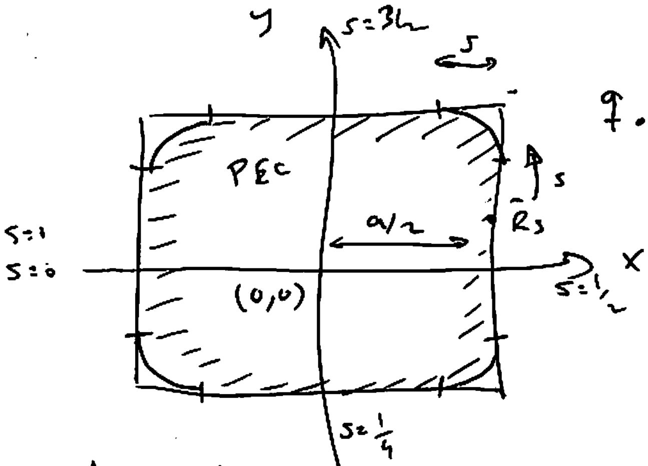
POTENTIAL INDUCED NEAR A WIRE BY A LINE OF CHARGE

19 - IX - 2020
SITGES



translationally invariant along z

$q \cdot (x_0, y_0) = \bar R_0$

q : charge per unit length along z

independent of s

$$V = \int dz \left[\frac{q}{\sqrt{|\bar R_s - \bar R_0|^2 + z^2}} + \oint \frac{ds'\,\rho(s')}{\sqrt{|\bar R_s - \bar R_{s'}|^2 + z^2}} \right]$$

$$\int_{-L}^{L} dz \frac{1}{\sqrt{\Delta^2 + z^2}} = 2 \ln\left[z + \sqrt{\Delta^2 + z^2}\right]\Big|_0^L$$

$L \to \infty$

$\oint ds\ \rho(s) = -q$, so that q is a photoelectron that leaves a hole behind, and the latter is fully screened

$$V = 2q \ln\left(\frac{L + \sqrt{|\bar R_s - \bar R_0|^2 + L^2}}{|\bar R_s - \bar R_0|}\right) + \oint ds'\ 2\rho(s') \ln\left[\frac{L + \sqrt{|\bar R_s - \bar R_{s'}|^2 + L^2}}{(\bar R_s - \bar R_{s'})}\right]$$

otherwise is $\neq$ for $L \to \infty$

$$\approx 2\left[\left(\ln(L) + \ln(2) + O\left(\frac{1}{L^2}\right)\right)\left(q + \underbrace{\oint ds\ 2\rho(s')}_{=-q}\right)\right] (= 0)$$

$$- 2q \ln|\bar R_s - \bar R_0| - 2\oint ds'\,\rho(s') \ln|\bar R_s - \bar R_{s'}|$$

$\ln\left(L + \sqrt{\Delta^2 + L^2}\right) =$

$= \ln(L) + \ln\left(1 + \sqrt{1 + \left(\frac{\Delta}{L}\right)^2}\right)$

$\simeq \ln(L) + \ln(2) + \frac{1}{4}\left(\frac{\Delta}{L}\right)^2$

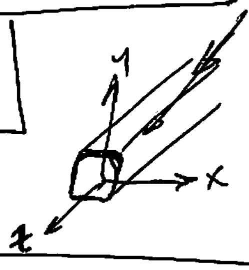
$\left(1 + \frac{1}{2}\left(\frac{\Delta}{L}\right)^2\right)^{1/2} \simeq 1 + \frac{1}{4}\left(\frac{\Delta}{L}\right)^2$

