Pole: $\sqrt{k_{xn}^2 + k_y^2} - k_p = 0$ $k_{xn} = \frac{\omega}{v} + \frac{2n\pi}{a}$

$\underline{n=0}$: $k_{x0} = \frac{\omega}{v} \Rightarrow \sqrt{\left(\frac{\omega}{v}\right)^2 + k_y^2} = k_p$

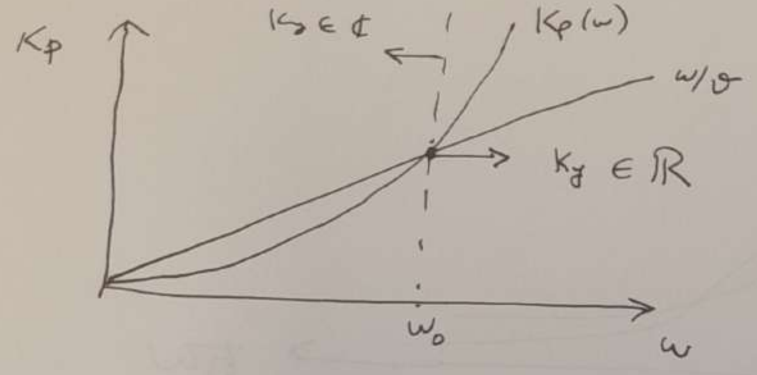
$\Leftrightarrow$ $k_y = \pm\sqrt{k_p^2 - (\omega/v)^2}$

$k_p > \frac{\omega}{v} - \frac{2\pi n}{a}$

$k_p + \frac{2\pi n}{a} > \frac{\omega}{v}$

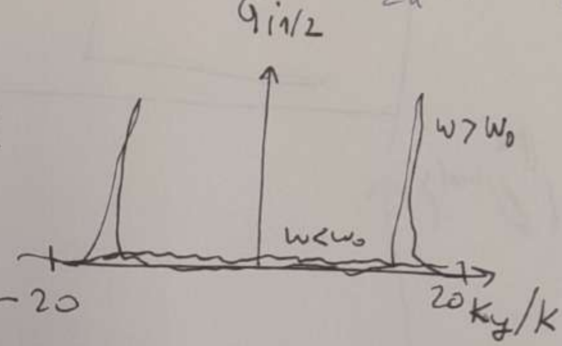
$\frac{2\pi n}{a} > \frac{\omega}{v} - k_p$

$n > \frac{a}{2\pi}\left(\frac{\omega}{v} - k_p\right)$



$\frac{k_p(\omega)}{k_0} = \frac{\varepsilon_1 + \varepsilon_2}{\alpha} \cdot \frac{\hbar\omega + i\gamma}{4E_F} = \frac{1}{\alpha}\frac{\hbar\omega + i\gamma}{2E_F}$

$\frac{\omega/v}{k_0} = \frac{c}{v}$

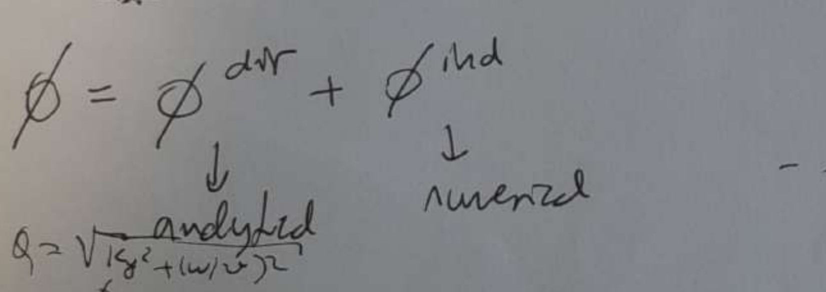


$\Rightarrow \frac{1}{\alpha}\frac{\hbar\omega_0}{2E_F} = \frac{c}{v} \Leftrightarrow \boxed{\omega_0 = \frac{\alpha c}{v} \cdot \frac{2E_F}{\hbar}}$

$\hbar\omega_0 = \frac{1}{137} \times \frac{c}{v} \times 2 \times 0.3 = 43\,\mathrm{meV}$

$\frac{\omega/v + 2n\pi/a}{k_0} = \frac{k_p(\omega)}{k_0}$

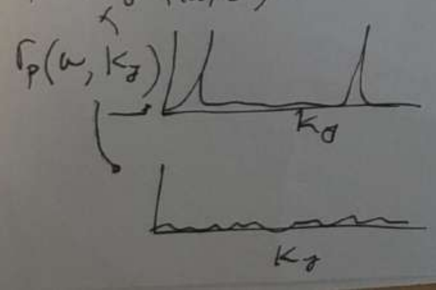
$\Rightarrow \omega_0^{(n)}$

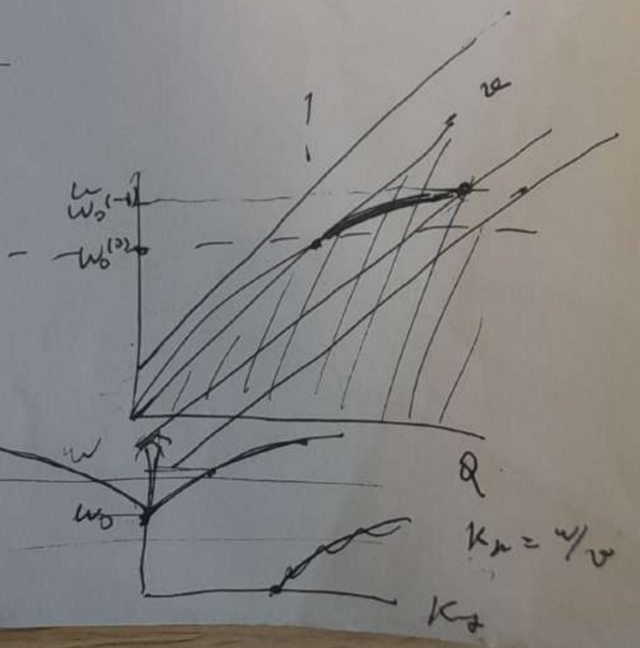


$\phi = \phi^{dir} + \phi^{ind}$

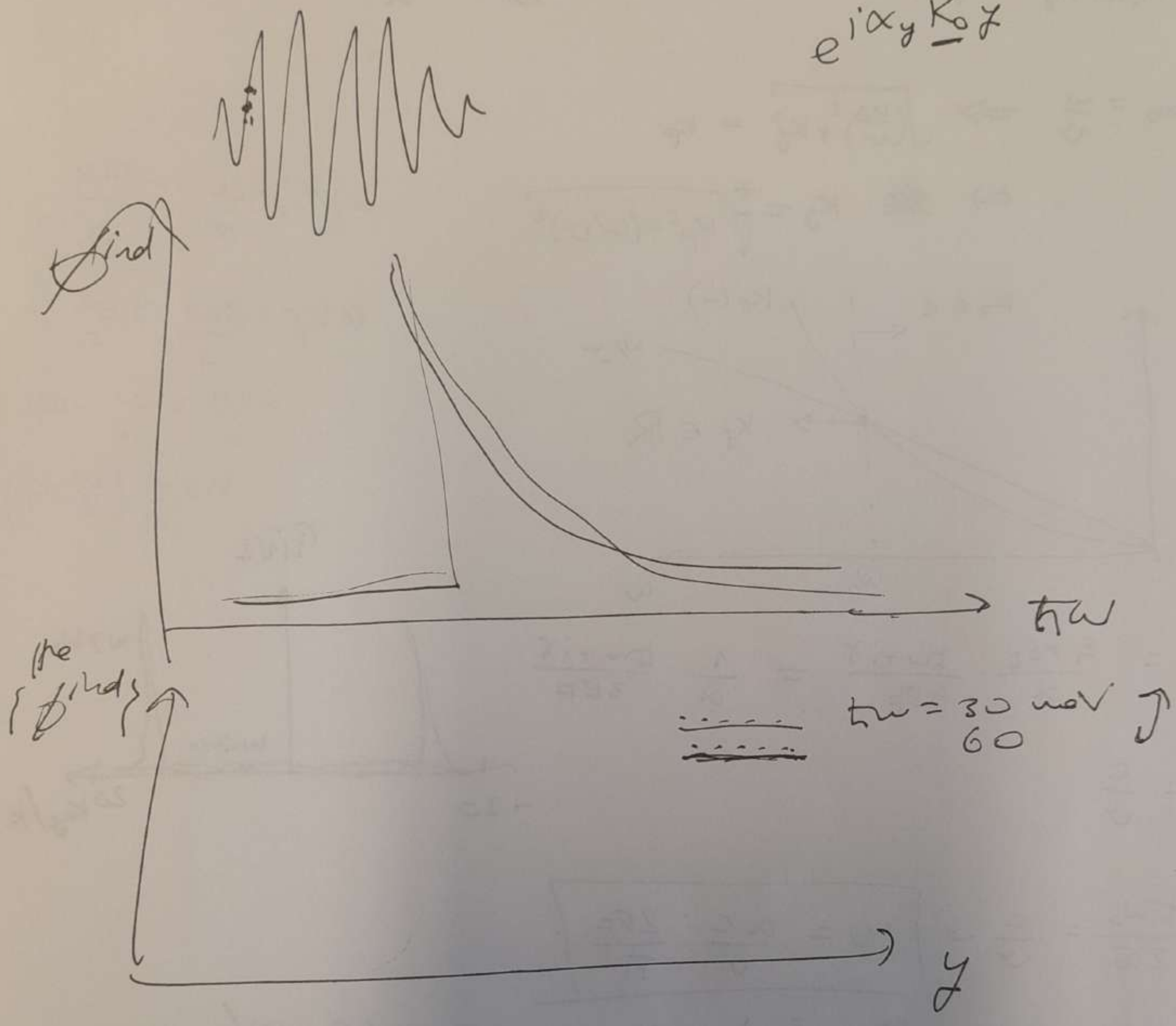
$\downarrow$ analytical $\downarrow$ numerical

