



$$\phi_0(\vec{r}, \omega) = -e \int dt e^{i\omega t} \frac{1}{|\vec{r} - (\vec{b} + \vec{v}t)|}$$

$$= -e \int dt \int \frac{d\vec{k}_{||}}{2\pi k_{||}} e^{i\omega t} e^{i(k_x(x-vt) + k_y(y-y_0)) - k_{||}(z+b)}$$

$$= -\frac{e}{v} \int \frac{dk_{||}}{k_{||}} e^{i k_{||} x + k_y(y-y_0) - k_{||}(z+b)}$$

$$\phi^{ext}(\vec{r}, \omega) = -\frac{e}{v} \int \frac{dk_{||}}{k_{||}} e^{i \vec{k}_{||} \cdot \vec{r} - i k_y y_0} \left[e^{-k_{||}(z+b)} - r_p e^{-k_{||}(b+2a-z)} \right]$$

$$\frac{1}{r} = \int \frac{d\vec{k}}{2\pi^2 k} e^{i \vec{k} \cdot \vec{r} - k|\theta|}$$

$$r_p = \frac{\epsilon - 1}{\epsilon + 1}$$

$$B = \frac{e^{-k_{||}b}}{e^{-k_{||}b} + A e^{-k_{||}b}}$$

$$1 + A = B$$

$$1 - A = \epsilon B$$

$$B = \frac{2}{1 + \epsilon}$$

$$A = \frac{1 - \epsilon}{1 + \epsilon}$$

$$\alpha_H = \frac{1}{k - g}$$

$$\bar{p} = \frac{e \alpha_H}{v} \int \frac{dk_y}{k_{||}} e^{-i k_y y_0} \left[e^{-k_{||}b} (i \vec{k}_{||} - \hat{z} k_{||}) - r_p e^{-k_{||}(b+2a)} (i \vec{k}_{||} + k_{||} \hat{z}) \right]$$

$$\phi_p(\vec{r}, \omega) = - \int \frac{d\vec{k}_{||}}{2\pi k_{||}} e^{i \vec{k}_{||} \cdot \vec{r}} \left[e^{-k_{||}|z|} \bar{p} (i \vec{k}_{||} - \text{sign}(z) k_{||} \hat{z}) - r_p e^{-k_{||}(2a-z)} \bar{p} (i \vec{k}_{||} - k_{||} \hat{z}) \right]$$

$$r_p \approx \frac{k_p}{k_{||} - k_p} e^{i \frac{\pi}{2}}$$

$$\int_{-\infty}^{\infty} dk_{||} \frac{J_n(k_p z) p(k_{||})}{k_{||} - k_p}$$

$$\approx \frac{1}{2} \int_{-\infty}^{\infty} dk_{||} \frac{H_n^{(1)}(k_p z) + H_n^{(2)}(k_p z) p(k_{||})}{k_{||} - k_p} = i \pi A_n^{(1)}(k_p z) p(k_p)$$

$\phi_p^{ext} = -\vec{p} \cdot \vec{\nabla} \frac{1}{r}$
 \vec{p} at $\vec{r}=0$
 $v_0 = 900$
 $a = 10 \text{ nm}$
 $E_F = 0.5 \text{ eV}$
 $\omega = 0.1 - 0.3 \text{ eV}$
 $V_C = \frac{1}{200}$