Pole: $\sqrt{k_{xn}^2+k_y^2} - k_p = 0$ $\qquad k_{xn} = \frac{\omega}{v} + \frac{2n\pi}{a}$

$\underline{n=0}$: $k_{x0} = \frac{\omega}{v} \Rightarrow \sqrt{\left(\frac{\omega}{v}\right)^2 + k_y^2} = k_p$

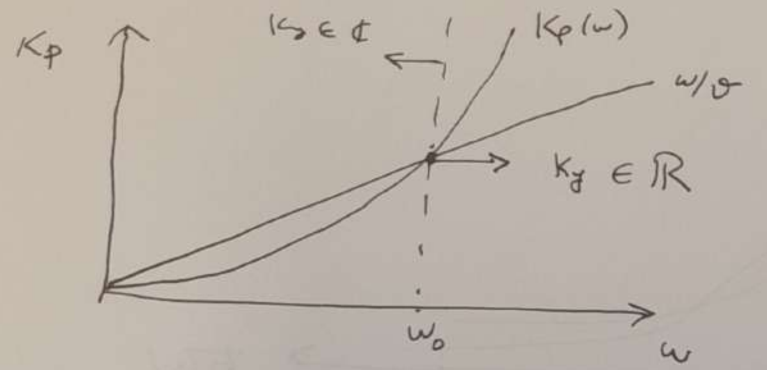
$\Leftrightarrow k_y = \pm\sqrt{k_p^2 - (\omega/v)^2}$

$k_p > \frac{\omega}{v} - \frac{2\pi n}{a}$

$k_p + \frac{2\pi n}{a} > \frac{\omega}{v}$

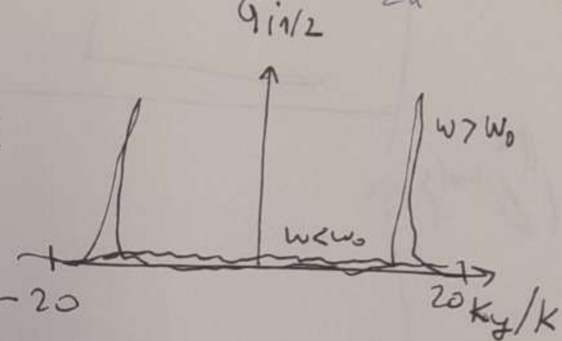
$\frac{2\pi n}{a} > \frac{\omega}{v} - k_p$

$n > \frac{a}{2\pi}\left(\frac{\omega}{v} - k_p\right)$



$\frac{k_p(\omega)}{k_0} = \frac{\varepsilon_1+\varepsilon_2}{\alpha} \cdot \frac{\hbar\omega + i\gamma}{4E_F} = \frac{1}{\alpha}\frac{\hbar\omega+i\gamma}{2E_F}$

$\frac{\omega/v}{k_0} = \frac{c}{v}$

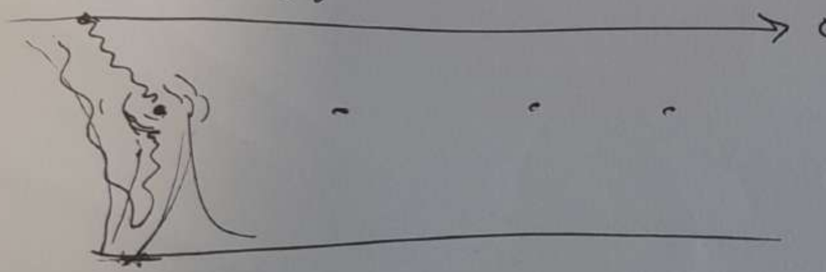


$\Rightarrow \frac{1}{\alpha}\frac{\hbar\omega_0}{2E_F} = \frac{c}{v} \Leftrightarrow \boxed{\omega_0 = \frac{\alpha c}{v} \cdot \frac{2E_F}{\hbar}}$

$\hbar\omega_0 = \frac{1}{137} \times \frac{c}{v} \times 2 \times 0.3 = 43\ \text{meV}$

$\frac{\omega/v + 2n\pi/a}{k_0} = \frac{k_p(\omega)}{k_0}$

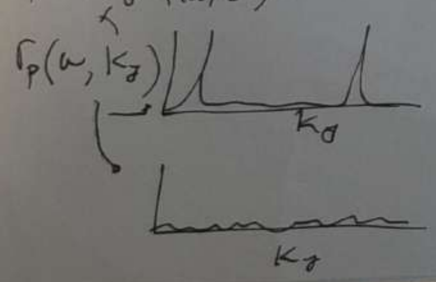
$\Rightarrow \omega_0^{(n)}$

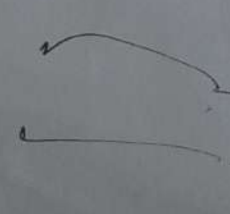


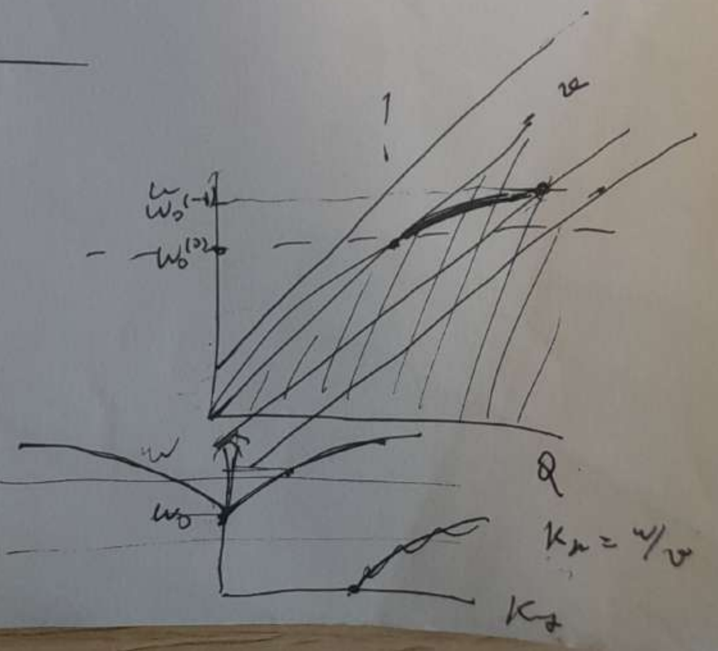
$\phi = \phi^{dir} + \phi^{ind}$

$\downarrow$ analytical $\qquad \downarrow$ numerical

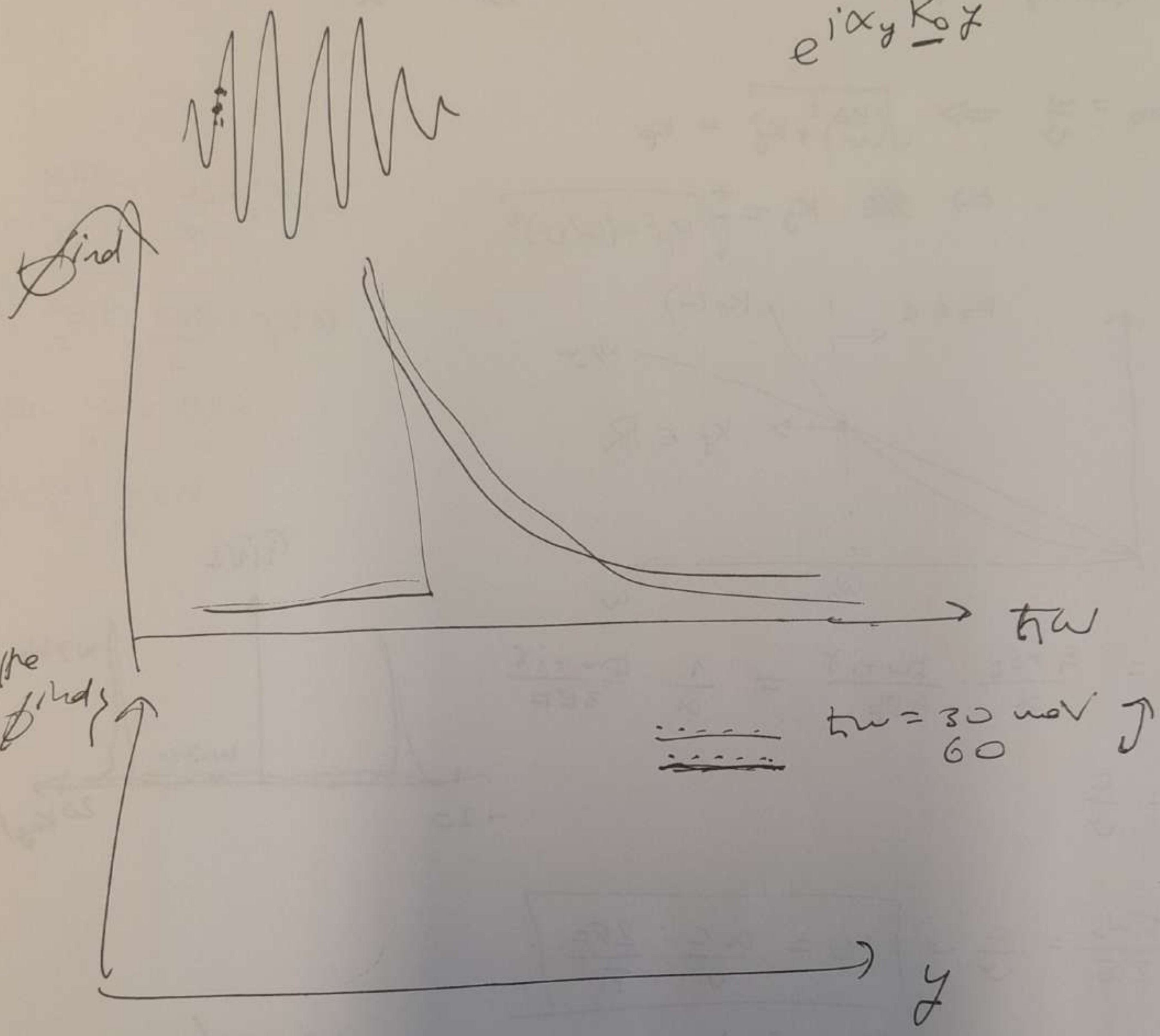
$Q = \sqrt{k_y^2 + (\omega/v)^2}$







$e^{i\alpha_y \underline{K_0} y}$



$$\int_0^{\infty} dk_{\parallel}\; f(k_{\parallel}) \cdot \frac{1}{k_{\parallel} - k_p} \qquad \mathrm{Im}\{k_p\} > 0$$

