CHLADNI PLATE ANALYSIS

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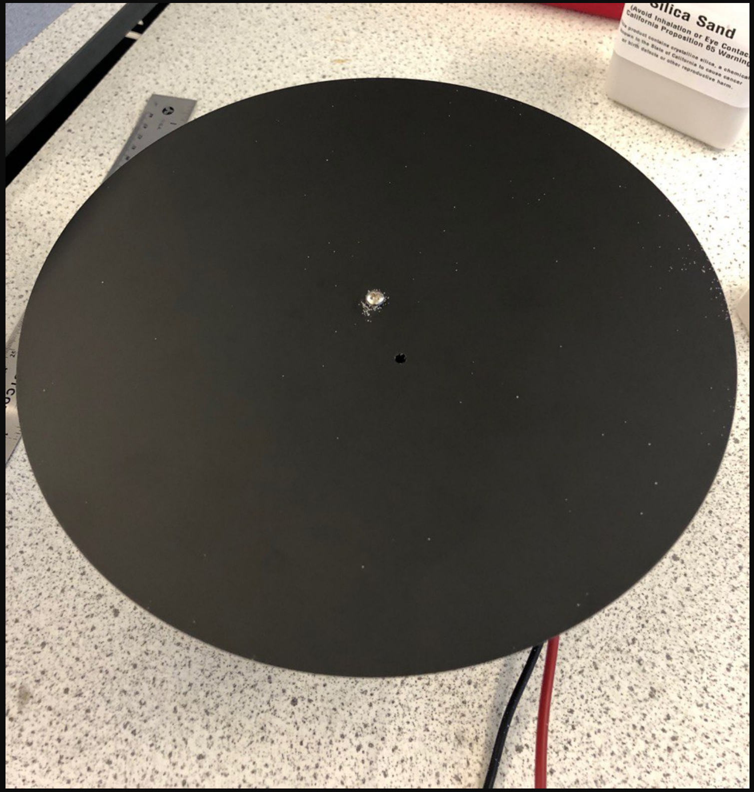
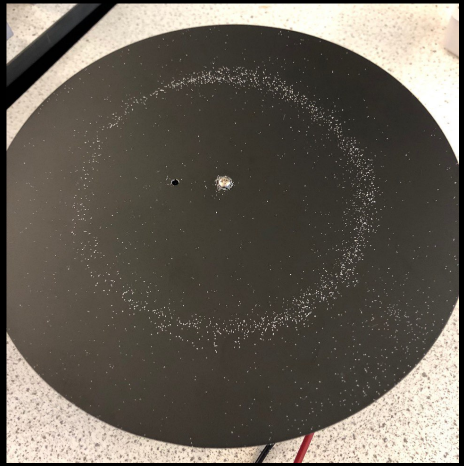
FALL 2019



