Dinámica no lineal aplicada a un láser

Mode-mode competition and unstable behavior in a homogeneously broadened ring laser, L. M. Narducci, J.R. Tredicce, L. A. Lugiato, N. B.Abraham and D. K. Bandy.

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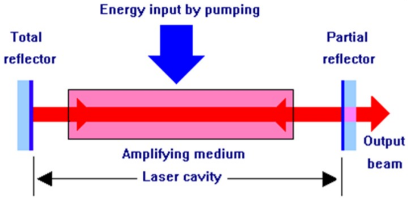
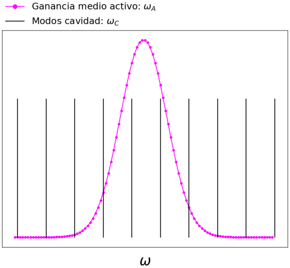
Índice

Cavidad de un láser



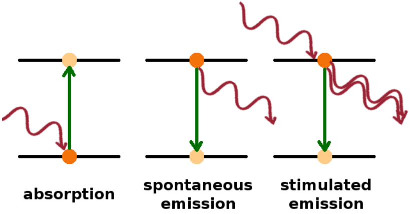


- Bombeo óptico
- Medio activo: aumentar la intensidad del láser

- ω_C: modos permitidos de la cavidad $(2\pi cn/\mathscr{L})$
- ω_A: linea atómica del medio activo $\rightarrow$ amplificación

Sistema de 2 niveles: Transiciones atómicas

- Absorción : $1 \to 2$
- Emisión espontánea : $2 \to 1$
- Emisión estimulada : $2 \to 1$



Con la emisión de fotones se aumenta la intensidad del láser

Sistema de ecuaciones

$\gamma_{\parallel}$ tasa inversión de población

k pérdidas de fotones

g coef. de prop. asociado a emisión estimulada y absorción

- $$\frac{dI}{dt} = -kI + gIN_2 - gIN_1$$
- $$\frac{dN_1}{dt} = \gamma_{\parallel} N_{o1} - gIN_1 + gIN_2 - \gamma_{\parallel} N_1$$
- $$\frac{dN_2}{dt} = \underbrace{\gamma_{\parallel} N_{o2}}_{\text{bombeo}} \underbrace{+gIN_1}_{\text{absorción}} \underbrace{-gIN_2}_{\substack{\text{emisión}\\\text{estimulada}}} \underbrace{-\gamma_{\parallel} N_2}_{\substack{\text{emisión}\\\text{espontánea}}}$$

Simplificar ecuaciones

1. Cambio de variable: $N = N_2 - N_1$

   $\bullet \dfrac{dN}{dt} = \gamma_{\parallel} N_o - 2gIN - \gamma_{\parallel} N \quad \bullet \dfrac{dI}{dt} = -kI + gIN = I(-k + gN)$

2. Adimensionalizar: $\tilde{t} = \gamma_{\parallel} t, \tilde{k} = k/\gamma_{\parallel}, \tilde{g} = gN_o/\gamma_{\parallel}$

3. Normalizar: $\tilde{N} = N/N_o, \tilde{I} = I/N_o, A = \tilde{g}/\tilde{k}$

Simplificar ecuaciones

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$$\bullet \frac{dN}{d\tilde{t}} = N_o - 2\frac{\tilde{g}}{N_o} IN - N \qquad \bullet \frac{dI}{d\tilde{t}} = I\Big(-\tilde{k} + \frac{\tilde{g}}{N_o} N\Big)$$

3. Normalizar: $\tilde{N} = N/N_o, \tilde{I} = I/N_o, A = \tilde{g}/\tilde{k}$

$$\boxed{\frac{d\tilde{N}}{d\tilde{t}} = 1 - 2\tilde{g}\tilde{I}\tilde{N} - \tilde{N} \qquad \frac{d\tilde{I}}{d\tilde{t}} = \tilde{I}\tilde{g}(-1/A + \tilde{N})}$$

Análisis de estabilidad lineal

Puntos fijos:

$$\tilde{N}_{pf}^{(1)} = 1 \qquad \tilde{I}_{pf}^{(1)} = 0$$

$$\tilde{N}_{pf}^{(2)} = 1/A \qquad \tilde{I}_{pf}^{(2)} = \frac{1}{2\tilde{g}}(A-1)$$

