Dipolo in- caso con kz

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1 Campo magnetico

Dipolo en el interior de un cilindro de radio R recubierto con grafeno (ver ref. Cuevas [2017] y apunte de Depine):

$$\mathbf{A}(\rho,\theta,z) = \sum_{m=-\infty}^{m=+\infty} \int_{-\infty}^{\infty} \mathrm{d}k_z e^{ik_z(z-z_D)} e^{im(\theta-\theta_D)} J_m(k_{t,1}\rho_<) H_m^{(1)}(k_{t,1}\rho_>) \frac{\omega}{2c}\mathbf{p}, \quad k_{t,1} = \sqrt{(\omega/c)^2\varepsilon_1 - k_z^2} \tag{1}$$

$$\mathbf{A}(\rho,\theta,z) = \sum_{m=-\infty}^{m=+\infty} \int_{-\infty}^{\infty} \mathrm{d}k_z e^{ik_z(z-z_D)} e^{im(\theta-\theta_D)} J_m(k_{t,1}\rho_D) H_m^{(1)}(k_{t,1}\rho) \frac{\omega}{2c}\mathbf{p} \quad \rho > \rho_D,$$

$$\mathbf{A}(\rho,\theta,z) = \sum_{m=-\infty}^{m=+\infty} \int_{-\infty}^{\infty} \mathrm{d}k_z e^{ik_z(z-z_D)} e^{im(\theta-\theta_D)} J_m(k_{t,1}\rho) H_m^{(1)}(k_{t,1}\rho_D) \frac{\omega}{2c}\mathbf{p} \quad \rho < \rho_D.$$

Las ultimas dos ecuaciones tienen sentido dado que Hankel diverge en el cero ($\rho < \rho_D$) y la de Bessel diverge en el infinito ($\rho > \rho_D$). La dependencia temporal del potencial y de los campos es $e^{-i\omega t}$. Con el potencial $\mathbf{A}$ se obtienen los campos (ref. Cuevas [2017]):

$$\mathbf{H}(\rho,\theta,z) = \vec{\nabla} \times \mathbf{A}(\rho,\theta,z) = \sum_{m=-\infty}^{m=+\infty} \int_{-\infty}^{\infty} \mathrm{d}k_z e^{ik_z z} e^{im\theta} [h_{\rho m}(\rho)\hat{\rho} + h_{\theta m}(\rho)\hat{\theta} + h_{zm}(\rho)\hat{z}], \tag{2}$$

$$\mathbf{E}(\rho,\theta,z) = \frac{ic}{\omega\varepsilon_1} \vec{\nabla} \times \mathbf{H}(\rho,\theta,z) = \sum_{m=-\infty}^{m=+\infty} \int_{-\infty}^{\infty} \mathrm{d}k_z e^{ik_z z} e^{im\theta} [e_{\rho m}(\rho)\hat{\rho} + e_{\theta m}(\rho)\hat{\theta} + e_{zm}(\rho)\hat{z}]. \tag{3}$$

Hallar las funciones $h_{\rho m}(\rho)$, $h_{\theta m}(\rho)$, $h_{zm}(\rho)$ para obtener los campos incidentes. Las funciones $e_{\rho m}(\rho)$, $e_{\theta m}(\rho)$, $e_{zm}(\rho)$ se pueden obtener a partir de las funciones h. El rotor en cilindricas es:

$$\vec{\nabla} \times \mathbf{A}(\rho,\theta,z) = \hat{\rho}\left(\frac{1}{\rho}\frac{\partial A_z}{\partial \theta} - \frac{\partial A_\theta}{\partial z}\right) + \hat{\theta}\left(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho}\right) + \hat{z}\left(\frac{1}{\rho}\frac{\partial(\rho A_\theta)}{\partial \rho} - \frac{1}{\rho}\frac{\partial A_\rho}{\partial \theta}\right).$$

Para $\rho > \rho_D$, al derivar respecto de la coordenada radial sólo se deriva la funcion de Bessel J y la función de Hankel se evalúa en ρ_D. Para $\rho < \rho_D$ se realiza lo inverso. Lo importante es que nunca se derivan las dos funciones simultaneamente (no quedan dos terminos al derivar respecto de ρ). Tener en cuenta (ver seccion A):

$$p_x = p_\rho \cos(\theta) - p_\theta \sin(\theta),\; p_y = p_\rho \sin(\theta) + p_\theta \cos(\theta) \tag{4}$$

$$p_\rho = p_x \cos(\theta) + p_y \sin(\theta),\; p_\theta = -p_x \sin(\theta) + p_y \cos(\theta) \tag{5}$$

$$\rightarrow \frac{\partial p_\rho}{\partial \theta} = -p_x \sin(\theta) + p_y \cos(\theta) = p_\theta$$

$$\rightarrow \frac{\partial p_\theta}{\partial \rho} = -\frac{\partial p_x}{\partial \rho} \sin(\theta) + \frac{\partial p_y}{\partial \rho} \cos(\theta) = -\cos(\theta)\sin(\theta) + \sin(\theta)\cos(\theta) = 0$$

Se va a usar en las siguientes subsecciones:

$$\frac{\partial p_\rho}{\partial \theta} = p_\theta \tag{6}$$

$$\frac{\partial p_\theta}{\partial \rho} = 0 \tag{7}$$

Conviene definir p_+, p_- a partir de p_x, p_y:

$$\begin{aligned} &p_+ = p_x + ip_y,\; p_- = p_x - ip_y, \\ &p_x = \frac{p_+ + p-}{2},\; p_y = \frac{p_+ - p-}{2i} = \frac{i(p_- - p+)}{2}. \end{aligned} \tag{8}$$

