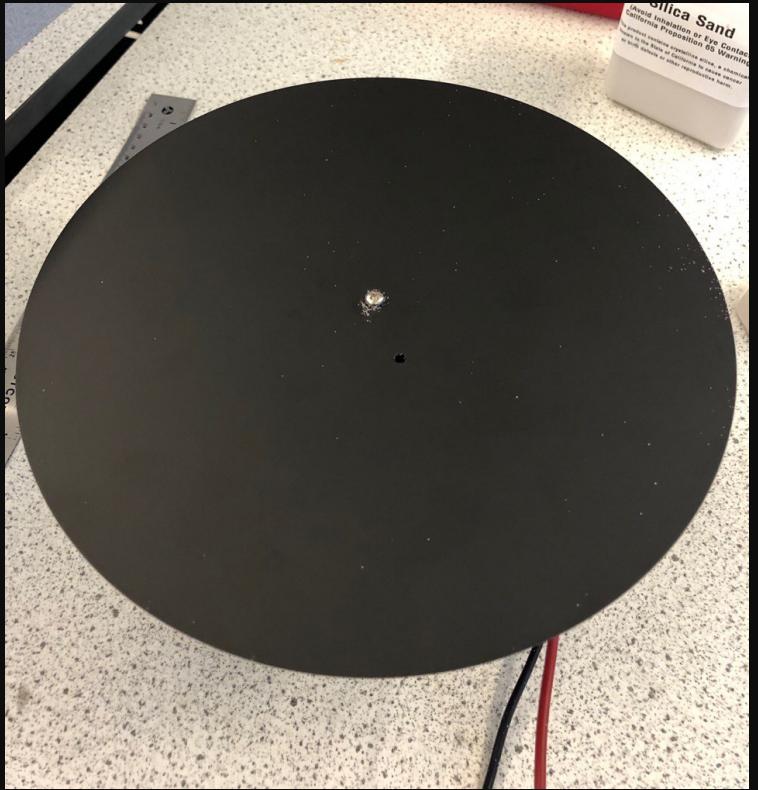


CHLADNI PLATE ANALYSIS

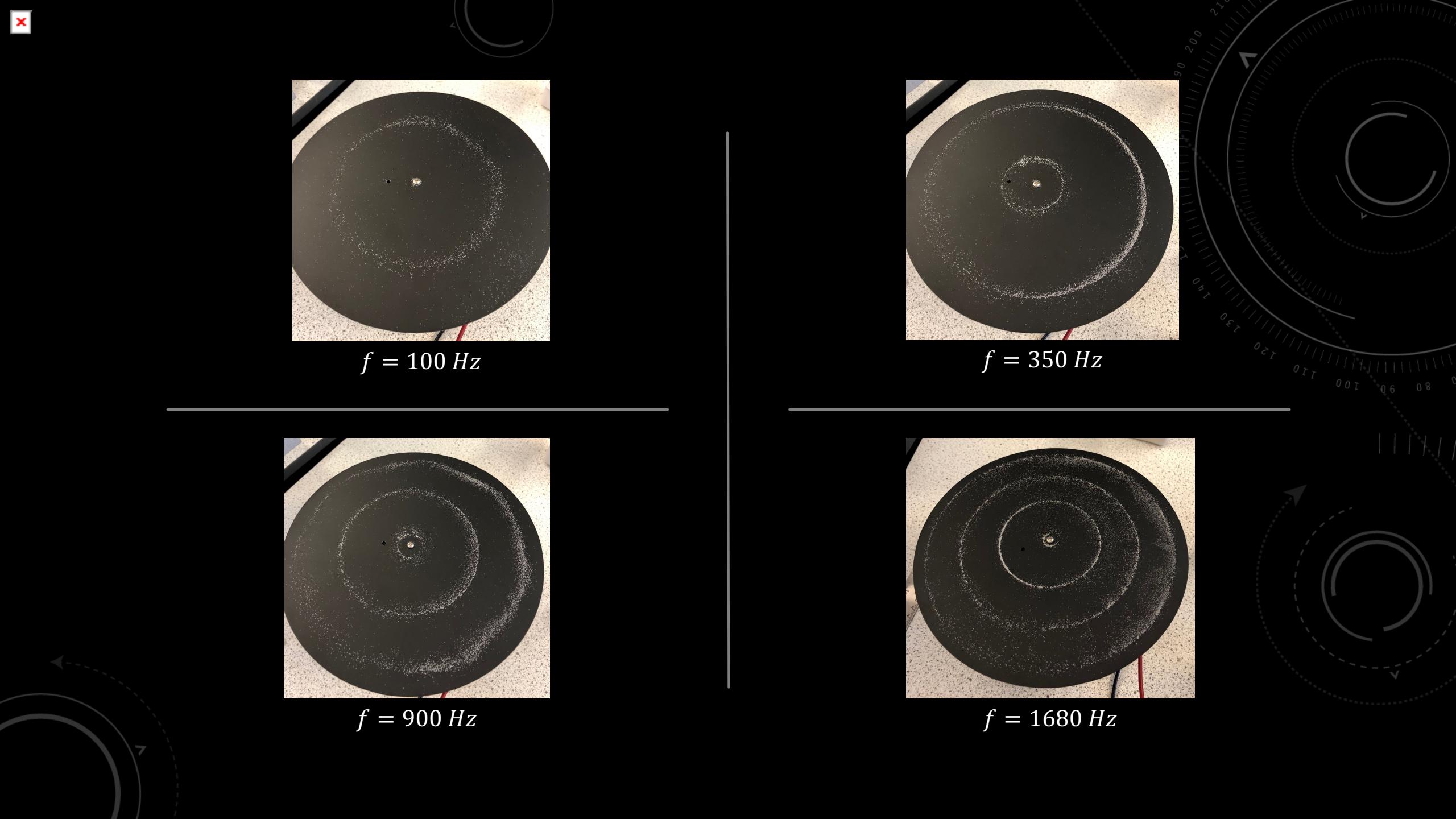
LEILA GHAFFARI

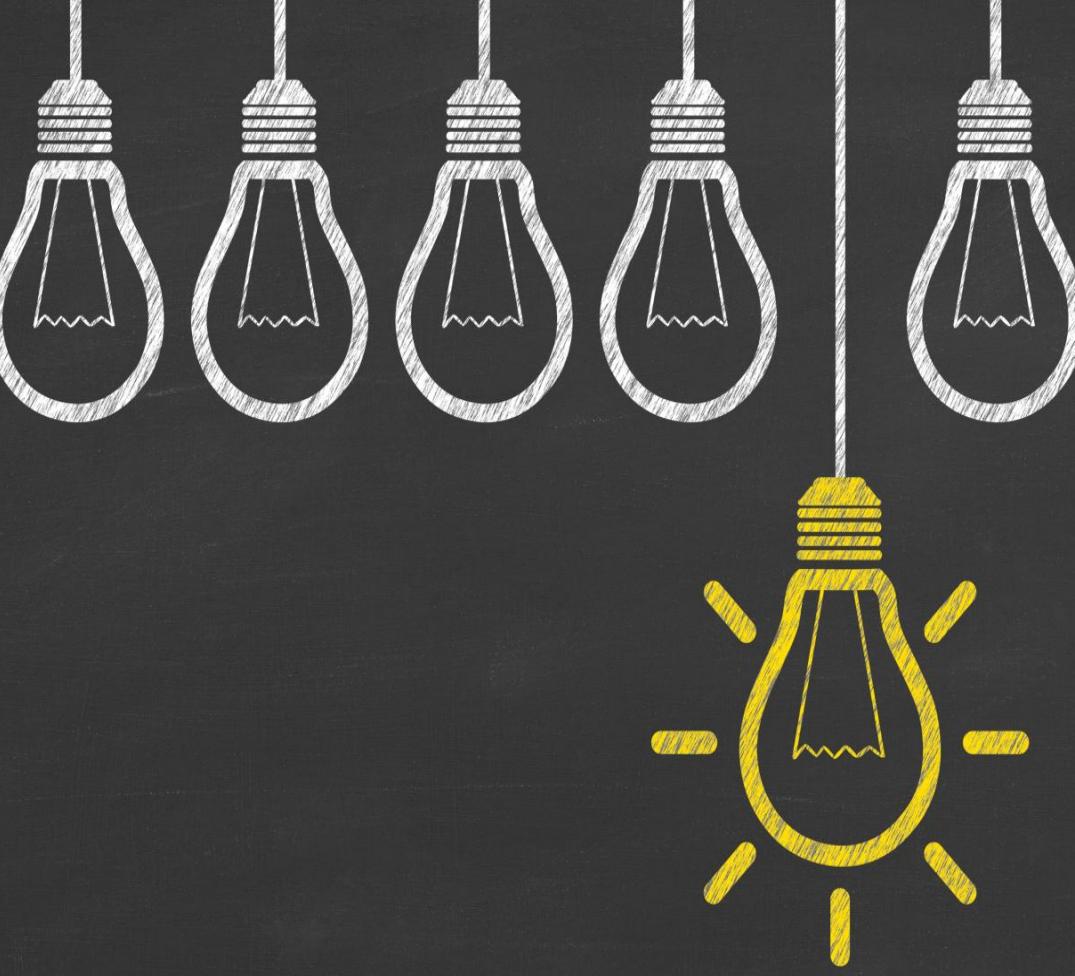
FALL 2019



EXPERIMENT







THEORETICAL MODEL

$$\nabla^4 u = -\beta \frac{\partial^2 u}{\partial t^2}$$

$$\beta = \frac{\rho h}{D}$$

$$D = \frac{Eh^3}{12(1-\nu^2)}$$

$$u(r, \theta, t) = \Psi_{mn}(r, \theta)T(t)$$

$$* \frac{d^2 T(t)}{dt^2} + \omega^2 T(t) = 0$$



$$T(t) = D_1 \cos(\omega t) + D_2 \sin(\omega t)$$

$$* \nabla^4 \Psi(r, \theta) - \lambda^4 \Psi(r, \theta) = 0, \quad \lambda^4 = \frac{\rho h \omega^2}{D}$$

$$\nabla^2 \Psi(r, \theta) - \lambda^2 \Psi(r, \theta) = 0$$

$$\nabla^2 \Psi(r, \theta) + \lambda^2 \Psi(r, \theta) = 0$$

$$\Psi_{mn}(r, \theta) = U_{mn}(r) (A_m \cos(m\theta) + B_m \sin(m\theta))$$

?

$m = 0$

Symmetric Modes

$$U_{nm}(\mathbf{r}) = C_{mn}^1 J_m(\lambda_{mn} r) + C_{mn}^2 Y_m(\lambda_{mn} r) + C_{mn}^3 I_m(\lambda_{mn} r) + C_{mn}^4 K_m(\lambda_{mn} r)$$

?

?