$Q = \sqrt{k_y^2 + (\omega/v)^2}$





$e^{i\alpha_y \underline{K_0} z}$



$$\int_0^{\infty} dK_{\parallel} \, f(K_{\parallel}) \cdot \frac{1}{K_{\parallel} - K_p} \qquad \mathrm{Im}\{K_p\} > 0$$

Line of Dipoles

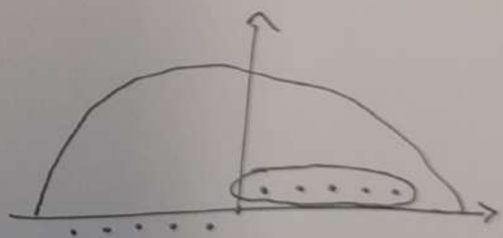
$$\int_{-\infty}^{+\infty} dK_y \, f(K_y) \cdot \frac{1}{\sqrt{K_{xn}^2 + K_y^2} - K_p}$$

$$= \int_{-\infty}^{+\infty} dK_y \, f(K_y) \, \frac{\sqrt{K_{xn}^2 + K_y^2} + K_p}{K_{xn}^2 + K_y^2 - K_p^2}$$

$$I_n = \int_{-\infty}^{+\infty} dK_y \, \overbrace{f(K_y) \cdot \sqrt{K_{xn}^2 + K_y^2} + K_p}^{\tilde{f}(K_y)} \cdot \left(\frac{1}{K_y + \sqrt{K_{xn}^2 - K_p^2}} - \frac{1}{K_y - \sqrt{K_{xn}^2 - K_p^2}} \right) \cdot \frac{1}{2\sqrt{K_{xn}^2 - K_p^2}}$$

$K_y = -\gamma_n \qquad \gamma_n = \gamma_n^r + i\gamma_n^i \qquad \Downarrow \; K_y = \gamma_n$

$$\gamma_n = \sqrt{K_{xn}^2 - K_p^2} = \sqrt{\left(\frac{\omega}{v} + \frac{2n\pi}{a}\right)^2 - K_p^2} = \pm K_y$$

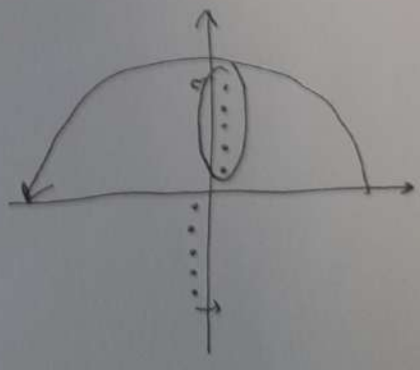


1) $|K_{xn}| > |K_p| \Rightarrow \gamma_n \in \mathbb{R}$

$\Downarrow \qquad \mathrm{Im}\{\gamma_n\} > 0$

$$\frac{\omega}{v} + \frac{2n\pi}{a} > K_p \Rightarrow n < \left(K_p - \frac{\omega}{v}\right) \cdot \frac{a}{2\pi} \equiv n_{max}$$

$$I_n = -2\pi i \, \tilde{f}(\gamma_n) \cdot \frac{1}{2\gamma_n}$$

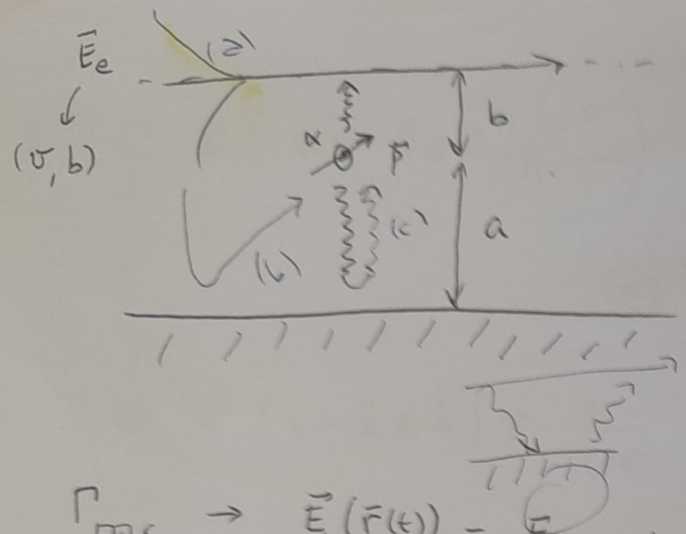


2) $|K_{xn}| < |K_p| \Rightarrow \gamma_n \in \mathbb{I}$

$K_y = \pm i \gamma_n' \qquad \gamma_n \equiv i\gamma_n'$

$$I_n = -2\pi i \, \tilde{f}(\gamma_n) \cdot \frac{1}{2\gamma_n}$$

$$I = \sum_n I_n = -2\pi i \sum_{n=0}^{\infty} \tilde{f}(\gamma_n) / 2\gamma_n$$



$$\vec{p} = \alpha \cdot \vec{E}_{tot} \qquad (1)$$

$$\underset{\vec{r}=\vec{0}}{E_{tot}^{(dip)}} = \bar{E}_e^{(a)} + \bar{E}_{e,ref}^{(b)} + \bar{E}_{dip,ref}^{(c)}(\vec{p}) \qquad (2)$$

$(2) \rightarrow (1) \Rightarrow \vec{p}$

$g\bar{p}$

$\vec{r} = \vec{v}t$

$\Gamma_{EELS} \rightarrow \vec{E}_{tot}(\vec{r}_e(t)) = \underbrace{\bar{E}_{e,ref}} + \underbrace{\bar{E}_{dip}(\vec{p})} + \underbrace{\bar{E}_{dip,ref}(\vec{p})}$

$\{p^- \tilde{r} + p^- \tilde{r} | \varepsilon^+ + \varepsilon^- p^-$

$2 z_e$ blue

$\Gamma_{EELS} = \Gamma_{\parallel} + \Gamma_{direct} + \Gamma_{ref}$

z_p blue, z electron

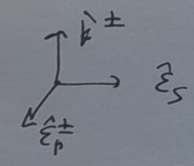
E_e

$k^2 p$ direct

$C_{\cdot}(b,w) \, \mathrm{Im}\{\alpha(w)\}$

$E_{e,ref} + E_{dip,ref}$

$k^2 p$ ref

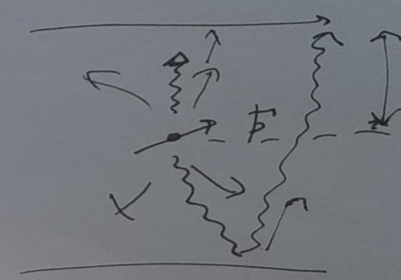


$$\mathbb{1} = \hat{k}^{\pm} \otimes \hat{k}^{\pm} + \hat{\varepsilon}_s \otimes \hat{\varepsilon}_s + \hat{\varepsilon}_p^{\pm} \otimes \hat{\varepsilon}_p^{\pm}$$

$$\vec{E}^{dip} \stackrel{-}{=} \cancel{k}\left(k^2 + \vec{\nabla} \otimes \vec{\nabla} \right) \vec{p} \underbrace{\left(\frac{e^{ikr}}{r} \right)}_{= \frac{1}{(2\pi)^2} \int \frac{d^2Q}{k_z} e^{i\vec{Q}\cdot\vec{R}} e^{ik_z z}}$$

$$\stackrel{-}{=} k^2 \left(1 - \hat{k} \otimes \hat{k} \right) \vec{p} \cdot \frac{1}{(2\pi)^2} \int \frac{d^2Q}{k_z} \ldots$$

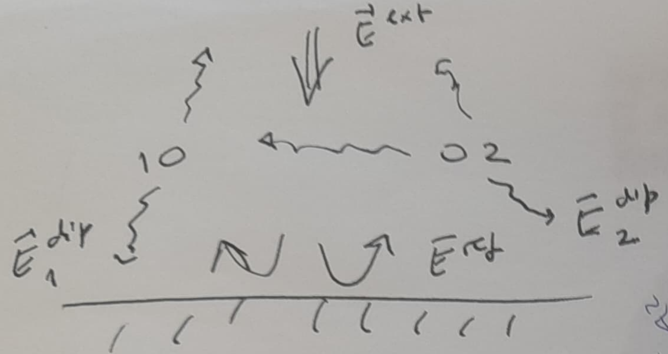
$$\stackrel{-}{=} \frac{k^2}{(2\pi)^2} \int \frac{d^2Q}{k_z} \left[\hat{\varepsilon}_s \otimes \hat{\varepsilon}_s + \hat{\varepsilon}_p^{\pm} \otimes \hat{\varepsilon}_p^{\pm} \right] \cdot \vec{p}\, e^{i\vec{Q}\cdot\vec{R}} e^{ik_z z}$$



$$\vec{E}^{dip}_{direct} = \frac{k^2}{(2\pi)^2} \int \frac{d^2Q}{k_z} \left[(\hat{\varepsilon}_s \cdot \vec{p})\, \hat{\varepsilon}_s + (\hat{\varepsilon}_p^{+} \cdot \vec{p})\, \hat{\varepsilon}_p^{+} \right] e^{i\vec{Q}\cdot\vec{R}} e^{ik_z(z+b)}$$

$$\vec{E}^{dip}_{ref} = \frac{k^2}{(2\pi)^2} \int \frac{d^2Q}{k_z} \left[(\hat{\varepsilon}_s \cdot \vec{p})\, \hat{\varepsilon}_s\, e^{ik_z(2a+b)} r_s + (\hat{\varepsilon}_p^{-} \cdot \vec{p})\, \hat{\varepsilon}_p^{+} e^{ik_z(2a+b)} r_p \right] e^{i\vec{Q}\cdot\vec{R}} e^{ik_z z}$$