Jacobiano del sistema:

$$DF = \begin{bmatrix} \dfrac{\partial \dot{\tilde{N}}}{\partial \tilde{N}} & \dfrac{\partial \dot{\tilde{N}}}{\partial \tilde{I}} \\ \dfrac{\partial \dot{\tilde{I}}}{\partial \tilde{N}} & \dfrac{\partial \dot{\tilde{I}}}{\partial \tilde{I}} \end{bmatrix} = \begin{bmatrix} -2\tilde{g}\tilde{I} - 1 & -2\tilde{g}\tilde{N} \\ \tilde{g}\tilde{I} & \tilde{g}(-1/A + \tilde{N}) \end{bmatrix}$$

Análisis de estabilidad lineal: Estabilidad de puntos fijos

- $\lambda_1^{(1)} = -1$

$$\Longrightarrow \boxed{\vec{x}_{PF}^{(1)} \text{ estable si } A < 1}$$

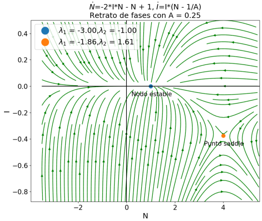
- $\lambda_2^{(1)} = \tilde{g}(1 - 1/A)$

- $\lambda_1^{(2)} = -\frac{A}{2} + \sqrt{\left(\frac{A}{2}\right)^2 - \tilde{g}(1 - 1/A)}$

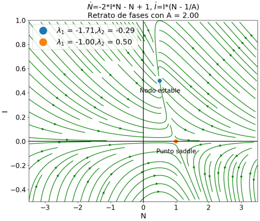
$$\Longrightarrow \boxed{\vec{x}_{PF}^{(2)} \text{ estable si } A > 1}$$

- $\lambda_2^{(2)} = -\frac{A}{2} - \sqrt{\left(\frac{A}{2}\right)^2 - \tilde{g}(1 - 1/A)}$

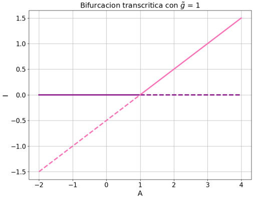
Bifurcación transcrítica + diagramas de flujos



(a) A = 0.25



(b) A = 2



(c) Diagrama de bif.

<u>Transcrítica</u>:
Cambio en la **estabilidad** de los puntos fijos al variar el parámetro

Formalismo de Maxwell-Bloch

Aproximación *slowly varying complex amplitude* para el campo eléctrico, se desprecian los campos transversales y $\sigma = 0$

$$E(z,t) = \mathscr{E}(z,t)e^{i(kz-\omega t)} + \text{c.c}$$

$$\mathscr{F}(z,t) \equiv \mu\mathscr{E}(z,t)/[2\hbar(\gamma_\perp\gamma_\parallel)^{1/2}]$$

$\mathscr{P}(z,t) \rightarrow$ Polarización compleja por átomo

$\mathscr{D}(z,t) \rightarrow$ Diferencia de prob. entre el excitado y el fundamental

$\gamma_\perp$ tasa de polarización

$\gamma_\parallel$ tasa de inversión de población

$\mathscr{L}$ longitud de la cavidad

L longitud del medio activo

Formalismo de Maxwell-Bloch

Las ecuaciones de Maxwell-Bloch son:

- $\dfrac{\partial \mathscr{F}}{\partial z} + \dfrac{1}{c}\dfrac{\partial \mathscr{F}}{\partial t} = -\alpha \mathscr{P}$ $\qquad \alpha$: ganancia/long. $\rightarrow \boxed{\alpha L}$

- $\dfrac{\partial \mathscr{P}}{\partial t} = \gamma_\perp[\mathscr{F}\mathscr{D} - (1+\tilde{\delta}_{AC})\mathscr{P}]$ $\qquad \boxed{\tilde{\delta}_{AC} = (\omega_A - \omega_C)/\gamma_\perp}$

- $\dfrac{\partial \mathscr{D}}{\partial t} = -\gamma_\parallel \left[\dfrac{1}{2}(\mathscr{F}^*\mathscr{P} + \mathscr{F}\mathscr{P}^*) + \mathscr{D} + 1\right]$