Se va a usar en las siguientes secciones $p_\theta(p_+, p_-)$, $p_\rho(p_+, p_-)$:

$$p_\theta = -p_x \sin(\theta) + p_y \cos(\theta) = -\frac{(p_+ + p-)}{2} \sin(\theta) + \frac{i(p_- - p+)}{2} \cos(\theta) =$$

$$p_\theta = \frac{p_+}{2} \Big\{ \underbrace{-\sin(\theta) - i\cos(\theta)}_{-ie^{-i\theta}} \Big\} + \frac{p_-}{2} \Big\{ \underbrace{-\sin(\theta) + i\cos(\theta)}_{ie^{i\theta}} \Big\} =$$

$$p_\theta = -\frac{ip_+ e^{-i\theta}}{2} + \frac{ip_- e^{i\theta}}{2} = \frac{\partial p_\rho}{\partial \theta} \tag{9}$$

$$p_\rho = p_x \cos(\theta) + p_y \sin(\theta) = \frac{(p_+ + p-)}{2} \cos(\theta) + \frac{i(p_- - p+)}{2} \sin(\theta) =$$

$$p_\rho = \frac{p_+}{2} \Big\{ \underbrace{\cos(\theta) - i\sin(\theta)}_{e^{-i\theta}} \Big\} + \frac{p_-}{2} \Big\{ \underbrace{\cos(\theta) + i\sin(\theta)}_{e^{i\theta}} \Big\} =$$

$$p_\rho = \frac{p_+ e^{-i\theta}}{2} + \frac{p_- e^{i\theta}}{2} \tag{10}$$

1.1 longitudinal h_{zm}

La coordenada z del rotor para $\rho < \rho_D$ es (se usa $J_m(k_{t,1}\rho)H_m^{(1)}(k_{t,1}\rho_D)$ en la fórmula para $\mathbf{A}$):

$$\mathbf{A}(\rho,\theta,z) = \sum_{m=-\infty}^{m=+\infty}\int_{-\infty}^{\infty} \mathrm{d}k_z e^{ik_z(z-z_D)} e^{im(\theta-\theta_D)} J_m(k_{t,1}\rho)H_m^{(1)}(k_{t,1}\rho_D)\frac{\omega}{2c}\mathbf{p}$$

$$(\vec{\nabla}\times\mathbf{A})_z = \left(\frac{1}{\rho}\frac{\partial(\rho A_\theta)}{\partial\rho} - \frac{1}{\rho}\frac{\partial A_\rho}{\partial\theta}\right) = \frac{1}{\rho}\left(A_\theta + \rho\frac{\partial A_\theta}{\partial\rho} - \frac{\partial A_\rho}{\partial\theta}\right) =$$

$$\frac{1}{\rho}\Bigg\{A_\theta + \underbrace{\rho\,\frac{\omega}{2c}\sum_{m=-\infty}^{m=+\infty}\int_{-\infty}^{\infty}\mathrm{d}k_z e^{ik_z(z-z_D)}e^{im(\theta-\theta_D)}H_m^{(1)}(k_{t,1}\rho_D)\left(J_m'(k_{t,1}\rho)k_{t,1}p_\theta + J_m(k_{t,1}\rho)\overbrace{\cancel{\frac{\partial p_\theta}{\partial\rho}}}^{=0}\right)}_{\partial_\rho A_\theta} +$$

$$\underbrace{-\frac{\omega}{2c}\sum_{m=-\infty}^{m=+\infty}\int_{-\infty}^{\infty}\mathrm{d}k_z e^{ik_z(z-z_D)}e^{im(\theta-\theta_D)}J_m(k_{t,1}\rho)H_m^{(1)}(k_{t,1}\rho_D)\Big(imp_\rho + \underbrace{\frac{\partial p_\rho}{\partial\theta}}_{p_\theta}\Big)}_{\partial_\theta A_\rho}\Bigg\} =$$

En el tercer y cuarto renglón se usaron Eq. 6, 7. Juntando los tres términos:

$$(\vec{\nabla}\times\mathbf{A})_z =$$
$$= \frac{\omega}{2c\rho}\sum_m \int e^{-i(m\theta_D + k_z z_D)}\mathrm{d}k_z e^{ik_z z}e^{im\theta}H_m^{(1)}(k_{t,1}\rho_D)\left[\cancel{p_\theta J_m(k_{t,1}\rho)} - (p_\rho im + \cancel{p_\theta})J_m(k_{t,1}\rho) + p_\theta\, k_{t,1}\rho J_m'(k_{t,1}\rho)\right] \tag{11}$$

Los términos tachados en rojo se cancelan entre si. Conviene usar una relación de recurrencia de la funcion Bessel para un modo n entero (ref. `https://www.math.usm.edu/lambers/mat415/lecture12.pdf` pag 3) para reescribir el último término de la Eq. 11 (el de la derivada J') en función de J:

$$J_n(x) = \pm J_{n\pm1}'(x) + \frac{n\pm1}{x}J_{n\pm1}(x) \to J_{n+1}'(x) = J_n(x) - \left(\frac{n+1}{x}\right)J_{n+1}(x), \tag{12}$$

$$k_{t,1}\rho J_m'(k_{t,1}\rho) = k_{t,1}\rho\left[J_{m-1}(k_{t,1}\rho) - \left(\frac{m}{k_{t,1}\rho}\right)J_m(k_{t,1}\rho)\right] = k_{t,1}\rho J_{m-1}(k_{t,1}\rho) - mJ_m(k_{t,1}\rho) \tag{13}$$