$\hat{\varepsilon}_s, \hat{\varepsilon}_p \to \varphi$



$$\bar{E}_j^{dip} = \beta_j \bar{p}_j$$

$$\begin{cases} \bar{p}_1 = \alpha_1 \vec{E}(\bar{r}_1) = \alpha_1 \left[\bar{E}^{ext} + \bar{E}^{ref} + \bar{E}_1^{dip} \tilde{r}_\sigma + \bar{E}_2^{dip} + \bar{E}_2^{dip} \bar{r}_\sigma \right] \\ \bar{p}_2 = \alpha_2 E(\bar{r}_2) = \alpha_2 \left[\quad " \quad " \quad \bar{E}_2^{dip} \bar{r}_\sigma + E_1^{dip} + \bar{E}_1^{dip} \bar{r}_\sigma \right] \end{cases}$$

$\alpha \bar{p}_1 \quad \alpha \bar{p}_2 \quad \alpha \bar{p}_2$

≈ 6

$$\begin{cases} \bar{p}_1 = \alpha_1 \left[\bar{E}^{ext}(1+\bar{r}_\sigma) + \beta_1 \bar{p}_1 + \beta_2 (1+\bar{r}_\sigma) \bar{p}_2 \right] \\ \bar{p}_2 = \alpha_2 \left[\qquad \right] \end{cases}$$

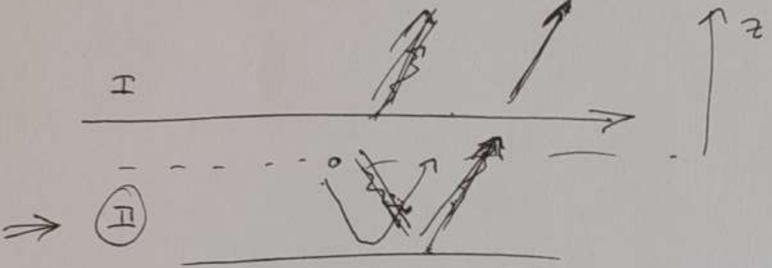
$$\bar{p}_1 = (\alpha_{eff}^1) \left[\bar{E}^{ext}(1+\bar{r}_\sigma) + \beta_2 (1+\bar{r}_\sigma) \bar{p}_2 \right] \qquad \alpha_{eff}^1 = \frac{1}{1/\alpha_1 - \beta_1}$$

$\phi_p^{dir} = -\bar{p}\cdot\bar{\nabla}\left(\frac{1}{|\bar r-\bar r_0|}\right)$

$\frac{1}{|\bar r-\bar r_0|} \int \frac{d^2\bar k_{\parallel}}{(2\pi)\, k_{\parallel}}\, e^{i\bar k_{\parallel}\cdot(\bar R-\bar R_0)}\, e^{-k_{\parallel}(z-z_0)}$

$\bar r = \bar R + z\hat z$

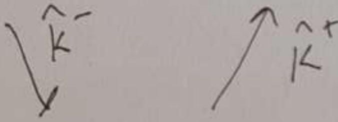
$= -i\bar p\cdot \int \frac{d^2\bar k_{\parallel}}{2\pi k_{\parallel}}\, \vec k\cdot e^{i\bar k_{\parallel}\cdot(\bar R-\bar R_0)}\, e^{-k_{\parallel}(z-z_0)}$



(I)

$\phi_p^{tot} = -i\bar p\cdot \int \frac{d^2\bar k_{\parallel}}{2\pi k_{\parallel}}\, \bar k^{+}\, \underline{e^{i\vec k_{\parallel}(\bar R-\bar R_0)}}\left(e^{-k_{\parallel}(z-z_0)} + \cancel{r_s\, e^{-k_{\parallel}(z-z_0)} + \dots r_p}\right)$

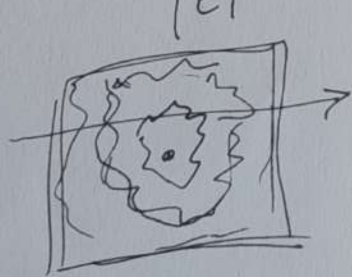
(II)

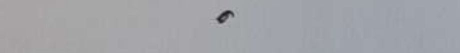


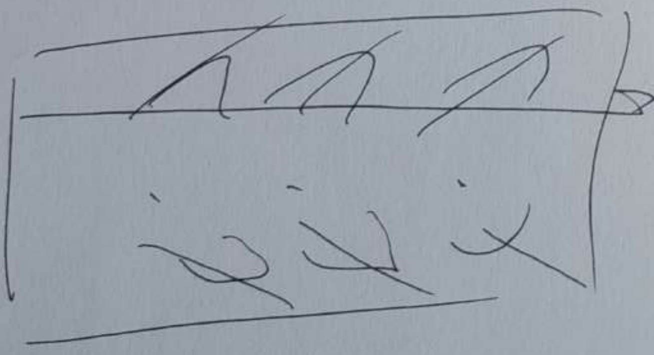
$\bar p$

$\phi_p \longrightarrow \vec E_p = -\bar\nabla\phi_p$

$|E|^2$







$\varepsilon_2 i \alpha_n + \varepsilon_1 i \alpha_n + 4\pi$

$r_p(\omega, k_{\parallel})$

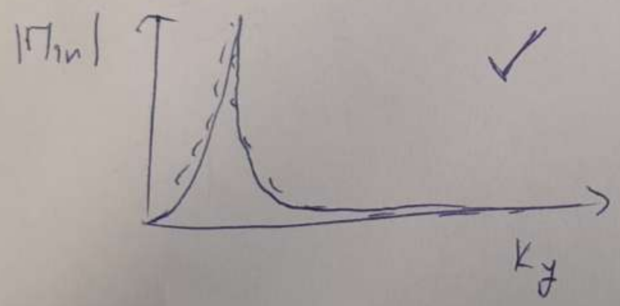
$k_{\parallel}^{(n)}$

$$M_{1n}(k_y) = k_{xn} \frac{e^{-2\sqrt{k_{xn}^2+k_y^2}\cdot z_p}}{\sqrt{k_{xn}^2+k_y^2}} \underbrace{r_p\left(\sqrt{k_{xn}^2+k_y^2}\right)} \cdot e^{i k_y y}$$

(1) → Fresnel

(2) ⇒ $\dfrac{R_p Q_p}{\sqrt{k_{xn}^2+k_y^2} - Q_p}$

n fixed



Numerical int (1) → I_1

Numerical int (2) → I_2

Analytical int. Pole → I_3

$\hbar\omega = 63$ meV

$k_{xn} < Q_p$

$\cancel{k_{xn} = \sqrt{k^2 - \left(k_y + \frac{2\pi n}{a}\right)^2}}$

$\frac{\omega}{c} > k_{\parallel} > \frac{\omega}{v} + \frac{2n\pi}{a}$

$\Rightarrow \frac{\omega}{c} > \frac{\omega}{v} + \frac{2n\pi}{a}$

$\Rightarrow \omega\left(1/c - 1/v\right) > \frac{2n\pi}{a} \Rightarrow \omega > \frac{2n\pi}{a}\left(\frac{1}{1/c - 1/v}\right)$

$k_{xn} = \frac{\omega}{v} + 2n\pi/a$

$k_{\parallel} = \sqrt{k_{xn}^2 + k_y^2} \Rightarrow k_y = \sqrt{k_{\parallel}^2 - k_{xn}^2} = \sqrt{k_{\parallel}^2 - \left(\frac{\omega}{v} + \frac{2n\pi}{a}\right)^2}$

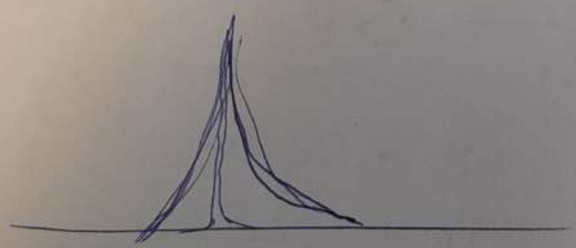
$$\int d^2\bar{K}_{//} \quad \begin{cases} \int_0^{2\pi} d\phi \int_0^{\infty} K_{//}\, dK_{//} \\ \int_{-\infty}^{+\infty} dK_x \int_{-\infty}^{+\infty} dK_y \end{cases}$$

$$\underline{\underline{K_{//} = Q_p}} = \sqrt{K_{xx}^2 + K_y^2} = Q_p$$

$$\Rightarrow K_{xx}^2 + K_y^2 = Q_p^2 \Rightarrow K_y = \sqrt{Q_p^2 - K_{xx}^2}$$

$$K_{xx} < Q_p$$

$$\underline{\underline{|\mathrm{Im}(Q_p)| << |\mathrm{Re}(Q_p)|}}$$



$$\frac{1}{x - x' + i\varepsilon} \approx P.V.\frac{1}{x - x'} - \underline{i\pi\,\delta(x - x')} \qquad \varepsilon \to 0^+$$

$$\int dk_y \frac{e^{-2\sqrt{k_{nx}^2+k_y^2}\,z_p}}{\sqrt{k_{nx}^2+k_y^2}} \underbrace{\left(\frac{R_p\, k_p}{\sqrt{k_{nx}^2+k_y^2} \cancel{+} - k_p}\right)} e^{ik_y y}$$

$$\text{P.V.}\ \frac{1}{\sqrt{k_{nx}^2+k_y^2}-k_p} \overset{+}{-}\ i\pi\,\delta\left[k_p-\sqrt{k_{nx}^2+k_y^2}\right]$$

neglecting P.V.

