$$\vec{E} = \frac{ie}{\pi}\int d^3\vec{k}\left[\frac{\vec{k}/\varepsilon - (\omega/c)\,\vec{v}/c}{k^2 - \varepsilon(\omega/c)^2}\right] e^{i\vec{k}\cdot\vec{r}}\,\delta(\omega - k_x v)$$

$\boxed{03/03/2023}$

$v = v\hat{x}$

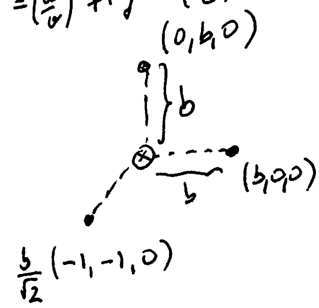
$$= \frac{ie}{\pi v}\int dk_y\,dk_z\left[\frac{\vec{k}/\varepsilon - (\omega/c)\,\vec{v}/c}{k^2 - \varepsilon(\omega/c)^2}\right] e^{i\vec{k}\cdot\vec{r}}$$

$\vec{k} = \left(\frac{\omega}{v}, k_y, k_z\right)$

$$= \frac{ie}{\pi v}\int dk_y\,dk_z \begin{pmatrix} \dfrac{(\omega/v)/\varepsilon - (\omega/c)\,v/c}{k^2 - \varepsilon(\omega/c)^2} \\[2ex] \dfrac{k_y/\varepsilon}{k^2 - \varepsilon(\omega/c)^2} \\[2ex] \dfrac{k_z\,\varepsilon}{k^2 - \varepsilon(\omega/c)^2} \end{pmatrix} e^{i\vec{k}\cdot\vec{r}}$$

$$\lambda_y^2 = k_x^2 + (\omega/v)^2 - \varepsilon(\omega/c)^2 = k^2 - \varepsilon(\omega/c)^2 - k_y^2$$

$$\lambda_z = k^2 - \varepsilon(\omega/c)^2 - k_z^2 = \left(\frac{\omega}{v}\right)^2 + k_y^2 - \varepsilon\left(\frac{\omega}{c}\right)^2$$



$$= \frac{ie}{\pi v}\begin{pmatrix} \left[\left(\frac{\omega}{v}\right)\frac{1}{\varepsilon} - \left(\frac{\omega}{c}\right)\frac{v}{c}\right]\displaystyle\int dk_y \int dk_z\,\frac{e^{i(\omega/v)x}\,e^{ik_y y}\,e^{ik_z z}}{k_z^2 + \lambda_z^2} \\[2ex] \dfrac{e^{i(\omega/v)x}}{\varepsilon}\displaystyle\int dk_y\,e^{ik_y y}\,k_y \int dk_z\,\frac{e^{ik_z z}}{k_z^2 + \lambda_z^2} \\[2ex] \dfrac{e^{i(\omega/v)x}}{\varepsilon}\displaystyle\int dk_z\,e^{ik_z z}\,k_z \int dk_y\,\frac{e^{ik_y y}}{k_y^2 + \lambda_y^2} \end{pmatrix}$$

$\boxed{(k_y, k_z) \Rightarrow (k_\parallel, \varphi)}$

$$\cancel{\int dk\,\frac{e^{ikx}\,k}{k^2+\lambda^2} = i\pi\,\mathrm{sign}(x)\,e^{-\lambda|x|}}$$

$$\int dk\,\frac{e^{ikx}}{k^2+\lambda^2} = \frac{\pi}{\lambda}\,e^{-\lambda x}$$

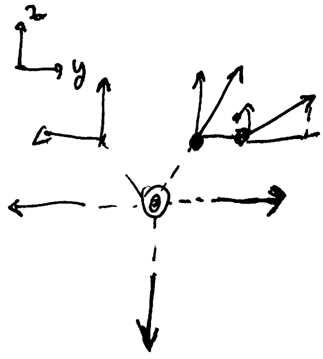
$$= \frac{ie}{\varepsilon v}\begin{pmatrix} \left[\left(\frac{\omega}{v}\right) - \varepsilon\left(\frac{\omega}{c}\right)\frac{v}{c}\right] e^{i(\omega/v)x}\displaystyle\int dk_y\,\frac{e^{-\lambda_z z}\,e^{ik_y y}}{\sqrt{\left(\frac{\omega}{v}\right)^2 - \varepsilon\left(\frac{\omega}{c}\right)^2 + k_y^2}} \\[2ex] e^{i(\omega/v)x}\displaystyle\int dk_y\,\frac{e^{-\lambda_z z}\,e^{ik_y y}\,k_y}{\sqrt{\left(\frac{\omega}{v}\right)^2 - \varepsilon\left(\frac{\omega}{c}\right)^2 + k_y^2}} \\[2ex] e^{i(\omega/v)x}\displaystyle\int dk_z\,\frac{e^{-\lambda_y y}\,e^{ik_z z}\,k_z}{\sqrt{\left(\frac{\omega}{v}\right)^2 - \varepsilon\left(\frac{\omega}{c}\right)^2 + k_z^2}} \end{pmatrix}$$