<u>CC</u> : $\mathscr{F}(z=0,t) = R\mathscr{F}(z=L, t-(\mathscr{L}-L)/c)$

Solución estacionaria + coordenadas polares

$$\mathscr{F}(z,t) = \mathscr{F}(z)_{st} e^{-i\delta\omega t}$$
$$\mathscr{P}(z,t) = \mathscr{P}(z)_{st} e^{-i\delta\omega t}$$
$$\mathscr{D}(z,t) = \mathscr{D}(z)_{st}$$

$$\mathscr{F}_{st}(z) = \rho(z)e^{i\theta(z)} \Rightarrow \frac{d\mathscr{F}_{st}}{dz} - \frac{i\delta\omega}{c}\mathscr{F}_{st} = \alpha\mathscr{F}_{st}(z)\frac{1-i\tilde{\Delta}}{1+\tilde{\Delta}^2+|\mathscr{F}_{st}(z)|^2}$$

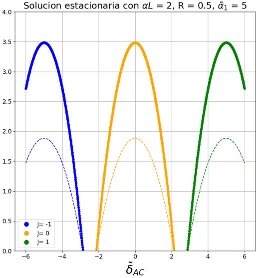
CC de $\theta \Rightarrow 2\pi j$

$$\tilde{\Delta}_j = \tilde{\delta}_{AC} - \delta\omega_j/\gamma_\perp \quad \frac{\delta\omega_j}{\gamma_\perp} = \frac{\tilde{k}\tilde{\delta}_{AC} + j\tilde{\alpha}_1}{1+\tilde{k}} \quad \boxed{\tilde{\alpha}_1 = \frac{2\pi c}{\mathscr{L}\gamma_\perp}} \quad \tilde{k} = \frac{c|\ln R|}{\mathscr{L}\gamma_\perp}$$

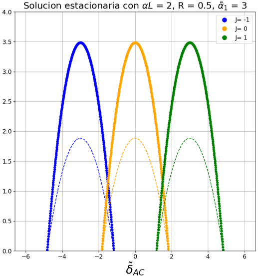
Estados estacionarios

$$\rho_j^2(L) = \frac{2}{1-R^2}\left[\alpha L - (1+\tilde{\Delta}_j^2)|\ln(R)|\right]$$

$$|\mathscr{F}_{st}^{(j)}|^2 \xrightarrow[\substack{R\to 1^- \\ \alpha L\to 0}]{} \left[2C - (1+\tilde{\Delta}_j^2)\right] \quad 2C \equiv \frac{\alpha L}{|\ln R|} < \infty$$



(a) Con $\tilde{\alpha}_1 = 5$



(b) Con $\tilde{\alpha}_1 = 3$

Cambio de variables: simplificar la condición de contorno

$$z' = z$$
$$t' = t + \frac{\mathscr{L} - L}{c}\frac{z}{L}$$

$$\Longrightarrow \underline{\text{CC}} : F(z=0, t') = F(z=L, t')$$

Nuevos campos: $F(z', t'), P(z', t'), D(z', t')$ con sus nuevas ecuaciones

$$k_n = \frac{2\pi n}{L} \qquad \boxed{\tilde{\alpha}_n = n\tilde{\alpha}_1} \qquad \delta\Omega = \delta\omega_j - j\alpha_1$$

Descomposición modal de los nuevos campos

$$F(z',t') = e^{-i\delta\Omega t'} \sum_{n=-\infty}^{\infty} e^{ik_n z'} e^{-i\alpha_n t'} f_n(t')$$

$$P(z',t') = e^{-i\delta\Omega t'} \sum_{n=-\infty}^{\infty} e^{ik_n z'} e^{-i\alpha_n t'} p_n(t')$$

$$D(z',t') = \sum_{n=-\infty}^{\infty} e^{ik_n z'} e^{-i\alpha_n t'} d_n(t')$$

$$F^*(z',t') = e^{i\delta\Omega t'} \sum_{n=-\infty}^{\infty} e^{-ik_n z'} e^{i\alpha_n t'} f_n^*(t')$$

$$P^*(z',t') = e^{i\delta\Omega t'} \sum_{n=-\infty}^{\infty} e^{-ik_n z'} e^{i\alpha_n t'} p_n^*(t')$$

