29-IV-05

$\boxed{\text{DIPOLE-DIPOLE INTERACTION NEAR A PLANAR SURFACE}}$

$n=1$

DIRECT FIELDS PRODUCED BY ELECTRIC AND MAGNETIC DIPOLES, p AND m,

$$\begin{cases} \vec{E}_0 = \left[(K^2 + \bar\nabla\bar\nabla)\vec{p} + ik\,\bar\nabla\times\vec{m}\right]\dfrac{e^{ikr}}{r} \\ \bar{H}_0 = \left[-ik\,\bar\nabla\times\bar{p} + (k^2+\bar\nabla\bar\nabla)\vec{m}\right]\dfrac{e^{ikr}}{r} \end{cases}$$

$$\vec{H} = \frac{1}{ik}\bar\nabla\times\bar{E}$$

$$\hat{k}\times\hat{\varepsilon}_s = -\hat{\varepsilon}_p$$

$$\hat{k}\times\hat{\varepsilon}_p = \hat{\varepsilon}_s$$

$$\hat{\varepsilon}_s = \hat{\varphi} = \frac{1}{Q}(-Q_y, Q_x, 0)$$

$$\hat{\varepsilon}_p = \hat{\theta} = \frac{1}{k}(sk_z\hat{Q}, -Q)$$

$$s = \operatorname{sign}\{z\}$$

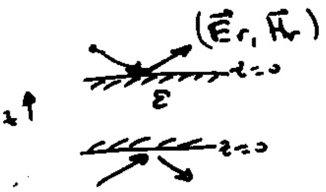
$$\vec{K} = (\vec{Q}, sk_z)$$

$$\begin{cases} \bar{E}_0 = \displaystyle\int \frac{d\vec{Q}}{(2\pi)^2}\frac{2\pi i}{k_z}e^{i\vec{k}\cdot\vec{r}}\left(\hat{\varepsilon}_s\alpha_s^0 + \hat{\varepsilon}_p\alpha_p^0\right) \\ \bar{H}_0 = \displaystyle\int \frac{d\vec{Q}}{(2\pi)^2}\frac{2\pi i}{k_z}e^{i\vec{k}\cdot\vec{r}}\left(\hat{\varepsilon}_s\alpha_p^0 - \hat{\varepsilon}_p\alpha_s^0\right) \end{cases}$$

$$\alpha_s^0 = \hat{\varepsilon}_s\cdot\left[k^2\bar{p} - \cancel{\bar{k}(\bar{k}\cdot\bar{p})}\right] + ik\,i(\bar{k}\times\bar{m})\cdot\hat{\varepsilon}_s$$

$$\alpha_s^0 = \frac{k^2}{Q}(-p_xQ_y + p_yQ_x) - \frac{skk_z}{Q}(m_xQ_x + m_yQ_y) + m_zkQ$$

$$\alpha_p^0 = \frac{k^2}{Q}(-m_xQ_y + m_yQ_x) + \frac{skk_z}{Q}(p_xQ_x + p_yQ_y) - p_zkQ$$



REFLECTED FIELDS ASSUMING THAT THE DIPOLES ARE $\begin{pmatrix}\text{ABOVE}\\\text{BELOW}\end{pmatrix}$ THE SURFACE $(z=0)$ (AND THE METAL ON THE OTHER SIDE)

$$\left.\begin{aligned} \alpha_s^r &= r_s\,\alpha_s^0 \\ \alpha_p^r &= r_p\,\alpha_p^0 \end{aligned}\right\} \text{ with } s = \begin{bmatrix}-1\\+1\end{bmatrix}$$

BUT $\bar{E}_r, \vec{H}_r$ HAVE $\hat{\varepsilon}_p, \vec{k}$ with $s = \begin{pmatrix}+1\\-1\end{pmatrix}$