Line of Dipoles

$$\int_{-\infty}^{+\infty} dk_y\; f(k_y) \cdot \frac{1}{\sqrt{k_{xn}^2 + k_y^2} - k_p}$$

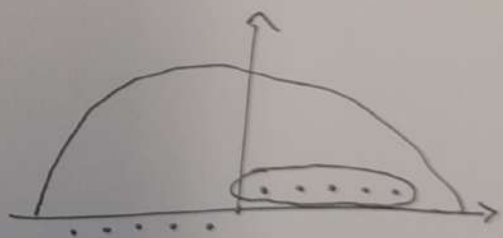
$$= \int_{-\infty}^{+\infty} dk_y\; f(k_y)\, \frac{\sqrt{k_{xn}^2 + k_y^2} + k_p}{k_{xn}^2 + k_y^2 - k_p^2}$$

$$I_n = \int_{-\infty}^{+\infty} dk_y\, \overbrace{f(k_y) \cdot \sqrt{k_{xn}^2 + k_y^2} + k_p}^{\tilde{f}(k_y)} \cdot \left(\frac{1}{k_y + \underbrace{\sqrt{k_{xn}^2 - k_p^2}}} - \frac{1}{\underbrace{k_y - \sqrt{k_{xn}^2 - k_p^2}}} \right) \cdot \frac{1}{2\sqrt{k_{xn}^2 - k_p^2}}$$

$k_y = -\gamma_n \Leftarrow \qquad \gamma_n = \gamma_n^r + i\gamma_n^i \qquad \Downarrow$

$k_y = \gamma_n$

$$\gamma_n = \sqrt{k_{xn}^2 - k_p^2} = \sqrt{\left(\frac{\omega}{v} + \frac{2n\pi}{a}\right)^2 - k_p^2} = \pm k_y$$

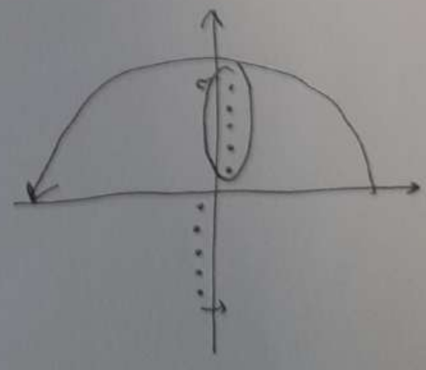


1) $|k_{xn}| > |k_p| \Rightarrow \gamma_n \in \mathbb{R}$

$\Downarrow \qquad \mathrm{Im}\{\gamma_n\} > 0$

$$\frac{\omega}{v} + \frac{2n\pi}{a} > k_p \Rightarrow n < \left(k_p - \frac{\omega}{v}\right) \cdot \frac{a}{2\pi} \equiv n_{max}$$

$$I_n = -2\pi i\; \tilde{f}(\gamma_n) \cdot \frac{1}{2\gamma_n}$$

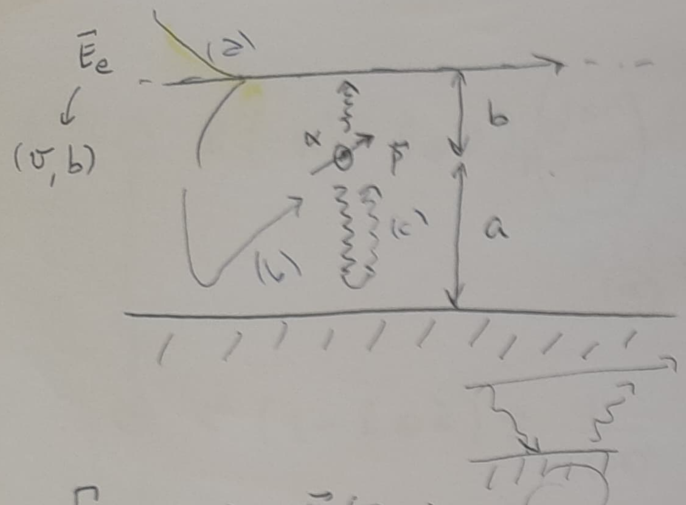


2) $|k_{xn}| < |k_p| \Rightarrow \gamma_n \in \mathbb{I}$

$k_y = \pm i\gamma_n' \qquad \gamma_n \equiv i\gamma_n'$

$$I_n = -2\pi i\; \tilde{f}(\gamma_n) \cdot \frac{1}{2\gamma_n}$$

$$I = \sum_n I_n = -2\pi i \sum_{n=0}^{\infty} \tilde{f}(\gamma_n) / 2\gamma_n$$



$$\vec{p} = \alpha \cdot \vec{E}_{tot} \qquad (1)$$

$$E_{tot}^{(dip)}\big|_{\bar{r}=\bar{0}} = \overset{(a)}{\bar{E}_e} + \overset{(b)}{\bar{E}_{e,ref}} + \overset{(c)}{\bar{E}_{dip,ref}}(\vec{p}) \qquad (2)$$

$(2) \rightarrow (1) \Rightarrow$ $\vec{p}$

$g\,\bar{p}$

$\bar{r} = \bar{v}t$

$\{\rho^- \tilde{r}_1 + \rho^- \tilde{r}_2 | \varepsilon^+ + \varepsilon^- \rho^-$

$2 z_e|_{zue}$

$$\Gamma_{EELS} \rightarrow \vec{E}_{tot}(\bar{r}_e(t)) = \underbrace{\bar{E}_{e,ref}} + \underbrace{\vec{E}_{dip}(\vec{p})} + \underbrace{\bar{E}_{dip,ref}(\vec{p})}$$

$$\Gamma_{EELS} = \Gamma_{\parallel} + \Gamma_{direct} + \Gamma_{ref}$$

$z_\rho|_{zue}, z_{electron}$

E_e

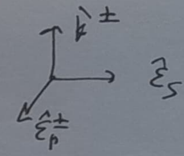
$k^2 \rho_{direct}$

$E_{e,ref} + E_{dip,ref}$

$k^2 \rho_{ref}$

$C_i(b\omega) \, Im\{\alpha(\omega)\}$

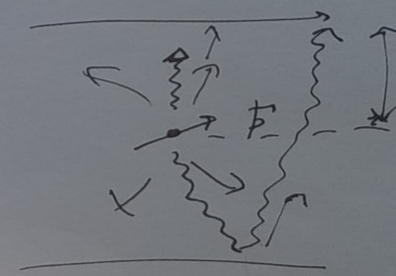
$$\vec{E}^{dip} \overset{\circ}{=} \cancel{\ast}\left(k^2 + \vec{\nabla}\otimes\vec{\nabla}\right)\vec{p}\underbrace{\left(\frac{e^{ikr}}{r}\right)}_{=\frac{1}{(2\pi)^2}\int \frac{d^2Q}{k_z} e^{i\vec{Q}\cdot\vec{R}} e^{ik_z z}}$$



$$\mathbb{1} = \hat{k}^{\pm}\otimes\hat{k}^{\pm} + \hat{\varepsilon}_s\otimes\hat{\varepsilon}_s + \hat{\varepsilon}_p^{\pm}\otimes\hat{\varepsilon}_p^{\pm}$$

$$\overset{\circ}{=} k^2\left(1 - \hat{k}\otimes\hat{k}\right)\vec{p}\cdot\frac{1}{(2\pi)^2}\int\frac{d^2Q}{k_z}\ldots$$

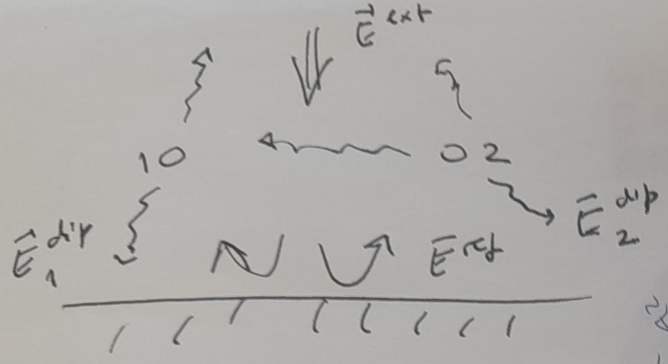
$$\overset{\circ}{=} \frac{k^2}{(2\pi)^2}\int\frac{d^2Q}{k_z}\left[\hat{\varepsilon}_s\otimes\hat{\varepsilon}_s + \hat{\varepsilon}_p^{\pm}\otimes\hat{\varepsilon}_p^{\pm}\right]\cdot\vec{p}\; e^{i\vec{Q}\cdot\vec{R}} e^{ik_z z}$$



$$\vec{E}^{dip}_{direct} = \frac{k^2}{(2\pi)^2}\int\frac{d^2Q}{k_z}\left[\left(\hat{\varepsilon}_s\cdot\vec{p}\right)\hat{\varepsilon}_s + \left(\hat{\varepsilon}_p^{+}\cdot\vec{p}\right)\hat{\varepsilon}_p^{+}\right] e^{i\vec{Q}\cdot\vec{R}} e^{ik_z(z+b)}$$

$$\vec{E}^{dip}_{ref} = \frac{k^2}{(2\pi)^2}\int\frac{d^2Q}{k_z}\left[\left(\hat{\varepsilon}_s\cdot\vec{p}\right)\hat{\varepsilon}_s e^{ik_z(2a+b)} r_s + \left(\hat{\varepsilon}_p^{-}\cdot\vec{p}\right)\hat{\varepsilon}_p^{+} e^{ik_z(2a+b)} r_p\right] e^{i\vec{Q}\cdot\vec{R}} e^{ik_z z}$$