$$= -i\pi\int dk_y \frac{e^{-2\sqrt{k_{nx}^2+k_y^2}\,z_p}}{\sqrt{k_{nx}^2+k_y^2}}\, e^{ik_y y}\,\delta\left[k_p-\sqrt{k_{nx}^2+k_y^2}\right] R_p \cancel{k_p} \qquad (*)$$

$$\delta\left[f(x)\right]=\sum_j \frac{\delta(x-x_j)}{|f'(x_j)|} \qquad f(x_j)=0 \qquad k_{nx}=\frac{\omega}{v}+\frac{2n\pi}{a}$$

$$k_p-\sqrt{k_{nx}^2+k_y^2}=0 \Leftrightarrow k_{nx}^2+k_y^2=k_p^2$$
$$\Leftrightarrow k_y^{(n)\pm}=\pm\sqrt{k_p^2-k_{nx}^2}$$
$$=\pm\sqrt{k_p^2-\left(\frac{\omega}{v}+\frac{2n\pi}{a}\right)^2}$$

$$k_p > \frac{\omega}{v}+\frac{2\pi n}{a} \Leftrightarrow n<\left(k_p-\frac{\omega}{v}\right)\frac{a}{2\pi}\equiv n_{max}$$

$$\Leftrightarrow n<k\left(\frac{k_p}{k}-\frac{c}{v}\right)\frac{a}{2\pi}$$

$$f'(k_y)=\frac{-k_y}{\sqrt{k_{nx}^2+k_y^2}} \Rightarrow \delta\left[f(k_y)\right]=\left(\cancel{\sum_{n=0}^{n_{max}}}\right)\sum_{v=\pm}\frac{\delta(k_y-k_y^{(n)v})}{\left|\frac{k_y^{(n)v}}{\sqrt{k_{nx}^2+k_y^{(n)v\,2}}}\right|}\Theta(n_{max}-n)$$

$$= k_p\left(\cancel{\sum_{n=0}^{n_{max}}}\right)\sum_{v=\pm}\frac{\delta(k_y-k_y^{(n)v})}{|k_y^{(n)v}|}\Theta(n_{max}-n)$$

$$(*) = \underline{-i\pi k_p R_p}\,\Theta(n_{max}-n)\sum_{v=\pm}\frac{e^{-2k_p z_p}}{|k_y^{(n)v}|}\, e^{ik_y^{(n)v}}$$

$$k_y^{(n)v} = v\, k_y^{(n)} \qquad k_y^{(n)}=\sqrt{k_p^2-k_{nx}^2}$$

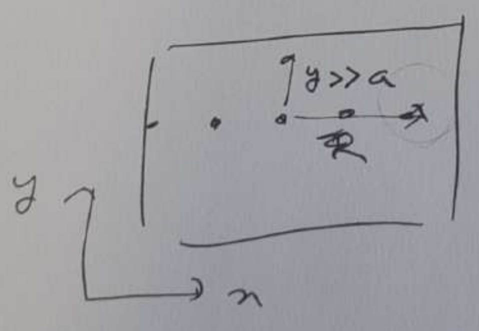
$$M_1 = \sum_n e^{i k_{xn} x} (-i\pi) k_\rho R_\rho \, \Theta(n_{max}-n) \sum_{\nu=\pm} \frac{e^{-2k_\rho z_\rho}}{|k_y^{(n)\nu}|} e^{i k_y^{(n)\nu} y}$$

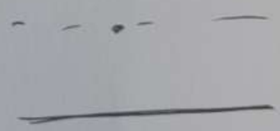
$$= \sum_{n=0}^{n_{max}} e^{i k_{xn} x} (-i\pi) k_\rho R_\rho \sum_{\nu=\pm} \frac{e^{-2k_\rho z_\rho}}{k_y^{(n)}} e^{i \nu k_y^{(n)} y}$$

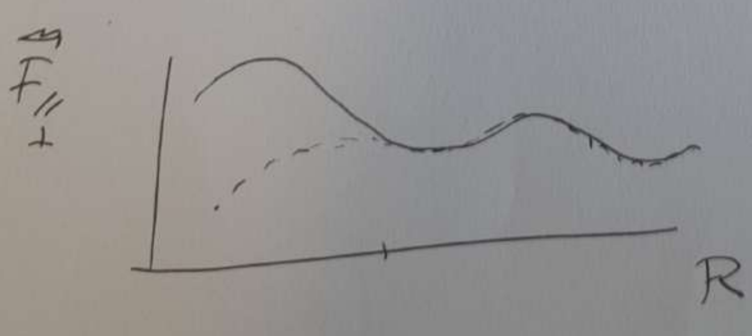
$$= \sum_{n=0}^{n_{max}} e^{i k_{xn} x} (-i\pi) k_\rho R_\rho \frac{e^{-2k_\rho z_\rho}}{k_y^{(n)}} \left(e^{i k_y^{(n)} y} + e^{-i k_y^{(n)} y} \right)$$

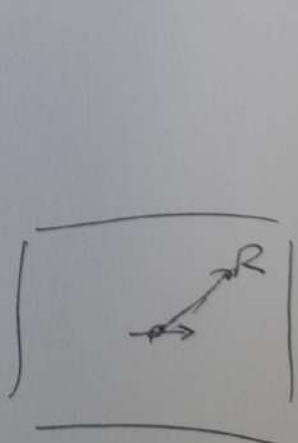
$$= \sum_{n=0}^{n_{max}} e^{i k_{xn} x} (-i2\pi) k_\rho R_\rho \frac{e^{-2k_\rho z_\rho}}{k_y^{(n)}} \cos\left(k_y^{(n)} y \right)$$

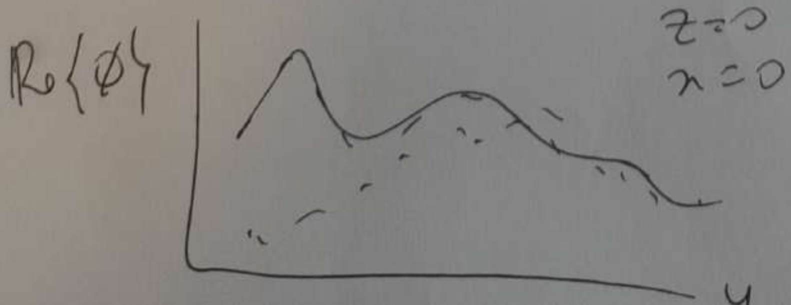
desde
$n=0$ ó desde
$-n_{max}$?









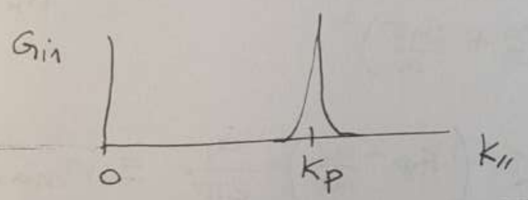


$$G_1 = \int_0^\infty dk_\parallel \, k_\parallel \, r_p \, J_0(k_\parallel R) \, e^{-k_\parallel(2z_p - z)} = \int_0^\infty dk_\parallel \, G_{i1}(k_\parallel)$$

$$r_p \simeq \frac{R_p k_p}{k_\parallel - k_p} \longrightarrow \text{P.V.}\left(\frac{R_p k_p}{k_\parallel - k_p}\right) - i\pi\delta(k - k_p) \cdot R_p k_p$$

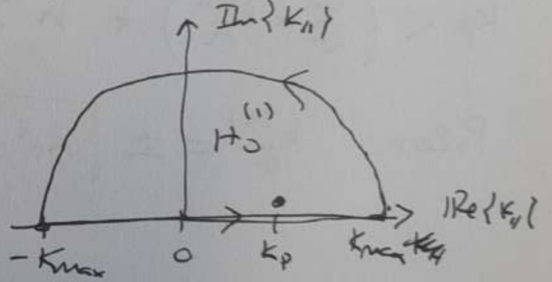
$$① = ⨍_0^\infty dk_\parallel \frac{k_\parallel R_p k_p}{k_\parallel - k_p} J_0(k_\parallel R) \, e^{-k_\parallel(2z_p - z)} -$$

$$- i\pi R_p k_p^2 J_0(k_p R) \, e^{-k_p(2z_p - z)}$$



$$② = $$

$$\approx \int_{-\infty}^{+\infty} dk_\parallel \, k_\parallel \, \underbrace{r_p}_{\frac{R_p k_p}{k_\parallel - k_p}} \, \overbrace{J_0(k_\parallel R)}^{= \frac{1}{2}\left[H_0^{(1)}(k_\parallel R) + H_0^{(2)}(k_\parallel R)\right]} \, e^{-k_\parallel(2z_p - z)}$$



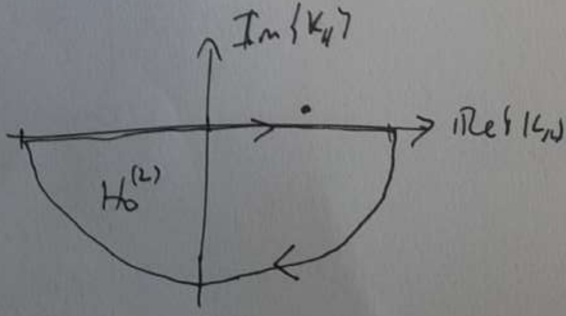
$k_{max} \to \infty$

$$= \int_{-\infty}^{+\infty} \frac{dk_\parallel}{2} k_\parallel \, r_p \, H_0^{(1)}(k_\parallel R) \, e^{-k_\parallel(2z_p - z)} + (\ldots) \to 0$$

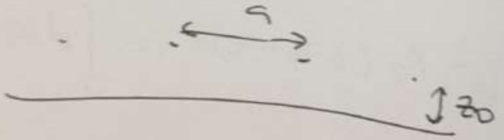
$$R[f(x), x_0] = \lim_{x \to x_0} (x - x_0) f(x)$$

$$= \frac{2\pi i}{2} \, k_p R_p k_p H_0^{(1)}(k_p R) \, e^{-k_p(2z_p - z)}$$

$$H_0^{(1)}(x) \xrightarrow[x \gg 1]{} \frac{e^{-x}}{\sqrt{x}} \quad (?)$$

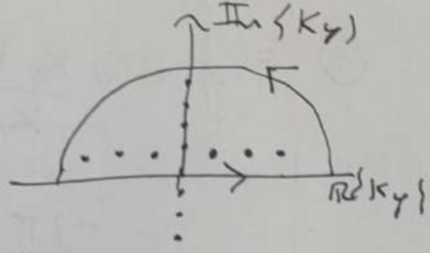


$$k_z \simeq i k_\parallel$$



$$\frac{R_p K_p}{\sqrt{K_{xn}^2+K_y^2}+K_p}$$

$$G_1 = K_{xn}\int_{-\infty}^{+\infty} dK_y \, \frac{e^{-2\sqrt{K_{xn}^2+K_y^2}\,z_p}}{\sqrt{K_{xn}^2+K_y^2}} \, r_p \, e^{i k_y y}$$