THE DIRECT PART IS

$$\vec{E}_0 = \frac{e^{ikr}}{r^3}\left[\left[(kr)^2 + ikr - 1\right]\vec{p} - \left[(kr)^2 + 3ikr - 3\right]\hat{r}(\bar{p}\cdot\hat{r}) + ik(ikr-1)(\bar{r}\times\bar{m})\right]$$

$$\bar{H}_0 = \frac{e^{ikr}}{r^3}\left[\left[(kr)^2 + ikr - 1\right]\bar{m} - \left[(kr)^2 + 3ikr - 3\right]\hat{r}(\bar{m}\cdot\hat{r}) - ik(ikr-1)(\bar{r}\times\bar{p})\right]$$

$\boxed{K\to\infty \text{ ASYMPTOTIC LIMIT}}$

$$\vec{\mathbf{E}} \simeq \underbrace{\vec{f}_E\frac{e^{ikr}}{r}}_{\text{free}} + \vec{E}_{sp}$$

$$\vec{H} \simeq \vec{f}_H\frac{e^{ikr}}{r} + \vec{H}_{sp}$$

$$\left.\begin{aligned} \vec{f}_E &= \hat{\varepsilon}_s\alpha_s + \hat{\varepsilon}_p\alpha_p \\ \vec{f}_H &= \hat{\varepsilon}_s\alpha_p - \hat{\varepsilon}_p\alpha_s \end{aligned}\right|_{\bar{k}=k\hat{r}}$$

$\Longrightarrow$ (1)

$$\bar p = \frac{\alpha_E}{\sqrt2}(\pm i\hat x - \hat y)$$
$$\bar m = \frac{\alpha_M}{\sqrt2}(\hat x \pm i\hat y)$$

$d_0 = 0$
$s_0 = 0$ (p)
$w_0 = \sqrt2$
$m = \pm 1$

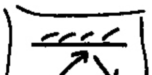
$$\bar H_\sigma = \frac{1}{ik}\bar D \times \vec E_\sigma = -i\,\bar E_{\tilde\sigma}$$

$$\bar p = \frac{\alpha_E}{\sqrt2}\left[\pm i(\cos\varphi\,\hat\rho - \sin\varphi\,\hat\varphi) - (\sin\varphi\,\hat\rho + \cos\varphi\,\hat\varphi)\right]$$
$$= \frac{\alpha_E}{\sqrt2}(\pm i\hat\rho - \hat\varphi)\,e^{\pm i\varphi}$$
$$\bar m = \frac{\alpha_M}{\sqrt2}\left[(\cos\varphi\,\hat\rho - \sin\varphi\,\hat\varphi) \pm i(\sin\varphi\,\hat\rho + \cos\varphi\,\hat\varphi)\right]$$
$$= \frac{\alpha_M}{\sqrt2}(\hat\rho \pm i\hat\varphi)\,e^{\pm i\varphi}$$

$\varphi = \varphi_Q$ HERE

$$\frac1Q(-p_x Q_y + p_y Q_x) = \frac{\alpha_E}{\sqrt2}(\mp i\sin\varphi_Q - \cos\varphi_Q) = -\frac{\alpha_E}{\sqrt2}e^{\pm i\varphi_Q}$$
$$\frac1Q(-m_x Q_y + m_y Q_x) = \frac{\alpha_M}{\sqrt2}(-\sin\varphi_Q \pm i\cos\varphi_Q) = \pm i\frac{\alpha_M}{\sqrt2}e^{\pm i\varphi_Q}$$
$$\frac1Q(p_x Q_x + p_y Q_y) = \frac{\alpha_E}{\sqrt2}(\pm i\cos\varphi_Q - \sin\varphi_Q) = \pm i\frac{\alpha_E}{\sqrt2}e^{\pm i\varphi_Q}$$
$$\frac1Q(m_x Q_x + m_y Q_y) = \frac{\alpha_M}{\sqrt2}(\cos\varphi_Q \pm i\sin\varphi_Q) = \frac{\alpha_M}{\sqrt2}e^{\pm i\varphi_Q}$$

$$\alpha_s^0 = -(\alpha_E k^2 + s k k_z \alpha_M)\frac{e^{\pm i\varphi_Q}}{\sqrt2} \qquad \alpha_s^r = -(\alpha_E k^2 + s k k_z \alpha_M)\, r_s \frac{e^{\pm i\varphi_Q}}{\sqrt2}$$
$$\alpha_p^0 = \pm i(\alpha_M k^2 + s k k_z \alpha_E)\frac{e^{\pm i\varphi_Q}}{\sqrt2} \qquad \alpha_p^r = \pm i(\alpha_M k^2 + s k k_z \alpha_E)\, r_p \frac{e^{\pm i\varphi_Q}}{\sqrt2}$$