$\hat{\varepsilon}_s, \hat{\varepsilon}_p \to \varphi$



$$\bar{E}_j^{dip} = \beta_j \bar{p}_j$$

≈ 6

$$\begin{cases} \bar{p}_1 = \alpha_1 \vec{E}(\bar{r}_1) = \alpha_1 \left[\bar{E}^{ext} + \bar{E}^{ref} + \underbrace{\bar{E}_1^{dip}}_{\propto \bar{p}_1} \tilde{r}_\sigma + \underbrace{\bar{E}_2^{dip}}_{\propto \bar{p}_2} + \underbrace{\bar{E}_2^{dip}}_{\propto \bar{p}_2} \bar{r}_\sigma \right] \\ \bar{p}_2 = \alpha_2 E(\bar{r}_2) = \alpha_2 \left[\quad " \quad " \quad \bar{E}_2^{dip} \bar{r}_\sigma + E_1^{dip} + \bar{E}_1^{dip} \bar{r}_\sigma \right] \end{cases}$$

$$\begin{cases} \bar{p}_1 = \alpha_1 \left[\bar{E}^{ext}(1+\bar{r}_\sigma) + \beta_1 \bar{p}_1 + \beta_2 (1+\bar{r}_\sigma) \bar{p}_2 \right] \\ \bar{p}_2 = \alpha_2 \left[\quad \right] \end{cases}$$

$$\frac{1}{1/\alpha_1 - \beta_1}$$

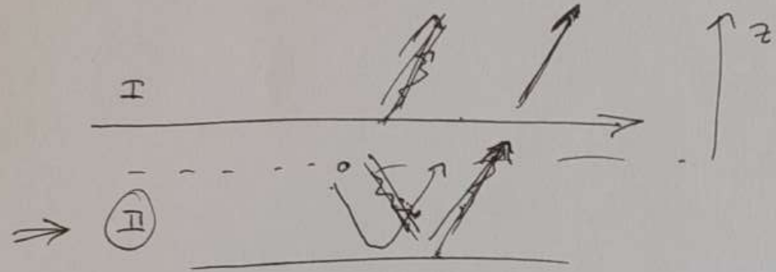
$$\begin{cases} \bar{p}_1 = (\alpha_{eff}^1) \left[\bar{E}^{ext}(1+\bar{r}_\sigma) + \beta_2 (1+\bar{r}_\sigma) \bar{p}_2 \right] \end{cases}$$

$$\phi_p^{dir} = -\mathbf{p}\cdot\bar\nabla\left(\frac{1}{|\bar r-\bar r_0|}\right)$$

$$\frac{1}{|\bar r-\bar r_0|} \int \frac{d^2\bar k_{\parallel}}{(2\pi)\, k_{\parallel}} e^{i\bar k_{\parallel}\cdot(\bar R-\bar R_0)} e^{-k_{\parallel}(z-z_0)}$$

$$\bar r = \bar R + z\hat z$$

$$= -i\mathbf{p}\cdot \int \frac{d^2\bar k_{\parallel}}{2\pi k_{\parallel}} \vec k\cdot e^{i\bar k_{\parallel}\cdot(\bar R-\bar R_0)} e^{-k_{\parallel}(z-z_0)}$$



(I)

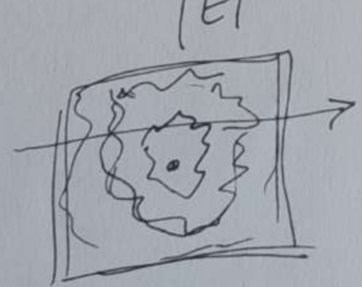
$$\phi_p^{tot} = -i\mathbf{p}\cdot\int\frac{d^2\bar k_{\parallel}}{2\pi k_{\parallel}} \bar k^{+} e^{i\vec k_{\parallel}(\bar R-\bar R_0)} \left(e^{-k_{\parallel}(z-z_0)} + \cancel{r_s e^{-k_{\parallel}(z-z_0)} + r_p \cdots} \right)$$

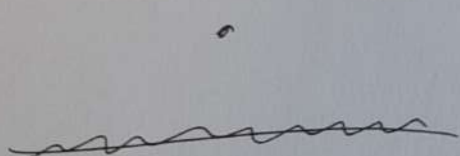
(II)

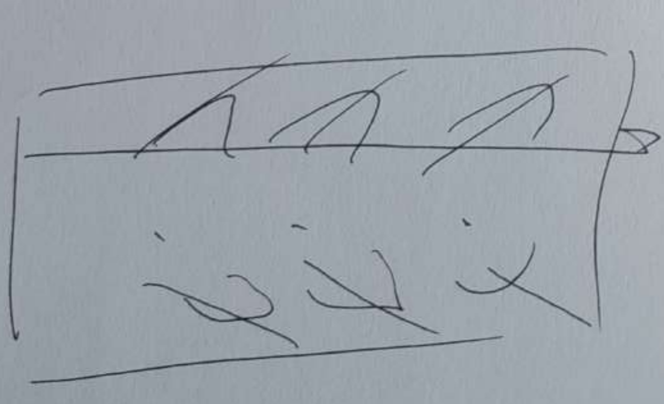
$\hat k^{-}$ $\hat k^{+}$

$\mathbf{p}$

$\phi_p \longrightarrow \quad \vec E_p = -\bar\nabla\phi_p$







$\varepsilon_2 i \alpha_n + \varepsilon_1 i \alpha_n + 4\pi$

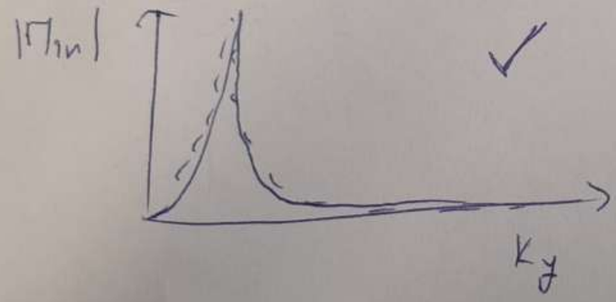
$r_p(\omega, k_{\parallel})$

$$M_{1n}(k_y) = k_{xn} \frac{e^{-2\sqrt{k_{xn}^2 + k_y^2} \cdot z_p}}{\sqrt{k_{xn}^2 + k_y^2}} \underbrace{r_p\left(\overbrace{\sqrt{k_{xn}^2 + k_y^2}}^{k_{\parallel}^{(n)}}\right)} \cdot e^{i k_y y}$$

(1) $\Rightarrow$ Fresnel

(2) $\Rightarrow \dfrac{R_p Q_p}{\sqrt{k_{xn}^2 + k_y^2} - Q_p}$

n fixed



✓

Numerical int (1) $\rightarrow I_1$
Numerical int (2) $\rightarrow I_2$
Analytic int. Pole $\rightarrow I_3$

$\hbar\omega = 63$ meV

$\underline{\underline{k_{xn} < Q_p}}$

$\cancel{k_{xn} = \sqrt{k^2 - (k_y + (\frac{2\pi n}{a})^2}}$

$\frac{\omega}{c} > k_{\parallel} > \frac{\omega}{v} + \frac{2n\pi}{a}$

$\Rightarrow \frac{\omega}{c} > \frac{\omega}{v} + \frac{2n\pi}{a}$

$\Rightarrow \omega(1/c - 1/v) > \frac{2n\pi}{a} \Rightarrow \underline{\underline{\omega}} > \frac{2n\pi}{a}\left(\frac{1}{1/c - 1/v}\right)$

$k_{xn} = \frac{\omega}{v} + 2n\pi/a$

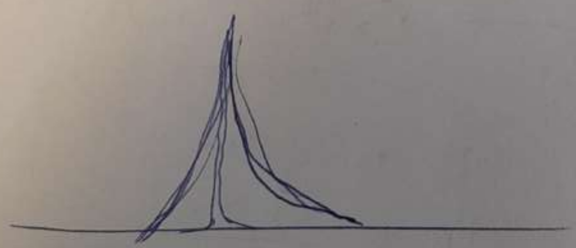
$k_{\parallel} = \sqrt{k_{xn}^2 + k_y^2} \Rightarrow k_y = \sqrt{k_{\parallel}^2 - k_{xn}^2} = \sqrt{k_{\parallel}^2 - \left(\frac{\omega}{v} + \frac{2n\pi}{a}\right)^2}$

$$\int d^2\bar{k}_{\parallel} \quad \begin{cases} \displaystyle\int_0^{2\pi} d\phi \int_0^{\infty} k_{\parallel}\, dk_{\parallel} \\ \displaystyle\int_{-\infty}^{+\infty} dk_x \int_{-\infty}^{+\infty} dk_y \end{cases}$$

$$\underline{\underline{k_{\parallel} = Q_p}} = \sqrt{k_{xx}^2 + k_y^2} = Q_p$$

$$\Rightarrow \quad k_{xx}^2 + k_y^2 = Q_p^2 \quad \Rightarrow \quad k_y = \sqrt{Q_p^2 - k_{xx}^2}$$

$$\underline{\underline{|\mathrm{Im}(Q_p)| << |\mathrm{Re}(Q_p)|}} \qquad k_{xx} < Q_p$$



$$\frac{1}{x - x' + i\varepsilon} \approx \mathrm{P.V.}\,\frac{1}{x - x'} - \underline{i\pi\,\delta(x - x')} \qquad \varepsilon \to 0^+$$

$$\int dk_y \frac{e^{-2\sqrt{k_{nx}^2+k_y^2}\,z_p}}{\sqrt{k_{nx}^2+k_y^2}} \left(\underbrace{\frac{R_p\, k_p}{\sqrt{k_{nx}^2+k_y^2}-k_p}} \right) e^{ik_y y}$$

$$\text{P.V.}\ \frac{1}{\sqrt{k_{nx}^2+k_y^2}-k_p} - i\pi\, \delta\left[k_p - \sqrt{k_{nx}^2+k_y^2}\right]$$

neglecting P.V.

