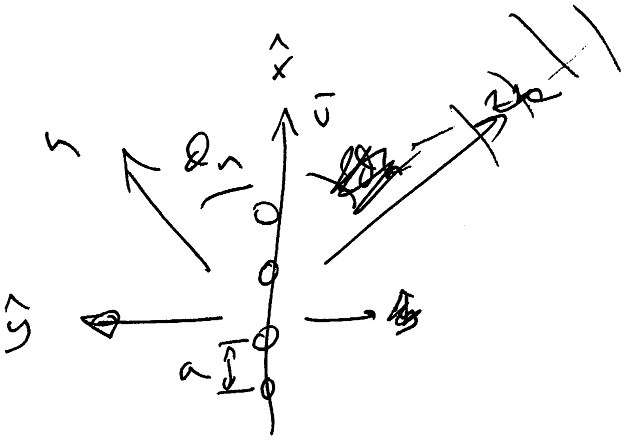
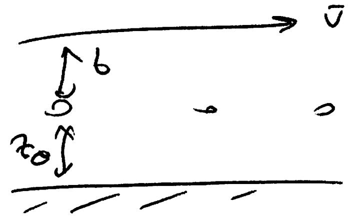


Sp  S-P normalization

12 - xii - 2022
ICFO
1



$$\phi(\bar r,t) = \int \frac{d\omega}{2\pi}\, e^{-i\omega t}\,\phi(\bar r,\omega)$$

$$\phi^\infty(\bar r,\omega) = \sum_n \phi_n^\infty(z,\omega)\, e^{iK_{xn}x}\, e^{iK_{yn}|y|}$$

$$K_{yn} = \sqrt{K_{sp}^2(\omega) - K_{xn}^2}$$

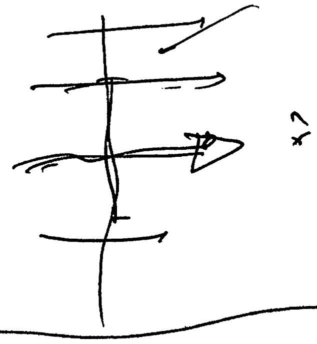
#331, p. 5193 →

$$\bar E_p = E_0(\hat x + i\,\mathrm{sign}\{z\}\,\hat z)\, e^{K_p(ix-|z|)}$$

$$I_p = \frac{\omega E_0^2}{2\pi |R_p| K_p^2}$$

Energy per unit time and transverse length

$$\frac{E\,L}{T\,L^2} = \frac{E}{L\,T}$$



$y > 0$

$$\bar E_\parallel^\infty(\bar r,\omega) = -\bar\nabla_\parallel \phi^\infty(\bar r,\omega)$$

$$= (-i)\sum_n \phi_n^\infty(z,\omega)\,(K_{xn}\hat x + K_{yn}\hat y)\, e^{i(K_{xn}x+K_{yn}y)}$$

$$\text{Emitted energy} \propto \int_{\text{plane}} \int dt \, |\vec{E}^{\infty}(\vec{r},t)|^2 =$$

$$= \int dt \int_{\text{plane}} \frac{d\omega}{2\pi} e^{-i\omega t} \vec{E}^{\infty}(\vec{r},\omega) \cdot \int \frac{d\omega'}{2\pi} \vec{E}(\vec{r},\omega') e^{i\omega' t}$$

$$= \int_{\text{plane}} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} |\vec{E}^{\infty}(\vec{r},\omega)|^2$$

$$= \int_{N-a} \int_0^{\infty} d\omega \, |\vec{E}^{\infty}(\vec{r},\omega)|^2$$

---

$$\boxed{\text{Emitted energy per unit of transverse length}} = \int_0^{\infty} d\omega \, \underbrace{\frac{\omega |\vec{E}_0^{\infty}(\vec{r},\omega)|^2}{2\pi |R_p| K_p^2(\omega)}}$$

$$\frac{E}{TL} T^2 = \frac{ET}{L} = \frac{E}{L}$$

$$\left[\int \frac{d\omega}{2\pi} E(\omega) e^{i\omega t}\right] = \frac{G}{L^2}$$