$K_{xn} = \frac{\omega}{v} + \frac{2n\pi}{a}$

Poles: $\sqrt{K_{xn}^2+K_y^2} = K_p$

$\Rightarrow \ K_y = \pm\sqrt{K_p^2 - K_{xn}^2}$

$= \pm\sqrt{K_p^2 - \left(\frac{\omega}{v}+\frac{2n\pi}{a}\right)^2}$

$K_y \in \mathbb{R}$:

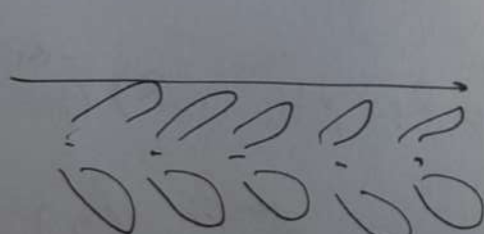
$K_p > \left(\frac{\omega}{v}+\frac{2n\pi}{a}\right) \Rightarrow n < \left(K_p - \frac{\omega}{v}\right)\cdot\frac{a}{2\pi} \equiv n_{max}$

$K_y \in \mathrm{Im}$:

$K_p < \left(\frac{\omega}{v}+\frac{2n\pi}{a}\right) \Rightarrow n > n_{max}$

Poles: $K_y^{(n)} = \pm\sqrt{K_p^2 - \left(\frac{\omega}{v}+\frac{2n\pi}{a}\right)^2}$

$(K_y - K_y^{(n)})\, f(K_y)$



$\rightarrow \frac{\omega}{v} + \frac{2n\pi}{a}$

$\varepsilon_2 i \alpha_n + \varepsilon_1 i \alpha_n + 4\pi$

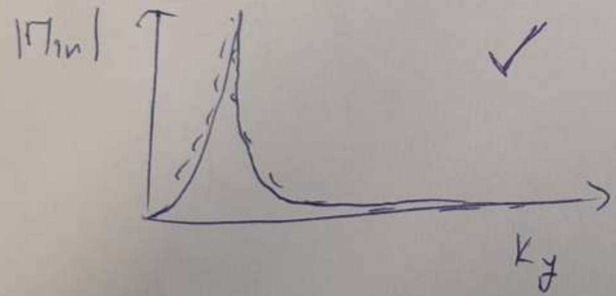
$r_p(\omega, k_\parallel)$

$$M_{1n}(k_y) = k_{xn} \frac{e^{-2\sqrt{k_{xn}^2 + k_y^2}\cdot z_p}}{\sqrt{k_{xn}^2 + k_y^2}} \underbrace{r_p\left(\overbrace{\sqrt{k_{xn}^2 + k_y^2}}^{k_\parallel^{(n)}}\right)} \cdot e^{i k_y y}$$

(1) $\Rightarrow$ Fresnel

(2) $\Rightarrow \dfrac{R_p Q_p}{\sqrt{k_{xn}^2 + k_y^2} - Q_p}$

n fixed



✓

Numerical int (1) $\rightarrow I_1$

Numerical int (2) $\rightarrow I_2$

Analytic int. Pole $\rightarrow I_3$

$\hbar\omega = 63$ meV

$\underline{\underline{k_{xn} < Q_p}}$

$\cancel{k_{xn} = \sqrt{k^2 - \left(k_y + \frac{2\pi n}{a}\right)^2}}$

$\frac{\omega}{c} > k_\parallel > \frac{\omega}{v} + \frac{2n\pi}{a}$

$\Rightarrow \frac{\omega}{c} > \frac{\omega}{v} + \frac{2n\pi}{a}$

$\Rightarrow \omega(1/c - 1/v) > \frac{2n\pi}{a} \Rightarrow \underline{\underline{\omega}} > \frac{2n\pi}{a}\left(\frac{1}{1/c - 1/v}\right)$

$k_{xn} = \frac{\omega}{v} + 2n\pi/a$

$k_\parallel = \sqrt{k_{xn}^2 + k_y^2} \Rightarrow k_y = \sqrt{k_\parallel^2 - k_{xn}^2} = \sqrt{k_\parallel^2 - \left(\frac{\omega}{v} + \frac{2n\pi}{a}\right)^2}$

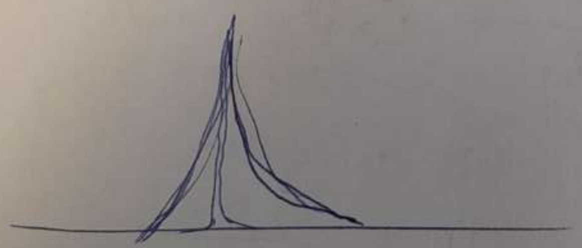
$$\int d^2\bar{k}_{\parallel} \;\rightarrow\; \int_0^{2\pi} d\phi \int_0^{\infty} k_{\parallel}\, dk_{\parallel}$$

$$\int d^2\bar{k}_{\parallel} \;\rightarrow\; \int_{-\infty}^{+\infty} dk_x \int_{-\infty}^{+\infty} dk_y$$

$$\underline{\underline{k_{\parallel} = Q_p}} = \sqrt{k_{xx}^2 + k_y^2} = Q_p$$

$$\Rightarrow \; k_{xx}^2 + k_y^2 = Q_p^2 \;\Rightarrow\; k_y = \sqrt{Q_p^2 - k_{xx}^2}$$

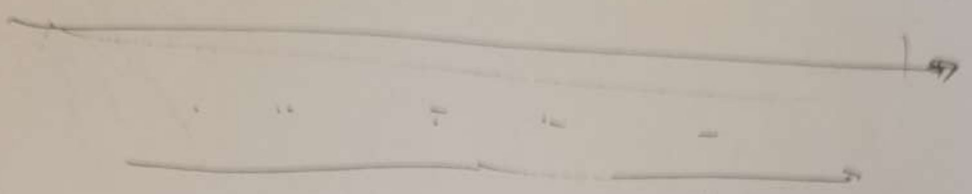
$$\underline{\underline{|\mathrm{Im}(Q_p)| \ll |\mathrm{Re}(Q_p)|}} \qquad k_{xx} < Q_p$$



$$\frac{1}{x - x' + i\varepsilon} \approx \mathrm{P.V.}\,\frac{1}{x - x'} - \underline{i\pi\,\delta(x - x')} \qquad \varepsilon \rightarrow 0^+$$

$$\frac{1}{L}\int_{-\infty}^{+\infty} dx \rightarrow \sum_n$$

Lunes 28



$$\frac{\Gamma_{EELS}}{L}$$

$$\delta\left(k_{xn} - \omega/v\right) = \delta\left(-\frac{2\pi n}{a}\right) = \frac{\delta(n)}{\left(\frac{2\pi}{a}\right)} = \frac{a}{2\pi}\delta(n)$$

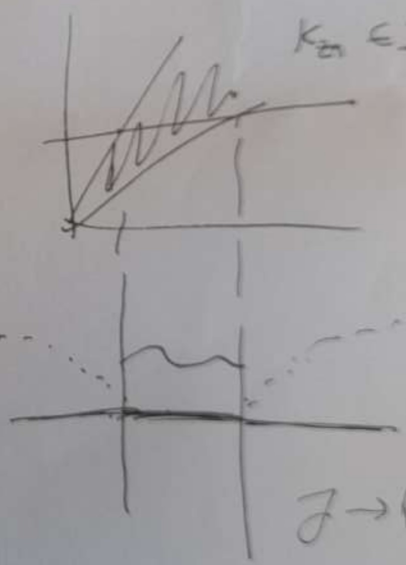
$2\omega/v - \frac{2\pi n}{a}$

$$= \frac{Na}{2\pi} \quad L$$

$$k_{xn} = \frac{\omega}{v} - \frac{2\pi n}{a}$$

$$\delta(f(x)) = \sum_i \frac{\delta(x - x_i)}{|f'(x_i)|}$$

$k_{zn} \in \mathbb{I}$



$$\int dk_{\parallel} \; r_p(k_{\parallel}, \omega)$$

$$-\mathrm{Im}\left\{ r_p(k_{\parallel}, \omega)\right\} \;\; " $$

$$-i\,\delta\left(k_{\parallel} - k_{\parallel j}\right)$$

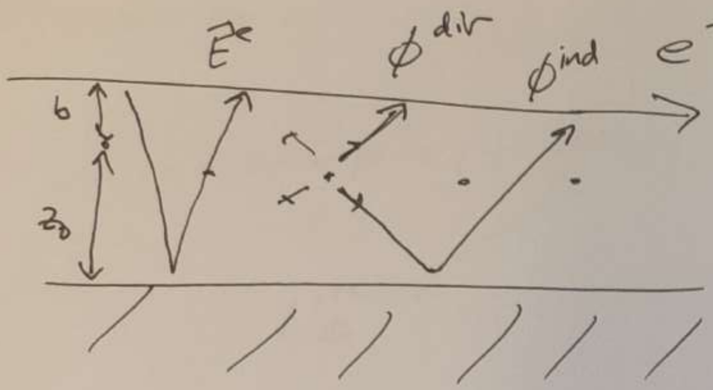
$J \rightarrow H^{(n)}$

$$r_p(k_{\parallel}, \omega) = \sum_j \frac{R_j k_j}{k_{\parallel} - k_j}$$

$$\approx \sum_j \mathrm{P.V.}\frac{R_j k_j}{k_{\parallel} - k_j} - i\pi \sum_j \delta(k_{\parallel} - k_j) R_j k_j$$