Forma útil de sacarse la derivada de J de encima. Reescribir el integrando de la Eq. 11 usando Eq. 12:

$$p_\theta\left[k_{t,1}\rho J_{m-1} - mJ_m\right] - p_\rho imJ_m$$

Usando $p_\rho = \frac{p_+e^{-i\theta}}{2} + \frac{p_-e^{i\theta}}{2}$, $p_\theta = -\frac{ip_+e^{-i\theta}}{2} + \frac{ip_-e^{i\theta}}{2}$ (Eqs. 10, 9) en la ecuacion anterior:

$$\left(-\frac{ip_+e^{-i\theta}}{2} + \frac{ip_-e^{i\theta}}{2}\right)\left[k_{t,1}\rho J_{m-1} - mJ_m\right] - \left(\frac{p_+e^{-i\theta}}{2} + \frac{p_-e^{i\theta}}{2}\right)imJ_m =$$

$$= \frac{p_+}{2}\left\{-ik_{t,1}\rho J_{m-1}e^{-i\theta} + \cancel{imJ_me^{-i\theta} - imJ_me^{-i\theta}}\right\} + \frac{p_-}{2}\left\{ik_{t,1}\rho J_{m-1}e^{i\theta} - imJ_me^{i\theta} - imJ_me^{i\theta}\right\} =$$

$$= \frac{ik_{t,1}\rho}{2}\Big\{-p_+J_{m-1}e^{-i\theta} + p_-J_{m-1}e^{i\theta}\Big\} - ip_-mJ_me^{i\theta} = -\frac{ip_+}{2}k_{t,1}\rho J_{m-1}e^{-i\theta} + \frac{ip_-}{2}e^{i\theta}\Big[\underbrace{k_{t,1}\rho J_{m-1} - 2mJ_m}_{(*)}\Big]. \tag{14}$$

Para reescribir $(*)$ se usa la siguiente propiedad (ref. `https://www.math.usm.edu/lambers/mat415/lecture12.pdf` pag 3)):

$$2nJ_n(x) = xJ_{n-1}(x) + xJ_{n+1}(x) \rightarrow (*) = k_{t,1}\rho J_{m-1} - 2mJ_m = -k_{t,1}\rho J_{m+1}$$

Reemplazando $(*)$ en Eq. 14:

$$-\frac{ip_+}{2}k_{t,1}\rho J_{m-1}e^{-i\theta} - \frac{ip_-}{2}k_{t,1}\rho J_{m+1}e^{i\theta} = -\frac{ik_{t,1}\rho}{2}\Big\{p_+J_{m-1}e^{-i\theta} + p_-J_{m+1}e^{i\theta}\Big\}$$

Agregar la sumatoria sobre m y los factores que dependen de m para absorber los términos $e^{-i\theta}$, $e^{+i\theta}$ con un cambio de variables:

$$-\sum_m e^{im(\theta-\theta_D)}H_m^{(1)}(k_{t,1}\rho_D)\frac{ik_{t,1}\rho}{2}\Big\{p_+J_{m-1}e^{-i\theta} + p_-J_{m+1}e^{i\theta}\Big\} =$$

$$= -\frac{ik_{t,1}\rho}{2}\sum_m \Big\{\underbrace{p_+e^{i(m-1)\theta}e^{-im\theta_D}H_m^{(1)}J_{m-1}}_{m'=m-1} + \underbrace{p_-e^{i(m+1)\theta}e^{-im\theta_D}H_m^{(1)}J_{m+1}}_{m''=m+1}\Big\} =$$

$$= -\frac{ik_{t,1}\rho}{2}\Big\{\sum_{m'} p_+e^{im'\theta}e^{-i(m'+1)\theta_D}H_{m'+1}J_{m'} + \sum_{m''} p_-e^{im''\theta}e^{-i(m''-1)\theta_D}H_{m''-1}J_{m''}\Big\} =$$

$$= -\frac{ik_{t,1}\rho}{2}\sum_m e^{im(\theta-\theta_D)}\Big\{p_+e^{-i\theta_D}H_{m+1}J_m + p_-e^{i\theta_D}H_{m-1}J_m\Big\} =$$

$$= -\frac{ik_{t,1}\rho}{2}\sum_m e^{im(\theta-\theta_D)}J_m(k_{t,1}\rho)\Big\{p_+e^{-i\theta_D}H_{m+1}(k_{t,1}\rho_D) + p_-e^{i\theta_D}H_{m-1}(k_{t,1}\rho_D)\Big\}$$

Reemplazando lo anterior en Eq. 11:

$$(\vec{\nabla}\times\mathbf{A})_z =$$
$$= -\frac{\omega}{2c\cancel{\rho}}\frac{ik_{t,1}\cancel{\rho}}{2}\sum_m \int \mathrm{d}k_z e^{ik_z(z-z_D)}e^{im(\theta-\theta_D)}J_m\Big\{p_+e^{-im\theta_D}H_{m+1} + p_-e^{im\theta_D}H_{m-1}\Big\}$$

$$\boxed{(\vec{\nabla}\times\mathbf{A})_z = -\frac{i\omega k_{t,1}}{4c}\sum_m \int \mathrm{d}k_z e^{ik_z(z-z_D)}e^{im(\theta-\theta_D)}J_m(k_{t,1}\rho)\Big\{p_+e^{-i\theta_D}H_{m+1}(k_{t,1}\rho_D) + p_-e^{i\theta_D}H_{m-1}(k_{t,1}\rho_D)\Big\}} \quad (15)$$

De la formula anterior se obtiene $h_{zm}(\rho)$ (Eq. 2) del campo $\mathbf{H}_{inc}$ para $\rho < \rho_D$:

$$h_{zm}(\rho) = \frac{i\omega}{4c}k_{t,1}e^{-im\theta_D - ik_zz_D}J_m(k_{t,1}\rho)\Big\{-p_+e^{-i\theta_D}H_{m+1}(k_{t,1}\rho_D) - p_-e^{i\theta_D}H_{m-1}(k_{t,1}\rho_D)\Big\} \qquad \rho < \rho_D.$$

Dado que las propiedades de las funciones son las mismas (ver Eq. 24) se obtuvo la formula de $h_{zm}(\rho)$ para $\rho > \rho_D$ simplemente intercambiando las funciones y sus respectivos argumentos:

$$h_{zm}(\rho) = \frac{i\omega}{4c}k_{t,1}e^{-im\theta_D - ik_zz_D}H_m^{(1)}(k_{t,1}\rho)\Big\{-p_+e^{-i\theta_D}J_{m+1}(k_{t,1}\rho_D) - p_-e^{i\theta_D}J_{m-1}(k_{t,1}\rho_D)\Big\} \qquad \rho > \rho_D.$$

Mismas formulas que el apunte de Depine.