$\Longrightarrow$ $\boxed{\begin{matrix}\alpha_M^{\parallel}\\ \alpha_E^{\parallel}\end{matrix}}$

FOR $\theta = \pi \Rightarrow$
$\Rightarrow Q = 0,\ k_z = k$
$r_p = -r_s = r$
$= \frac{\sqrt\varepsilon - 1}{\sqrt\varepsilon + 1}$
$\Rightarrow i\alpha_s = \alpha_p =$
$= i k^2 \frac{e^{i\varphi}}{\sqrt2}\cdot$
$\cdot[\alpha_M(1+r) - \alpha_E(1-r)]$

$\boxed{m=1}$ $\Rightarrow$
$$\begin{cases}\alpha_s = \dfrac{e^{i\varphi_Q}}{\sqrt2}k\left[\alpha_M k_z(1-r_s) - \alpha_E k(1+r_s)\right]\\[2mm] \alpha_p = \dfrac{e^{i\varphi_Q}}{\sqrt2}k\, i\left[\alpha_M k(1+r_p) - \alpha_E k_z(1-r_p)\right]\end{cases}$$

$\boxed{\text{TO COMPARE with ONE.m}}$

$\Longrightarrow$ (2)

(p) $d_0 = k$, $s_0 = 0$, $m = 0$, $w_0 = 1$ $\to$ $\vec p = \alpha_E^{\perp}\hat z$, $\vec m = 0$ $\to$ $\alpha_s = 0$, $\alpha_p = -\alpha_E^{\perp} k Q (1 + r_p)$

$\Rightarrow$ $\boxed{\begin{matrix}\alpha_M = 0\\ \alpha_E = 1\\ \text{AND } \bar p = \frac{1}{\sqrt2}(i\hat x - \hat y)\end{matrix}}$ $\Rightarrow I^{(1)}_{free} = \int_{\ominus} d\Omega\ \underbrace{|\alpha_s\hat\varepsilon_s + \alpha_p\hat\varepsilon_p|^2}_{\underset{(r\to\infty)}{= \hat r\cdot(\vec E_{free}\times\bar H^*_{free})}} = \frac{k^2}{2}2\pi\int_{\pi/2}^{\pi}\sin\theta\, d\theta\,\left(k^2|1+r_s|^2 + k_z^2|1-r_p|^2\right)$

$\boxed{\hat\varepsilon_p \times \hat\varepsilon_s = \hat k}$

$r_s = r_p = 0 \Rightarrow I^{(1)}_{free} = \pi k^2\int^0 d\mu\, k^2(1+\mu^2) = \frac{4\pi k^4}{3}$

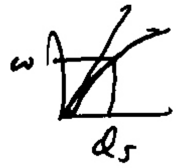
SP PART

$$\int d\vec{Q} = \int_0^\infty Q\,dQ \int d\varphi_Q$$

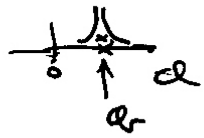
$$\int d\varphi_Q \to J_n(QR)$$

$$r_p \simeq 2\,\frac{(K_z^s)^2}{Q_s}\left(\frac{\varepsilon^2}{1-\varepsilon^2}\right)\frac{1}{(Q-Q_s)}, \qquad Q_s = K\sqrt{\frac{\varepsilon}{\varepsilon+1}}$$

$\mathrm{Im}\{K_z^s\} > 0$



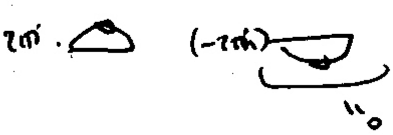
$\int_0^\infty \to \simeq \int_{-\infty}^{\infty}$ AND ASSUMING ANALYTICAL CONTINUITY



$$J_n(QR) = \frac{1}{2}\left(H_n^{(1)} + H_n^{(2)}\right)$$

$H_n^{(1)} \sim e^{iQR}$, $H_n^{(2)} \sim e^{-iQR}$

$\leftarrow \mathrm{Im}\{Q_s\} > 0$



$2\pi i$, $(-2\pi i)$, "0"