EXPERIMENT

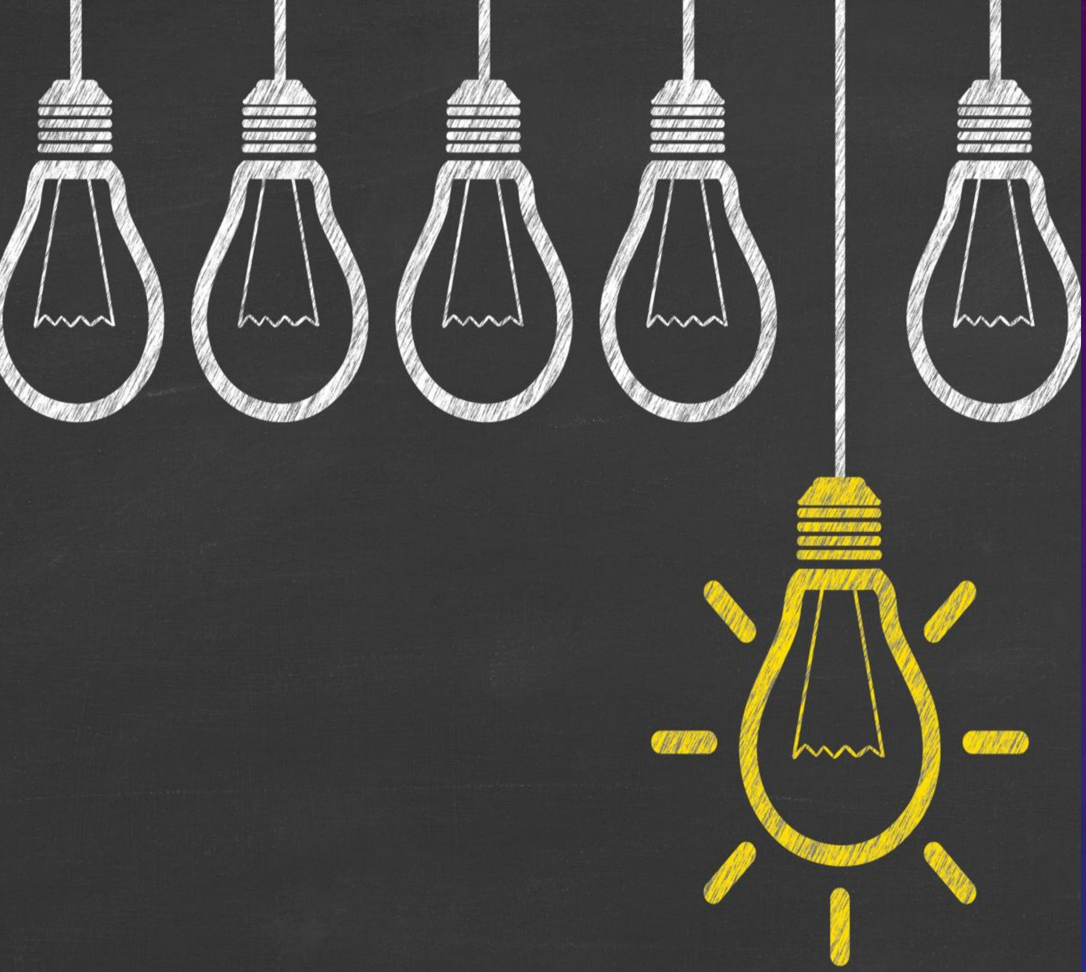


$f = 100\ Hz$

$f = 350\ Hz$

$f = 900\ Hz$

$f = 1680\ Hz$



THEORETICAL MODEL

$$\boldsymbol{\nabla^4 u = -\beta \frac{\partial^2 u}{\partial t^2}}$$

$$\beta = \frac{\rho\, h}{D}$$

$$D = \frac{E h^3}{12(1 - \nu^2)}$$

$$u(r,\theta,t) = \Psi_{mn}(r,\theta)T(t)$$

- $$\frac{d^2T(t)}{dt^2} + \omega^2 T(t) = 0$$

  $\longrightarrow$ $T(t) = D_1 \,\mathrm{Cos}(\omega t) + D_2 \,\mathrm{Sin}(\omega t)$

- $$\nabla^4\Psi(r,\theta) - \lambda^4\Psi(r,\theta) = 0, \qquad \lambda^4 = \frac{\rho h \omega^2}{D}$$

$$\nabla^2\Psi(r,\theta) - \lambda^2\Psi(r,\theta) = 0 \qquad \nabla^2\Psi(r,\theta) + \lambda^2\Psi(r,\theta) = 0$$

$$\Psi_{mn}(r,\theta) = \underset{?}{U_{mn}(r)}\underbrace{(A_m\cos(m\theta) + B_m\sin(m\theta))}_{\substack{m=0 \\ \text{Symmetric Modes}}}$$

$$U_{nm}(r) = \overset{?}{C^{1}_{mn}} J_m(\underset{?}{\lambda_{mn}} r) + \overset{?}{C^{2}_{mn}} Y_m(\underset{?}{\lambda_{mn}} r) + \overset{?}{C^{3}_{mn}} I_m(\underset{?}{\lambda_{mn}} r) + \overset{?}{C^{4}_{mn}} K_m(\underset{?}{\lambda_{mn}} r)$$

BOUNDARY CONDITIONS

Boundary Conditions: Set I (Full Disk)

- $\left. u(r,\theta,t)\right|_{r=0} = finite$
- $\left. \frac{1}{r}\frac{\partial u}{\partial r}\right|_{r=0} = finite$
- $\left. \frac{\partial^2 u}{\partial r^2}\right|_{r=R} = 0$
- $\left. \frac{\partial^3 u}{\partial r^3}\right|_{r=R} = 0$