1.2 angular $h_{\theta m}$

La coordenada θ del rotor para $\rho > \rho_D$ es (se usa $J_m(k_{t,1}\rho)H_m^{(1)}(k_{t,1}\rho_D)$ en la fórmula para $\mathbf{A}$ Eq. 1):

$$\mathbf{A}(\rho,\theta,z) = \sum_{m=-\infty}^{m=+\infty}\int_{-\infty}^{\infty} \mathrm{d}k_z e^{ik_z(z-z_D)} e^{im(\theta-\theta_D)} J_m(k_{t,1}\rho)H_m^{(1)}(k_{t,1}\rho_D)\frac{\omega}{2c}\mathbf{p}$$

$$(\vec{\nabla}\times\mathbf{A})_\theta = \left(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho}\right) = \sum_{m=-\infty}^{m=+\infty}\int_{-\infty}^{\infty} \mathrm{d}k_z ik_z\, e^{ik_z(z-z_D)} e^{im(\theta-\theta_D)} J_m(k_{t,1}\rho)H_m^{(1)}(k_{t,1}\rho_D)\frac{\omega}{2c}p_\rho +$$

$$-\underbrace{\sum_{m=-\infty}^{m=+\infty}\int_{-\infty}^{\infty} \mathrm{d}k_z e^{ik_z(z-z_D)} e^{im(\theta-\theta_D)} J'_m(k_{t,1}\rho)H_m^{(1)}(k_{t,1}\rho_D)\frac{\omega k_{t,1}}{2c}p_z}_{\partial_\rho A_z} =$$

$$(\vec{\nabla}\times\mathbf{A})_\theta = \frac{\omega}{2c}\sum_{m=-\infty}^{m=+\infty}\int_{-\infty}^{\infty} e^{-im\theta_D - ik_z z_D} \mathrm{d}k_z e^{ik_z z} e^{im\theta} H_m^{(1)}(k_{t,1}\rho_D)\Big[J'_m(k_{t,1}\rho)k_{t,1}p_z + J_m(k_{t,1}\rho)ik_z p_\rho\Big]. \tag{16}$$

Usar la fórmula de $p_\rho = \dfrac{p_+ e^{-i\theta}}{2} + \dfrac{p_- e^{i\theta}}{2}$ (Eq. 10):

$$J'_m(k_{t,1}\rho)k_{t,1}p_z + J_m(k_{t,1}\rho)ik_z\left\{\frac{p_+}{2}e^{-i\theta} + \frac{p_-}{2}e^{i\theta}\right\}$$

Reemplazando en Eq. 16:

$$(\vec{\nabla}\times\mathbf{A})_\theta = \frac{\omega}{2c}\sum_{m=-\infty}^{m=+\infty}\int_{-\infty}^{\infty} e^{-im\theta_D - ik_z z_D}\, \mathrm{d}k_z e^{ik_z z} e^{im\theta} H_m^{(1)}(k_{t,1}\rho_D)\Big[J'_m(k_{t,1}\rho)k_{t,1}p_z + J_m(k_{t,1}\rho)ik_z\left\{\frac{p_+}{2}e^{-i\theta} + \frac{p_-}{2}e^{i\theta}\right\}\Big] =$$

$$= \frac{\omega}{4c}\sum_{m=-\infty}^{m=+\infty}\int_{-\infty}^{\infty} e^{-im\theta_D - ik_z z_D}\, \mathrm{d}k_z e^{ik_z z} e^{im\theta} H_m^{(1)}(k_{t,1}\rho_D)\Big[2J'_m(k_{t,1}\rho)k_{t,1}p_z + J_m(k_{t,1}\rho)ik_z\left\{p_+e^{-i\theta} + p_-e^{i\theta}\right\}\Big] =$$

$$= \frac{\omega}{4c}\int_{-\infty}^{\infty} \mathrm{d}k_z\, e^{ik_z(z-z_D)}\sum_m\Big\{2k_{t,1}p_z e^{im\theta}e^{-im\theta_D}H_m^{(1)}(k_{t,1}\rho_D)J'_m(k_{t,1}\rho) + ik_z p_+ \underbrace{e^{i(m-1)\theta}e^{-im\theta_D}H_m^{(1)}(k_{t,1}\rho_D)J_m(k_{t,1}\rho)}_{m'=m-1} +$$

$$+ ik_z p_- \underbrace{e^{i(m+1)\theta}e^{-im\theta_D}H_m^{(1)}(k_{t,1}\rho_D)J_m(k_{t,1}\rho)}_{m''=m+1}\Big\} =$$

$$= \frac{\omega}{4c}\int_{-\infty}^{\infty} \mathrm{d}k_z\, e^{ik_z(z-z_D)}\sum_m\Big\{2k_{t,1}p_z e^{im\theta}e^{-im\theta_D}H_m^{(1)}(k_{t,1}\rho_D)J'_m(k_{t,1}\rho) +$$

