Graph Theory Overview

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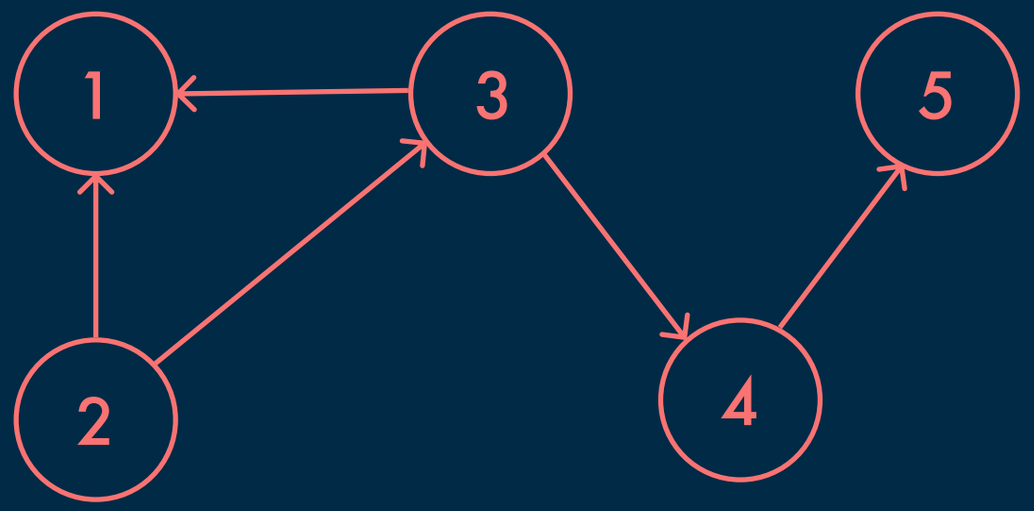
Graph Theory

In mathematics, graph theory is the study of graphs, being mathematical structures used to model paired interrelations between objects.

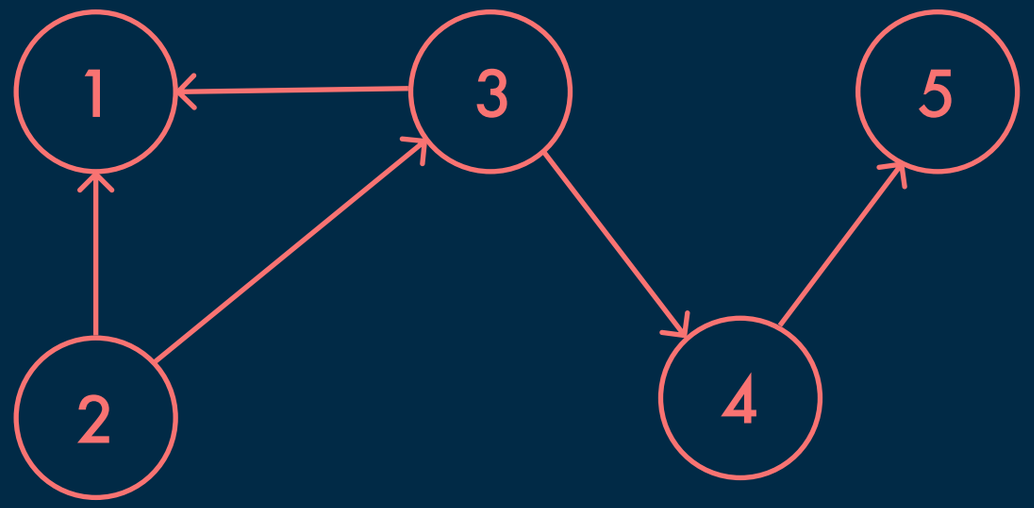
In terms of this context, a graph is composed of vertices (also called nodes or points) that are joined by edges (also called links or lines).



Directed Graph

A distinct type of graph, where edges link a pair of vertices asymmetrically.