$\rightarrow$ ecuaciones para las amplitudes modales

Linealizacion alrededor de estado estacionario + Ansatz

$$\chi_n = \chi_n^{(j)} \delta_{n,j} + \delta\chi_n$$
$$d_n = d_n^{(j)} \delta_{n,0} + \delta d_n$$

$$\begin{bmatrix} \delta f_{j+n}(t') \\ \delta f^*_{j-n}(t') \\ \delta p_{j+n}(t') \\ \delta p^*_{j-n}(t') \\ \delta d_n(t') \end{bmatrix} = e^{\lambda t'} \begin{bmatrix} \delta f_{j+n}(0) \\ \delta f^*_{j-n}(0) \\ \delta p_{j+n}(0) \\ \delta p^*_{j-n}(0) \\ \delta d_n(0) \end{bmatrix} \rightarrow \begin{bmatrix} \dot{\delta} f_{j+n} \\ \dot{\delta} f^*_{j-n} \\ \dot{\delta} p_{j+n} \\ \dot{\delta} p^*_{j-n} \\ \dot{\delta} d_n \end{bmatrix} = \lambda e^{\lambda t} \begin{bmatrix} \delta f_{j+n}(0) \\ \delta f^*_{j-n}(0) \\ \delta p_{j+n}(0) \\ \delta p^*_{j-n}(0) \\ \delta d_n(0) \end{bmatrix} =$$

$$= \boxed{\lambda \begin{bmatrix} \delta f_{j+n}(t') \\ \delta f^*_{j-n}(t') \\ \delta p_{j+n}(t') \\ \delta p^*_{j-n}(t') \\ \delta d_n(t') \end{bmatrix} = A \begin{bmatrix} \delta f_{j+n}(t') \\ \delta f^*_{j-n}(t') \\ \delta p_{j+n}(t') \\ \delta p^*_{j-n}(t') \\ \delta d_n(t') \end{bmatrix}}$$

Sistema matricial adimensionalizado

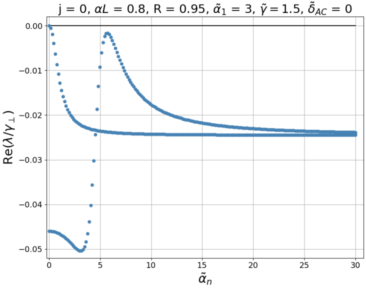
$$\tilde{A} = \frac{1}{\gamma_\perp}\begin{bmatrix} i\delta\Omega_j - \kappa & 0 & -\kappa 2C & 0 & 0 \\ 0 & -i\delta\Omega_j - \kappa & 0 & -\kappa 2C & 0 \\ \gamma_\perp d_0^{(j)} & 0 & -\gamma_\perp \tilde{\Delta}_{j,n}^{+} & 0 & \gamma_\perp f_j^{(j)} \\ 0 & \gamma_\perp d_0^{(j)} & 0 & -\gamma_\perp \tilde{\Delta}_{j,n}^{-} & \gamma_\perp (f_j^{(j)})^* \\ -\frac{\gamma_\parallel}{2}(p_j^{(j)})^* & -\frac{\gamma_\parallel}{2} p_j^{(j)} & -\frac{\gamma_\parallel}{2}(f_j^{(j)})^* & -\frac{\gamma_\parallel}{2} f_j^{(j)} & i\tilde{\alpha}_n - \gamma_\parallel \end{bmatrix}$$

$$\tilde{\Delta}_{j,n}^{+} = [1 + i(\tilde{\Delta}_j - \tilde{\alpha}_n)]$$
$$\tilde{\Delta}_{j,n}^{-} = [1 - i(\tilde{\Delta}_j + \tilde{\alpha}_n)]$$

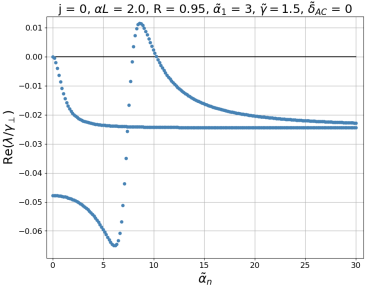
Análisis de estabilidad lineal con $\tilde{\delta}_{AC} = 0$