$\ln(1 + x) \simeq x$

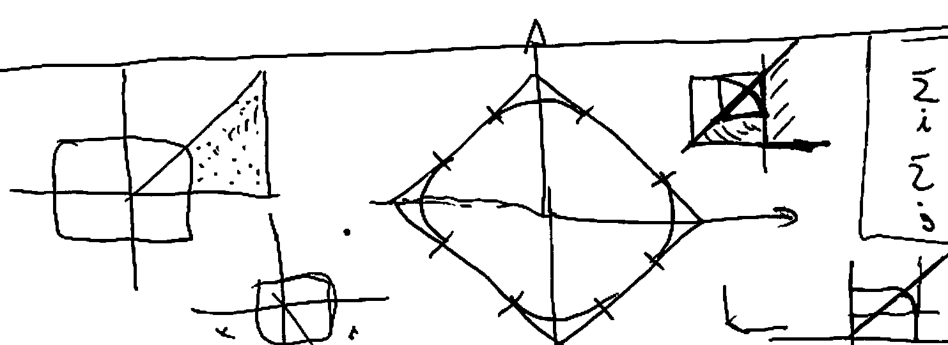
$\rho(s) \equiv n(s)\, q$

$$\boxed{q \ln|\bar R_s - \bar R_0|^2 + q \oint ds'\ n(s') \ln|\bar R_s - \bar R_{s'}|^2 = -V}$$

$$\boxed{\oint ds'\ n(s') = -1}$$



Total potential $\longrightarrow \frac{1}{q} V(R) = -\left(\ln|\bar R - \bar R_0|^2\right) - \oint ds' \left[n(s')\right] \ln|\bar R - \bar R_{s'}|^2$



$$\boxed{\begin{aligned} &\sum_i n_i\,\Delta s_i = -1 \\ &\sum_j n_j\, M_{ij} + \frac{V}{q} = -\ln d_i^2 \end{aligned}}$$

$$\left[\begin{array}{c|c} M & +1 \\ \hline \Delta s & 0 \end{array}\right] \begin{bmatrix} n \\ \frac{V}{q} \end{bmatrix} = \begin{bmatrix} -\ln d^2 \\ -1 \end{bmatrix}$$

HEAT CAPACITY OF A DEGENERATE 3D ELECTRON GAS

21-23-IX-2020 SITGES

$$f_E = \frac{1}{e^{(E-\mu)\beta}+1} \qquad \beta = \frac{1}{k_B T}$$

$$n = \frac{2}{V}\sum_k f_E = \frac{2}{V}\frac{4\pi}{3}k_F^3\frac{V}{(2\pi)^3} = \frac{k_F^3}{3\pi^2} \quad \text{indep. of } T$$

$$\int_0^\infty k^2 dk\, f_E = \frac{k_F^3}{3} \Rightarrow \mu(T) \qquad E = \frac{\hbar^2 k^2}{2m^*}$$

$$\left[\frac{2}{V}\sum_k \to \frac{2\cdot 4\pi}{\cancel{V}}\frac{\cancel{V}}{(2\pi)^3}\int_0^\infty k^2 dk = \frac{1}{\pi^2}\int_0^\infty k^2 dk\right] \qquad k^2 dk = \frac{k\, dk^2}{2}$$

$$\int_0^\infty \frac{\sqrt{E}\, dE}{e^{(E-\mu)\beta}+1} = \frac{2E_F^{3/2}}{3} \qquad E/E_F = \theta$$

$\boxed{\text{CHEMICAL POTENTIAL}}$

$$\int_0^\infty \frac{\sqrt{\theta}\, d\theta}{e^{(\theta-\frac{\mu}{E_F})E_F\beta}+1} = \frac{2}{3} \Rightarrow \mu = E_F\, M\left[\frac{E_F}{k_B T}\right]$$

$$\int_0^\infty \frac{\sqrt{\theta}\, d\theta}{e^{(\theta-M)x^{-1}}+1} = \frac{2}{3} \Rightarrow M(x^{-1}) \qquad \boxed{x^{-1} = \frac{E_F}{k_B T}}$$

$\boxed{\text{HEAT CAPACITY}}$

$$Q = \frac{2}{V}\sum_k E\left[f_E - \theta(E_F - E)\right] \qquad \boxed{x = \frac{k_B T}{E_F}}$$

$$= \frac{1}{\pi^2}\frac{1}{2}\left(\frac{2m^*}{\hbar^2}\right)^{3/2}\int_0^\infty \sqrt{E}\, dE\; E\left[f_E - \theta(E_F - E)\right]$$

$$= \frac{(2m^*)^{3/2}}{2\pi^2\hbar^3}E_F^{5/2}\int_0^\infty \theta^{3/2}d\theta\left[\frac{1}{e^{(\theta-M)x^{-1}}+1} - \theta(1-\theta)\right]$$

$$= \frac{(2m^*)^{3/2}}{2\pi^2\hbar^3}E_F^{5/2}\underbrace{\left[\int_0^\infty \frac{\theta^{3/2}d\theta}{e^{(\theta-M)x^{-1}}+1} - \frac{2}{5}\right]} \to \left[\frac{E}{L^3}\right] \text{ OK}$$