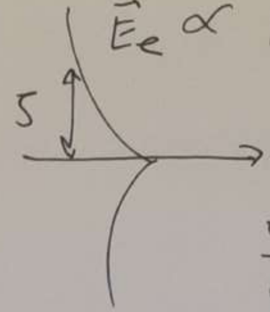
$$E = \int dk_{\parallel} F(k_{\parallel})\, r_p(k_{\parallel}) = \sum_j \mathrm{P.V.} \int dk_{\parallel} F(k_{\parallel})\, r_p(k_{\parallel}) - i\pi \sum_j F(k_j) R_j k_j$$

20 de Julio



$\vec{E}_e \propto K_0\left(\frac{\omega b}{\gamma v}\right)$

TO CHECK



$\varsigma \gg (b+z_0)\times 2$
we neglect

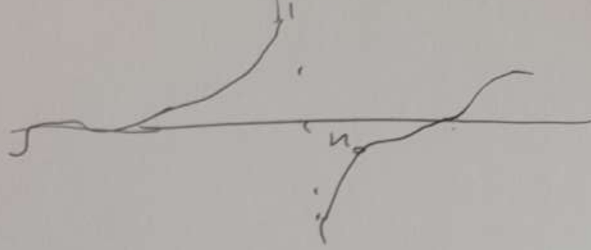
$\varsigma \sim v/\omega$

$$\Gamma_{EELS} = \Gamma^{e}_{EELS} + \underline{\underline{\Gamma^{dir}_{EELS}}} + \underline{\underline{\Gamma^{ind}_{EELS}}}$$

Lunes 25 de Julio

$$\fint_a^b dx \, \frac{f(x)}{x-x_0} = \int_a^{x_0} dx \, \frac{f(x)}{x-x_0} + \int_{x_0}^b dx \, \frac{f(x)}{x-x_0}$$

$a < x_0 < b$

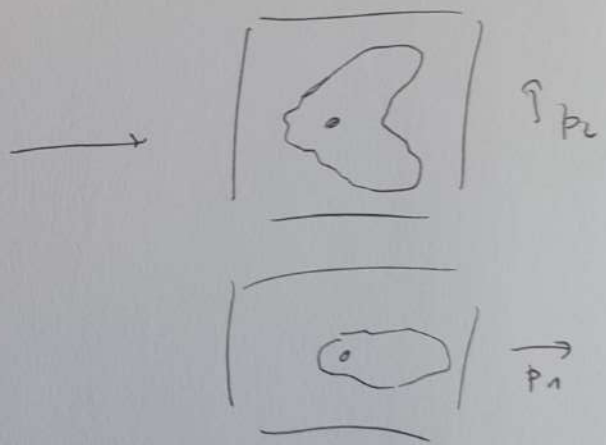


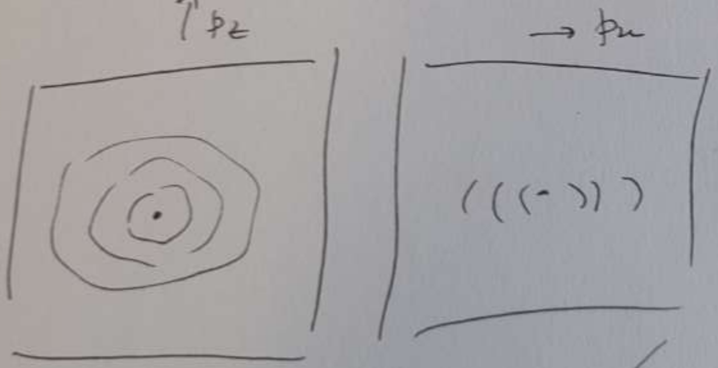
$$\fint_a^b dx \, \frac{f(x)}{x-x_0} =$$

$$= \fint_a^b dx \, \frac{f(x)-f(x_0)}{x-x_0} + \underbrace{\fint_a^b dx \, \frac{f(x_0)}{x-x_0}}$$

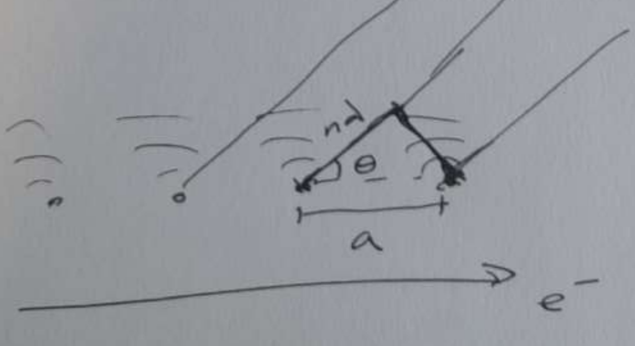
$$\int dk_x \, \bar{\phi}(\,) \, e^{---} = 0$$

$$\phi^{\infty} = \underbrace{\sum_{j=-N}^{N} \phi_1(\vec{r}_j) \, e^{-aj\,\omega/v}}$$





v/c



$$e^{i k_{xn} x} \, e^{i\sqrt{k_p^2-k_{xn}^2}\, y} = e^{i\vec{k}\cdot\vec{r}}$$

$$\vec{k} = \left(k_{xn} , \sqrt{k_p^2-k_{xn}^2}\right)$$

$$\theta_n = \arctan\left(\frac{\sqrt{k_p^2-k_{xn}^2}}{k_{xn}}\right)$$

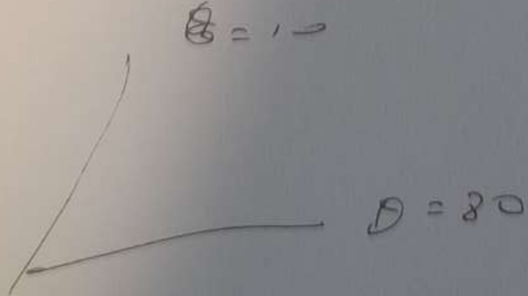
$$k_{xn} = k_p \cos(\theta_n)$$

$$\Rightarrow \theta_n = \arccos\left(\frac{k_{xn}}{k_p}\right)$$

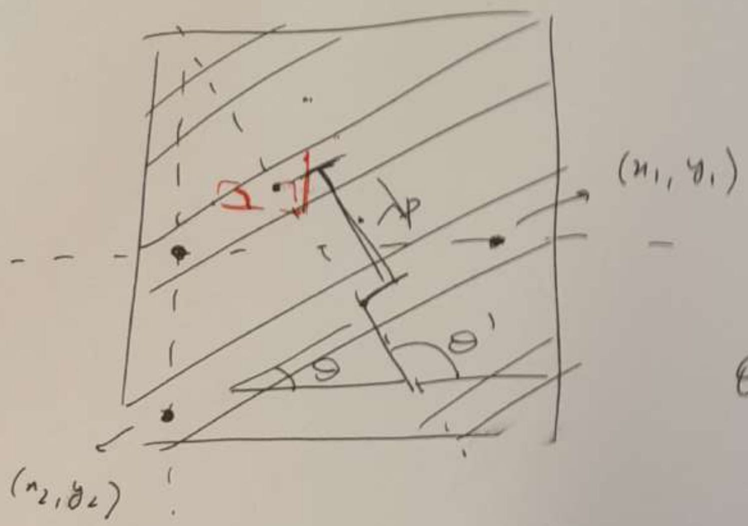
$$= \arccos\left(\frac{\omega/v - 2n\pi/a}{k_p}\right) \qquad k_p = \frac{2\pi}{\lambda_p}$$

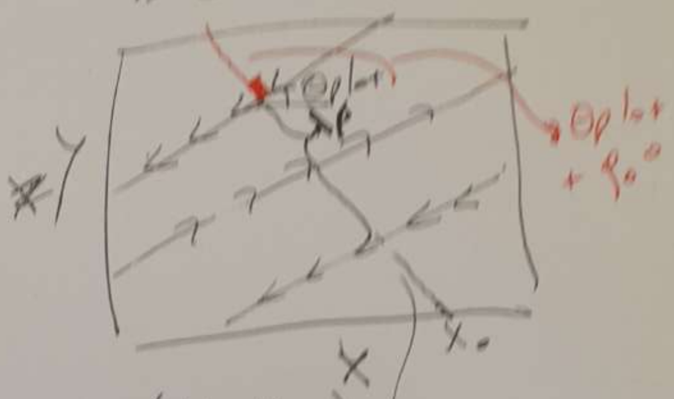
$$= \arccos\left(\frac{\omega \cdot \lambda_p}{2\pi v} - n\frac{\lambda_p}{a}\right)$$

$$= \arccos\left(\lambda_p\left(\frac{\omega}{2\pi v} - \frac{n}{a}\right)\right)$$



20 de septiembre



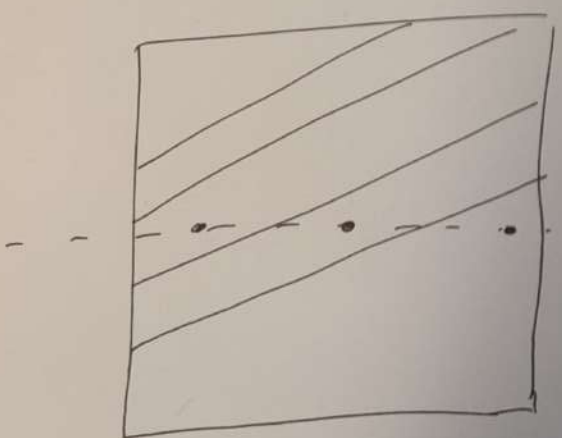


$$\theta = \arctan\left(\frac{y_1 - y_2}{x_1 - x_2}\right)$$

$y = (-x + x_0)$

~~$y = (-m x + x_0)$~~

$y = mx + b$



$(0, 1.5)$

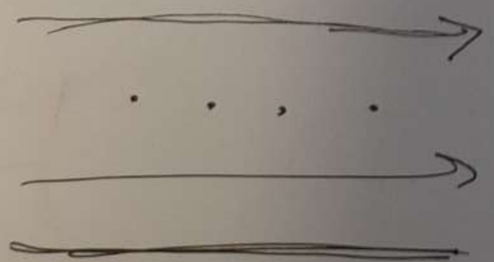
$(0.25, 1.0)$

$$\sqrt{0.25^2 + 0.5^2}$$

$\approx 0.56\ \mu m$

$$\vec{K} = \left(K_x, \sqrt{K_p^2 - K_x^2}\right)$$

$$K = \sqrt{\cancel{K_x^2} + K_p^2 - \cancel{K_x^2}} = K_p \rightarrow \underline{\lambda = \lambda_p}$$



$$e^{i k_{xn} x} \, e^{i\sqrt{k_p^2 - k_{xn}^2}\, y} = e^{i\vec{k}\cdot\vec{r}}$$