Estable sii $\mathrm{Re}(\lambda_i) < 0 \; \forall i, n$

Inestabilidad dada por λ amplitud



(a) Con $\alpha L = 0.8$

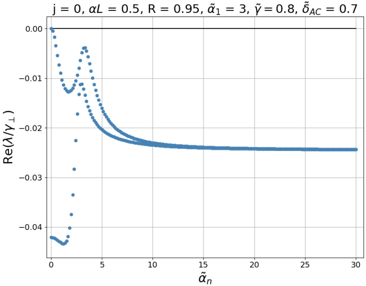


(b) Con $\alpha L = 2$

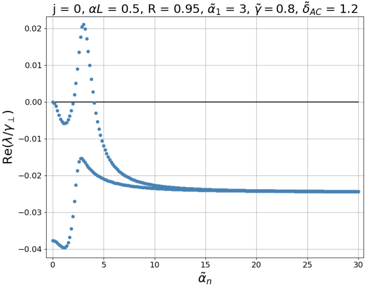
Análisis de estabilidad lineal con $\tilde{\delta}_{AC} \neq 0$

Si aumento $\tilde{\delta}_{AC}$ requiero menor ganancia para inestabilidad

Inestabilidad dada por λ fase



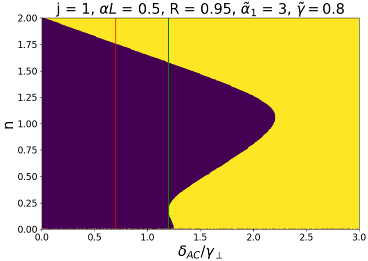
(a) Con $\tilde{\delta}_{AC} = 0.7$



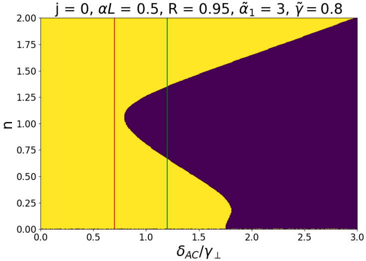
(b) Con $\tilde{\delta}_{AC} = 1.2$

Análisis de estabilidad lineal

amarillo $\mathrm{Re}(\lambda_i) < 0 \; \forall i$ violeta $\mathrm{Re}(\lambda_i) > 0$



(a) $\mathrm{j} = 1$



(b) $\mathrm{j} = 0$

Conclusiones

- Se pudo modelar la física de un láser como un balance de poblaciones y con el formalismo de Maxwell-Bloch
- Se calculó la matriz adimensional A de 5$\times$5 y sus autovalores
- Se lograron reproducir los gráficos de la segunda y tercera secciones del paper
- Se observó el comportamiento esperado de $\tilde{\delta}_{AC}$ y $\tilde{\alpha}_1$
- Se graficaron mapas de color mostrando las zonas de estabilidad y de inestabilidad

Muchas gracias!!!

Sistema de ecuaciones

- $\dfrac{dI}{dt} = -kI + gIN_2 - gIN_1$
- $\dfrac{dN_1}{dt} = -gIN_1 + gIN_2 + \gamma_{\|} N_2 + R_1$
- $\dfrac{dN_2}{dt} = gIN_1 - gIN_2 - \gamma_{\|} N_2 + R_2$

$R_2 \equiv \gamma_{\|} N_{o2}$ y $R_1 \equiv \gamma_{\|}(N_{o1} - N_T)$ con $N_T \equiv N_1 + N_2$.

- $\dfrac{dN_2}{dt} = gIN_1 - gIN_2 - \gamma_{\|} N_2 + \gamma_{\|} N_{o2}$
- $\dfrac{dN_1}{dt} = -gIN_1 + gIN_2 - \gamma_{\|} N_1 + \gamma_{\|} N_{o1}$

Sistema de ecuaciones

Sea $N_T \equiv N_1 + N_2$ y $N_{oT} \equiv N_{o1} + N_{o2}$:

$$\frac{dN_T}{dt} = \gamma_{\parallel}(N_{oT} - N_T)$$
$$\implies N_T(t) = N_{oT}\left(1 - Ae^{-\gamma_{\parallel} t}\right)$$

Me queda que N_T =cte si y sólo si A=0, o equivalentemente si $N_T(t=0) = N_{oT} = N_{o1} + N_{o2}$, o equivalentemente $R_1 + R_2 = 0 \iff R_2 = -R_1$.

