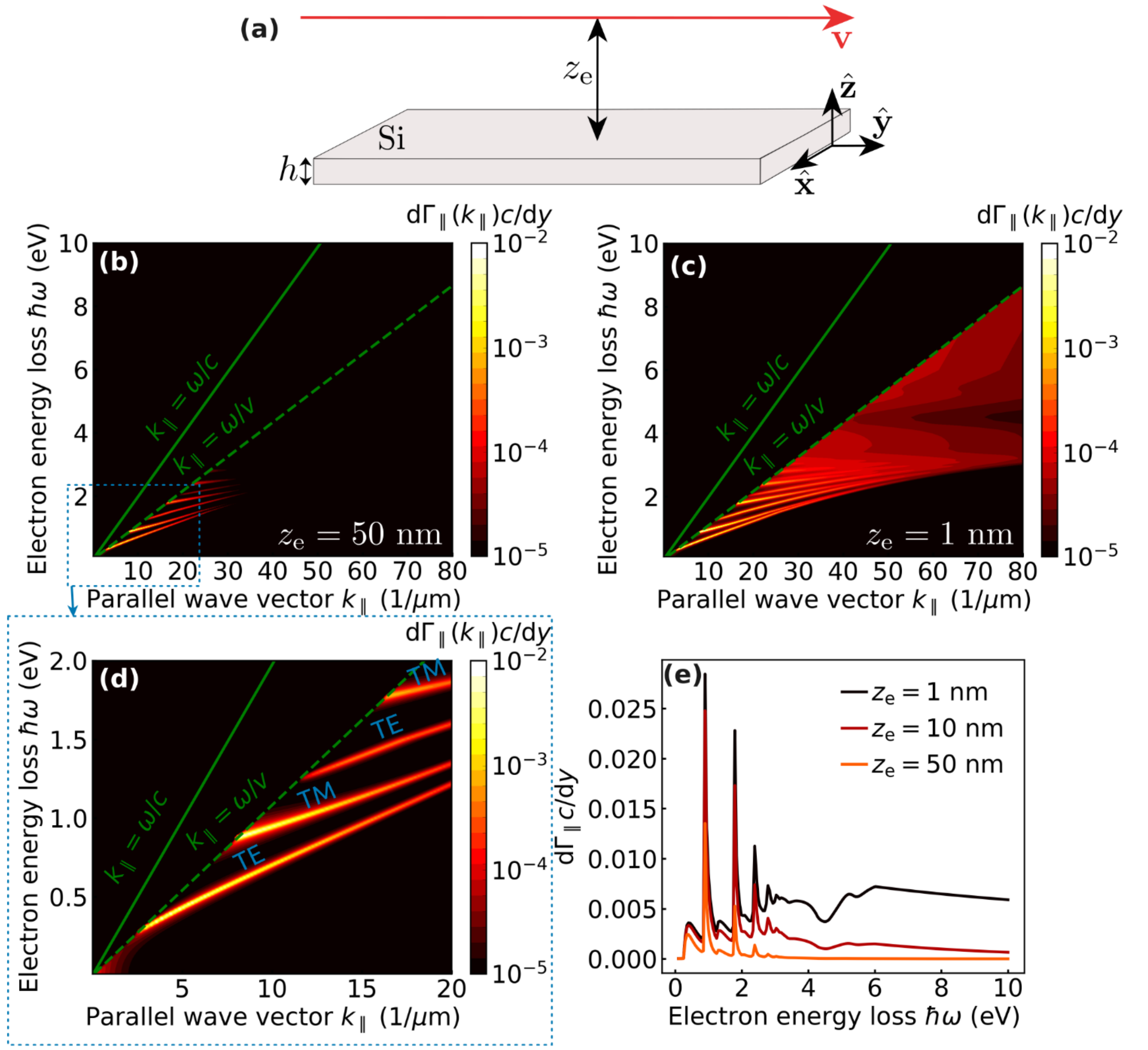
Fig. 1:

$$\frac{d\Gamma}{dy}(k_\parallel,\omega) = \frac{2e^2}{\pi\hbar v^2}\frac{k}{k_\parallel^2}\mathrm{Re}\left\{k_{z1}\mathrm{e}^{2\mathrm{i}k_{z1}z_\mathrm{e}}\left[\left(\frac{k_x v}{k_{z1}c}\right)^2 r^\mathrm{s}_{123}(k_\parallel) - \frac{1}{\epsilon_1}r^\mathrm{p}_{123}(k_\parallel)\right]\right\} \qquad \frac{d\Gamma}{dy}(\mathbf{r},\omega) = \frac{2e^2}{\pi\hbar v^2}\int_0^\infty \frac{\mathrm{d}k_x}{k_\parallel^2}\mathrm{Re}\left\{k_{z1}\mathrm{e}^{2\mathrm{i}k_{z1}z_\mathrm{e}(\mathbf{r})}\left[\left(\frac{k_x v}{k_{z1}c}\right)^2 r^\mathrm{s}_{123}(k_\parallel) - \frac{1}{\epsilon_1}r^\mathrm{p}_{123}(k_\parallel)\right]\right\},$$




$$r^\nu_{123} = r^\nu_{12} + \frac{t^\nu_{12}t^\nu_{21}r^\nu_{23}\mathrm{e}^{2\mathrm{i}k_{z2}h}}{1 - r^\nu_{21}r^\nu_{23}\mathrm{e}^{2\mathrm{i}k_{z2}h}},$$

$\epsilon_2(\omega) + \mathrm{i}10^{-1}$ Si from Aspnes

FIG. 3. **Electron coupling to a waveguide mode**. **(a)** Schematic representation of an electron beam moving with velocity $\mathbf{v} = v\hat{\mathbf{y}}$ at a distance $z_\mathrm{e}$ from a laterally infinite Si surface with a thickness $h$ embedded in vacuum. The electron excites the waveguide modes supported by the surface. **(b)** Wave-vector-resolved electron energy loss (EEL) probability per unit length described in Eq. (1b) (multiplied by the velocity of light $c$) as a function of the parallel wave vector $k_\parallel$ and the electron energy loss $\hbar\omega$, for $z_\mathrm{e} = 50$nm. The light (in vacuum) and electron dispersion relation lines are marked in solid and dashed green lines, respectively. **(c)** Same as (b) for an electron-surface distance of $z_\mathrm{e} = 1$ nm. **(d)** Close up of the region marked in a blue dashed square in (b). We identify each of the TM and TE modes supported by the waveguide with a label. **(e)** Total EEL probability (i.e., integrated over wave vector as defined in Eq. (1a)) as a function of the electron energy loss $\hbar\omega$, for different values of the electron-surface distance $z_\mathrm{e}$. In all panels, the thickness of the surface is $h = 0.2$ $\mu$m and the energy of the electron is $E_\mathrm{e} = 100$ keV.