In polar coordinates: $k_y = k_\parallel \cos(\varphi)$ $\quad k_z = k_\parallel \sin(\varphi)$

$$dk_y\, dk_y = k_\parallel\, dk_\parallel\, d\varphi$$

$\vec{k}_\parallel \cdot \vec{R} = k_\parallel R \cos(\varphi-\varphi_R)$

$\vec{R} = R\,(\cos(\varphi_R), \sin(\varphi_R))$



$$\vec{E} = \frac{ie}{\pi v \varepsilon}\begin{bmatrix} \left[\left(\frac{\omega}{v}\right) - \varepsilon\left(\frac{\omega}{c}\right)\frac{v}{c}\right] e^{i(\omega/v)x}\int dk_\parallel\, k_\parallel \int d\varphi \cdot \dfrac{e^{ik_\parallel R\cos(\varphi-\varphi_R)}}{\left(\frac{\omega}{v}\right)^2 + k_\parallel^2 - \varepsilon\left(\frac{\omega}{c}\right)^2} \\ e^{i(\omega/v)x}\int dk_\parallel\, k_\parallel \int d\varphi\, \dfrac{e^{ik_\parallel R\cos(\varphi-\varphi_R)}\cos(\varphi)k_\parallel}{\left(\frac{\omega}{v}\right)^2 + k_\parallel^2 - \varepsilon\left(\frac{\omega}{c}\right)^2} \\ e^{i(\omega/v)x}\int dk_\parallel\, k_\parallel \int d\varphi\, \dfrac{e^{ik_\parallel R\cos(\varphi-\varphi_R)}\sin(\varphi)k_\parallel}{\left(\frac{\omega}{v}\right)^2 + k_\parallel^2 - \varepsilon\left(\frac{\omega}{c}\right)^2} \end{bmatrix}$$

$$\frac{\omega}{v}\left(1-\varepsilon\frac{v^2}{c^2}\right) = \frac{\omega}{v}\frac{1}{\gamma^2}$$

$$= \frac{2iee^{i(\omega/v)x}}{v\varepsilon}\begin{bmatrix} \left[\left(\frac{\omega}{v}\right) - \varepsilon\left(\frac{\omega}{c}\right)\left(\frac{v}{c}\right)\right] \int dk_\parallel\, \dfrac{k_\parallel\, J_0(k_\parallel R)}{\left(\frac{\omega}{v}\right)^2 + k_\parallel^2 - \varepsilon\left(\frac{\omega}{c}\right)^2} \\ i\cos(\varphi_R)\int dk_\parallel\, k_\parallel^2\, \dfrac{J_1(k_\parallel R)}{\left(\frac{\omega}{v}\right)^2 + k_\parallel^2 - \varepsilon\left(\frac{\omega}{c}\right)^2} \\ i\sin(\varphi_R)\int dk_\parallel\, k_\parallel^2\, \dfrac{J_1(k_\parallel R)}{\left(\frac{\omega}{v}\right)^2 + k_\parallel^2 - \varepsilon\left(\frac{\omega}{c}\right)^2} \end{bmatrix}$$

$$\gamma = 1\Big/\sqrt{1-\varepsilon\frac{v^2}{c^2}}$$

$\sqrt{E_y^2+E_z^2}$ for the same $|\vec{R}-\vec{R}_0|$

$$\int dk_\parallel \frac{k_\parallel\, J_0(k_\parallel R)}{\left(\frac{\omega}{v}\right)^2 + k_\parallel^2 - \varepsilon\left(\frac{\omega}{c}\right)^2} = K_0\left(\left[\left(\frac{\omega}{v}\right)^2 - \varepsilon\left(\frac{\omega}{c}\right)^2\right] R\right)$$

$$\int dk_\parallel \frac{k_\parallel^2\, J_1(k_\parallel R)}{\left(\frac{\omega}{v}\right)^2 + k_\parallel^2 - \varepsilon\left(\frac{\omega}{c}\right)^2} = \left[\left(\frac{\omega}{v}\right)^2 - \varepsilon\left(\frac{\omega}{c}\right)^2\right] K_1\left(\left[\left(\frac{\omega}{v}\right)^2 - \varepsilon\left(\frac{\omega}{c}\right)^2\right] R\right)$$

$$\frac{\omega^2}{v^2}\left(1 - \varepsilon\left(\frac{v}{c}\right)^2\right) \equiv a = \frac{\omega^2}{v^2\gamma^2}$$

$$= \frac{2iee^{i(\omega/v)x}}{v\varepsilon}\begin{bmatrix} \frac{\omega}{v} K_0(aR) \\ i\cos(\varphi_R)\, a\, K_1(aR) \\ i\sin(\varphi_R)\, a\, K_1(aR) \end{bmatrix} = \frac{2e\omega}{\gamma^2 v^2 \varepsilon} e^{i(\omega/v)x}\begin{bmatrix} iK_0\left(\frac{\omega}{v}R/\gamma^2\right) \\ -K_1\left(\frac{\omega}{v}\frac{R}{\gamma^2}\right)\cos(\varphi_R) \\ -K_1\left(\frac{\omega}{v}\frac{R}{\gamma^2}\right)\sin(\varphi_R) \end{bmatrix}$$

$$E_y = \frac{e}{2\pi v}\int \frac{dk_y}{\sqrt{(\omega/v)^2 + k_y^2}}\, e^{i(\omega/v)x}\, e^{ik_y y}\, e^{i\sqrt{\left(\frac{\omega}{c}\right)^2 - \left(\frac{\omega}{v}\right)^2 - k_y^2}\,|z-b|}\, i k_y$$

$$E_z = \frac{-e}{2\pi v}\int dk_y\, e^{i(\omega/v)x}\, e^{ik_y y}\, e^{i\sqrt{\left(\frac{\omega}{c}\right)^2 - \left(\frac{\omega}{v}\right)^2 - k_y^2}\,|z-b|}$$

$$\sqrt{\left(\frac{\omega}{c}\right)^2 - \left(\frac{\omega}{v}\right)^2 - k_y^2} \approx i\sqrt{k_y^2 + \left(\frac{\omega}{v}\right)^2}$$