Análisis de estabilidad lineal: Primer punto fijo

$$DF = \begin{bmatrix} \dfrac{\partial \dot{\tilde{N}}}{\partial \tilde{N}} & \dfrac{\partial \dot{\tilde{N}}}{\partial \tilde{I}} \\ \\ \dfrac{\partial \dot{\tilde{I}}}{\partial \tilde{N}} & \dfrac{\partial \dot{\tilde{I}}}{\partial \tilde{I}} \end{bmatrix} = \begin{bmatrix} -2\tilde{g}\tilde{I} - 1 & -2\tilde{g}\tilde{N} \\ \\ \tilde{g}\tilde{I} & \tilde{g}(-1/A + \tilde{N}) \end{bmatrix}$$

$$DF\Big|_{N_{pf}^{(1)}, I_{pf}^{(1)}} = DF\Big|_{1,0} = \begin{bmatrix} -1 & -2\tilde{g} \\ \\ 0 & -\tilde{g}/A + \tilde{g} \end{bmatrix}$$

$$\boxed{\lambda_1^{(1)} = -1 \qquad \lambda_2^{(1)} = \tilde{g}(1 - 1/A)}$$

Análisis de estabilidad lineal: Segundo punto fijo

$$DF\Big|_{N_{pf}^{(2)},I_{pf}^{(2)}} = DF\Big|_{\frac{1}{A},\frac{A-1}{2\tilde{g}}} = \begin{bmatrix} -A & -\dfrac{2\tilde{g}}{A} \\ \dfrac{A-1}{2} & 0 \end{bmatrix}$$

$$\boxed{\begin{aligned} \lambda_1^{(2)} &= -\frac{A}{2} + \sqrt{\left(\frac{A}{2}\right)^2 - \tilde{g}(1-1/A)} \\ \lambda_2^{(2)} &= -\frac{A}{2} - \sqrt{\left(\frac{A}{2}\right)^2 - \tilde{g}(1-1/A)} \end{aligned}}$$

Solución estacionaria

$$\mathscr{F}(z,t) = \mathscr{F}(z)_{st} e^{-i\delta\omega t} \rightarrow \tilde{\Delta} = \tilde{\delta}_{AC} - \delta\omega/\gamma_{\perp}$$
$$\mathscr{P}(z,t) = \mathscr{P}(z)_{st} e^{-i\delta\omega t}$$
$$\mathscr{D}(z,t) = \mathscr{D}(z)_{st}$$

$$\mathscr{P}_{st}(z) = -\mathscr{F}_{st}(z)\frac{1 - i\tilde{\Delta}}{1 + \tilde{\Delta}^2 + |\mathscr{F}_{st}(z)|^2}$$
$$\mathscr{D}_{st}(z) = -\mathscr{F}_{st}(z)\frac{1 + \tilde{\Delta}}{1 + \tilde{\Delta}^2 + |\mathscr{F}_{st}(z)|^2}$$

$\underline{\text{CC}}$: $\mathscr{F}_{st}(z=0) = R\mathscr{F}_{st}(z=L)e^{i\delta\omega(\mathscr{L}-L)/c}$

Coordenadas polares

$$\mathscr{F}_{st}(z) = \rho(z)e^{i\theta(z)} \rightarrow \frac{d\mathscr{F}_{st}}{dz} - \frac{i\delta\omega}{c}\mathscr{F}_{st} = \alpha\mathscr{F}_{st}(z)\frac{1 - i\tilde{\Delta}}{1 + \tilde{\Delta}^2 + |\mathscr{F}_{st}(z)|^2}$$

$$\rho'(z) = \frac{\alpha\rho(z)}{1 + \tilde{\Delta}^2 + |\rho(z)|^2}$$

$$\theta'(z) = -\frac{\alpha\tilde{\Delta}}{1 + \tilde{\Delta}^2 + |\rho(z)|^2} + \frac{\delta\omega}{c}$$

$$\underline{\mathrm{CC}}: \qquad \begin{aligned} &\rho(z=0) = R\rho(z=L) \\ &\theta_j(z=L) - \theta(z=0) = -\delta\omega(\mathscr{L} - L)/c + 2\pi j \end{aligned}$$