Table 1. Natural Frequency (Hz) for set I				
Theoretical Data	117.61	607	1409.80	2531.87
Experimental Data	100	350	900	1680

$$U_{mn}(r) = C_{mn}^{1} J_m(\lambda_{mn} r) + \cancelto{0}{C_{mn}^{2}} Y_m(\lambda_{mn} r) + C_{mn}^{3} I_m(\lambda_{mn} r) + \cancelto{0}{C_{mn}^{4}} K_m(\lambda_{mn} r)$$

Boundary Conditions: Set II (Annular Disk)

- $\boldsymbol{u(r,\theta,t)}\Big|_{r=R_i} = 0$
- $\left.\dfrac{\partial \boldsymbol{u}}{\partial \boldsymbol{r}}\right|_{r=R_i} = 0$
- $\left.\dfrac{\partial^2 \boldsymbol{u}}{\partial \boldsymbol{r}^2}\right|_{r=R} = 0$
- $\left.\dfrac{\partial^3 \boldsymbol{u}}{\partial \boldsymbol{r}^3}\right|_{r=R} = 0$

Table 2. Natural Frequency (Hz) for set II				
Theoretical Data	42.32	324.56	991.52	1980.55
Experimental Data	100	350	900	1680

$$U_{mn}(r) = C_{mn}^1 J_m(\lambda_{mn} r) + C_{mn}^2 Y_m(\lambda_{mn} r) + C_{mn}^3 I_m(\lambda_{mn} r) + C_{mn}^4 K_m(\lambda_{mn} r)$$

WHAT COULD BE THE PROBLEM?!

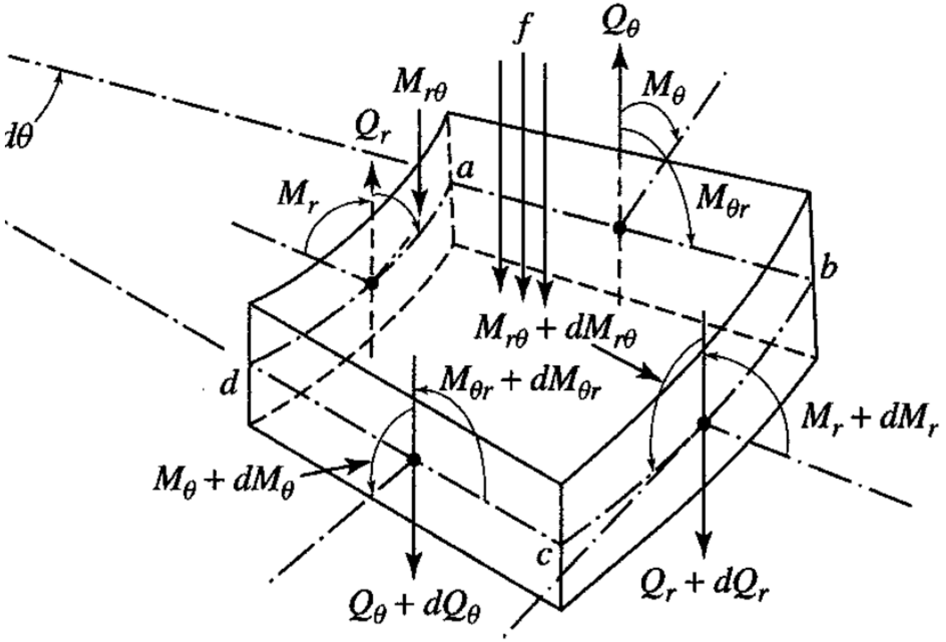
Moment Force

$$M_r(r,\theta,t) = -D\left[\frac{\partial^2 u}{\partial r^2} + \frac{\nu}{r}\frac{\partial u}{\partial r} + \frac{\nu}{r^2}\frac{\partial^2 u}{\partial \theta^2}\right]$$

Shear Force

$$V_r(r,\theta,t) = -D\left[\frac{\partial}{\partial r}\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}\right) + \frac{1-\nu}{r^2}\frac{\partial^2}{\partial \theta^2}\left(\frac{\partial u}{\partial r} - \frac{u}{r}\right)\right]$$

$$D\nabla^4 u + \rho h \frac{\partial^2 u}{\partial t^2} = f$$



Boundary Conditions: Set III (Full Disk)