$$= -i\pi \int dk_y \frac{e^{-2\sqrt{k_{nx}^2+k_y^2}\,z_p}}{\sqrt{k_{nx}^2+k_y^2}} e^{ik_y y}\, \delta\left[k_p - \sqrt{k_{nx}^2+k_y^2}\right] R_p\, k_p \qquad (*)$$

$$\delta[f(x)] = \sum_j \frac{\delta(x-x_j)}{|f'(x_j)|} \qquad f(x_j)=0 \qquad k_{nx} = \frac{\omega}{v} + \frac{2n\pi}{a}$$

$$k_p - \sqrt{k_{nx}^2+k_y^2} = 0 \Leftrightarrow k_{nx}^2 + k_y^2 = k_p^2$$

$$\Leftrightarrow k_y^{(n)\pm} = \pm\sqrt{k_p^2 - k_{nx}^2}$$

$$= \pm\sqrt{k_p^2 - \left(\frac{\omega}{v}+\frac{2n\pi}{a}\right)^2}$$

$$k_p > \frac{\omega}{v} + \frac{2\pi n}{a} \Leftrightarrow n < \left(k_p - \frac{\omega}{v}\right)\frac{a}{2\pi} \equiv n_{max}$$

$$\Leftrightarrow n < k\left(\frac{k_p}{k} - \frac{c}{v}\right)\frac{a}{2\pi}$$

$$f'(k_y) = \frac{-k_y}{\sqrt{k_{nx}^2+k_y^2}} \Rightarrow \delta[f(k_y)] = \cancel{\sum_{n=0}^{n_{max}}} \sum_{v=\pm} \frac{\delta(k_y - k_y^{(n)v})}{\left|\frac{k_y^{(n)v}}{\sqrt{k_{nx}^2+k_y^{(n)v\,2}}}\right|} \Theta(n_{max}-n)$$

$$= k_p \cancel{\sum_{n=0}^{n_{max}}} \sum_{v=\pm} \frac{\delta(k_y - k_y^{(n)v})}{|k_y^{(n)v}|} \Theta(n_{max}-n)$$

$$(*) = -i\pi k_p R_p\, \Theta(n_{max}-n) \sum_{v=\pm} \frac{e^{-2k_p z_p}}{|k_y^{(n)v}|} e^{ik_y^{(n)v}}$$

$$k_y^{(n)v} = v\, k_y^{(n)} \qquad k_y^{(n)} = \sqrt{k_p^2 - k_{nx}^2}$$

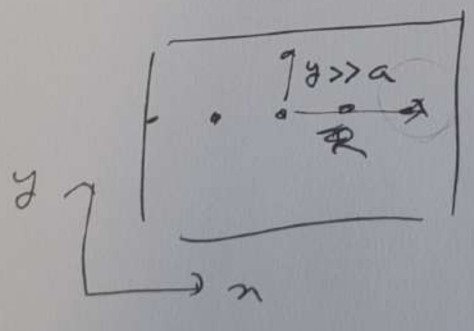
$$M_1 = \sum_n e^{ik_{xn}x} (-i\pi) k_\rho R_\rho \, \Theta(n_{max}-n) \sum_{\nu=\pm} \frac{e^{-2k_\rho z_\rho}}{|k_y^{(n)\nu}|} e^{ik_y^{(n)\nu} y}$$

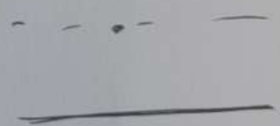
$$= \sum_{n=0}^{n_{max}} e^{ik_{xn}x} (-i\pi) k_\rho R_\rho \sum_{\nu=\pm} \frac{e^{-2k_\rho z_\rho}}{k_y^{(n)}} e^{i\nu k_y^{(n)} y}$$

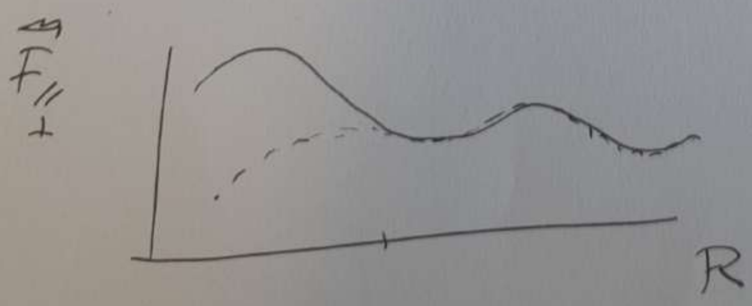
$$= \sum_{n=0}^{n_{max}} e^{ik_{xn}x} (-i\pi) k_\rho R_\rho \frac{e^{-2k_\rho z_\rho}}{k_y^{(n)}} \left(e^{ik_y^{(n)} y} + e^{-ik_y^{(n)} y} \right)$$

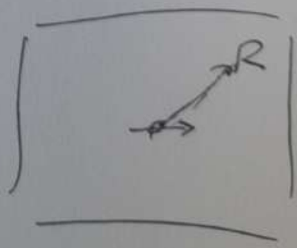
$$= \sum_{n=0}^{n_{max}} e^{ik_{xn}x} (-i2\pi) k_\rho R_\rho \frac{e^{-2k_\rho z_\rho}}{k_y^{(n)}} \cos\left(k_y^{(n)} y\right)$$

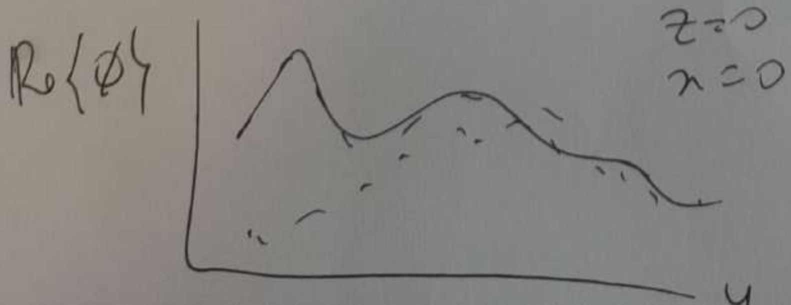
desde $n=0$ ó desde $-n_{max}$?









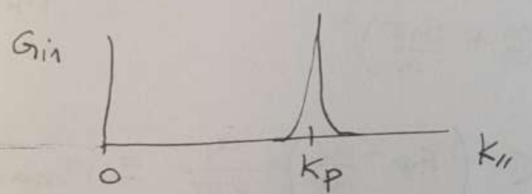


$$G_1 = \int_0^\infty dK_{\parallel}\, K_{\parallel}\, r_p\, J_0(K_{\parallel} R)\, e^{-K_{\parallel}(2z_p - z)} = \int_0^\infty dK_{\parallel}\, G_{i1}(K_{\parallel})$$

$$r_p \simeq \frac{R_p K_p}{K_{\parallel} - K_p} \longrightarrow \mathrm{P.V.}\left(\frac{R_p K_p}{K_{\parallel} - K_p}\right) - i\pi\delta(K - K_p)\, R_p K_p$$

① $$= \int_0^\infty dK_{\parallel}\, \frac{K_{\parallel} R_p K_p}{K_{\parallel} - K_p}\, J_0(K_{\parallel} R)\, e^{-K_{\parallel}(2z_p - z)} -$$

$$- i\pi R_p K_p^2\, J_0(K_p R)\, e^{-K_p(2z_p - z)}$$

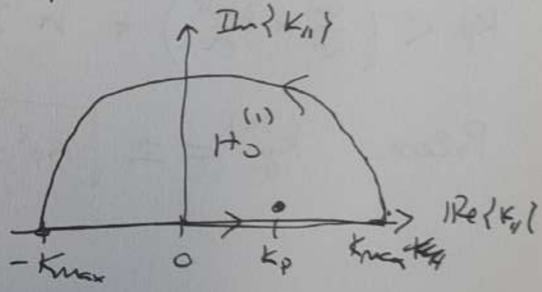


②

$$\approx \int_{-\infty}^{+\infty} dK_{\parallel}\, K_{\parallel}\, r_p\, J_0(K_{\parallel} R)\, e^{-K_{\parallel}(2z_p - z)}$$

$$J_0(K_{\parallel} R) = \frac{1}{2}\left[H_0^{(1)}(K_{\parallel} R) + H_0^{(2)}(K_{\parallel} R)\right]$$

$$r_p \rightarrow \frac{R_p K_p}{K_{\parallel} - K_p}$$



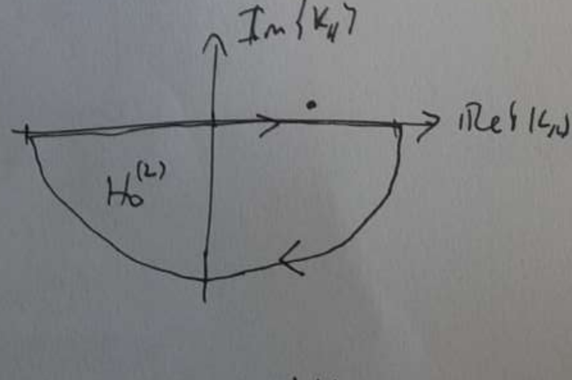
$K_{max} \to \infty$

$$= \int_{-\infty}^{+\infty} \frac{dK_{\parallel}\, K_{\parallel}}{2}\, r_p\, H_0^{(1)}(K_{\parallel} R)\, e^{-K_{\parallel}(2z_p - z)} + (\dots) \to 0$$