Cambio de variables

Nuevas campos:

$$F(z',t') = \mathscr{F}(z',t')e^{z' \ln R/L}$$
$$P(z',t') = \mathscr{P}(z',t')e^{z' \ln R/L}$$
$$D(z',t') = \mathscr{D}(z',t')$$

Nuevas ecuaciones:

$$\frac{\partial F}{\partial t'} + \frac{cL}{\mathscr{L}}\frac{\partial F}{\partial z'} = -k(F + 2CP)$$
$$\frac{\partial P}{\partial t'} = \gamma_{\perp}[FD - (1 + \bar{\delta}_{AC})P]$$
$$\frac{\partial P}{\partial t'} = -\gamma_{\|}\left[\frac{1}{2}(F^*P + FP^*) + D + 1\right]$$

Ecuaciones para las amplitudes modales

$$\dot{f}_n = i\delta\Omega f_n - \kappa(f_n + 2Cp_n)$$
$$\dot{f}_n^* = -i\delta\Omega f_n^* - \kappa(f_n + 2Cp_n^*)$$

$$\dot{p}_n = \gamma_\perp \left[\sum_{n'} f_{n'} d_{n-n'} - p_n \left(1 + i \left(\tilde{\delta}_{AC} - \frac{\tilde{\Omega}}{\gamma_\perp} - \tilde{\alpha}_n \right) \right) \right]$$

$$\dot{p}_n^* = \gamma_\perp \left[\sum_{n'} f_{n'}^* d_{n-n'}^* - p_n^* \left(1 - i \left(\tilde{\delta}_{AC} - \frac{\tilde{\Omega}}{\gamma_\perp} - \tilde{\alpha}_n \right) \right) \right]$$
$$\dot{d}_n = i\alpha_n d_n - \gamma_\parallel \left[\frac{1}{2} \sum_{n'} (f_{n'}^* p_{n+n'} + f_{n'} p_{n'-n}^*) + d_n + \delta_{n,0} \right]$$

Ecuaciones de movimiento para las amplitudes modales

$$\mathscr{P}_{st}(z) = -\mathscr{F}_{st}(z)\frac{1 - i\tilde{\Delta}}{1 + \tilde{\Delta}^2 + |\mathscr{F}_{st}(z)|^2}$$

$$p_n^{(j)} = -f_n^{(j)}\frac{1 - i\tilde{\Delta}_j}{1 + \tilde{\Delta}_j^2 + |f_n^{(j)}|^2}$$

$$\mathscr{D}_{st}(z) = -\mathscr{F}_{st}(z)\frac{1 + \tilde{\Delta}}{1 + \tilde{\Delta}^2 + |\mathscr{F}_{st}(z)|^2}$$

$$d_n^{(j)} = -\frac{1 + \tilde{\Delta}_j}{1 + \tilde{\Delta}_j^2 + |f_n^{(j)}|^2}\delta_{n,0}$$

$$|\mathscr{F}_{st}^{(j)}|^2 \xrightarrow[R \to 1^-]{} \left[2C - (1 + \tilde{\Delta}_j^2)\right]$$

$$f_n^{(j)} = [2C - (1 + \tilde{\Delta}_j^2)]^{1/2}\delta_{n,j}$$

Linealizacion de las ecuaciones de amplitudes modales

$$\chi_n = \chi_n^{(j)}\delta_{n,j} + \delta\chi_n$$
$$d_n = d_n^{(j)}\delta_{n,0} + \delta d_n$$

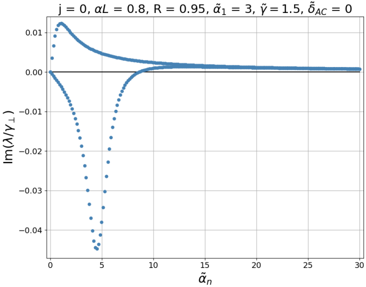
$$\dot{\delta} f_{j+n} = i\delta\Omega_j \delta f_{j+n} - \kappa(\delta f_{j+n} + 2C\delta p_{j+n})$$
$$\dot{\delta} f^*_{j-n} = -i\delta\Omega_j \delta f^*_{j-n} - \kappa(\delta f^*_{j-n} + 2C\delta p^*_{j-n})$$
$$\dot{\delta} p_{j+n} = \gamma_\perp \left[f_j^{(j)}\delta d_n + \delta f_{j+n} d_0^{(j)} - 1[1 + i(\tilde{\Delta}_j - \tilde{\alpha}_n)]\delta p_{j+n} \right]$$
$$\dot{\delta} p^*_{j-n} = \gamma_\perp \left[f_j^{(j)*}\delta d_n + \delta f_{j-n} d_0^{(j)} - 1[1 - i(\tilde{\Delta}_j + \tilde{\alpha}_n)]\delta p^*_{j-n} \right]$$
$$\dot{\delta} d_n = i\alpha_n \delta d_n - \gamma_\parallel \Big[\frac{1}{2}(f_j^{(j)*}\delta p_{j+n} + p_j^{(j)}\delta f^*_{j-n} + f_j^{(j)}\delta p^*_{j-n} +$$
$$(p_j^{(j)})^*\delta f_{j+n}) + \delta d_n \Big]$$

