Theoretical Study of a Cylindrical Graphene-based Localized Surface Plasmon Spaser

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Results from a cylinder with an active medium and covered with a graphene monolayer are presented. Using electromagnetically rigorous numerical simulations, we obtained the critical values of the optical gain needed to compensate losses. In the dielectric, non-dispersive case the contribution of the active medium compensates the intrinsic losses of the graphene plasmons while for the dispersive, metallic-like interior, the active medium also has to compensate its intrinsic ohmic losses

Theory

The longitudinal magnetic field H_z (p polarization):

$$H_z^{(1)}(\rho,\varphi) = \sum_{n=-\infty}^{\infty} c_n J_n(k_1\rho) e^{in\varphi} \qquad \rho < R$$

$$H_z^{(2)}(\rho,\varphi) = \sum_{n=-\infty}^{\infty} [A_o i^n J_n(k_2\rho) + a_n H_n^{(1)}(k_2\rho)] e^{in\varphi} \qquad \rho > R$$

For the field H_z, we applied the boundary conditions [1] and we obtained the formulas for a_n, c_n. The denominator of a_n gives the fully-retarded dispersion relation:

$$\mu_2 h_n - \mu_1 j_n + \frac{4\pi i}{c}\frac{\omega}{c} R \mu_1 \mu_2 j_n h_n = 0$$

For the conductivity of the graphene we used the Kubo's formula:

$$\sigma(\omega) = \frac{ie^2\mu_c}{\pi\hbar^2(\omega + i\gamma_c)} + \frac{e^2}{4\hbar}\left[\Theta(\hbar\omega - 2\mu_c) - \frac{i}{\pi}\ln\left|\frac{\hbar\omega + 2\mu_c}{\hbar\omega - 2\mu_c}\right|\right]$$

In the quasi-static approximation (QE), the functions:

$$j_n(k_1R) = \frac{J_n'(k_1R)}{k_1 R J_n(k_1R)} \qquad h_n(k_2R) = \frac{H_n'^{(1)}(k_2R)}{k_2 R H_n^{(1)}(k_2R)}$$

have the limits when x is small:

$$j_n(x) \to n/x^2 \qquad h_n(x) \to -n/x^2$$

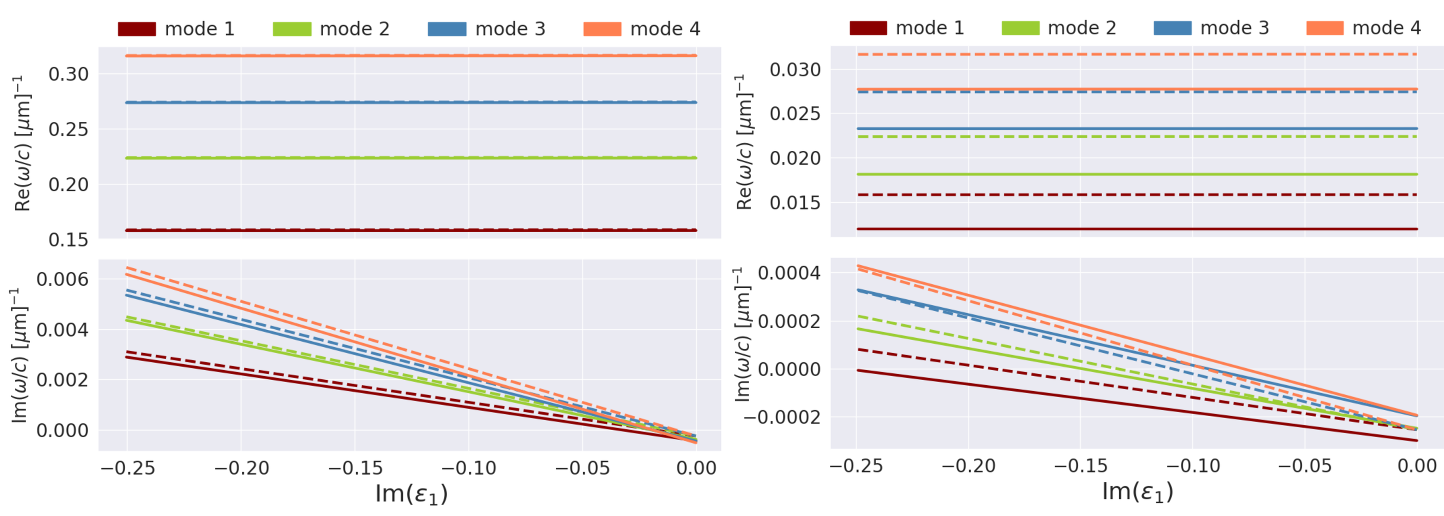
Results: Complex poles and critical values