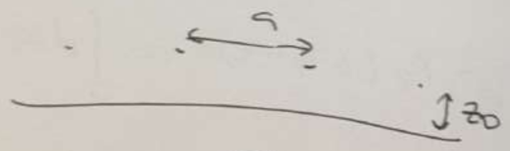
$$R\,[f(x), x_0] = \lim_{x \to x_0} (x - x_0) f(x)$$

$$= \frac{2\pi i}{2}\, K_p R_p K_p\, H_0^{(1)}(K_p R)\, e^{-K_p(2z_p - z)}$$

$$H_0^{(1)}(x) \xrightarrow[x \gg 1]{} \frac{e^{-x}}{\sqrt{x}} \quad (?)$$



$k_z \simeq i K_{\parallel}$



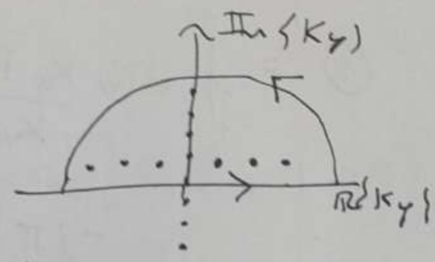
$$\frac{R_p K_p}{\sqrt{K_{xn}^2 + K_y^2} + K_p}$$

$$G_1 = K_{xn} \int_{-\infty}^{+\infty} dK_y \, \frac{e^{-2\sqrt{K_{xn}^2 + K_y^2}\, z_p}}{\sqrt{K_{xn}^2 + K_y^2}} \, r_p \, e^{i K_y y}$$

Poles: $\sqrt{K_{xn}^2 + K_y^2} = K_p$

$\Rightarrow \quad K_y = \pm \sqrt{K_p^2 - K_{xn}^2}$

$\quad = \pm \sqrt{K_p^2 - \left(\frac{\omega}{v} + \frac{2n\pi}{a}\right)^2}$



$K_{xn} = \frac{\omega}{v} + \frac{2n\pi}{a}$

$K_y \in \mathbb{R}$:

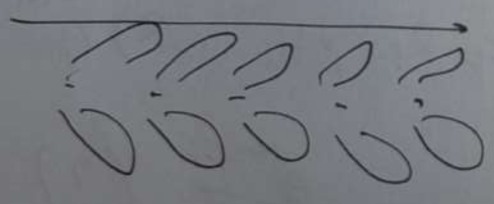
$K_p > \left(\frac{\omega}{v} + \frac{2n\pi}{a}\right) \Rightarrow n < \left(K_p - \frac{\omega}{v}\right) \cdot \frac{a}{2\pi} \equiv n_{max}$

$K_y \in \mathrm{Im}$:

$K_p < \left(\frac{\omega}{v} + \frac{2n\pi}{a}\right) \Rightarrow n > n_{max}$

Poles: $K_y^{(n)} = \pm \sqrt{K_p^2 - \left(\frac{\omega}{v} + \frac{2n\pi}{a}\right)^2}$

$\left(K_y - K_y^{(n)}\right) f(K_y)$



$\rightarrow \frac{\omega}{v} + \frac{2n\pi}{a}$

$\varepsilon_2 i \alpha_n + \varepsilon_1 i \alpha_n + 4\pi$

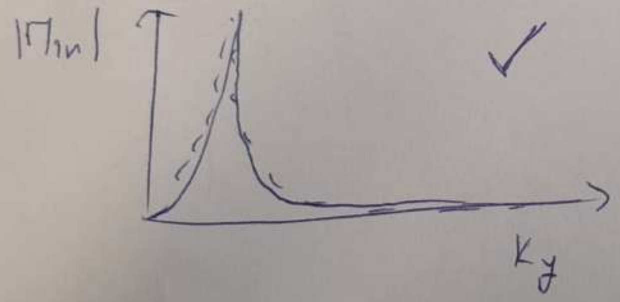
$r_p(\omega, k_{\parallel})$

$$M_{1n}(k_y) = k_{xn} \frac{e^{-2\sqrt{k_{xn}^2 + k_y^2}\cdot z_p}}{\sqrt{k_{xn}^2 + k_y^2}} \, r_p\Big(\underbrace{\sqrt{k_{xn}^2 + k_y^2}}_{k_{\parallel}^{(n)}}\Big) \cdot e^{i k_y y}$$

(1) $\Rightarrow$ Fresnel

(2) $\Rightarrow \dfrac{R_p Q_p}{\sqrt{k_{xn}^2 + k_y^2} - Q_p}$

n fixed



✓

Numerical int (1) $\rightarrow I_1$
Numerical int (2) $\rightarrow I_2$
Analytic int. Pole $\rightarrow I_3$

$\hbar\omega = 63$ meV

$\underline{k_{xn} < Q_p}$

$\cancel{k_{xn} = \sqrt{k^2 - (k_y + \frac{2\pi n}{a})^2}}$

$\frac{\omega}{c} > k_{\parallel} > \frac{\omega}{v} + \frac{2n\pi}{a}$

$\Rightarrow \frac{\omega}{c} > \frac{\omega}{v} + \frac{2n\pi}{a}$

$\Rightarrow \omega(1/c - 1/v) > \frac{2n\pi}{a} \Rightarrow \underline{\omega} > \frac{2n\pi}{a}\left(\frac{1}{1/c - 1/v}\right)$

$k_{xn} = \frac{\omega}{v} + 2n\pi/a$

$k_{\parallel} = \sqrt{k_{xn}^2 + k_y^2} \Rightarrow k_y = \sqrt{k_{\parallel}^2 - k_{xn}^2} = \sqrt{k_{\parallel}^2 - \left(\frac{\omega}{v} + \frac{2n\pi}{a}\right)^2}$

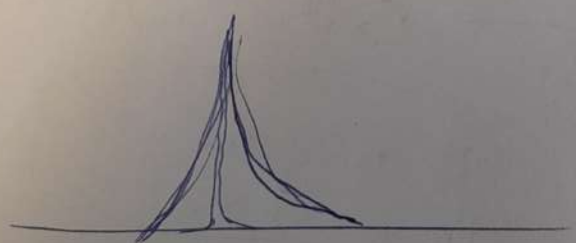
$$\int d^2\bar{k}_{//} \;\to\; \int_0^{2\pi} d\phi \int_0^{\infty} k_{//}\, dk_{//}$$

$$\int d^2\bar{k}_{//} \;\to\; \int_{-\infty}^{+\infty} dk_x \int_{-\infty}^{+\infty} dk_y$$

$$\underline{\underline{k_{//} = Q_p}} = \sqrt{k_{xx}^2 + k_y^2} = Q_p$$

$$\Rightarrow \quad k_{xx}^2 + k_y^2 = Q_p^2 \quad \Rightarrow \quad \cancel{k_p}\; k_y = \sqrt{Q_p^2 - k_{xx}^2}$$

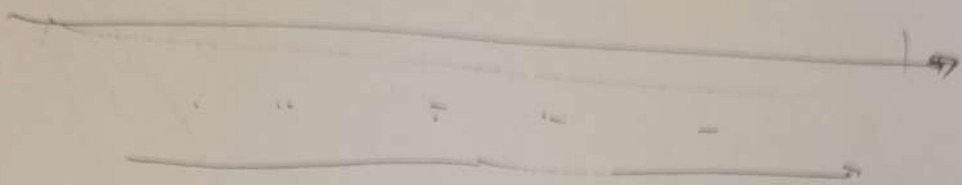
$$\underline{\underline{|\mathrm{Im}(Q_p)| \ll |\mathrm{Re}(Q_p)|}} \qquad k_{xx} < Q_p$$



$$\frac{1}{x - x' + i\varepsilon} \approx P.V.\, \frac{1}{x - x'} - \underline{i\pi\,\delta(x - x')} \qquad \varepsilon \to 0^+$$

$\frac{1}{L}\int_{-\infty}^{+\infty} dx \to \sum_n$

Lunes 18



$\frac{\Gamma_{EELS}}{L}$

$2\omega/v - \frac{2\pi n}{a}$

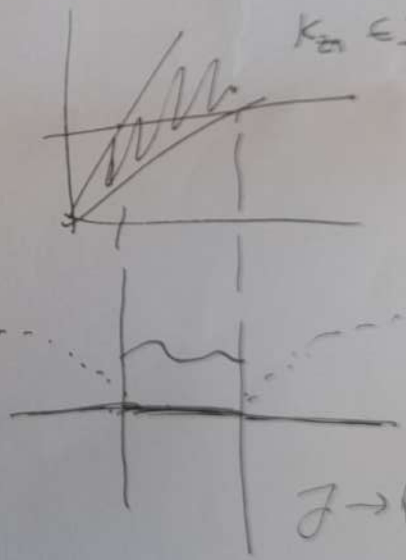
$\delta(k_{xn}-\omega/v) = \delta\left(-\frac{2\pi n}{a}\right) = \frac{\delta(n)}{\left(\frac{2\pi}{a}\right)} = \frac{a}{2\pi}\delta(n)$

$= \frac{Na}{2\pi} \quad L$

$k_{xn} = \frac{\omega}{v} - \frac{2\pi n}{a}$

$\delta(f(x)) = \sum_i \frac{\delta(x-x_i)}{|f'(x_i)|}$

$k_{zn} \in \mathbb{R}$



$\int dk_\| \; r_p(k_\|,\omega)$

$-\mathrm{Im}\{r_p(k_\|,\omega)\}$

$-i\,\delta(k_\| - k_{\|j})$

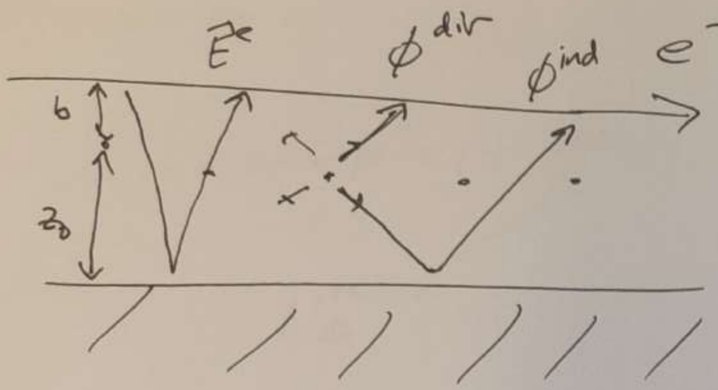
$J \to H^{(n)}$

$r_p(k_\|,\omega) = \cancel{\mathrm{P.V.}} \sum_j \frac{R_j k_j}{k_\| - k_j}$

$\approx \sum_j \mathrm{P.V.}\frac{R_j k_j}{k_\| - k_j} - i\pi \sum_j \delta(k_\| - k_j) R_j k_j$