$$+ ik_z p_+ e^{im'\theta}e^{-i(m'+1)\theta_D}H_{m'+1}^{(1)}(k_{t,1}\rho_D)J_{m'+1}(k_{t,1}\rho) + ik_z p_- e^{im''\theta}e^{-i(m''-1)\theta_D}H_{m''-1}^{(1)}(k_{t,1}\rho_D)J_{m''-1}(k_{t,1}\rho)\Big\} =$$

$$= \frac{\omega}{4c}\int_{-\infty}^{\infty} \mathrm{d}k_z\, e^{ik_z(z-z_D)}\sum_m\Big\{2k_{t,1}p_z e^{im\theta}e^{-im\theta_D}H_m J'_m +$$

$$+ ik_z p_+ e^{im\theta}e^{-i(m+1)\theta_D}H_{m+1}J_{m+1} + ik_z p_- e^{im\theta}e^{-i(m-1)\theta_D}H_{m-1}J_{m-1}\Big\} =$$

$$\boxed{= \frac{\omega}{4c}\int_{-\infty}^{\infty} \mathrm{d}k_z\, e^{ik_z(z-z_D)}\sum_m e^{im(\theta-\theta_D)}\Big\{2k_{t,1}p_z H_m J'_m + ik_z p_+ e^{-i\theta_D}H_{m+1}J_{m+1} + ik_z p_- e^{i\theta_D}H_{m-1}J_{m-1}\Big\}.}$$

Recordar que Hankel esta evaluado en $\rho = \rho_D$ (por eso la derivada que aparece es la de Bessel y no la de Hankel). De la formula anterior se obtiene la de $h_{\theta m}(\rho)$ (Eq. 2) del campo $\mathbf{H}_{inc}$ para $\rho < \rho_D$:

$$h_{\theta m}(\rho) = \frac{i\omega}{4c} e^{-im\theta_D - ik_z z_D} \Big\{ -2ik_{t,1} p_z H_m J_m' + k_z p_+ e^{-i\theta_D} H_{m+1} J_{m+1} + k_z p_- e^{i\theta_D} H_{m-1} J_{m-1} \Big\} \qquad \rho < \rho_D,$$

con $H_m = H_m(k_{t,1}\rho_D)$, $J_m = J_m(k_{t,1}\rho)$. Dado que las propiedades de las funciones son las mismas (ver Eq. 24) se obtuvo la formula de $h_{\theta m}(\rho)$ para $\rho > \rho_D$ simplemente intercambiando las funciones y sus respectivos argumentos:

$$h_{\theta m}(\rho) = \frac{i\omega}{4c} e^{-im\theta_D - ik_z z_D} \Big\{ -2ik_{t,1} p_z H_m' J_m + k_z p_+ e^{-i\theta_D} H_{m+1} J_{m+1} + k_z p_- e^{i\theta_D} H_{m-1} J_{m-1} \Big\} \qquad \rho > \rho_D,$$

con $H_m = H_m(k_{t,1}\rho)$, $J_m = J_m(k_{t,1}\rho_D)$. Se parece a la formula del apunte de Depine salvo por un factor i que no aparece en el primer termino con p_z y por la derivada H_m'.

1.3 radial $h_{\rho m}$

$$\mathbf{A}(\rho,\theta,z) = \sum_{m=-\infty}^{m=+\infty} \int_{-\infty}^{\infty} \mathrm{d}k_z e^{ik_z(z-z_D)} e^{im(\theta-\theta_D)} J_m(k_{t,1}\rho) H_m^{(1)}(k_{t,1}\rho_D) \frac{\omega}{2c}\mathbf{p}$$

$$(\vec{\nabla}\times\mathbf{A})_\rho = \left(\frac{1}{\rho}\frac{\partial A_z}{\partial\theta} - \frac{\partial A_\theta}{\partial z}\right) =$$

$$= \frac{1}{\rho}\frac{\omega}{2c}\sum_m \int_{-\infty}^{\infty} \mathrm{d}k_z e^{ik_z(z-z_D)} e^{im(\theta-\theta_D)} J_m(k_{t,1}\rho) H_m^{(1)}(k_{t,1}\rho_D) \Big\{ imp_z - \rho ik_z p_\theta \Big\}$$

Usar la formula $p_\theta = -\dfrac{ip_+e^{-i\theta}}{2} + \dfrac{ip_-e^{i\theta}}{2}$ (Eq. 9) en la ecuacion anterior: (con el factor i^2 cambia de signo)

$$(\vec{\nabla}\times\mathbf{A})_\rho = \frac{\omega}{4c\rho}\sum_m \int_{-\infty}^{\infty} \mathrm{d}k_z e^{ik_z(z-z_D)} e^{im(\theta-\theta_D)} J_m(k_{t,1}\rho) H_m^{(1)}(k_{t,1}\rho_D) \Big\{ 2imp_z - \rho k_z(p_+e^{-i\theta} - p_-e^{i\theta}) \Big\} =$$

$$= \frac{\omega}{4c\rho}\int_{-\infty}^{\infty} \mathrm{d}k_z\, e^{ik_z(z-z_D)} \sum_m H_m^{(1)}(k_{t,1}\rho_D) J_m(k_{t,1}\rho) \Big\{ 2imp_z e^{im\theta} e^{-im\theta_D} - \rho k_z p_+ \underbrace{e^{i(m-1)\theta} e^{-im\theta_D}}_{m'=m-1} +$$

$$+ \rho k_z p_- \underbrace{e^{i(m+1)\theta} e^{-im\theta_D}}_{m''=m+1} \Big\} =$$

$$= \frac{\omega}{4c\rho}\int_{-\infty}^{\infty} \mathrm{d}k_z\, e^{ik_z(z-z_D)} \sum_m \Big\{ 2imH_m J_m p_z e^{im\theta} e^{-im\theta_D} - \rho k_z p_+ H_{m+1} J_{m+1} e^{im\theta} e^{-i(m+1)\theta_D} +$$

$$+ \rho k_z p_- H_{m-1} J_{m-1} e^{im\theta} e^{-i(m-1)\theta_D} \Big\} =$$

$$= \boxed{\frac{\omega}{4c}\int_{-\infty}^{\infty} \mathrm{d}k_z\, e^{ik_z(z-z_D)} e^{im(\theta-\theta_D)} \sum_m \Big\{ 2imH_m J_m \frac{p_z}{\rho} - k_z p_+ H_{m+1} J_{m+1} e^{-i\theta_D} + k_z p_- H_{m-1} J_{m-1} e^{i\theta_D} \Big\}}$$