Non-dispersive medium: an active medium with an electric permeability $\varepsilon_1 = \mathrm{Re}(\varepsilon_1) + \mathrm{iIm}(\varepsilon_1)$

Under the QE approximation, we have an analytical solution:

$$\omega_n = \sqrt{\frac{\omega_{on}^2}{\varepsilon_1 + \varepsilon_2} - \left(\frac{\gamma_c}{2}\right)^2} - i\frac{\gamma_c}{2} \approx \frac{\omega_{on}}{\sqrt{\varepsilon_1 + \varepsilon_2}} - i\frac{\gamma_c}{2} \quad \text{with} \quad \omega_{on}^2 = \frac{4e^2\mu_c n}{\hbar^2 R}$$

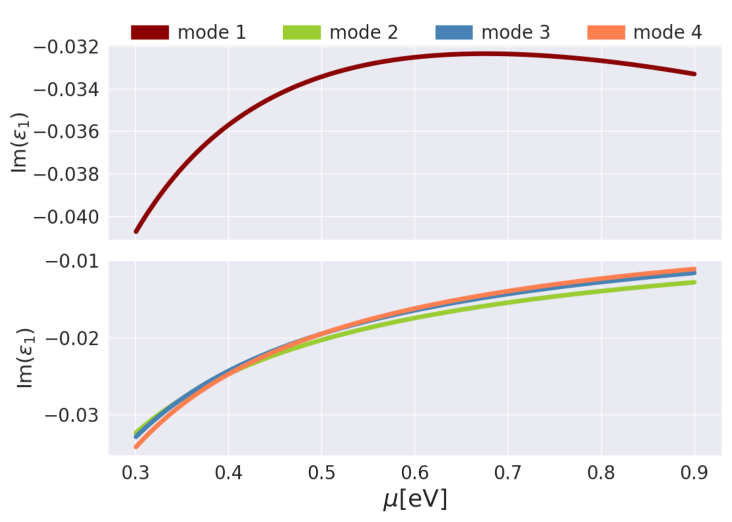
Numerical solutions (line) and QE approximation (dashed line):



R = 0.5 μm　　R = 50 μm

For bigger R values, higher $|\mathrm{Im}(\varepsilon_1)|$ values are required

Critical values:

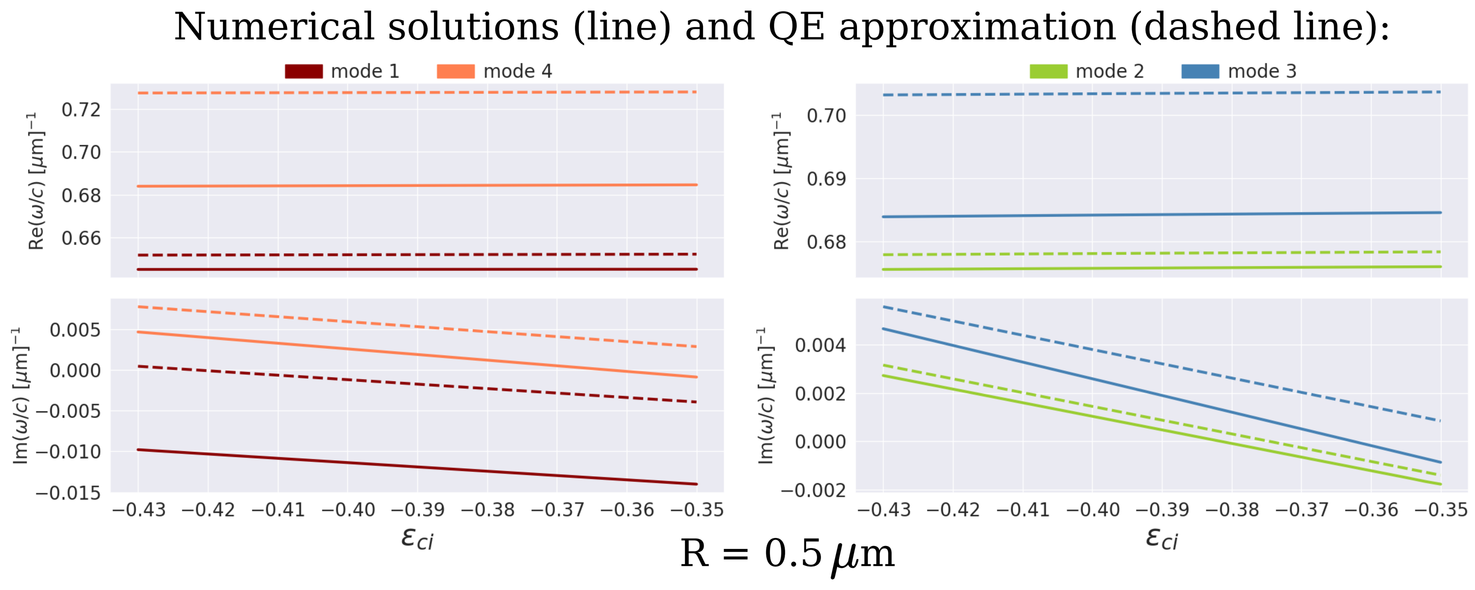


The dipolar mode needs a higher $|\mathrm{Im}(\varepsilon_1)|$ to compensate losses

Dispersive medium: a nanocrystal with electric permeability described by the Drude-Lorentz model, infiltrated by a dye (active medium), with electric permeability $\varepsilon_{cr} + i\varepsilon_{ci}$

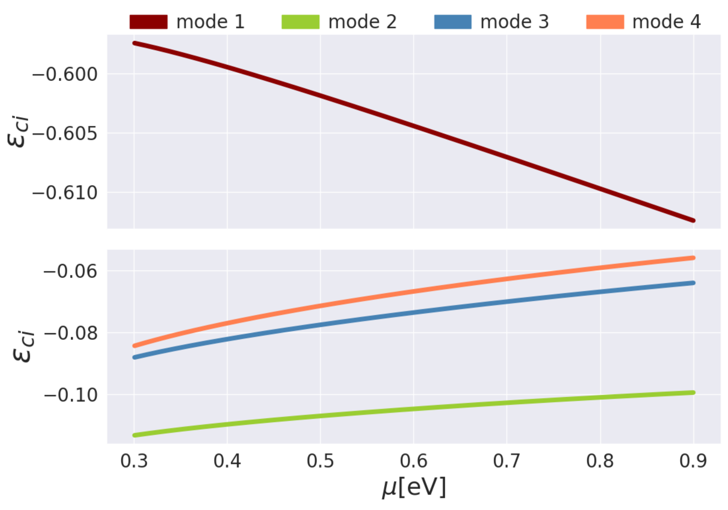
Under the QE approximation, we have an analytical solution: $\omega_{1n} = \sqrt{\dfrac{\omega_p^2 + \omega_{0n}^2}{\varepsilon'_\infty + \varepsilon_2 + i\varepsilon_{ci}}}$ $\qquad \omega_{2n} = \dfrac{\omega_p^2\gamma_m + \omega_{0n}^2\gamma_c}{2(\omega_p^2 + \omega_{0n}^2)}$

Numerical solutions (line) and QE approximation (dashed line):



R = 0.5 μm

Critical values:



In the dispersive medium, the $|\varepsilon_{ci}|$ value increases because the active medium has to compensate not only the losses of the graphene but also the losses of the nanocrystal.

Conclusions

Results from a cylindrical active core coated with a **graphene** monolayer were presented. We solved numerically the **dispersion relation** for different values of the optical gain of the interior medium and we found the critical gain value which minimizes the Im(ω/c). The numerical results obtained were consistent with the QE approximation. We plotted the **critical values** vs the μ_c of the graphene (tunable parameter). We paid particular attention to the **spaser behavior** of the absorption, scattering and extinction cross-sections near the critical values.

The cases studied were:

1) Non-dispersive active medium

2) Dispersive medium: a mix of a nanocrystal and a dye (active medium)

In summary:

- We applied a **rigorous, non retarded method** to study the modal characteristics of localized surface plasmons of a dielectric active wire coated with a graphene monolayer.

- We gave **approximate analytical expressions** for the dependence of resonance frequencies and decay rates with the gain coefficient of the core, the radius of the cylinder and the constitutive parameters of the system.