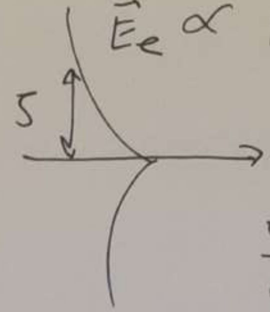
$E = \int dk_\| \, F(k_\|)\, r_p(k_\|) = \sum_j \mathrm{P.V.}\int d_\| \, F(k_\|)\, r_p(k_\|) - i\pi \sum_j F(k_j) R_j k_j$

§ 20 de Julio



$\vec{E}_e \propto K_0\left(\frac{\omega b}{v}\right)$

TO CHECK



$\varsigma \gg (b+z_0)\times 2$

we neglect

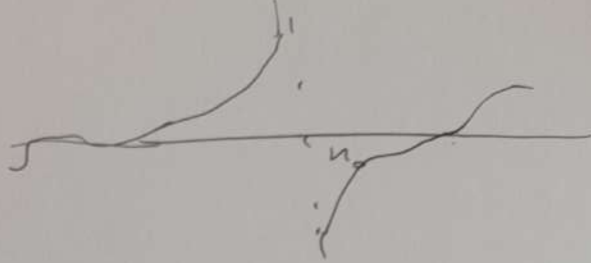
$\varsigma \sim v/\omega$

$$\Gamma_{EELS} = \Gamma^{e}_{EELS} + \underline{\underline{\Gamma^{dir}_{EELS}}} + \underline{\underline{\Gamma^{ind}_{EELS}}}$$

Lunes 25 de Julio

$$-\!\!\!\!\!\!\int_a^b dx \, \frac{f(x)}{x-x_0} = \int_a^{x_0} dx \, \frac{f(x)}{x-x_0} + \int_{x_0}^{b} dx \, \frac{f(x)}{x-x_0}$$

$a < x_0 < b$

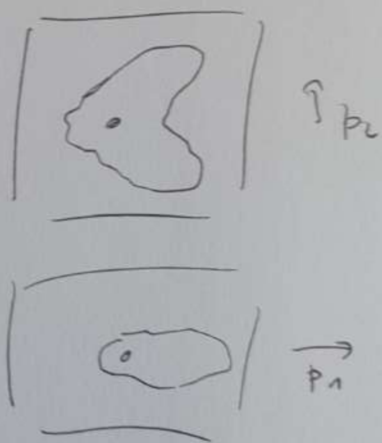


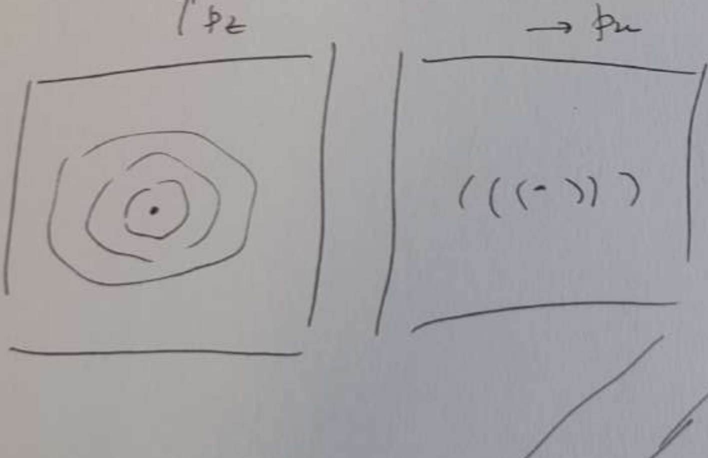
$$-\!\!\!\!\!\!\int_a^b dx \, \frac{f(x)}{x-x_0} =$$

$$= -\!\!\!\!\!\!\int_a^b dx \, \frac{f(x)-f(x_0)}{x-x_0} + -\!\!\!\!\!\!\int_a^b dx \, \frac{f(x_0)}{x-x_0}$$

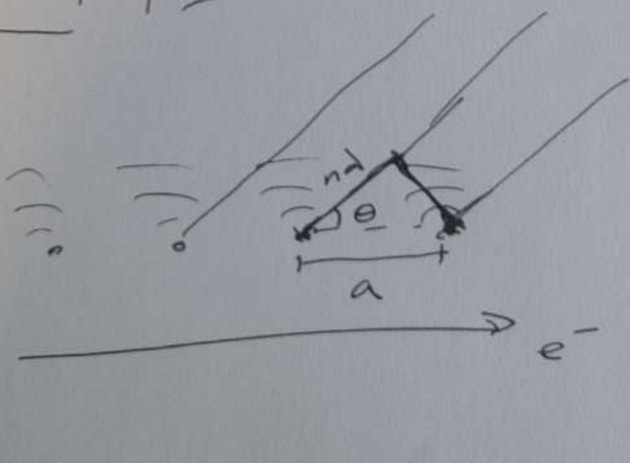
$$\int dk_x \, \bar{\phi}(\;)\, e^{---} = 0$$

$$\phi^{\infty} = \sum_{j=-N}^{N} \phi_1(\vec{r}_j)\, e^{-aj k_x v}$$





v/c



$$e^{i k_{xn} x} \, e^{i\sqrt{k_p^2 - k_{xn}^2}\, y} = e^{i\vec{k}\cdot\vec{r}}$$

$$\vec{k} = \left(k_{xn} \, , \sqrt{k_p^2 - k_{xn}^2}\right)$$

$$\theta_n = \arctan\left(\frac{\sqrt{k_p^2 - k_{xn}^2}}{k_{xn}}\right)$$

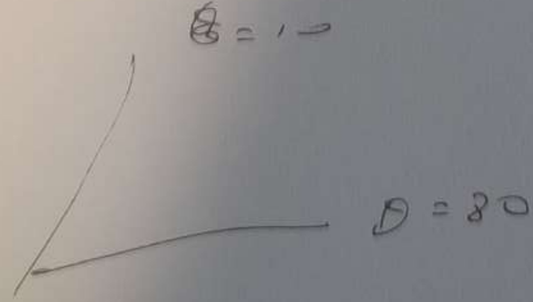
$$k_{xn} = k_p \cos(\theta_n)$$

$$\Rightarrow \theta_n = \arccos\left(\frac{k_{xn}}{k_p}\right)$$

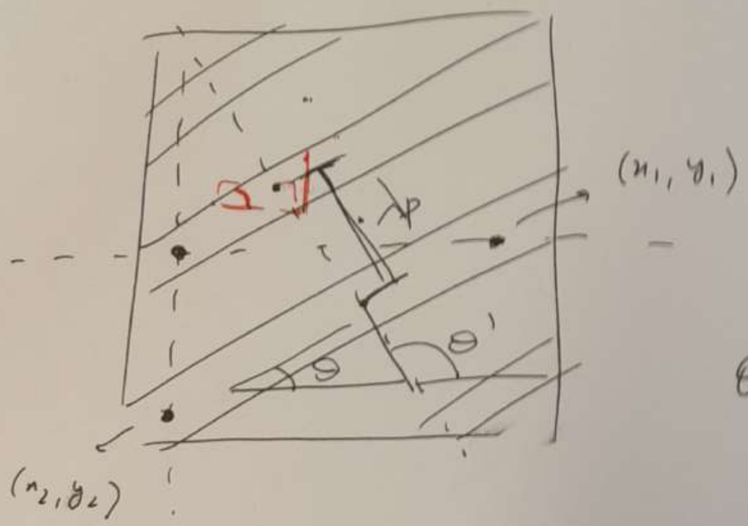
$$= \arccos\left(\frac{\omega/\vartheta - 2n\pi/a}{k_p}\right) \qquad k_p = \frac{2\pi}{\lambda p}$$

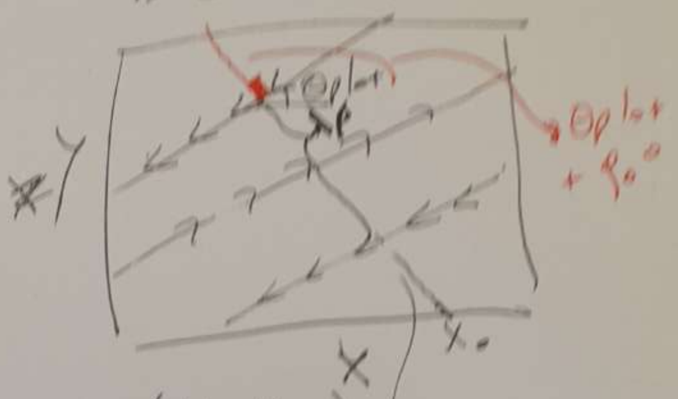
$$\cancel{\cos(\theta) na} \qquad = \arccos\left(\frac{\omega \cdot \lambda p}{2\pi\vartheta} - n\frac{\lambda p}{a}\right)$$

$$= \arccos\left(\lambda p\left(\frac{\omega}{2\pi\vartheta} - \frac{n}{a}\right)\right)$$