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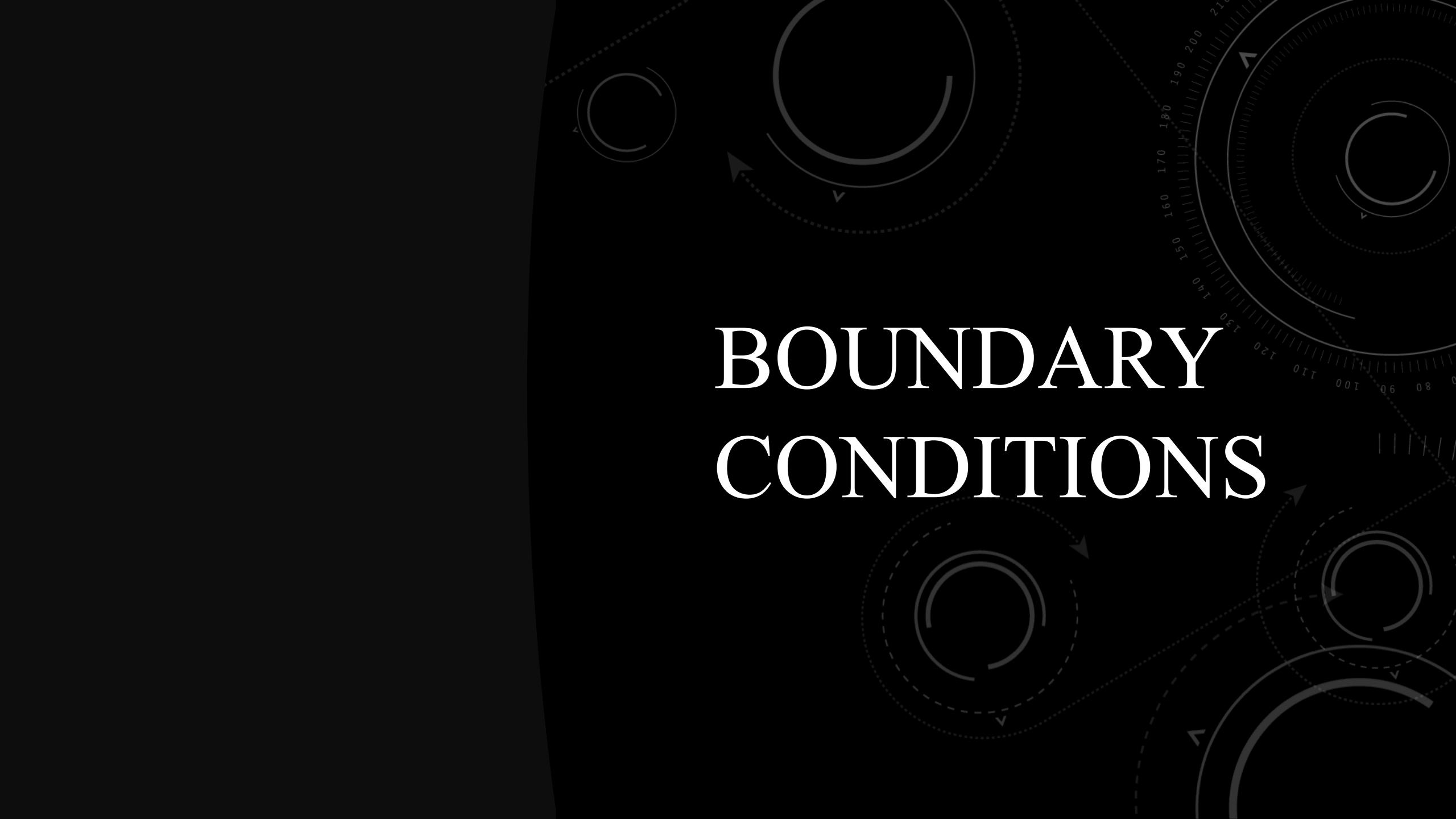
?

?

?

?

BOUNDARY CONDITIONS

The background features a complex arrangement of concentric circles and arcs in various shades of gray. Some circles have arrows pointing clockwise or counter-clockwise around them. A prominent feature is a large circle in the upper center with a thick gray arc at the top. Dotted lines connect some of the circles. In the lower right, there's a cluster of circles with dashed outlines and arrows, suggesting a more dynamic or turbulent flow pattern.

Boundary Conditions: Set I (Full Disk)

$$* u(r, \theta, t) \Big|_{r=0} = \text{finite}$$

$$* \frac{1}{r} \frac{\partial u}{\partial r} \Big|_{r=0} = \text{finite}$$

$$* \frac{\partial^2 u}{\partial r^2} \Big|_{r=R} = 0$$

$$* \frac{\partial^3 u}{\partial r^3} \Big|_{r=R} = 0$$

$$U_{mn}(r) = C_{mn}^1 J_m(\lambda_{mn} r) + C_{mn}^2 Y_m(\lambda_{mn} r) + C_{mn}^3 I_m(\lambda_{mn} r) + C_{mn}^4 K_m(\lambda_{mn} r)$$

0

0

Table 1. Natural Frequency (Hz) for set I

Theoretical Data	117.61	607	1409.80	2531.87
Experimental Data	100	350	900	1680

Boundary Conditions: Set II (Annular Disk)

$$* u(r, \theta, t) \Big|_{r=R_i} = 0$$

$$* \frac{\partial u}{\partial r} \Big|_{r=R_i} = 0$$

$$* \frac{\partial^2 u}{\partial r^2} \Big|_{r=R} = 0$$

$$* \frac{\partial^3 u}{\partial r^3} \Big|_{r=R} = 0$$

Table 2. Natural Frequency (Hz) for set II

Theoretical Data	42.32	324.56	991.52	1980.55
Experimental Data	100	350	900	1680

$$U_{mn}(r) = C_{mn}^1 J_m(\lambda_{mn} r) + C_{mn}^2 Y_m(\lambda_{mn} r) + C_{mn}^3 I_m(\lambda_{mn} r) + C_{mn}^4 K_m(\lambda_{mn} r)$$

WHAT COULD BE THE PROBLEM?!