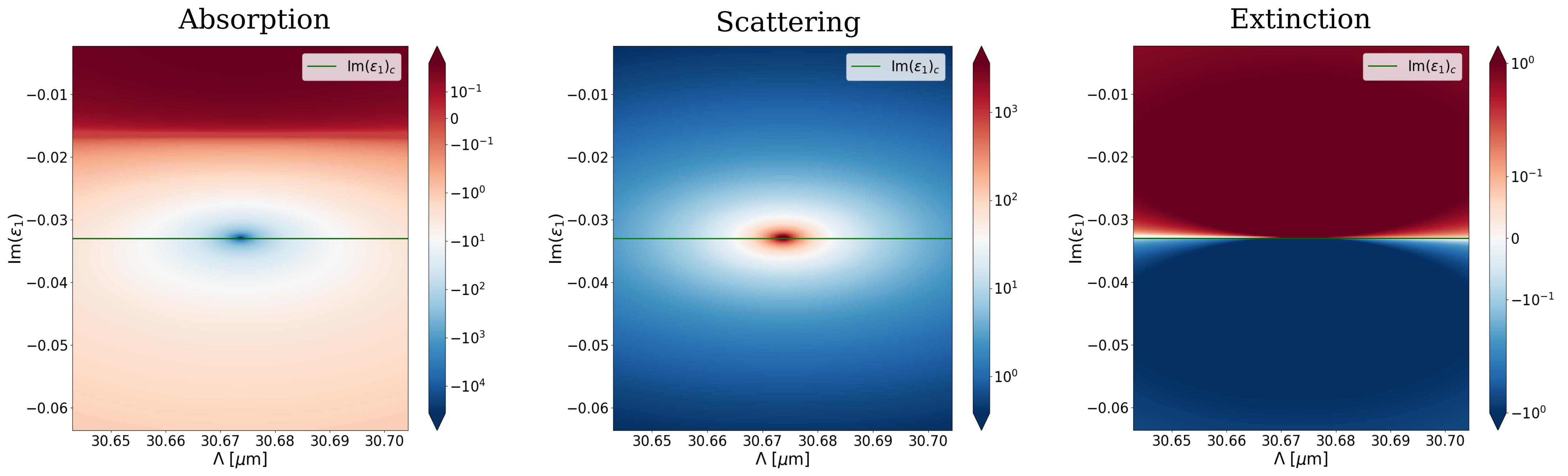
- We illustrated the **non-radiative transfer** between the active medium and the localized surface plasmons of graphene.

- We focused on **spacing conditions** and **tunability**

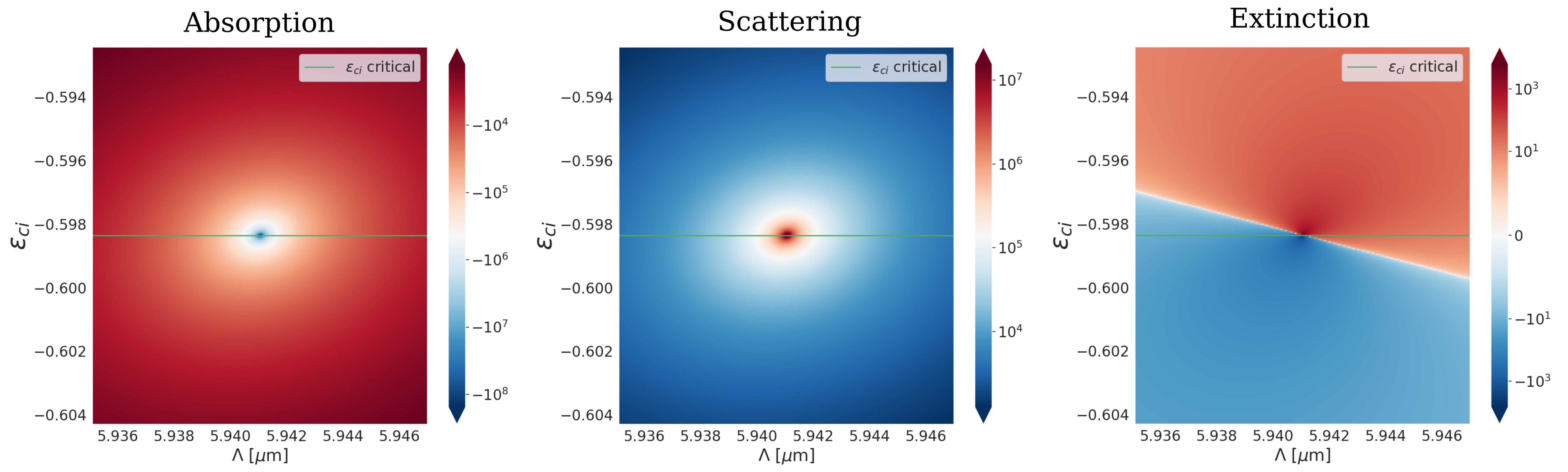
Results: Cross-sections

We plotted the absorption, scattering and extinction cross-sections near the critical values for both cases: non-dispersive medium and dispersive medium. For the critical value (green line) we observed minimization of absorption, maximization of scattering and anulation of extinction (change of sign).

Non-dispersive medium



Dispersive medium



References

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