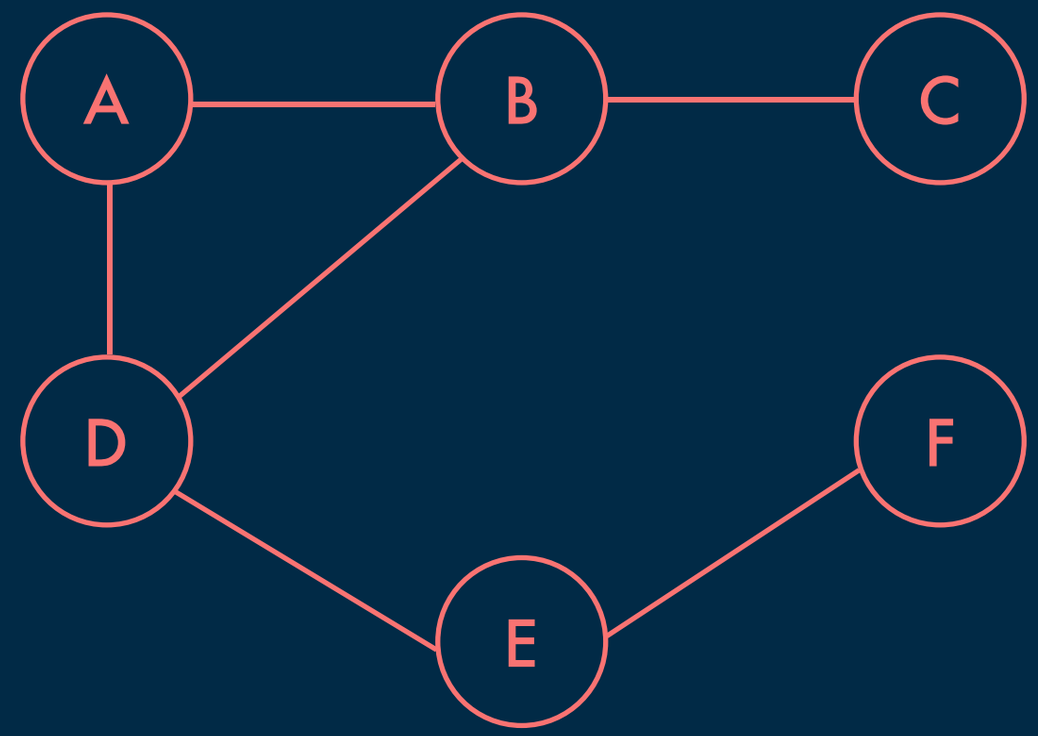
A Directed Graph G(V,E) is considered as a set of edges and vertices, where each edge has a direction.



Undirected Graph

A distinct type of graph, where edges link a pair of vertices symmetrically.

An Undirected Graph G(V,E) is considered as a set of edges and vertices, where each edge is bidirectional.



*edge (U, V) is equivalent to the edge (V,U)