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$$= \arccos\left(\frac{\omega/v - 2n\pi/a}{k_p}\right) \qquad k_p = \frac{2\pi}{\lambda_p}$$

$$= \arccos\left(\frac{\omega \cdot \lambda_p}{2\pi v} - n\frac{\lambda_p}{a}\right)$$

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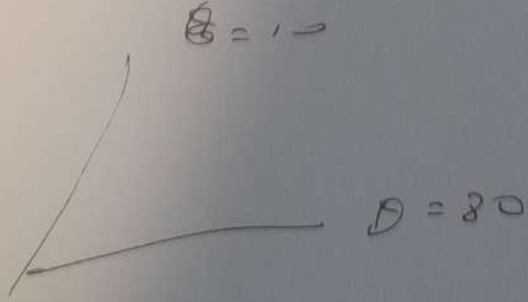
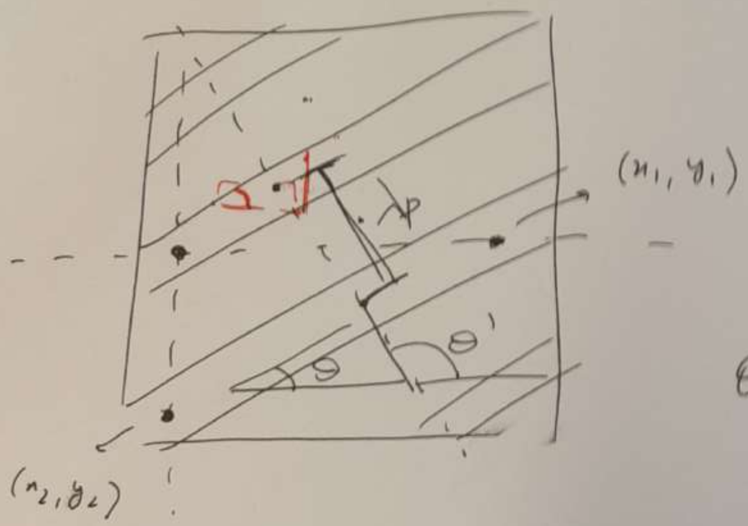
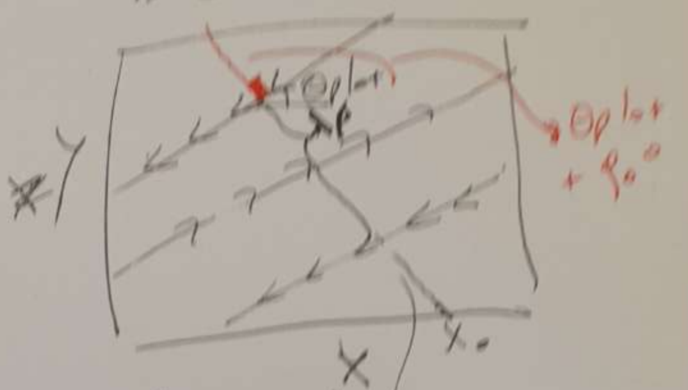


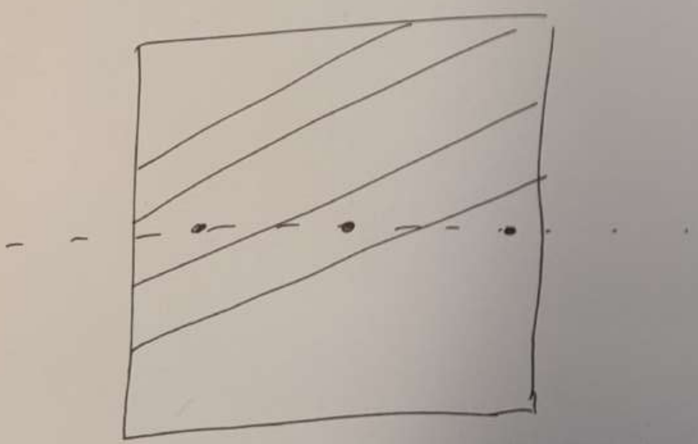
20 de septiembre





$$\theta = \arctan\left(\frac{y_1 - y_2}{x_1 - x_2}\right)$$

$y = mx + b$



$(0, 1.5)$

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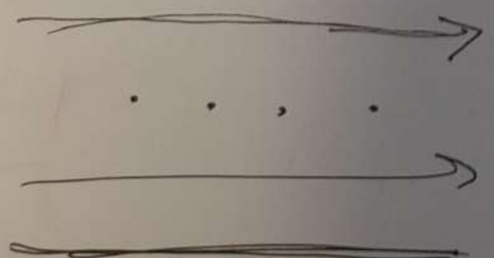
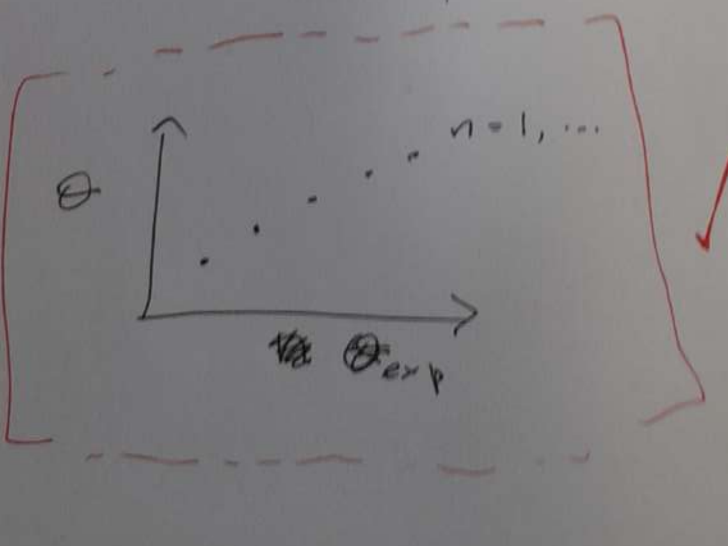
$$K = \sqrt{\cancel{K_x^2} + K_p^2 - \cancel{K_x^2}} = K_p \rightarrow \underline{\lambda = \lambda_p}$$





$$\cos(\theta) = \frac{\lambda_p}{2\pi}\left[\frac{\omega}{v} + \frac{2\pi n}{a}\right] \qquad \lambda_p = \frac{2\pi}{k_p} = f(\omega)$$

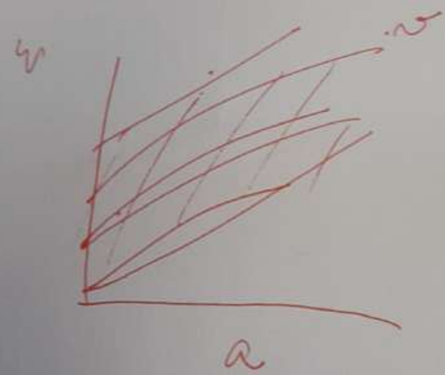
$$\Rightarrow -1 \le \frac{\lambda_p}{2\pi}\left[\frac{\omega}{v} + \frac{2n\pi}{a}\right] \le 1$$

$$\Leftrightarrow \omega_{min} \le \omega \le \omega_{max}$$

$$\underline{f_n(v,a)}$$

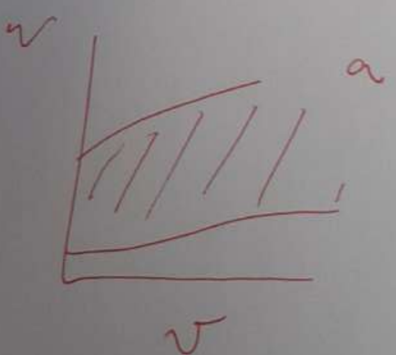
$$k_p = A\cdot\omega^2 \Rightarrow \lambda_p = \frac{2\pi}{A\omega^2}$$

$$-1 \le \frac{2\pi}{\ldots} \ldots$$



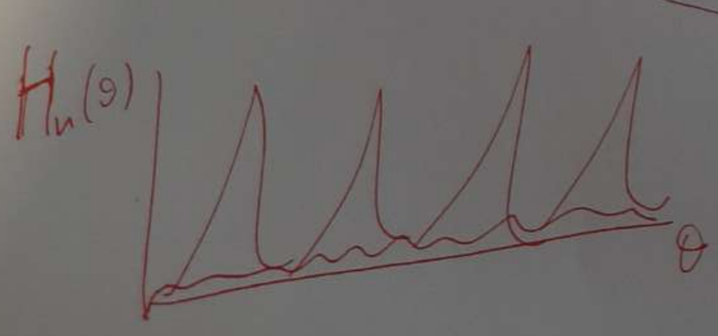
$$\phi^{(\Gamma)} = \sum_n \int dy\, F_n(y)$$