So, THE PRESCRIPTION IS TO DO THE $\int d\varphi_Q$ INTEGRAL, TO REPLACE $r_s \to 0$, $r_p \to 2\,\frac{K_z^2}{Q}\,\frac{\varepsilon^2}{1-\varepsilon^2}$,

$Q \to Q_s$, $\quad J_n \to \frac{1}{2} H_n^{(1)}$, $\quad \int_0^\infty dQ \to 2\pi i$, $\quad (\mathrm{Im}\{K_z\} > 0)$.

$$\int d\varphi_Q\, e^{i\vec{Q}\cdot\vec{R}} = 2\pi J_0(QR)$$

$$\int d\varphi_Q\, Q_x\, e^{i\vec{Q}\cdot\vec{R}} = 2\pi i\, Q \cos\varphi\, J_1(QR)$$

$$\int d\varphi_Q\, Q_y\, e^{i\vec{Q}\cdot\vec{R}} = 2\pi i\, Q \sin\varphi\, J_1$$

$$Q_x^2 = \pi Q^2\left[\cos^2\varphi\,(-J_2+J_0) + \sin^2\varphi\,(J_2+J_0)\right]$$

$$Q_y^2 = \pi Q^2\left[\cos^2\varphi\,(J_2+J_0) + \sin^2\varphi\,(-J_2+J_0)\right]$$

$$Q_x Q_y = -2\pi Q^2 \cos\varphi \sin\varphi\, J_2$$

$$\int d\varphi_Q\, e^{i\varphi_Q}\, e^{i\vec{Q}\cdot\vec{R}} = 2\pi i\, e^{i\varphi_R} J_1(QR)$$

$$\int d\varphi_Q\, e^{i\varphi_Q}\, e^{i\vec{Q}\cdot\vec{R}}\, \vec{Q} = 2\pi\, \vec{\nabla}_R\, e^{i\varphi_R} J_1(QR) \ominus$$

$$\cos\varphi = \frac{e^{i\varphi}+e^{-i\varphi}}{2} \qquad \cos^2\varphi = \frac{1}{4}\left(e^{2i\varphi}+e^{-2i\varphi}+2\right)$$

$$\sin\varphi = \frac{e^{i\varphi}-e^{-i\varphi}}{2i} \qquad \sin^2\varphi = -\frac{1}{4}\left(e^{2i\varphi}+e^{-2i\varphi}-2\right)$$

$$\sin\varphi\cos\varphi = \frac{1}{4i}\left(e^{2i\varphi}-e^{-2i\varphi}\right)$$

$$e^{im\varphi} J_m(QR) = j_m$$

$$\int d\varphi\, e^{i\vec{Q}\cdot\vec{R}} \Rightarrow \begin{cases} 1 \to 2\pi j_0 \\ Q_x \to 2\pi i\, Q\, \frac{1}{2}(j_1 + j_{-1}) \\ Q_y \to 2\pi i\, Q\, \frac{1}{2i}(j_1 - j_{-1}) \end{cases}$$

$\ominus$ $2\pi\left[Q\, J_1'(QR)\,\hat{R} + \frac{i}{R} J_1(QR)\,\hat{\varphi}\right] \cdot e^{i\varphi_R}$

$$Q_x^2 \to \frac{\pi Q^2}{2}\left(-j_2 - j_{-2} + 2 j_0\right), \quad Q_y^2 \to \frac{\pi Q^2}{2}\left(j_2 + j_{-2} + 2 j_0\right), \quad Q_x Q_y \to \frac{-\pi Q^2}{4i}\left(j_2 - j_{-2}\right)$$