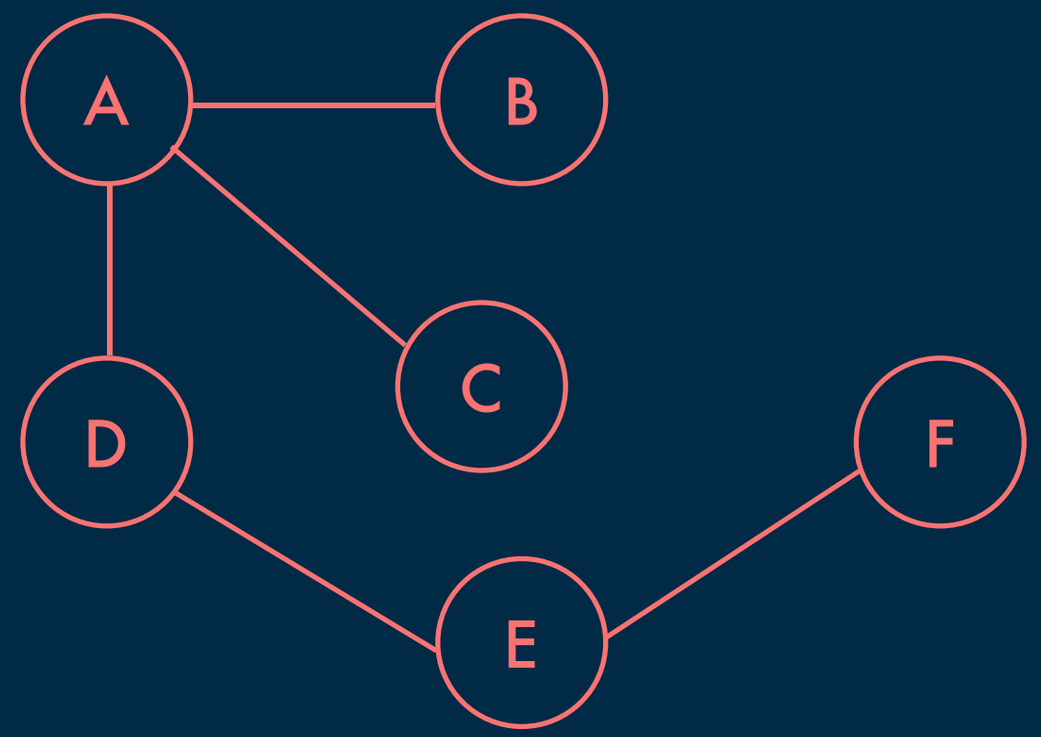
Graph Types

Trees

An undirected graph where two vertices form a pairwise relationship by exactly one path.

A graph is considered a tree if:

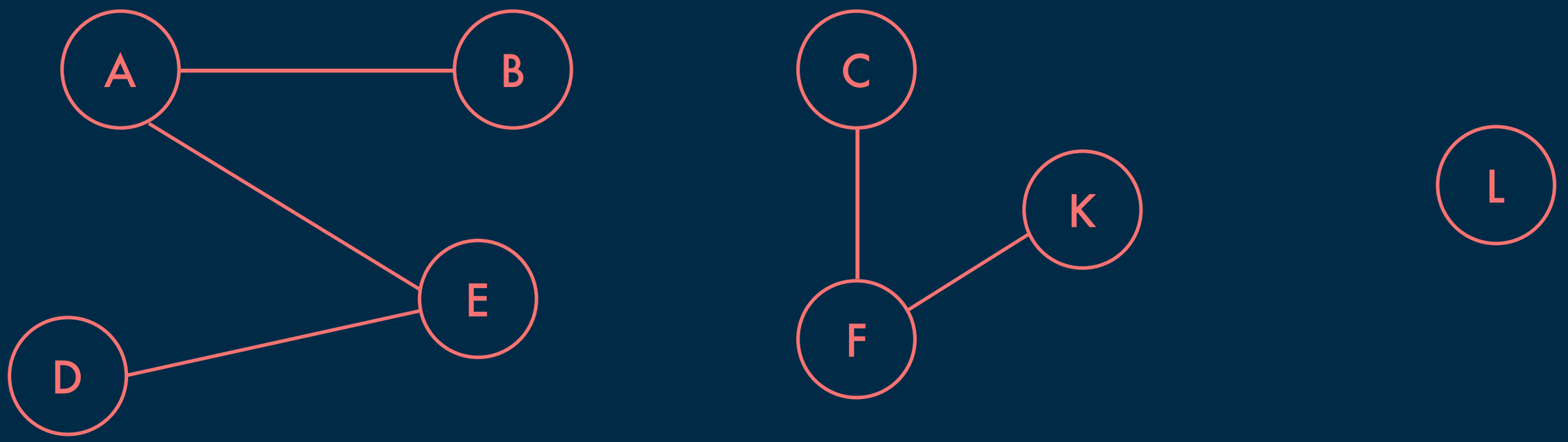
1. It is connected & contains no cycles(acyclic)
2. Has no simple cycles and has n-1 edges
3. Any two vertices can be merely connected by a unique simple path



Forest

An undirected graph where each two vertices are linked by at most one path. Forest is equivalent to:

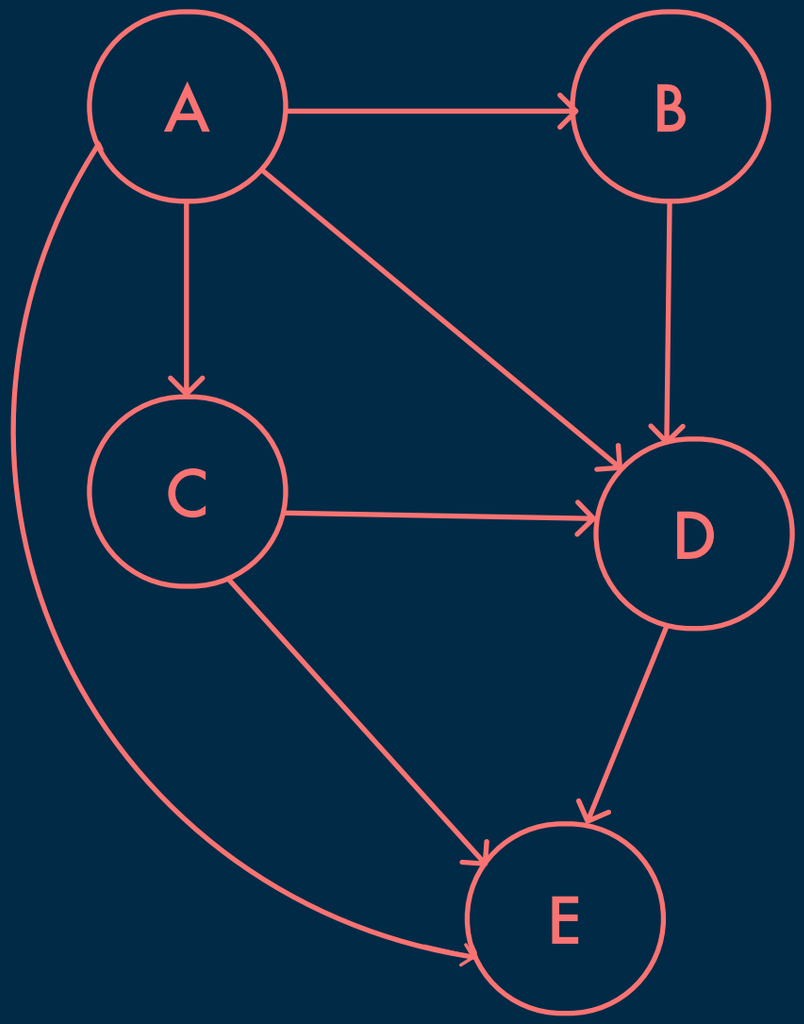
1. Acyclic undirected graph
2. Disjoint union of trees.



Directed Acyclic Graph

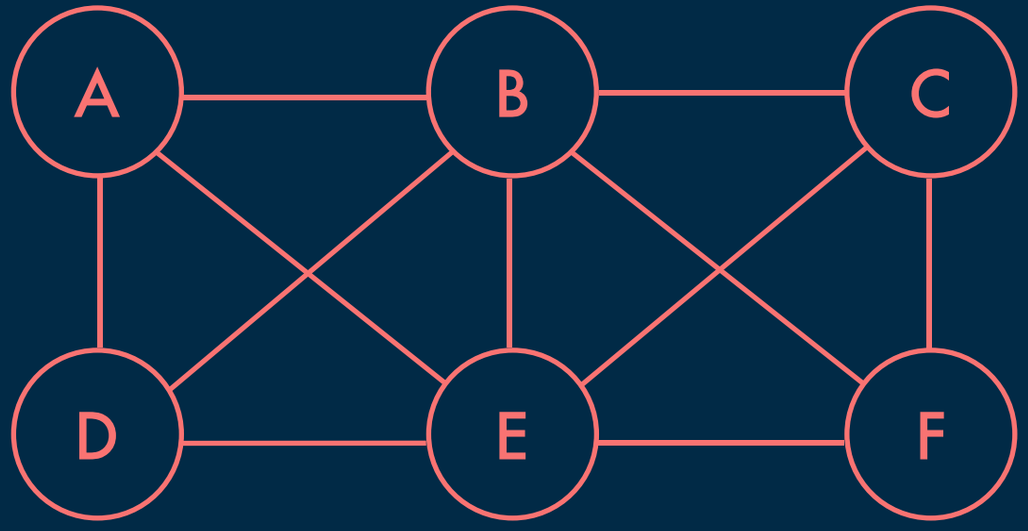
A directed graph with no directed cycles.