De la formula anterior se obtiene la de $h_{\theta m}(\rho)$ (Eq. 2) del campo $\mathbf{H}_{inc}$:

$$h_{\rho m}(\rho) = \frac{i\omega}{4c} e^{-im\theta_D - ik_z z_D} \Big\{ 2m\frac{p_z}{\rho} H_m J_m + ik_z p_+ e^{-i\theta_D} H_{m+1} J_{m+1} - ik_z p_- e^{i\theta_D} H_{m-1} J_{m-1} \Big\}.$$

Con $H_m = H_m(k_{t,1}\rho_D)$, $J_m = J_m(k_{t,1}\rho)$ para $\rho < \rho_D$ y con $H_m = H_m(k_{t,1}\rho)$, $J_m = J_m(k_{t,1}\rho_D)$ para $\rho > \rho_D$.

2 Campo eléctrico

Las formulas del campo electrico deberian coincidir con las del apunte de Depine salvo por un factor $-$ porque ellos usaron la convencion $e^{+i\omega t}$. Ellos usaron $\mathbf{E} = -\frac{ic}{\omega\varepsilon_1}\vec{\nabla}\times\mathbf{H}$ mientras que aca usamos $\mathbf{E} = +\frac{ic}{\omega\varepsilon_1}\vec{\nabla}\times\mathbf{H}$.

Las funciones $e_{\rho m}(\rho)$, $e_{\theta m}(\rho)$, $e_{zm}(\rho)$ se pueden obtener a partir de las funciones h.

$$\frac{ic}{\omega\varepsilon_1}\vec{\nabla}\times\mathbf{H}_{inc}(\rho,\theta,z) = \frac{ic}{\omega\varepsilon_1}\hat{\rho}\left(\frac{1}{\rho}\frac{\partial H_z}{\partial\theta} - \frac{\partial H_\theta}{\partial z}\right) + \frac{ic}{\omega\varepsilon_1}\hat{\theta}\left(\frac{\partial H_\rho}{\partial z} - \frac{\partial H_z}{\partial\rho}\right) + \frac{ic}{\omega\varepsilon_1}\hat{z}\left(\frac{1}{\rho}\frac{\partial(\rho H_\theta)}{\partial\rho} - \frac{1}{\rho}\frac{\partial H_\rho}{\partial\theta}\right) =$$

$$= \frac{ic\hat{\rho}}{\omega\varepsilon_1}\left(\frac{1}{\rho}\text{"}imH_z\text{"} - \text{"}ik_zH_\theta\text{"}\right) + \frac{ic\hat{\theta}}{\omega\varepsilon_1}\left(\text{"}ik_zH_\rho\text{"} - \frac{\partial h_{zm}(\rho)}{\partial\rho}\right) + \frac{ic\hat{z}}{\omega\varepsilon_1}\left(\frac{H_\theta}{\rho} + \frac{\partial H_\theta}{\partial\rho} - \frac{1}{\rho}\frac{\partial H_\rho}{\partial\theta}\right)$$

Parte radial:

$$e_{\rho m}(\rho) = \frac{ic}{\omega\varepsilon_1}\left[\frac{imh_{zm}(\rho)}{\rho} - ik_zh_{\theta m}(\rho)\right]. \tag{17}$$

Parte angular:

$$e_{\theta m}(\rho) = \frac{ic}{\omega\varepsilon_1}\left[ik_zh_{\rho m}(\rho) - \frac{\partial h_{zm}(\rho)}{\partial\rho}\right]. \tag{18}$$

Parte longitudinal:

$$e_{zm}(\rho) = \frac{ic}{\omega\varepsilon_1}\left[\frac{h_{\theta m}(\rho)}{\rho} + \frac{\partial h_{\theta m}(\rho)}{\partial\rho} - \frac{im}{\rho}h_{\rho m}(\rho)\right]. \tag{19}$$

2.1 longitudinal e_{zm}

Recordar que $H_m = H_m(k_{t,1}\rho_D)$, $J_m = J_m(k_{t,1}\rho)$ (se deriva J nada mas):

$$e_{zm}(\rho) = \frac{ic}{\omega\varepsilon_1}\left[\frac{h_{\theta m}(\rho)}{\rho} + \frac{\partial h_{\theta m}(\rho)}{\partial\rho} - \frac{im}{\rho}h_{\rho m}(\rho)\right],$$

- $h_{\theta m}(\rho) = \frac{i\omega}{4c}e^{-im\theta_D - ik_zz_D}\Big\{-2ik_{t,1}p_zH_mJ'_m + k_zp_+e^{-i\theta_D}H_{m+1}J_{m+1} + k_zp_-e^{i\theta_D}H_{m-1}J_{m-1}\Big\},$