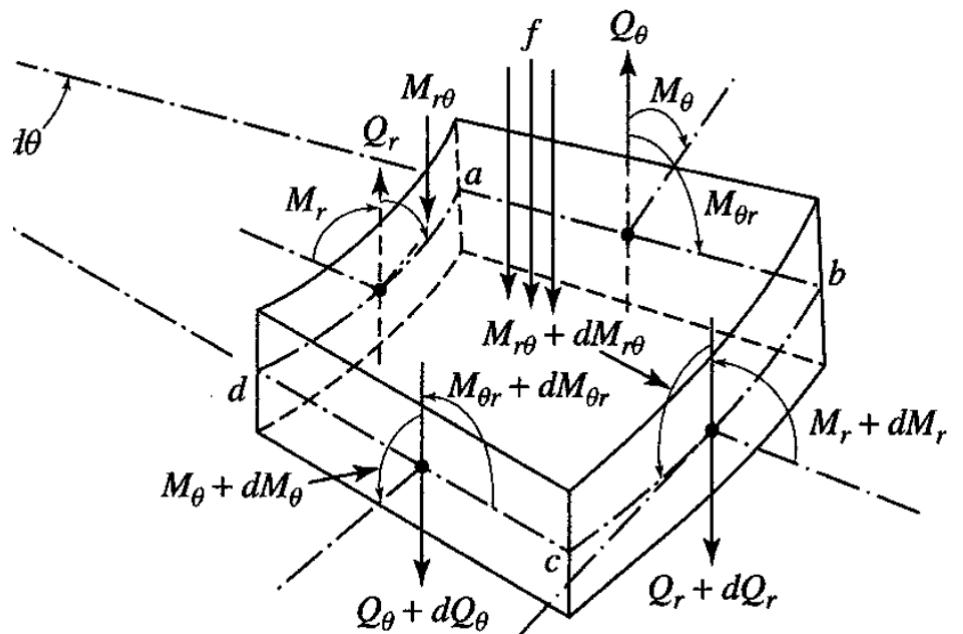
Moment Force

$$M_r(r, \theta, t) = -D \left[\frac{\partial^2 u}{\partial r^2} + \frac{v}{r} \frac{\partial u}{\partial r} + \frac{v}{r^2} \frac{\partial^2 u}{\partial \theta^2} \right]$$

Shear Force

$$V_r(r, \theta, t) = -D \left[\frac{\partial}{\partial r} \left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} + \frac{1}{r^2} \frac{\partial^2 u}{\partial \theta^2} \right) + \frac{1-v}{r^2} \frac{\partial^2}{\partial \theta^2} \left(\frac{\partial u}{\partial r} - u \right) \right]$$

$$D\nabla^4 u + \rho h \frac{\partial^2 u}{\partial t^2} = f$$



Boundary Conditions: Set III (Full Disk)

$$* \ u(r, \theta, t) \Big|_{r=0} = \text{finite}$$

$$* \frac{1}{r} \frac{\partial u}{\partial r} \Big|_{r=0} = \text{finite}$$

$$* \ M_r(r, \theta, t) \Big|_{r=R} = -D \left[\frac{\partial^2 u}{\partial r^2} + \frac{\nu \partial u}{r \partial r} + \frac{\nu}{r^2} \frac{\partial^2 u}{\partial \theta^2} \right]_{r=R} = 0$$

$$* \ V_r(r, \theta, t) \Big|_{r=R} = -D \left[\frac{\partial}{\partial r} \left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} + \frac{1}{r^2} \frac{\partial^2 u}{\partial \theta^2} \right) + \frac{1-\nu}{r^2} \frac{\partial^2}{\partial \theta^2} \left(\frac{\partial u}{\partial r} - \frac{u}{r} \right) \right]_{r=R} = 0$$