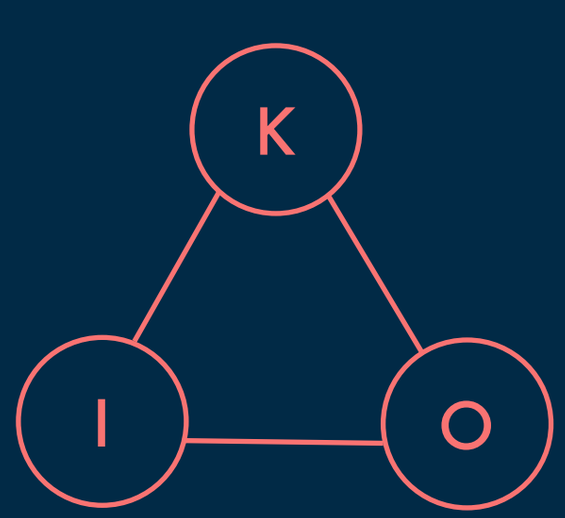
DAG consists of vertices and edges (also called arcs), where each edge that is directed from one vertex to another will never form a connection that creates a closed loop.



Complete Graph

An undirected graph where every pair of distinct vertices is connected by a pair of unique edges (one in each direction).





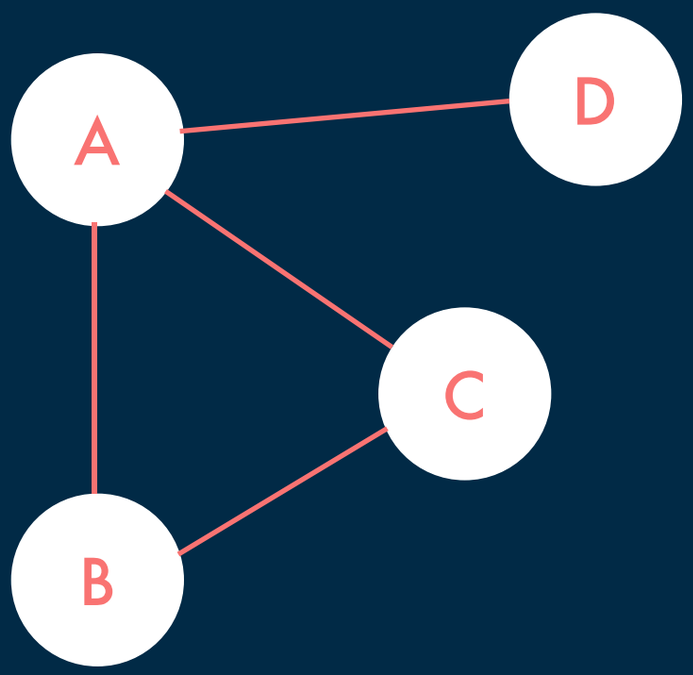
Representation of Graphs

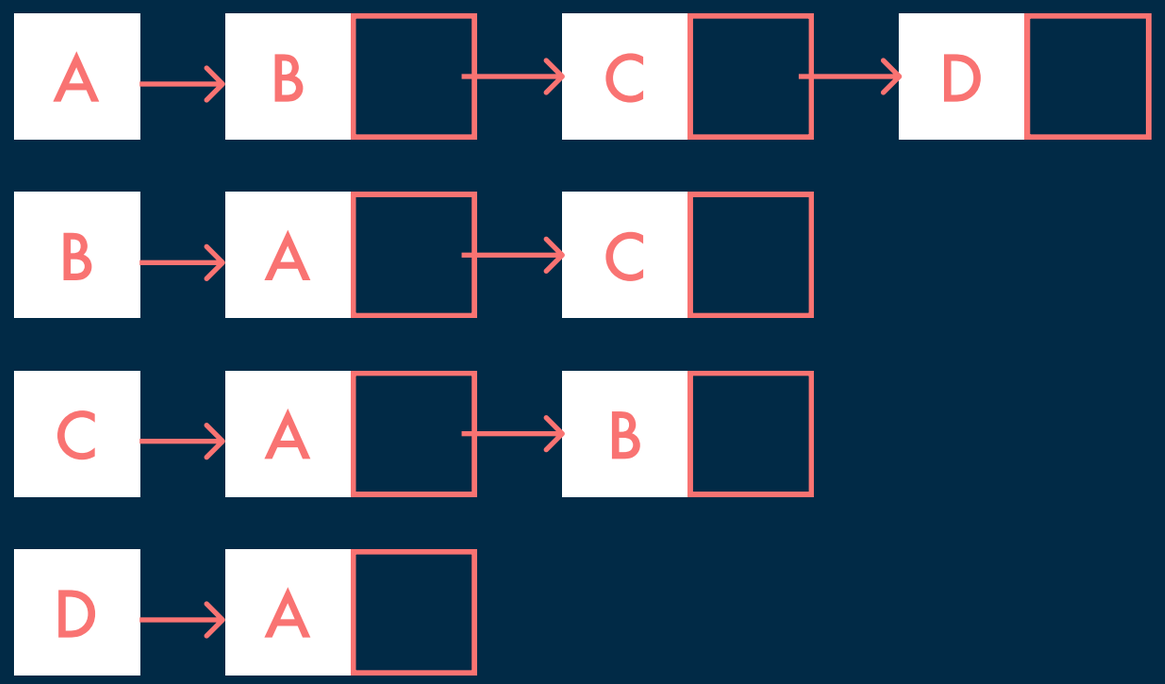
Using Programming Language

Adjacency List

A representation of a graph as an array of linked lists.

The index of the array represents a vertex and each element in its linked list represents the other vertices that form an edge with the vertex.





Adjacency List

Pros +

An adjacency list is efficient in terms of storage because we only need to store the values for the edges. For a sparse graph with millions of vertices and edges, this can mean a lot of saved space.

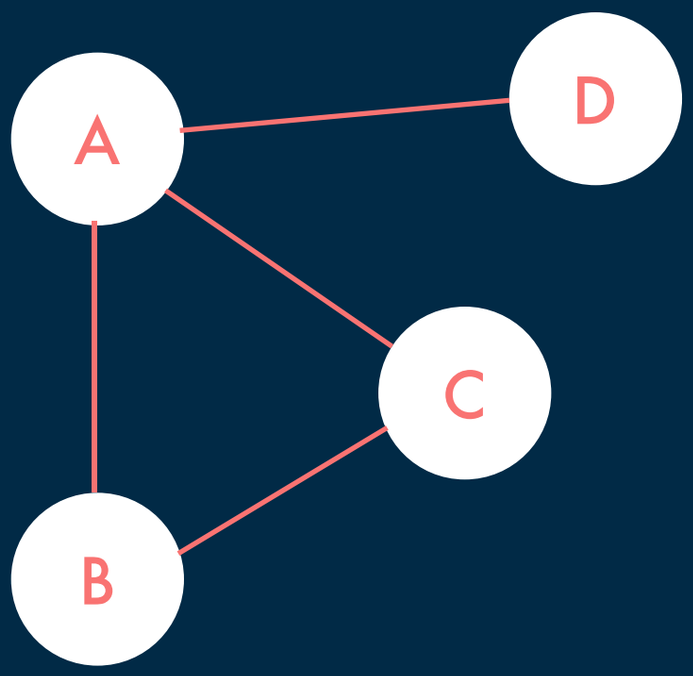
Cons -

An adjacency list is not efficient for edge weight lookup. It is slow – O(E), as we need to find it in linear time.

Adjacency Matrix

A a way of representing a graph G = {V, E} as a matrix of booleans.

The size of the matrix is VxV where V is the number of vertices in the graph and the value of an entry Aij is either 1 or 0 depending on whether there is an edge from vertex i to vertex j.



	A	B	C	D
A	0	1	1	1
B	1	0	1	C
C	1	1	0	0
D	1	0	0	0

Adjacency Matrix

Pros +

The basic operations like adding an edge, removing an edge and checking whether there is an edge from vertex i to vertex j are extremely time efficient, constant time operations.

Cons -

The VxV space requirement of the adjacency matrix makes it a memory hog.

Applications

Graph Theory Applications

1. Finding communities in networks, such as social media (friend/ connection recommendations).
2. Ranking and ordering hyperlinks in search engines.
3. Google Maps/GPS to find shortest path to the destination.
4. DNA sequencing.
5. Computer network security.

and so on...

Thank You !