- ※ $\boldsymbol{u(r,\theta,t)}\Big|_{r=0} = \boldsymbol{finite}$
- ※ $\dfrac{1}{\boldsymbol{r}}\dfrac{\partial \boldsymbol{u}}{\partial \boldsymbol{r}}\Big|_{r=0} = \boldsymbol{finite}$
- ※ $\boldsymbol{M_r(r,\theta,t)}\Big|_{r=R} = -\boldsymbol{D}\left[\dfrac{\partial^2 \boldsymbol{u}}{\partial \boldsymbol{r}^2} + \dfrac{\boldsymbol{\nu}}{\boldsymbol{r}}\dfrac{\partial \boldsymbol{u}}{\partial \boldsymbol{r}} + \dfrac{\boldsymbol{\nu}}{\boldsymbol{r}^2}\dfrac{\partial^2 \boldsymbol{u}}{\partial \boldsymbol{\theta}^2}\right]_{r=R} = 0$
- ※ $\boldsymbol{V_r(r,\theta,t)}\Big|_{r=R} = -\boldsymbol{D}\left[\dfrac{\partial}{\partial \boldsymbol{r}}\left(\dfrac{\partial^2 \boldsymbol{u}}{\partial \boldsymbol{r}^2} + \dfrac{1}{\boldsymbol{r}}\dfrac{\partial \boldsymbol{u}}{\partial \boldsymbol{r}} + \dfrac{1}{\boldsymbol{r}^2}\dfrac{\partial^2 \boldsymbol{u}}{\partial \boldsymbol{\theta}^2}\right) + \dfrac{1-\boldsymbol{\nu}}{\boldsymbol{r}^2}\dfrac{\partial^2}{\partial \boldsymbol{\theta}^2}\left(\dfrac{\partial \boldsymbol{u}}{\partial \boldsymbol{r}} - \dfrac{\boldsymbol{u}}{\boldsymbol{r}}\right)\right]_{r=R} = 0$

Table 3. Natural frequency (Hz) for set III				
Theoretical Data	147.17	624.91	1425.06	2545.91
Experimental Data	100	350	900	1680

$$U_{mn}(r) = C_{mn}^{1} J_m(\lambda_{mn} r) + \cancelto{0}{C_{mn}^{2}} Y_m(\lambda_{mn} r) + C_{mn}^{3} I_m(\lambda_{mn} r) + \cancelto{0}{C_{mn}^{4}} K_m(\lambda_{mn} r)$$

Boundary Conditions: Set IV (Annular Disk)

Table 4. Natural frequency (Hz) for set IV

Theoretical Data	61.75	347.05	1007.94	1995.29
Experimental Data	100	350	900	1680

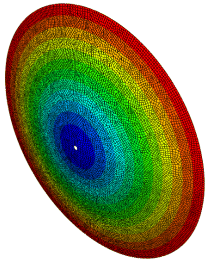
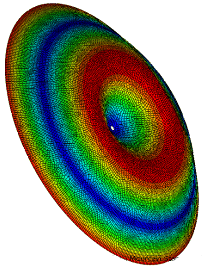
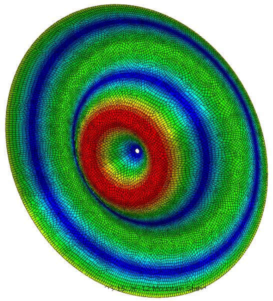
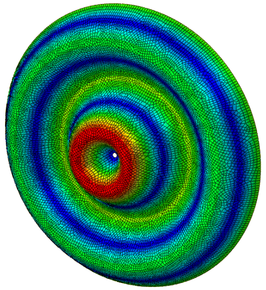
- ※ $\boldsymbol{u}(\boldsymbol{r},\boldsymbol{\theta},\boldsymbol{t})\Big|_{r=R_i} = 0$
- ※ $\frac{\partial \boldsymbol{u}}{\partial \boldsymbol{r}}\Big|_{r=R_i} = 0$
- ※ $\boldsymbol{M_r}(\boldsymbol{r},\boldsymbol{\theta},\boldsymbol{t})\Big|_{r=R} = -\boldsymbol{D}\left[\frac{\partial^2 \boldsymbol{u}}{\partial \boldsymbol{r}^2} + \frac{\boldsymbol{\nu}}{\boldsymbol{r}}\frac{\partial \boldsymbol{u}}{\partial \boldsymbol{r}} + \frac{\boldsymbol{\nu}}{\boldsymbol{r}^2}\frac{\partial^2 \boldsymbol{u}}{\partial \boldsymbol{\theta}^2}\right]_{r=R} = 0$
- ※ $\boldsymbol{V_r}(\boldsymbol{r},\boldsymbol{\theta},\boldsymbol{t})\Big|_{r=R} = -\boldsymbol{D}\left[\frac{\partial}{\partial \boldsymbol{r}}\left(\frac{\partial^2 \boldsymbol{u}}{\partial \boldsymbol{r}^2} + \frac{1}{\boldsymbol{r}}\frac{\partial \boldsymbol{u}}{\partial \boldsymbol{r}} + \frac{1}{\boldsymbol{r}^2}\frac{\partial^2 \boldsymbol{u}}{\partial \boldsymbol{\theta}^2}\right) + \frac{1-\boldsymbol{\nu}}{\boldsymbol{r}^2}\frac{\partial^2}{\partial \boldsymbol{\theta}^2}\left(\frac{\partial \boldsymbol{u}}{\partial \boldsymbol{r}} - \frac{\boldsymbol{u}}{\boldsymbol{r}}\right)\right]_{r=R} = 0$