$\boxed{z<0}$

$$\vec{E}_{sp} = \overbrace{\frac{K_z^2}{Q}\frac{\varepsilon^2}{1-\varepsilon^2}}^{r'_p}\ \overbrace{\frac{2\pi Q}{K_z}\frac{2\pi}{(2\pi)^2}}^{/d\varphi\ \text{etc}}\ \overbrace{2\pi}^{/dQ}\ \overbrace{\left(\frac{1}{2}\right)}^{J\to\frac{1}{2}H}\ \overbrace{\frac{1}{Q}}^{\hat{\varepsilon}_p}\ \overbrace{\frac{iQ}{\sqrt{2}}}^{\alpha_p}\cdot e^{i\varphi}$$

$\alpha_m = 0$, $\alpha_E^{\phi} = 1$

$z<0$

$\left(\begin{matrix} s=-1 \\ m\ \hat{\varepsilon}_p\end{matrix}\right)$

$$Q = \kappa\sqrt{\frac{\varepsilon}{1+\varepsilon}} \Big\} \cdot \Big[-K_z\left[Q\, H_1^{(1)'}(QR)\,\hat{R} + \frac{i}{R} H_1^{(1)}(QR)\,\hat{\varphi}\right] - Q^2 i\, H_1^{(1)}(QR)\,\hat{z}\Big]\cdot e^{iK_z|z|}$$

$$= -\frac{2\pi}{\sqrt{2}}\frac{\varepsilon^2}{1-\varepsilon^2}\kappa K_z^2\ \vec{E}^{H-}_{m=1,\sigma=p,Q,1}\ e^{iK_z|z|}$$

$$\bar{H}_{\sigma} = -\frac{i}{\kappa}\bar{\nabla}\times\bar{E}_{\sigma} = -i\sqrt{\varepsilon}\ \bar{E}_{\tilde{\sigma}}$$

$$\bar{H}_{sp} = \frac{2\pi i}{\sqrt{2}}\frac{\varepsilon^2}{1-\varepsilon^2}\kappa K_z^2\ \bar{E}^{H-}_{m=1,\sigma=s,Q,1}\ e^{iK_z|z|}$$

$$H^{(1)}_1 \underset{(QR\gg1)}{\simeq} \sqrt{\frac{2}{\pi QR}}\, e^{iQR}\cdot e^{-i\pi/2}e^{-i\pi/4}$$

$$H_1^{(1)'} \simeq i\, H_1^{(1)}$$

$$\mathrm{Re}\{\vec{E}_{sp}\times\bar{H}^*_{sp}\} \underset{R\to\infty}{\to} |e^{iK_z|z|}|^2\, C\cdot \mathrm{Re}\left\{\frac{i^2}{\kappa}(K_z\hat{R}+Q\hat{z})\times\hat{\varphi}\right\} =$$

$$= -C\frac{1}{\kappa}\left(\mathrm{Re}\{K_z\}\hat{z} - \mathrm{Re}\{Q\}\hat{R}\right)$$

$$C = \left|\frac{2\pi}{\sqrt{2}}\frac{\varepsilon^2}{1-\varepsilon^2}K_z^2\kappa\sqrt{\frac{2}{\pi QR}}e^{iQR}\right|^2$$

$$= \frac{1}{R}|e^{iK_z|z|}|^2\, A\left(\hat{R} - \frac{\mathrm{Re}\{K_z\}}{\mathrm{Re}\{Q\}}\hat{z}\right)$$

$z>0$

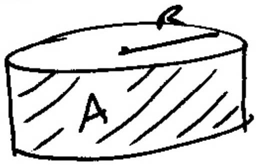
$$\vec{E}_{sp} = -\frac{2\pi}{\sqrt{2}}\frac{\varepsilon^2}{1-\varepsilon^2}\kappa K_z^2\ \vec{E}^{H+}_{m=1,\sigma=p,Q,\varepsilon}\ \frac{t_p}{r_p}\ e^{iK'_z|z|}$$

$$\hat{R}\times\hat{\varphi} = \hat{z} \qquad \hat{z}\times\hat{\varphi} = -\hat{R}$$

