cdot \left(\frac{\boldsymbol{U} \otimes \boldsymbol{U}}{\rho} + P \boldsymbol{I}_3 \right) &= 0 \\ \frac{\partial E}{\partial t} + \nabla \cdot \left(\frac{(E+P)\boldsymbol{U}}{\rho} \right) &= 0 \end{aligned} \tag{1}$$

ρ = Volume Mass Density
$\boldsymbol{U}$ = Momentum Density, $\boldsymbol{U} = \rho \boldsymbol{u}$ and $\boldsymbol{u}$ = the vector velocity field
E = Total Energy Density, $E = \rho e$ and e = Total Energy
P = Pressure

$$P = (\gamma - 1)\left(E - \rho \frac{\boldsymbol{U} \cdot \boldsymbol{U}}{2} \right) \tag{2}$$

$\gamma = c_p / c_v$

Description of Technical Tasks

Exact Solutions

$\boldsymbol{u} = \bar{\boldsymbol{u}} + \delta \boldsymbol{u}$
$T = \bar{T} + \delta T$
where $\bar{\rho} = 1$, $\bar{P} = 1$, and $\bar{\boldsymbol{u}} = (1, 1, 0)$ represent the mean flow

$$(\delta u_1,\, \delta u_2) = \frac{\epsilon}{2\pi}\, e^{0.5(1-r^2)}\, (-\bar{y},\, \bar{x})\,, \tag{3}$$

$$\delta T = -\frac{(\gamma - 1)\, \epsilon^2}{8\, \gamma\, \pi^2}\, e^{1-r^2} \tag{4}$$

$(\bar{x},\, \bar{y}) = (x - x_c,\, y - y_c)$, $(x_c,\, y_c) =$ Center of the Domain

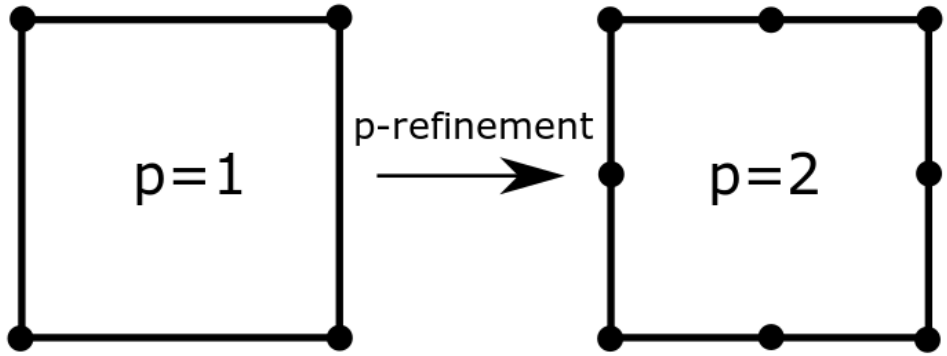
$r^2 = \bar{x}^2 + \bar{y}^2$

$\epsilon =$ Vortex strength

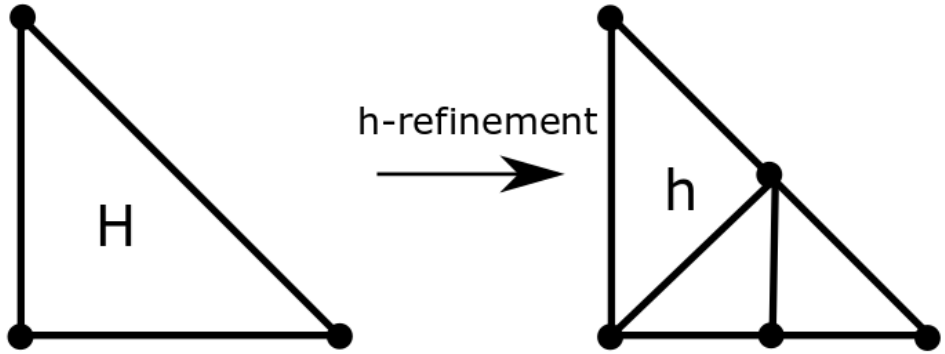
Description of Technical Tasks

Verification Test

$||Exact\ Solution - Numerical\ Result||@p$

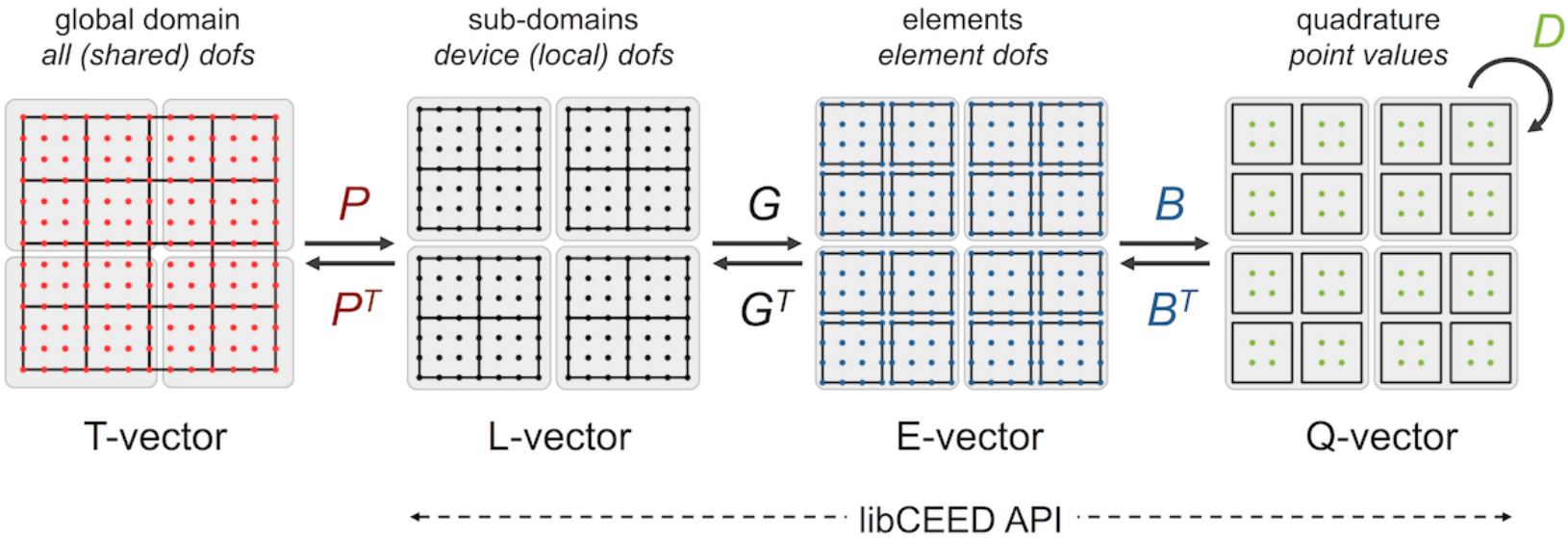


$||Exact\ Solution - Numerical\ Result||@h$



Description of Risks

$$A = P^T G^T B^T D B G P$$



Currently, PETSc lacks proper periodic BCs and this might affect the **accuracy** and **stability** of the results.

Question