$$U_{mn}(r) = C_{mn}^1 J_m(\lambda_{mn} r) + C_{mn}^2 Y_m(\lambda_{mn} r) + C_{mn}^3 I_m(\lambda_{mn} r) + C_{mn}^4 K_m(\lambda_{mn} r)$$

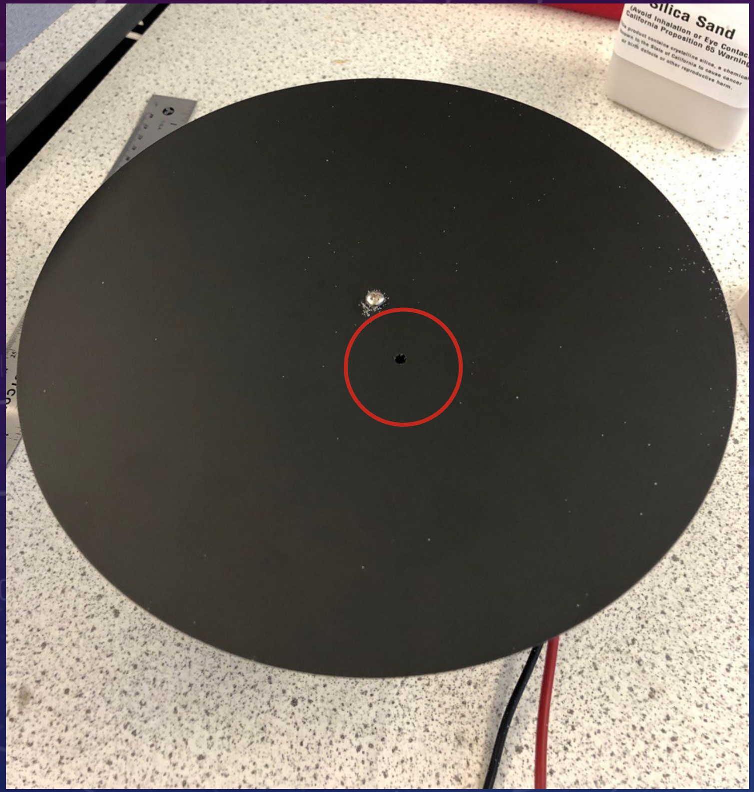
WHERE DOES THE ERROR COME FROM?