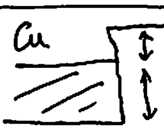
$$N(x^{-1}) \to \frac{\pi^2}{6}x^2,\ x \ll 1 \quad \text{OK}$$

N, M

$x = \frac{k_B T}{E_F}$

$M(0) = 1$

$N(0) = 0$



Cu

$\Phi = 4.7\,\text{eV}$

$E_F = 9.73\,\text{eV}$

(diff. from the value 7.0 eV with free bands)

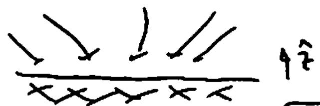
$$E_F = \frac{\hbar^2 k_F^2}{2m^*} = \frac{\hbar}{2m^*}(3\pi^2 n)^{2/3}$$

Cu $4\,e^-/a^3 = n$, $a = 3.597\,\text{Å}$

$m^* = 1.01 \Rightarrow E_F = 9.73\,\text{eV}$

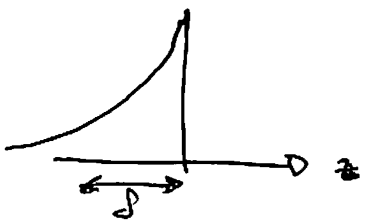
SKIN-DEPTH, ETC

25-ix-2020
ΛTGE



$$k_z' = \sqrt{k^2 \varepsilon - q^2} \simeq k\sqrt{\varepsilon}$$

$q \lesssim k \qquad |\varepsilon| \gg 1 \quad \Rightarrow$



$$|E|^2 = |E_0 e^{+ik\sqrt{\varepsilon} z}|^2$$

Absorption density $\rightarrow$ $\mathrm{Im}\{\varepsilon\} |E|^2 \frac{\omega}{2\pi}$

$$Q = \mathrm{Im}\{\varepsilon\} \left|\frac{E}{E^{ext}}\right|^2 \frac{\omega}{2\pi} |E^{ext}|^2 \sqrt{\pi}\,\Delta \ominus$$

heat per unit volume

$[E/L^3]$ or

$$I = I_0 e^{-(t/\Delta)^2}$$

$$F = \int dt\, I = \Delta\sqrt{\pi}\, I_0$$

$$I_0 = \frac{c}{2\pi} |E^{ext}|^2$$

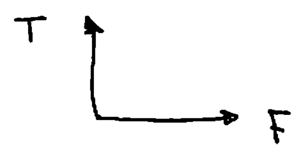
$I = \frac{I_0}{2} \rightarrow (t/\Delta)^2 = \ln 2 \rightarrow t = \Delta\sqrt{\ln 2}$

$\Rightarrow \quad FWHM = 2\Delta\sqrt{\ln 2}$

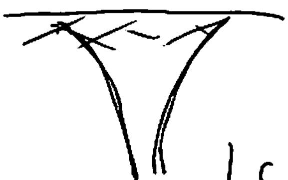
$$\ominus \quad \frac{\omega}{c} \mathrm{Im}\{\varepsilon\} \left|\frac{E}{E^{ext}}\right|^2 F$$

$\hbar\omega = 1.57$ eV

$\mathrm{Im}\{\varepsilon_{Cu}\} = 2.484$



electron heat conductivity



$k = k_0 T_e \qquad k_0 = 401 \frac{W}{mK}$ at $300\,K$

characteristic time of cooling

$$\tau \sim \frac{C_e \delta^2}{k}$$

~~$[Q] \;\; \Delta \;\; \tau\, k \;\; \Delta \;\; \tau \sim \Delta\, \frac{Q}{k_0}$~~

1 - iV - 2021
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PARAMETRIZATION OF RECTANGULAR WIRE