- $h_{\rho m}(\rho) = \frac{i\omega}{4c}e^{-im\theta_D - ik_zz_D}\Big\{2m\frac{p_z}{\rho}H_mJ_m + ik_zp_+e^{-i\theta_D}H_{m+1}J_{m+1} - ik_zp_-e^{i\theta_D}H_{m-1}J_{m-1}\Big\}$

$$e_{zm}(\rho) = \frac{ic}{\omega\varepsilon_1}\frac{i\omega}{4c}\Bigg[-2ik_{t,1}p_zH_m\left(\frac{J'_m}{\rho} + k_{t,1}J''_m\right) + k_zp_+e^{-i\theta_D}H_{m+1}\left(\frac{J_{m+1}}{\rho} + k_{t,1}J'_{m+1}\right) + k_zp_-e^{i\theta}H_{m-1}\left(\frac{J_{m-1}}{\rho} + k_{t,1}J'_{m-1}\right) +$$

$$-\frac{2im^2p_z}{\rho^2}H_mJ_m - i^2m\frac{k_z}{\rho}p_+e^{-i\theta_D}H_{m+1}J_{m+1} + i^2m\frac{k_z}{\rho}p_-e^{i\theta_D}H_{m-1}J_{m-1}\Bigg]e^{-im\theta_D - ik_zz_D}$$

Juntar los terminos con p_z, p_+, p_-:

$$e_{zm}(\rho) = \frac{ic}{\omega\varepsilon_1}\frac{i\omega}{4c}\Bigg[-2ip_z H_m\Bigg(\underbrace{\frac{J'_m k_{t,1}}{\rho} + k_{t,1}^2 J''_m + \frac{m^2 J_m}{\rho^2}}_{(1)}\Bigg) + k_z p_+ e^{-i\theta_D} H_{m+1}\Bigg(\underbrace{\frac{J_{m+1}}{\rho} + k_{t,1}J'_{m+1} + \frac{mJ_{m+1}}{\rho}}_{(2)}\Bigg) +$$

$$+ k_z p_- e^{i\theta_D} H_{m-1}\Bigg(\underbrace{\frac{J_{m-1}}{\rho} + k_{t,1}J'_{m-1} - \frac{mJ_{m-1}}{\rho}}_{(3)}\Bigg)\Bigg] e^{-im\theta_D - ik_z z_D}$$

Simplificar el (2) y (3):

$$(2) = \frac{J_{m+1}}{\rho} + k_{t,1}J'_{m+1} + \frac{mJ_{m+1}}{\rho} = \frac{1}{\rho}\left[J_{m+1} + k_{t,1}\rho J'_{m+1} + mJ_{m+1}\right] = \frac{1}{\rho}\Bigg[\cancel{J_{m+1}} + \underbrace{k_{t,1}\rho J_m \cancel{-(m} \cancel{+1)J_{m+1}}}_{k_{t,1}\rho J'_{m+1}} \cancel{+mJ_{m+1}}\Bigg] = k_{t,1}J_m$$

$$(3) = \frac{J_{m-1}}{\rho} + k_{t,1}J'_{m-1} - \frac{mJ_{m-1}}{\rho} = \frac{1}{\rho}\left[J_{m-1} + k_{t,1}\rho J'_{m-1} - mJ_{m-1}\right] = \frac{1}{\rho}\Bigg[\cancel{J_{m-1}} \underbrace{-k_{t,1}\rho J_m \cancel{+(m} \cancel{-1)J_{m-1}}}_{k_{t,1}\rho J'_{m-1}} \cancel{-mJ_{m-1}}\Bigg] = -k_{t,1}J_m$$

Para simplificar (1) primero hay que reescribir J''_m y J'_m en terminos de J_{m+1}, J_m:

$$J''_m(z) = [J'_m(z)]' = \left[-J_{m+1}(z) + \frac{m}{z}J_m(z)\right]' = -J'_{m+1}(z) - \frac{m}{z^2}J_m(z) + \frac{m}{z}J'_m(z) =$$

$$= -\left[J_m(z) - \left(\frac{m+1}{z}\right)J_{m+1}(z)\right] - \frac{m}{z^2}J_m(z) + \frac{m}{z}\left[-J_{m+1}(z) + \frac{m}{z}J_m(z)\right] =$$

$$= J_m(z)\left\{-1 \cancel{-\frac{m^2}{z^2}} + \cancel{\frac{m^2}{z^2}}\right\} + J_{m+1}(z)\left\{\frac{m+1}{z} - \frac{m}{z}\right\} = -J_m(z) + \frac{1}{z}J_{m+1}(z)$$

Reemplazando en (1):

$$(1) = \frac{J'_m k_{t,1}}{\rho} + k_{t,1}^2 J''_m + \frac{m^2 J_m}{\rho^2} = \frac{k_{t,1}}{\rho}\Big[\underbrace{\cancel{-J_{m+1}} + \frac{m}{k_{t,1}\rho}J_m}_{J'_m}\Big] + k_{t,1}^2\Big[\underbrace{-J_m + \cancel{\frac{1}{k_{t,1}\rho}J_{m+1}}}_{J''_m}\Big] + \frac{m^2 J_m}{\rho^2} = J_m\left\{\frac{m(m+1)}{\rho^2} - k_{t,1}^2\right\}$$