Ansatz

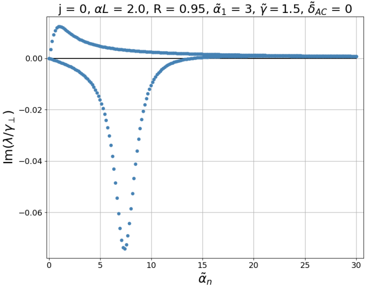
$$\begin{bmatrix} \delta f_{j+n}(t') \\ \delta f^*_{j-n}(t') \\ \delta p_{j+n}(t') \\ \delta p^*_{j-n}(t') \\ \delta d_n(t') \end{bmatrix} = e^{\lambda t'} \begin{bmatrix} \delta f_{j+n}(0) \\ \delta f^*_{j-n}(0) \\ \delta p_{j+n}(0) \\ \delta p^*_{j-n}(0) \\ \delta d_n(0) \end{bmatrix} \rightarrow \begin{bmatrix} \dot{\delta} f_{j+n} \\ \dot{\delta} f^*_{j-n} \\ \dot{\delta} p_{j+n} \\ \dot{\delta} p^*_{j-n} \\ \dot{\delta} d_n \end{bmatrix} = \lambda e^{\lambda t} \begin{bmatrix} \delta f_{j+n}(0) \\ \delta f^*_{j-n}(0) \\ \delta p_{j+n}(0) \\ \delta p^*_{j-n}(0) \\ \delta d_n(0) \end{bmatrix} =$$

$$= \lambda \begin{bmatrix} \delta f_{j+n}(t') \\ \delta f^*_{j-n}(t') \\ \delta p_{j+n}(t') \\ \delta p^*_{j-n}(t') \\ \delta d_n(t') \end{bmatrix} = A \begin{bmatrix} \delta f_{j+n}(t') \\ \delta f^*_{j-n}(t') \\ \delta p_{j+n}(t') \\ \delta p^*_{j-n}(t') \\ \delta d_n(t') \end{bmatrix}$$

Análisis de estabilidad lineal: partes imaginarias



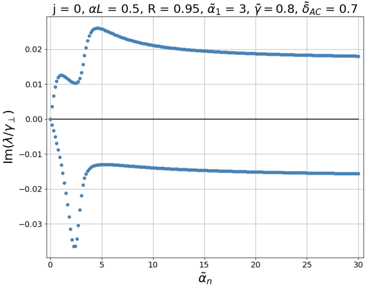
(a) Con $\alpha L = 0.8$



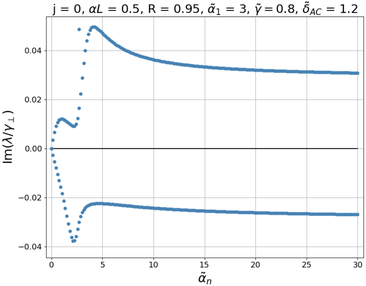
(b) Con $\alpha L = 2$

Figure: R $= 0.95$, $\tilde{\alpha}_1 = 3$, $\tilde{\gamma} = 1.5$, $\tilde{\delta}_{AC} = 0$

Análisis de estabilidad lineal: partes imaginarias



(a) Con $\tilde{\delta}_{AC} = 0.7$



(b) Con $\tilde{\delta}_{AC} = 1.2$

Figure: $\alpha L = 0.5, \mathrm{R} = 0.95,\ \tilde{\alpha}_1 = 3,\ \tilde{\gamma} = 0.8$