From #331: $P^{scat} = I_p \int R\, d\varphi \, \frac{1}{|k_p R|} \left| \frac{E^{scat}_{\hat z}}{E_0} \right|^2$ 07/03/2023

$$= \left(\frac{2\pi |R_p|}{\varepsilon_1} \right)^2 \frac{I_p}{|E_0|^2} k_p^5 \left(\frac{|p_\parallel|^2}{2} + |p_z|^2 \right)$$

$$\Rightarrow I_p(\omega) = \frac{\varepsilon_1 \omega |E_0|^2}{2\pi |R_p| k_p^2}$$

Total emitted energy: $\boxed{I_p = \int_0^\infty d\omega \, I_p(\omega) = \int_0^\infty d\omega \, \frac{\varepsilon_1 \omega |E_0|^2}{2\pi |R_p| k_p^2}}$

$$\phi^\infty_{ref,n} = \phi \, e^{i k_{x,n} x} e^{i k_{y,n} |y|} e^{-k_p (2z_0 - z)}$$

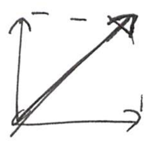
$$\bar E^\infty_{ref,n} = -\bar\nabla \phi_{ref,n} = -i \bar k_n \phi \, e^{i k_{x,n} x} e^{i k_{y,n} |y|} e^{-k_p (2z_0 - z)}$$

$$\bar k_n = (k_{x,n}, k_{y,n}, i k_p)$$

$$|\bar E^\infty_{ref,n}(\bar r_p)| = |\bar k_n| |\phi|$$

$$|\bar k_n| = \sqrt{\bar k_n \cdot \bar k_n^*} = \sqrt{(|k_{x,n}|^2 + |k_{ny}|^2 + |k_p|^2)} = \sqrt{2 k_p^2} = \sqrt{2}\, k_p$$

$k_{y,n}$



$k_{x,n}$

$$\bar E^\infty_{ref} = -i \bar k_n \phi \, e^{i k_p r} e^{-k_p (2z_0 - z)}$$

$$\downarrow$$

$$\underbrace{k_p \hat r}_{k_{nx} \hat x + k_{ny} \hat y} + i k_p \hat z$$

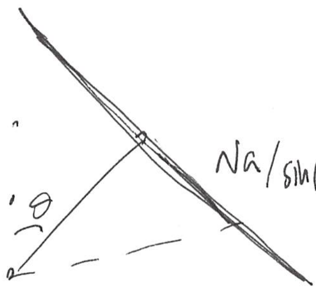
$\Downarrow$

$$\boxed{\begin{aligned} E_0 &= \sqrt{2}\, k_p \cdot \frac{R_p k_p}{2\pi a} [p_x, p_y, p_z] \cdots \\ &= \sqrt{2}\, k_p |\phi^\infty(0,\omega)| \end{aligned}}$$

$\Downarrow$

$$\cancel{I_p = \int d\omega} \quad \frac{\cancel{2} \omega \cancel{k_p^2}}{\cancel{2} \pi |R_p| \cancel{k_p^2}} \cdot |\phi^\infty(0,\omega)|^2 = \frac{\omega}{\pi |R_p|} |\phi^\infty(0,\omega)|^2$$

$$\frac{\Gamma_n(\omega)}{N} = \frac{I_p(\omega)}{\hbar \omega} \frac{\cancel{a}}{\sin\theta} = \frac{1}{\hbar \pi |R_p|} |\phi^\infty(0,\omega)|^2$$



$Na/\sin(\theta)$

$$-\bar\nabla \phi_{ref} = E_0 (\hat x + i \, \mathrm{sign}(z) \hat z) \, e^{k_p (i x - |z|)} \qquad e^{i \vec k_p \cdot \vec R - k_p |z - z_0|}$$

$$= E_0 \frac{1}{\sqrt{2} k_p} (k_{xn} \hat x + k_{ny} \hat y + i k_p \, \mathrm{sign}(z) \hat z) \rightarrow -i \vec k_n$$

$$E_0 / k_p = \phi \Rightarrow E_0 = k_p \phi$$