20 de septiembre

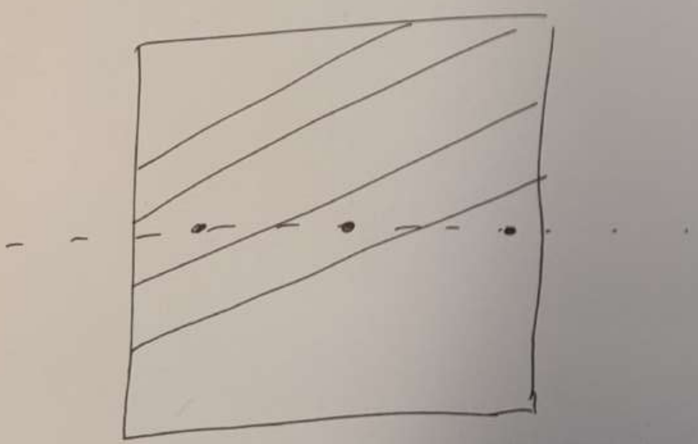




$$\theta = \arctan\left(\frac{y_1 - y_2}{x_1 - x_2}\right)$$

$y = (x + x_0)$

$y = mx + b$



$(0, 1.5)$

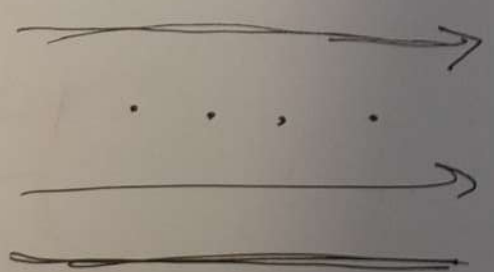
$(0.25, 1.0)$

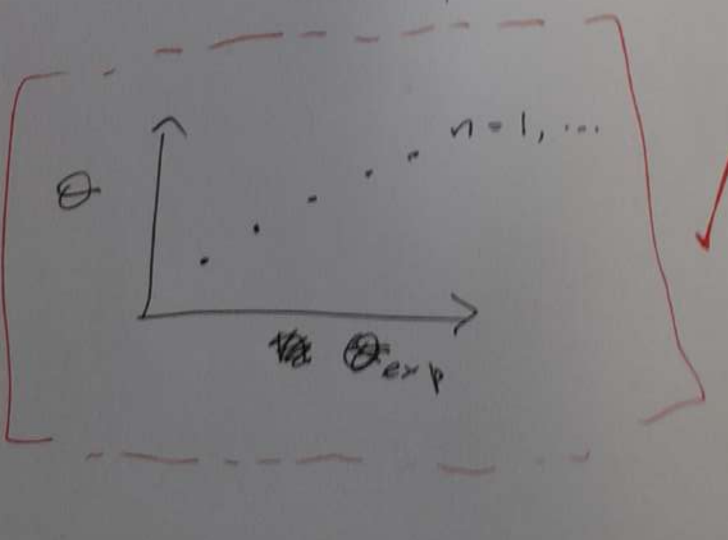
$$\sqrt{0.25^2 + 0.5^2}$$

$\approx 0.56\ \mu m$

$$\vec{K} = \left(K_x, \sqrt{K_p^2 - K_x^2}\right)$$

$$K = \sqrt{\cancel{K_x^2} + K_p^2 - \cancel{K_x^2}} = K_p \rightarrow \underline{\lambda = \lambda_p}$$





$$e^{i k_{xn} x} \, e^{i\sqrt{k_p^2 - k_{xn}^2}\, y} = e^{i\vec{k}\cdot\vec{r}}$$

$$\vec{k} = \left(k_{xn} \, , \sqrt{k_p^2 - k_{xn}^2}\right)$$

$$\theta_n = \arctan\left(\frac{\sqrt{k_p^2 - k_{xn}^2}}{k_{xn}}\right)$$

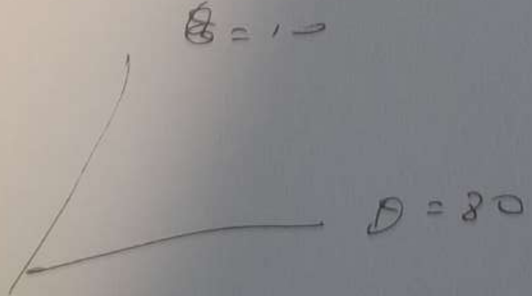
$$k_{xn} = k_p \cos(\theta_n)$$

$$\Rightarrow \theta_n = \arccos\left(\frac{k_{xn}}{k_p}\right)$$

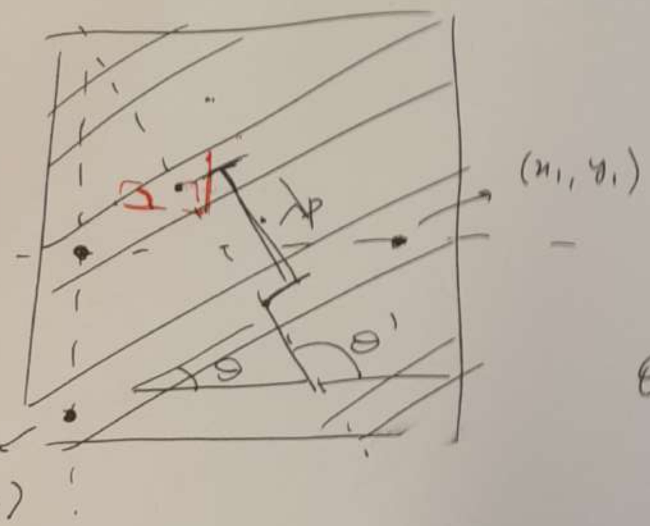
$$= \arccos\left(\frac{\omega/\vartheta - 2n\pi/a}{k_p}\right) \qquad k_p = \frac{2\pi}{\lambda p}$$

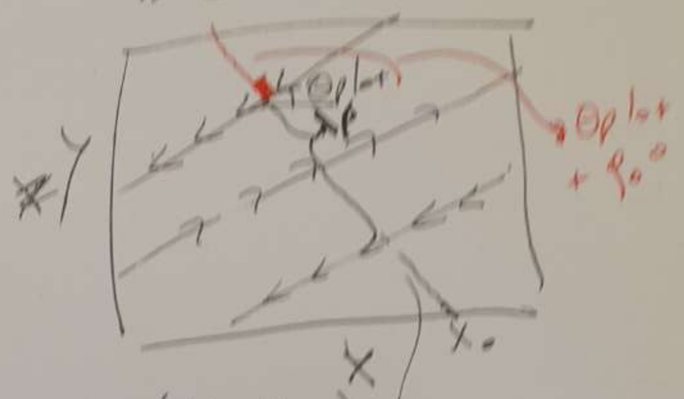
$$= \arccos\left(\frac{\omega \cdot \lambda p}{2\pi \vartheta} - n\frac{\lambda p}{a}\right)$$

$$= \arccos\left(\lambda p \left(\frac{\omega}{2\pi\vartheta} - \frac{n}{a}\right)\right)$$



20 de septiembre



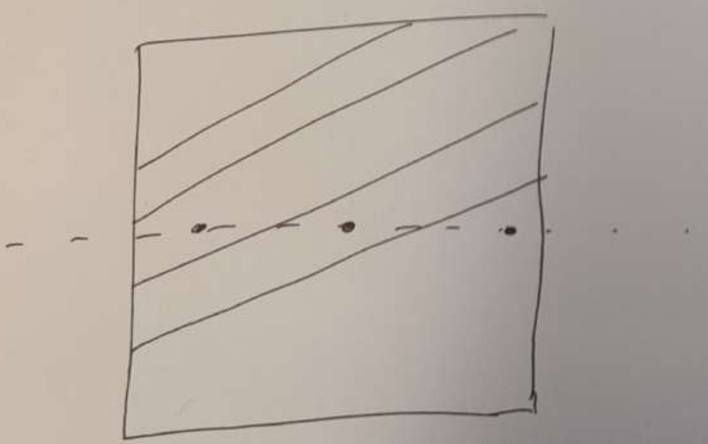


$$\theta = \arctan\left(\frac{y_1 - y_2}{x_1 - x_2}\right)$$

$y = (x + x_0)$

~~$y = (-m x + x_0)$~~

$y = mx + b$



$(0, 1.5)$

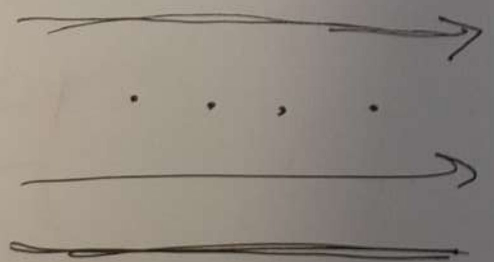
$(0.25, 1.0)$

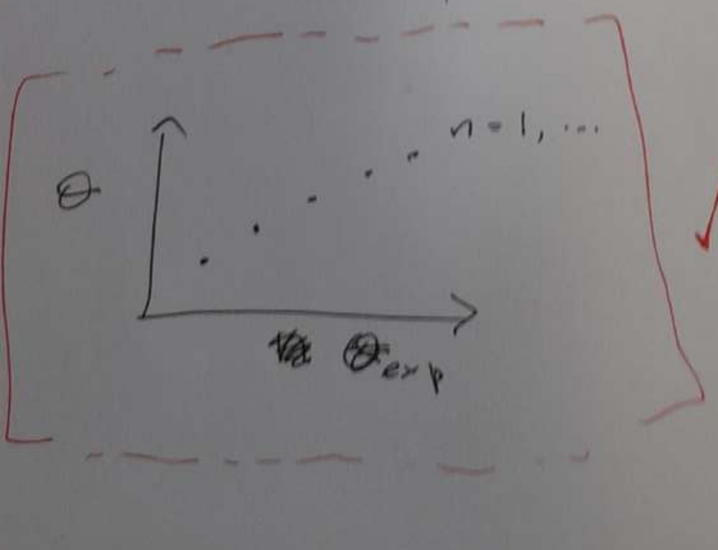
$$\sqrt{0.25^2 + 0.5^2}$$

$\approx 0.56\ \mu m$

$$\vec{K} = \left(K_x, \sqrt{K_p^2 - K_x^2}\right)$$

$$K = \sqrt{\cancel{K_x^2} + K_p^2 - \cancel{K_x^2}} = K_p \rightarrow \underline{\lambda = \lambda_p}$$





$$\cos(\theta) = \frac{\lambda_p}{2\pi}\left[\frac{\omega}{v} + \frac{2\pi n}{a}\right] \qquad \lambda_p = \frac{2\pi}{k_p} = f(\omega)$$