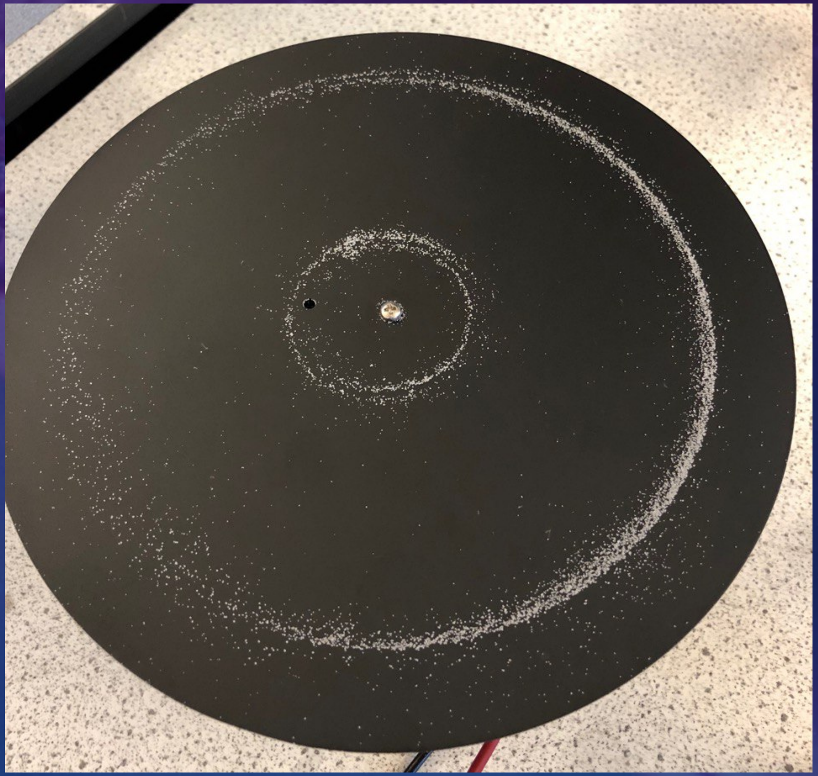
Table 5. Natural frequency (Hz) for set IV

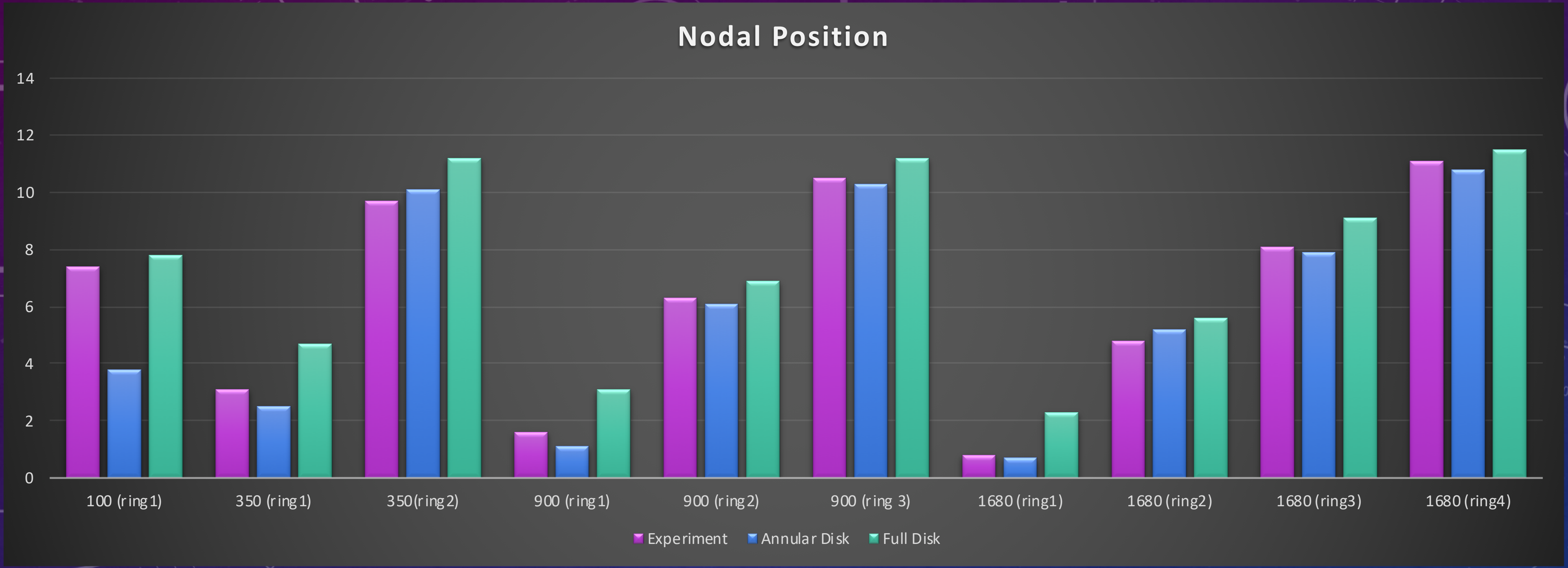
Analytical Solution	61.75	347.05	1007.94	1995.29
Numerical Solution	61.679	346.40	1006.2	1992.5
Experimental Data	100	350	900	1680

POSSIBLE ERRORS IN THE EXPERIMENT







WHERE ARE THE RINGS?