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Response to reviews and description of changes made to the manuscript.

We thank reviewer 2 for helping us improve the presentation of the paper and ward off potential misconception. We give below a response to his recommendations and criticisms, together with a summary of the changes made in the revised manuscript.

Report of Referee 2:

The paper concerns calculation of lasing condition and lasing frequencies of the system consisting of dielectric or plasmonic core coated by graphene monolayer. It is found the lasing frequencies and critical gain at which total losses in the structure are compensated. It is calculated scattering and absorption cross sections. The paper concerns important topic about nanolasers based on graphene. However, in the present form the paper contains very much technical details without its physical relevance. As a consequence, several points presented below remain unclear.

The main question is the influence of the spontaneous emission on the lasing. The results which the authors obtained, as I suppose, are enough to evaluate β-factor and Purcell factor. They enable to understand the contribution of the spontaneous emission into total emission energy. It is particularly important for the case of core made from plasmonic material because in this case the contribution of spontaneous emission both below and above threshold is dominant, see [Opt. Express 20(14), 15309–15325 (2012)].

Response to the referee:
Our calculations are within what we could call “linear electrodynamics without saturation”. What we are referring to as saturation effects come from the fact that the complete population inversion of the active medium can no longer be sustained close to the gain-loss compensation condition. As shown in the reference suggested by the referee (and others), including saturation effects basically involve solving self-consistent rate equations which typically considers the population of excited and ground states of the active material, the spatial distribution of electromagnetic fields, spontaneous emission (Purcell-enhanced or not), and stimulated emission. The inclusion of saturation has many effects, the main one is that fields do not go to infinite for any finite value of the external pumping mechanism, whatever it is. Despite this, the strategy of using linear electrodynamics without saturation is still useful to correctly predict the mode that will be lasing (or spasing) and the frequency at which this occurs. Moreover, it can estimate the minimum gain required to observe an important enhancement of near and far-fields. See for example Refs. [Arnlod2015, Natarov2019]. The gain-loss compensation condition discussed in our work (referenced in the previous version of the manuscript as "lasing threshold"), does not necessarily mean that most emitted photons are the result of stimulated emission since, as a calculation including saturation shows, spontaneous emission can still be very important (especially if Purcell effects are relevant in the system studied).[khurgin2012] Requiring that most emitted photons come from spontaneous emission is a more strict condition that demands larger gain values than the ones informed in our manuscript. This (probably more distinctive) characteristic of lasing (or spasing) is what many authors typically also call lasing threshold [khurgin2012,khurgin2020].

To avoid confusions:
1) We replaced the term “lasing threshold” with “gain threshold” all along with the manuscript.
2) We modified section 2B. (previously “Lasing thresholds” now “Gain thresholds”) to include the concepts above discussed. In particular,
2.1) we changed the sentence “The interplay between the electromagnetic field profile given by a certain mode, the dynamics of the population inversion, and even the external field is what

ultimately determines the intensity of the electromagnetic fields of optically active systems...”
by
“Including saturation effects basically involve solving self-consistent rate equations which typically consider the population of excited and ground states of the active material, the spatial distribution of electromagnetic fields, spontaneous emission (Purcell-enhanced or not), and the rate of stimulated emission.”
2.2) We removed the sentence
“However, we are still not taking into account saturation effects, and thus the true intensity of electromagnetic fields at the lasing condition is beyond the scope of this work.”
2.3) We added the paragraph
“Despite its limitations, the strategy used in the present work is still useful to correctly predict the mode that will be lasing (or spasing) and the frequency at which this occurs. Moreover, it can help to estimate the minimum gain required to observe an important enhancement of near and far-fields, see for example Refs. \cite{arnold2015,nosich}. The gain-loss compensation condition discussed in our work, also referenced as the gain threshold,\cite{passarelli2019} does not necessarily mean that most emitted photons are the result of stimulated (coherent) emission since, as calculations including saturation show, spontaneous (incoherent) emission can still be very important (especially if Purcell effects are relevant in the system studied). Requiring that most emitted photons come from spontaneous emission is a more strict condition that demands larger gain values than the ones informed here.\cite{khurgin2012,khurgin2021}”

Report of Referee 2:

The second point is the comparison of the full solution of the eigenfrequencies and eigen field distribution with solution obtained from the quasistatic approximation. The authors shortly mentioned that quasistatic approximation works well for the higher multipole modes and is not applicable for the dipole mode due to high radiation losses. It is interesting, how the curves in Figs 2 and 6 change in the quasistatic approximation. In addition, the authors should discuss in what case - dielectric or metallic – quasistatic approximation works better.

Response to the referee:
Following the recommendation of Referee 2, and in order to indicate that the quasistatic approximation works better in the dielectric than in the metallic case, in section 3B we have added the sentence:

“However, the variation in the real part of the eigenfrequency is greater here than in the non-dispersive case (Figure 2), in agreement with the fact that obtaining Eq. (16) involves the use of approximations (in particular expanding in powers of …) that are not invoked to obtain Eq. (12)”.

Report of Referee 2:

- it is necessary to briefly describe the type of active medium for which the obtained value of critical gain can be realized

Response to the referee:
Following the recommendation of Referee 2, we have added the following sentence in the introductory part of section 3:

“Regarding the values of the constitutive parameters of the interior medium, and taking into account that our main purpose is to demonstrate spaser characteristics --such as gain thresholds and tunability-- of graphene-coated active wires but not to reproduce the exact behavior of a given material, in the non-dispersive case we have chosen values that are representative of Si- or Ge-based transparent dielectric materials, whereas in the dispersive case we have chosen constitutive parameter values that can be attained using semiconductor plasmonic nanocrystals [19-21]."

We added a new reference [21], a recent review (2020) comparing properties of plasmonic materials, including examples of semiconductor nanocrystals.

Report of Referee 2:

- at p. 4 on right column the word "approximation" is repeated twice.

Response to the referee:
We thank the reviewer for pointing out this problem which has been corrected in the revised version.

The following references have been added in the new version of the manuscript

21. M. M. L. Wang, M. Hasanzadeh Kafshgari, "Optical properties and applications of plasmonic-metal nanoparticles," Adv. Funct. Mater. **30**, 2005400 (2020).

32. J. B. Khurgin and G. Sun, "Injection pumped single mode surface plasmon generators: threshold, linewidth, and coherence," Opt. Express **20**, 15309–15325 (2012).

33. J. B. Khurgin and M. A. Noginov, "How do the Purcell factor, the Q-factor, and the beta factor affect the laser threshold?" Laser & Photonics Rev. **15**, 2000250 (2021).