Table 3. Natural frequency (Hz) for set III

Theoretical Data	147.17	624.91	1425.06	2545.91
Experimental Data	100	350	900	1680

$$U_{mn}(r) = C_{mn}^1 J_m(\lambda_{mn} r) + C_{mn}^2 Y_m(\lambda_{mn} r) + C_{mn}^3 I_m(\lambda_{mn} r) + C_{mn}^4 K_m(\lambda_{mn} r)$$

Boundary Conditions: Set IV (Annular Disk)

$$* \quad u(r, \theta, t) \Big|_{r=R_i} = 0$$

$$* \quad \frac{\partial u}{\partial r} \Big|_{r=R_i} = 0$$

$$* \quad M_r(r, \theta, t) \Big|_{r=R} = -D \left[\frac{\partial^2 u}{\partial r^2} + \frac{\nu}{r} \frac{\partial u}{\partial r} + \frac{\nu}{r^2} \frac{\partial^2 u}{\partial \theta^2} \right]_{r=R} = 0$$

$$* \quad V_r(r, \theta, t) \Big|_{r=R} = -D \left[\frac{\partial}{\partial r} \left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} + \frac{1}{r^2} \frac{\partial^2 u}{\partial \theta^2} \right) + \frac{1-\nu}{r^2} \frac{\partial^2}{\partial \theta^2} \left(\frac{\partial u}{\partial r} - \frac{u}{r} \right) \right]_{r=R} = 0$$

Table 4. Natural frequency (Hz) for set IV

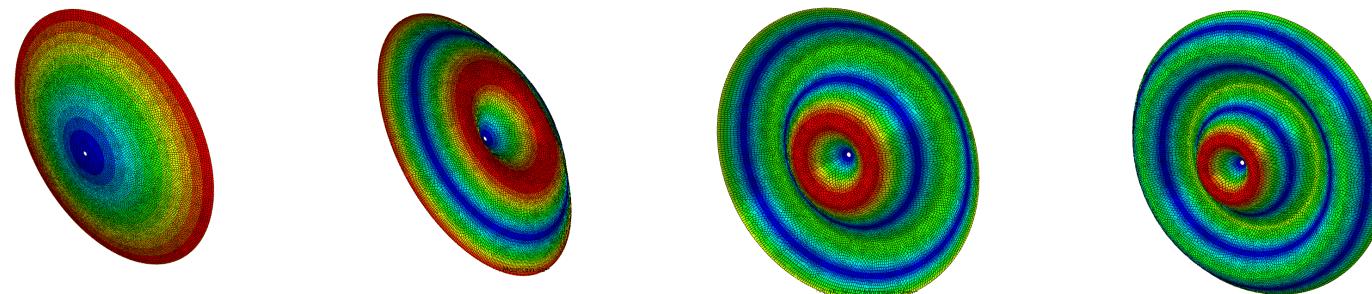
Theoretical Data	61.75	347.05	1007.94	1995.29
Experimental Data	100	350	900	1680

$$U_{mn}(r) = C_{mn}^1 J_m(\lambda_{mn} r) + C_{mn}^2 Y_m(\lambda_{mn} r) + C_{mn}^3 I_m(\lambda_{mn} r) + C_{mn}^4 K_m(\lambda_{mn} r)$$

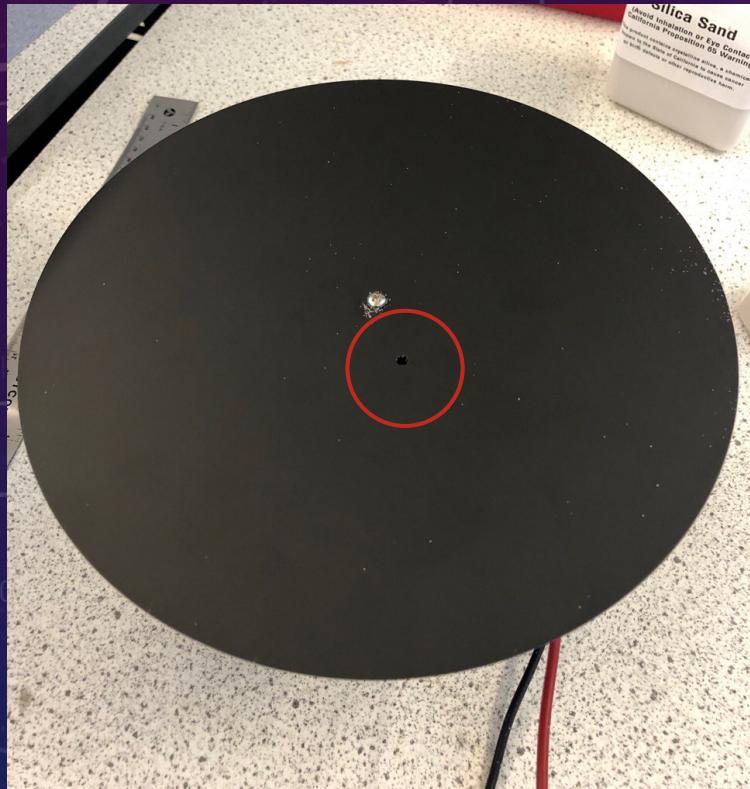
WHERE DOES THE ERROR COME FROM?