$$= \sum_n \int d\theta\, F_n(\theta)$$



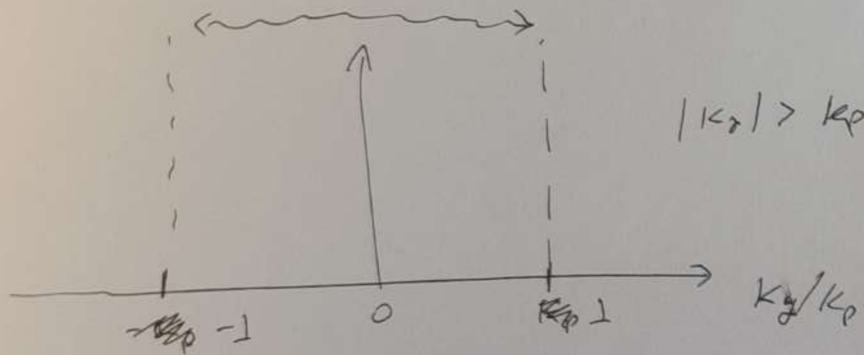
$$E^{(\vec{r})} = \sum_n \int d\theta\, G_n(\theta)$$

$$\Gamma = \sum_n \int d\theta\, \underline{H_n(\theta)}$$



$$\Gamma = \frac{2}{\hbar}\,\mathrm{Im}\{\vec{A}^* \cdot \vec{E}_{ind,n}\}$$

$\boxed{\Gamma_{SPE,n}}$



$|k_y| > k_p$

$$F = \int dk_y \, f(k_y)$$
$$= \int_{-k_p}^{k_p} dk_y \, f(k_y) + I$$

Γ

$$r_p = \frac{k_p R_p}{k_{\parallel} - k_p}$$

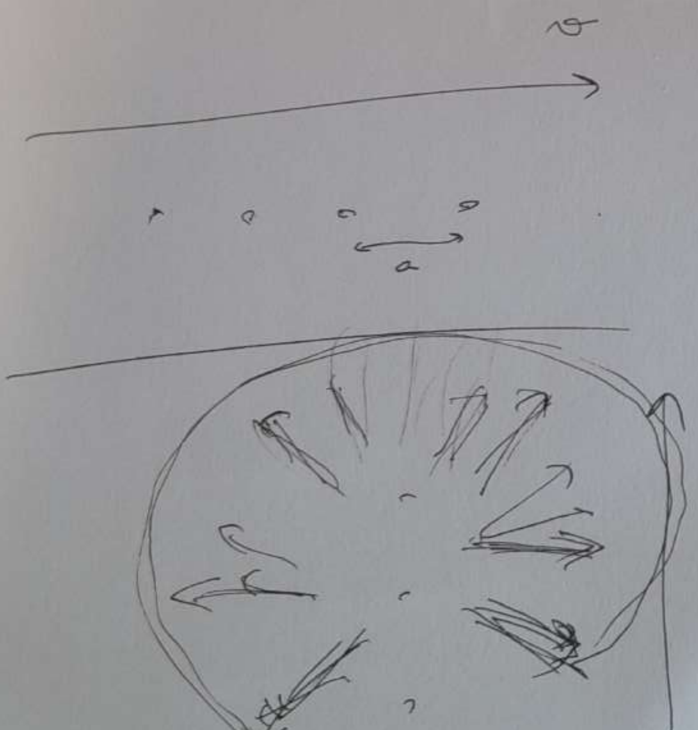
$$\sqrt{k_{xn}^2 + k_y^2} = k_p$$
$$\Downarrow$$
$$k_{xn} = k_p \cos(\theta_n)$$
$$k_y = k_p \sin(\theta_n)$$

R_p, k_p

#331

$r_p = \text{—}$

710nm



$$C\,\mathrm{Im}\{\vec{p}^{*}\cdot\overleftrightarrow{G}\cdot\vec{p}\}$$

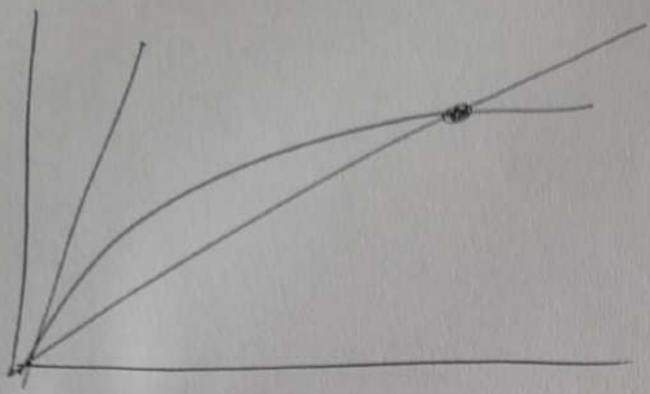
$$C\begin{bmatrix} p_x^{*} & p_y^{*} & p_z^{*}\end{bmatrix}\begin{bmatrix} G_{\parallel} & & \\ & G_{\parallel} & \\ & & G_{\perp}\end{bmatrix}\begin{bmatrix} p_x \\ p_y \\ p_z\end{bmatrix}$$

$$= C\,G_{\parallel}\underbrace{\left(|p_x|^2+|p_y|^2\right)}_{\underline{|p_{\parallel}|^2}} + G_{\perp}\,\underline{|p_z|^2}$$

$$C\,G_{\parallel} = \Gamma / |p_{\parallel}|^2$$

$$C\,G_{\perp} = \Gamma / |p_{\perp}|^2$$

Wed 19-10-2022



$\hbar\omega = 3\text{ eV}$

$k_\parallel = 3000\ \mu m^{-1}$

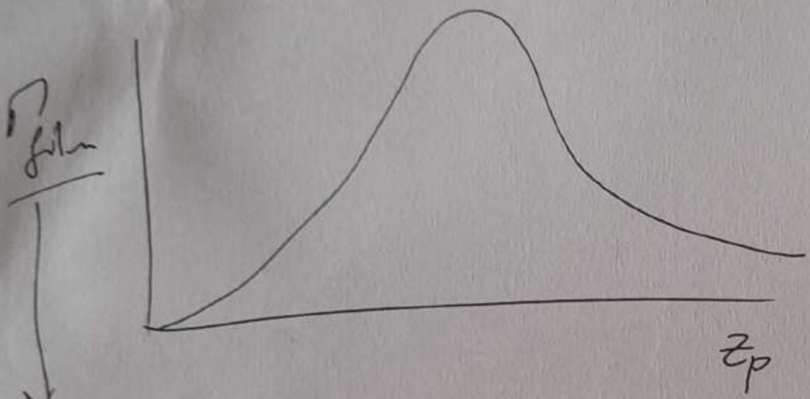
$$\lambda_p = \frac{2\pi\ \mu m}{3000} = 2\text{ nm}$$

$d = 10\text{ nm}$

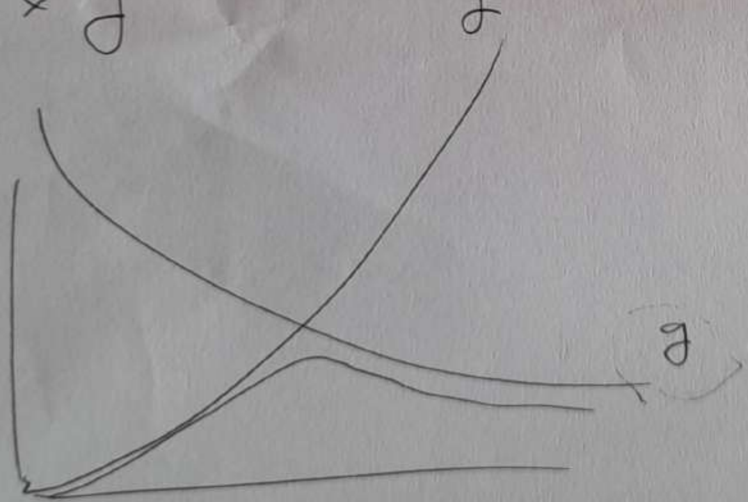
$d = 100\text{ nm} \rightarrow \lambda_p = 20\text{ nm}$

$\lambda_p \gg d$

when pole approx works



$f \times g$



$e^{-k_p z_p}$

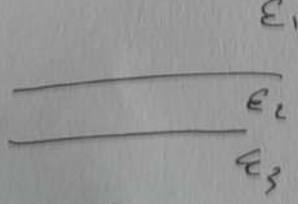
$\downarrow$

$2\pi/\lambda_p$

$z_p^{opt} \sim \lambda_p$

$$z_p \sim \frac{\lambda_p}{1000}$$

λ_p



$$\frac{\lambda_p}{d} = \ell\text{—} \qquad r_p = (\ldots) \qquad \frac{1}{r_p} = 0$$

Drude for Silver

$$\omega_D = \frac{\hbar u\, d}{m^*}$$

Javier notes

$\hbar\omega = \sqrt{AK_\parallel}$

$A = \frac{d\, \hbar^2 \omega_{bonk}^2}{\varepsilon_1 + \varepsilon_3}$

→ $d = 1um$

$\lambda_p = 20um = \frac{2\pi}{k_\parallel}$

↳ $d = 10um$ (problem)

$$\omega_p = \sqrt{\frac{ne^2}{\varepsilon_0 m^*}} \rightarrow \omega_p^2 = \frac{ne^2}{\varepsilon_0 m^*} = \frac{4\pi ne^2}{m}$$

$$\omega_D = \hbar \cdot d \cdot \frac{\omega_p^2}{e^2} \cdot \underbrace{\varepsilon_0}_{4\pi \text{ ó } 1/4\pi}$$

$$\frac{1}{\alpha \cdot c}$$

$$\omega_D = \frac{d \cdot \omega_p^2}{\alpha \cdot c} \cdot \underbrace{\varepsilon_0}_{4\pi \text{ ó } 1/4\pi}$$

ω_p^2

$\frac{\text{metro}}{\text{metro}}$ seg $\frac{1}{seg^2} \rightarrow \frac{1}{seg}$

$$\omega_D = \frac{\omega_p^2}{4\pi c} d$$

$$\omega_D = \frac{\omega_p^2 \cdot d}{4\pi \alpha \cdot c} \quad \text{ó} \quad \omega_D = \frac{\omega_p^2 \cdot d}{\cancel{4\pi}\, \alpha \cdot c} 4\pi$$