#331 p.5193 $\quad [E_0] = \dfrac{G}{L^2}$

$$[E(\omega)] = \frac{T\,G}{L^2}$$

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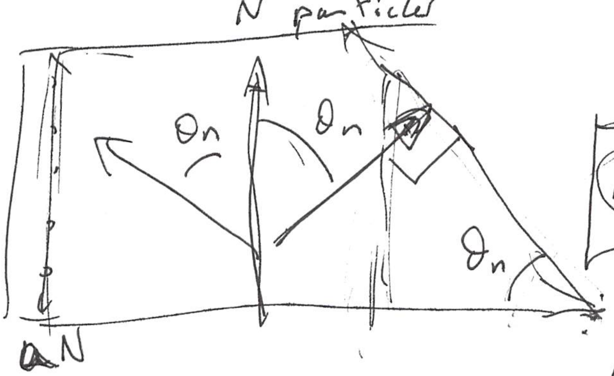
N particles

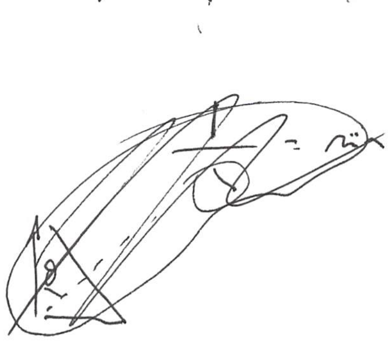



$$\cancel{\Gamma_n = \frac{1}{\hbar\omega} \int_0^{\infty} d\omega \frac{1}{\hbar} \frac{\omega |\vec{E}_0^{\infty}(\vec{r},\omega)|^2}{2\pi}}$$

$$\boxed{N \Gamma_n(\omega) = \frac{1}{\hbar} \frac{1}{\hbar\omega} \left( \frac{\omega |\vec{E}_0^{\infty}(\vec{r},\omega)|^2}{2\pi |R_p| K_p^2} \right) \frac{aN}{\sin\theta_n}}$$

$$\boxed{\text{total Emitted energy}} = \sum_n \int_0^{\infty} d(\omega\hbar) \Gamma_n(\omega) \hbar\omega N$$

$$\boxed{\text{total excited sp's}} = \sum_n \int_0^{\infty} d\omega \, \Gamma_n(\omega) N$$



\# sp's per unit frequency (energy) per particle

# 331, eq 1 $\rightarrow$ $\underline{E_0}\,(\hat{x} + i\,\mathrm{sign}\,kz\,t\,\hat{z})\,e$ ———

$|E_0|^2 \Longrightarrow |\phi_n^\infty(0,n)|^2_{z=0}\,\underbrace{(K_{xn}^2 + K_{yn}^2)}$

$$\underbrace{K_p^2 - \cancel{K_{xn}^2} + \cancel{K_{y,n}^2}}_{=\,K_p^2}$$

SO's per unit energy per particle

$$\boxed{\Gamma_n(\omega)} = \frac{1}{\hbar^2\cancel{\omega}}\frac{\cancel{\omega}\,|\phi_n^\infty(0,\omega)|^2}{2\pi(R_p)\cancel{K_p^2}}\cancel{K_p^2}\;a$$

$$= \boxed{\frac{a}{2\pi\hbar^2(R_p)}\,|\phi_n^\infty(0,\omega)|^2}$$

CGS

$[\phi(t)] = \frac{C}{L}$

$[\phi(\omega)] = \frac{C\,T}{L}$

$[\;] = \frac{L}{E^2T^2}\,\frac{C^2T^2}{L^2}$

$= \frac{L}{E^2T^2}\,\frac{E\,L\,T^2}{L^2}$

$= \frac{1}{E}$