$$\vec{H}_{sp} = \frac{2\pi i}{\sqrt{2}}\frac{\varepsilon^2}{1-\varepsilon^2}\kappa K_z^2\ \vec{E}^{H+}_{m=1,\sigma=s,Q,\varepsilon}\frac{\sqrt{\varepsilon}}{\cancel{\sqrt{\varepsilon}}}e^{iK'_z|z|}$$

$$\boxed{A = 4\pi^2\left|\frac{\varepsilon^2}{1-\varepsilon^2}\right|^2|K_z|^4\kappa\,|e^{iQR}|^2\ \frac{\mathrm{Re}\{Q\}}{|Q|}}$$

$$= \frac{2K_z\sqrt{\varepsilon}}{\varepsilon K_z - K'_z} = \frac{1}{\sqrt{\varepsilon}} \qquad K'_z = -\varepsilon K_z$$



$$I_{sp} = \int_A \hat{R}\cdot\mathrm{Re}\{\vec{E}_{sp}\times\vec{H}^*_{sp}\}\, dR\, R\, dz\ 2\pi$$

$$= 2\pi\ A\left[\frac{1}{2\,\mathrm{Im}\{K_z\}} + \mathrm{Re}\left\{\frac{Q/|Q|}{\varepsilon}\right\}\frac{1}{2\,\mathrm{Im}\{K'_z\}}\right]$$

$$I^{(1)}_{sp} = 4\pi^2\left|\frac{\varepsilon^2}{1-\varepsilon^2}\right|^2|K_z|^4\kappa\ |e^{iQR}|^2\left[\frac{\mathrm{Re}\{Q\}}{|Q|}\frac{1}{\mathrm{Im}\{K_z\}} + \mathrm{Re}\left\{\frac{(Q/|Q|)}{\varepsilon}\right\}\frac{1}{\mathrm{Im}\{K'_z\}}\right]$$

$$|e^{iK_z|z|}| = e^{-\mathrm{Im}\{K_z\}|z|}$$

$$Q = \kappa\frac{\sqrt{\varepsilon}}{\sqrt{1+\varepsilon}} \to K'_z = \kappa\sqrt{\varepsilon - \frac{\varepsilon}{\varepsilon+1}} = \frac{\varepsilon\kappa}{\sqrt{\varepsilon+1}}$$

$$K_z = \kappa\sqrt{1-\frac{\varepsilon}{\varepsilon+1}} = \frac{-\kappa}{\sqrt{\varepsilon+1}} \leftarrow$$

$\mathrm{Re}\{\sqrt{\varepsilon+1}\} > 0$
$\mathrm{Im}\{\sqrt{\varepsilon+1}\} > 0$
$\hookrightarrow \mathrm{Im}\{K_z\} > 0$
$\mathrm{Re}\{K_z\} < 0$

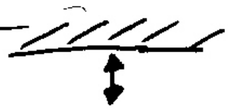
$\varepsilon\to-\infty,\ Q\in\mathbb{R}$

$$I^{(1)}_{sp} \to \frac{4\pi^2\kappa^4}{|\varepsilon|^{3/2}}$$

$Q_0 = K$
$S_0 = 0$
$m = 0$
$w_0 = 1$

$$\rightarrow \quad I^{(1)}_{free} = K^2\, 2\pi \int_{\pi/2}^{\pi} \sin\vartheta \, d\vartheta \quad Q^2\, |1+r_p|^2$$

$(\alpha_E^{\perp} = 1)$



$$\vec{E}_{SP} = \frac{2 K_z^2}{Q} \frac{\varepsilon^2}{1-\varepsilon^2} \frac{2Q}{K_z} \frac{2\pi}{(2\pi)^2} (2\pi) \frac{i}{2} \frac{1}{KQ} (-KQ) \cdot$$

$$\left(-K_z\, i \hat{R} Q\, H_1^{(1)} - Q^2 H_0^{(1)} \hat{z}\right)$$

$$= -2\pi \frac{\varepsilon^2}{1-\varepsilon^2} K K_z Q \; \vec{E}^{H-}_{m=0,\sigma=p,Q,1}$$

$$I^{(2)}_{SP} = 2\, I^{(1)}_{SP} \left|\frac{Q}{K_z}\right|^2$$