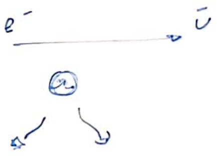
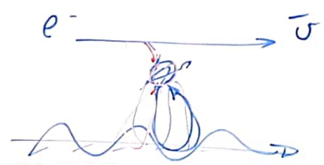




$\Gamma_{CL} = \sim |\alpha|^2 \leftarrow \sim \mathrm{Im}\{\alpha\} = \Gamma_{EELS}$

(K_1, K_0)

$\vec{I}(-e\vec{d}) = \vec{p} \qquad \vec{E} = k^2\left[\vec{p} - (\vec{p}\cdot\hat{r})\hat{r}\right]\frac{e^{ikr}}{r}\, e^{-i\omega t} + c.c.$

$kr \gg 1$

$\gamma = \frac{1}{\hbar\omega}\int d\Omega\, \hat{r}\cdot\vec{S}_{Poyn} = \frac{c}{4\pi}\, 2\int d\Omega\, |\vec{E}|^2 = \frac{1}{\hbar\omega}\frac{c}{2\pi}k^4|p|^2\int d\Omega\,(1-(\hat{p}\cdot\hat{r})^2)$

$\frac{4k^3}{3\hbar\pi}|p|^2 \qquad 4\pi\int_0^1 d\mu\,(1-\mu^2) \qquad 1-\frac{1}{3}=\frac{2}{3}$

j, 0

$\alpha_{xx}(\omega) = \frac{2e^2}{\hbar}\sum_j |\langle j|x|0\rangle|^2\left(\frac{1}{\omega_j-\omega_0-\omega-i\gamma/2} + \frac{1}{\omega_j-\omega_0+\omega+i\gamma/2}\right)$

$\alpha = \frac{3}{2k^3}t_1^E$

$k = \omega/c$

$\mathrm{Im}\{-1/\alpha\} \geq \frac{2k^3}{3}$

$\mathrm{Im}\{\alpha\} \geq \frac{2k^3}{3}|\alpha|^2$

j, 0

$\to \alpha(\omega) \simeq \frac{(2e^2/\hbar)\, d_{10}^2}{\omega_{10}-\omega-i\gamma/2}$

$\gamma = \frac{4k^3}{3\hbar\pi}e^2 d_{10}^2$

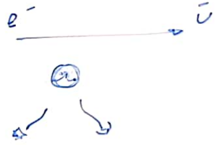
$\mathrm{Im}\{\alpha\} \simeq \frac{2e^2}{\hbar}d_{10}^2\,\delta(\omega_{10}-\omega)$

$\gamma \ll \omega_{10}$

$\int d\omega\, \Gamma_{EELS} \ll 1$

$\vec{p} = \alpha\left(\vec{E}^{ext} + \vec{E}^{ext}_r + \underbrace{\vec{E}^{dip}_r}_{G\vec{p}}\right) \to \vec{p} = \frac{1}{1/\alpha - G}\left(\vec{E}^{ext} + \vec{E}^{ext}_r\right)$

$\vec{E}^{ind} \to \Delta\Gamma_{EELS}$



$\vec R = (x, y)$

$z > 0$

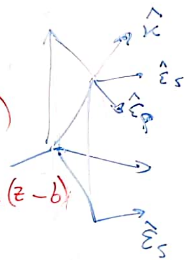
$z < 0$

$$\bar E^{ext} = \frac{ie}{\pi}\int \frac{d^3\bar q}{(2\pi)^3}\, \frac{\bar q - \kappa \bar v/c}{q^2 - \kappa^2 - i0^+}\, e^{i\bar q\cdot(\bar r - \bar b) - i\omega t}\, \delta(\omega - \bar q\cdot\bar v)$$

$$= \frac{ie}{\pi}\, 2\pi i \int \frac{d^2\bar Q}{(2\pi)^3}\, \frac{(\bar k^\pm - \kappa \bar v/c)}{2k_z}\, e^{i\bar Q\cdot\bar R - i\omega t}\, \delta(\omega - \bar k^\pm\cdot\bar v)\, e^{\pm i k_z(z-b)}$$

$$\bar k^\pm = \bar Q \pm k_z \hat z$$

$\pm$ $\uparrow$ $z > 0$, $\downarrow$ $z < 0$



$$q_z = \pm\sqrt{\kappa^2 - Q^2 + i0^+} = \pm k_z \qquad q_x^2 + q_y^2$$

$$\bar E^{ind}_{dir} = (\kappa^2\bar p + (\bar p\cdot\bar\nabla)\bar\nabla)\, \frac{e^{i\kappa r}}{r}$$

$$= (\kappa^2\bar p + (\bar p\cdot\bar\nabla)\bar\nabla) \int \frac{d^2\bar Q}{(2\pi)^2}\, \frac{2\pi i}{k_z}\, e^{i\bar Q\cdot\bar R + i k_z|z|}$$

$$= \kappa^2 \int \frac{d^2\bar Q}{(2\pi)^2}\, \frac{2\pi i}{k_z}\, e^{i\bar Q\cdot\bar R + i k_z b}\, \left[(\hat\varepsilon_p^+\cdot\bar p)\,\hat\varepsilon_p^+ + (\hat\varepsilon_s\cdot\bar p)\,\hat\varepsilon_s\right]$$

$$k_z = \sqrt{\kappa^2 - Q^2 + i0^+} \qquad \mathrm{Im}\{k_z\} > 0$$

$Q < \kappa$: $k_z = \sqrt{\kappa^2 - Q^2}$

$Q > \kappa$: $k_z = i\sqrt{Q^2 - \kappa^2}$

$$\bar E^{ind} = \kappa^2 \int \frac{d^2\bar Q}{(2\pi)^2}\, \frac{2\pi i}{k_z}\, e^{i\bar Q\cdot\bar R}\, e^{i k_z b} \Big[\big((\hat\varepsilon_p^+\cdot\bar p) - (\hat\varepsilon_p^-\cdot\bar p)\, r_p\, e^{2ik_z z_0}\big)(\hat x\cdot\hat\varepsilon_p^+) + \big((\hat\varepsilon_s\cdot\bar p) + (\hat\varepsilon_s\cdot\bar p)\, r_s\, e^{2ik_z z_0}\big)(\hat\varepsilon_s\cdot\hat x)\Big]$$

$$\int \frac{dx}{v}\, e^{iQ_x x - i\omega x/v}$$

$$\bar R = (x, 0)$$

$$\Delta\Gamma_{EELS} = \frac{e}{\pi\hbar\omega}\, \frac{\kappa^2}{2\pi}\, \frac{1}{v}\, 2\pi\, \mathrm{Re}\left\{\int_{-\infty}^{\infty} \frac{dQ_y}{k_z}\, e^{ik_z b}\, \square\right\}$$

$$\int_{-\infty}^{\infty} dQ_y \to 2\int_0^\infty dQ_y$$

$$\bar Q = \left(\frac{\omega}{v}, Q_y\right)$$

$$\hat\varepsilon_s = \frac{1}{Q}(-Q_y, Q_x)$$

$$\hat\varepsilon_p^\pm = \frac{1}{\kappa Q}(\bar Q k_z - Q^2 \hat z)$$

$$\bar p = \alpha\Big(\bar E^{ext} + \bar E^{ext}_r + \underbrace{\bar E^{dip}_r}_{G\bar p}\Big) \to \bar p = \frac{1}{1/\alpha - G}\left(\bar E^{ext} + \bar E^{ext}_r\right)$$

$$\to \bar E^{ind} \to \Delta\Gamma_{EELS}$$