Table 5. Natural frequency (Hz) for set IV

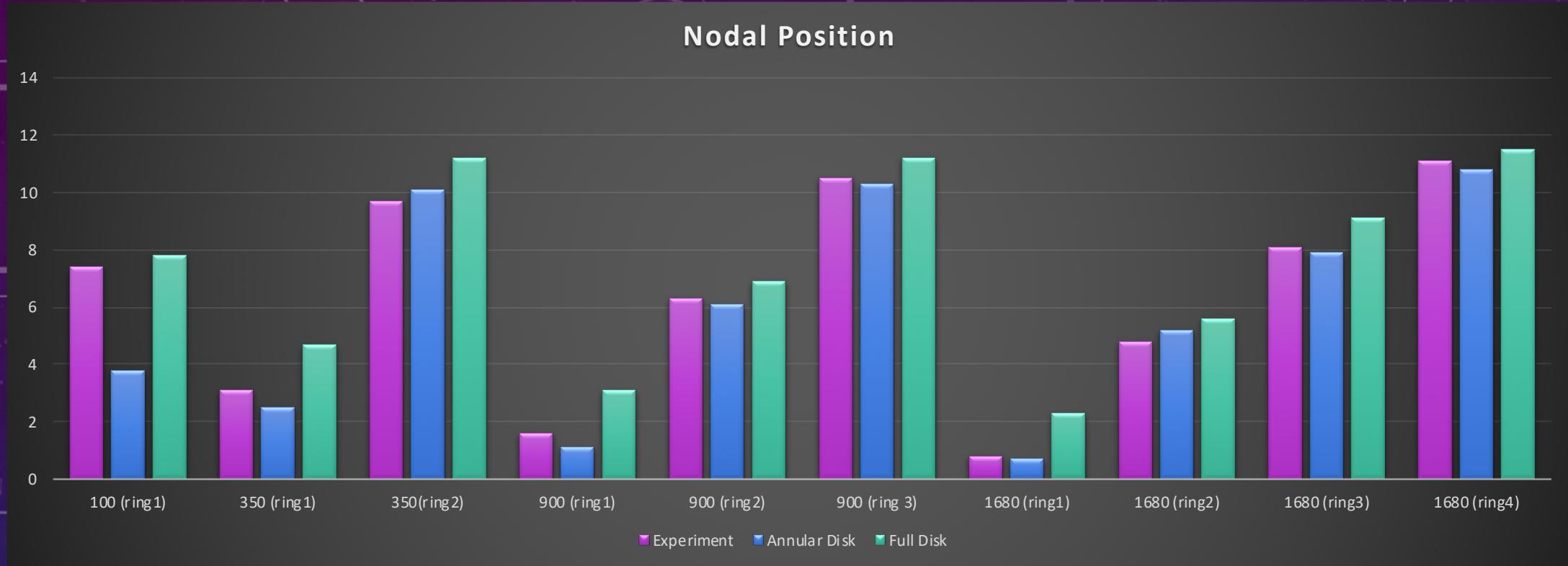
Analytical Solution	61.75	347.05	1007.94	1995.29
Numerical Solution	61.679	346.40	1006.2	1992.5
Experimental Data	100	350	900	1680



POSSIBLE ERRORS IN THE EXPERIMENT



Nodal Position



WHERE ARE THE RINGS?