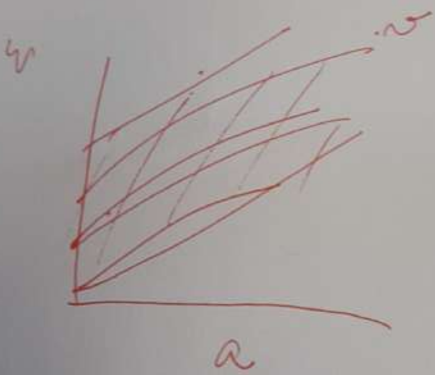
$$\Rightarrow \quad -1 \le \frac{\lambda_p}{2\pi}\left[\frac{\omega}{v} + \frac{2n\pi}{a}\right] \le 1$$

$$\Leftarrow \quad \omega_{min} \le \omega \le \omega_{max}$$

$$\underline{f_n(v, a)}$$

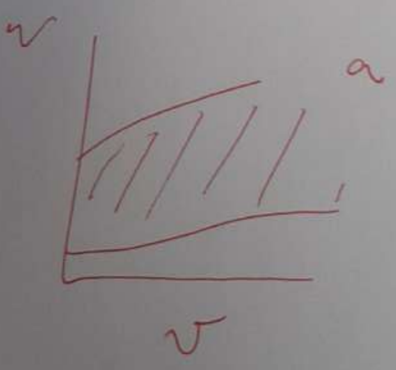
$$k_p = A \cdot \omega^2 \quad \Rightarrow \quad \lambda_p = \frac{2\pi}{A\omega^2}$$

$$-1 \le \frac{2\pi}{\ } \ldots$$



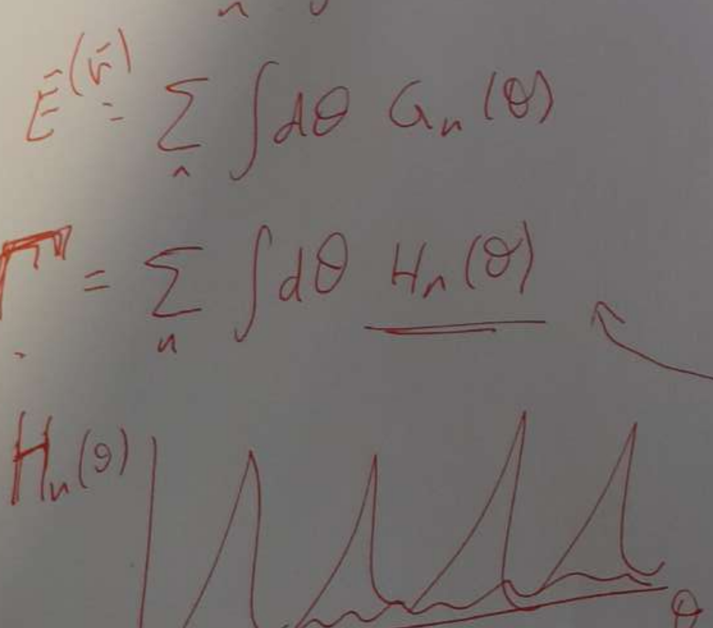
$$\phi^{(\vec{r})} = \sum_n \int dy \, F_n(y)$$

$$= \sum_n \int d\theta \, F_n(\theta)$$



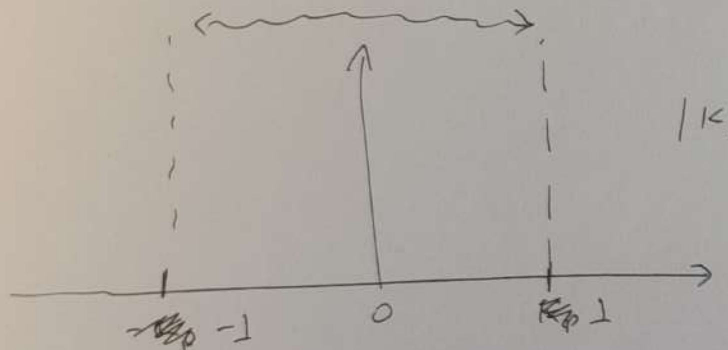
$$\vec{E}^{(\vec{r})} = \sum_n \int d\theta \, G_n(\theta)$$

$$\Gamma = \sum_n \int d\theta \, \underline{H_n(\theta)}$$



$$\Gamma = \frac{2}{\hbar} \, \mathrm{Im}\{\vec{\mu}^* \cdot \vec{E}_{ind,n}\}$$

$\Gamma_{SPE,n}$



$$F = \int dk_y \, f(k_y)$$

$$= \int_{-k_p}^{k_p} dk_y \, f(k_y) + I$$

Γ

$$r_p = \frac{k_p R_p}{k_{\parallel} - k_p}$$

$$\sqrt{k_{xn}^2 + k_y^2} = k_p$$

$\Downarrow$

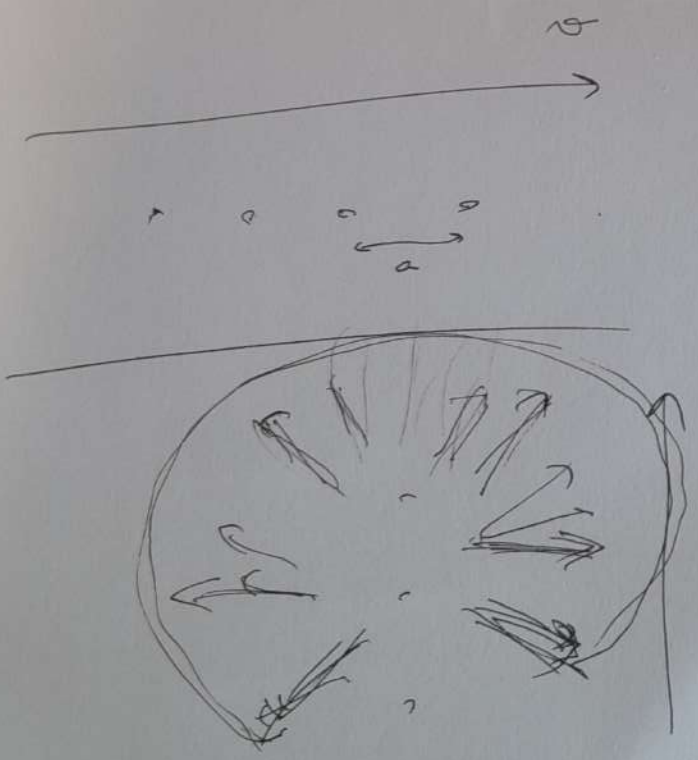
$k_{xn} = k_p \cos(\theta_n)$
$k_y = k_p \sin(\theta_n)$

R_p, k_p

#331

$r_p =$

$\sim 10\,nm$



$\sim e^{-z_p/\lambda_p}$

$z_p^{opt} \sim \lambda_p \propto d$

$\Gamma(z_p d)$

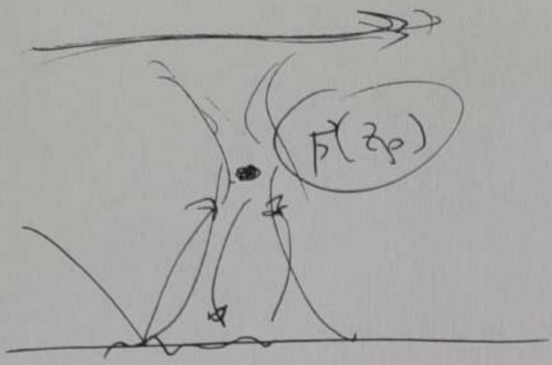
$\lambda_p \propto d$

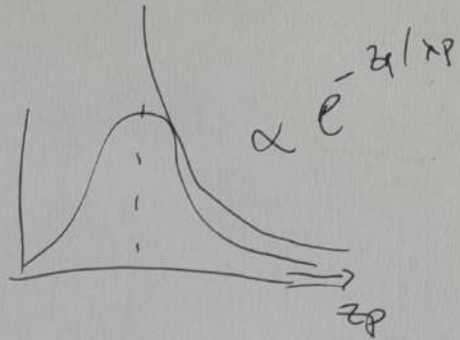
$d \uparrow \quad z_p^{opt} \ll \lambda_p \quad ?$

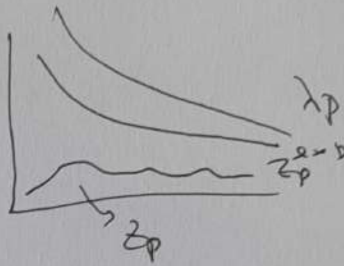
LBW
(sólo el 1° modo)

4

$\lambda_p = 2\pi / \mathrm{Re}\{k_p\}$







$$\Gamma = \frac{2}{\hbar} |\vec{p}|^2 \, \mathrm{Im}\{G_{self}\}$$

$$\frac{\Gamma}{|\vec{p}|^2} = \frac{2}{\hbar} \mathrm{Im}\{G_{self}\}$$

optimal z_p

$$\varepsilon_{\parallel} \to \sigma_{\parallel} \longrightarrow \eta = \frac{i\sigma(\omega)}{D\omega\underline{\underline{\varepsilon}}}$$ ~ env.

$\Gamma_{film} = \Gamma_{sp} + \Gamma_{others}$ 30/11/2022

(13) $\quad (13) - \int_0^1 \dots$

$\Gamma = \Gamma_{sp} + \Gamma_{rad} + \Gamma_{other}$

Γ_{sp}

Γ_{sp} + radiation

Silver $\simeq 1$

$\frac{2}{\hbar} p^* \mathrm{Im}\{E_{ind}\}$