Reemplazando en $e_{zm}(\rho)$ y sacando factor comun J_m:

$$e_{zm}(\rho) = \frac{ic}{\omega\varepsilon_1}\frac{i\omega}{4c}J_m\left[-2ip_z H_m\left\{\frac{m(m+1)}{\rho^2} - k_{t,1}^2\right\} + k_z k_{t,1} p_+ e^{-i\theta_D} H_{m+1} - k_z k_{t,1} p_- e^{i\theta_D} H_{m-1}\right] e^{-im\theta_D - ik_z z_D}$$

$$e_{zm}(\rho) = -\frac{1}{4\varepsilon_1}J_m\left[-2ip_z H_m\left\{\frac{m(m+1)}{\rho^2} - k_{t,1}^2\right\} + k_z k_{t,1} p_+ e^{-i\theta_D} H_{m+1} - k_z k_{t,1} p_- e^{i\theta_D} H_{m-1}\right] e^{-im\theta_D - ik_z z_D}$$

$$e_{zm}(\rho) = \frac{k_{t,1}}{4\varepsilon_1}e^{-im\theta_D - ik_z z_D} J_m(k_{t,1}\rho)\left[2ip_z H_m(k_{t,1}\rho_D)\left\{\frac{m(m+1)}{\rho^2 k_{t,1}} - k_{t,1}\right\} - k_z p_+ e^{-i\theta_D} H_{m+1}(k_{t,1}\rho_D) + k_z p_- e^{i\theta_D} H_{m-1}(k_{t,1}\rho_D)\right]$$

$$e_{zm}(\rho) = \frac{k_{t,1}}{4\varepsilon_1}e^{-im\theta_D - ik_z z_D} H_m(k_{t,1}\rho)\left[2ip_z J_m(k_{t,1}\rho_D)\left\{\frac{m(m+1)}{\rho^2 k_{t,1}} - k_{t,1}\right\} - k_z p_+ e^{-i\theta_D} J_{m+1}(k_{t,1}\rho_D) + k_z p_- e^{i\theta_D} J_{m-1}(k_{t,1}\rho_D)\right]$$

para $\rho < \rho_D$ y $\rho > \rho_D$, respectivamente. Diferencia con la formula del apunte de Depine: aca sobra el $\frac{m(m+1)}{\rho^2}$. Recordar que tiene que haber un signo $-$ de diferencia con la formula de Depine.

2.2 angular $e_{\theta m}$

Recordar que $H_m = H_m(k_{t,1}\rho_D)$, $J_m = J_m(k_{t,1}\rho)$ (se deriva J nada mas):

$$e_{\theta m}(\rho) = \frac{ic}{\omega\varepsilon_1}\left[ik_z h_{\rho m}(\rho) - \frac{\partial h_{zm}(\rho)}{\partial\rho}\right],$$

- $h_{\rho m}(\rho) = \frac{i\omega}{4c}e^{-im\theta_D - ik_z z_D}\Big\{2m\frac{p_z}{\rho}H_m J_m + ik_z p_+ e^{-i\theta_D}H_{m+1}J_{m+1} - ik_z p_- e^{i\theta_D}H_{m-1}J_{m-1}\Big\},$

- $h_{zm}(\rho) = \frac{i\omega}{4c}e^{-im\theta_D - ik_z z_D}J_m\Big\{-p_+ e^{-i\theta_D}H_{m+1} - p_- e^{i\theta_D}H_{m-1}\Big\},$

$$e_{\theta m}(\rho) = \frac{ic}{\omega\varepsilon_1}\frac{i\omega}{4c}e^{-im\theta_D - ik_z z_D}\Big\{2mp_z\frac{ik_z}{\rho}H_m J_m + p_+ e^{-i\theta_D}\left(-k_z^2 H_{m+1}J_{m+1} + k_{t,1}J'_m H_{m+1}\right) +$$

$$+p_- e^{i\theta_D}\left(k_z^2 H_{m-1}J_{m-1} + k_{t,1}J'_m H_{m-1}\right)\Big\} =$$

$$e_{\theta m}(\rho) = \frac{ic}{\omega\varepsilon_1}\frac{i\omega}{4c}e^{-im\theta_D - ik_z z_D}\Big\{2mp_z\frac{ik_z}{\rho}H_m J_m + p_+ e^{-i\theta_D}H_{m+1}\left(-k_z^2 J_{m+1} + k_{t,1}J'_m\right) + p_- e^{i\theta_D}H_{m-1}\left(k_z^2 J_{m-1} + k_{t,1}J'_m\right)\Big\}$$

no se parece a la formula del apunte de Depine

3 Campos totales

Se agregan los campos del dipolo en los campos longitudinales del medio 1 (principio de superposicion) porque $\rho_D < R$ (el dipolo está en el interior del cilindro):

$$E_z^{(1)}(\rho,\phi,z,t) = \sum_{n=-\infty}^{\infty}[A_n^{(1)}J_n(k_{t,1}\rho) + e_{zm}(\rho)]e^{in\phi}e^{i(k_z z-\omega t)} \qquad \rho < R$$

$$E_z^{(2)}(\rho,\phi,z,t) = \sum_{n=-\infty}^{\infty}[B_o i^n J_n(k_{t,2}\rho) + B_n^{(2)}H_n^{(1)}(k_{t,2}\rho)]e^{in\phi}e^{i(k_z z-\omega t)} \qquad \rho > R$$

$$H_z^{(1)}(\rho,\phi,z,t) = \sum_{n=-\infty}^{\infty}[C_n^{(1)}J_n(k_{t,1}\rho) + h_{zm}(\rho)]e^{in\phi}e^{i(k_z z-\omega t)} \qquad \rho < R$$

$$H_z^{(2)}(\rho,\phi,z,t) = \sum_{n=-\infty}^{\infty}[A_o i^n J_n(k_{t,2}\rho) + D_n^{(2)}H_n^{(1)}(k_{t,2}\rho)]e^{in\phi}e^{i(k_z z-\omega t)} \qquad \rho > R$$

Se deduce de lo anterior que las unidades de los campos del dipolo son las mismas unidades que la de los coeficientes. Los campos del dipolo $e_{zm}(\rho)$ y $h_{zm}(\rho)$ son diferentes para $\rho > \rho_D$ y $\rho < \rho_D$ (siendo ρ_D la posicion del dipolo). Se utilizan las siguientes formulas para obtener los campos transversales:

$$E_\rho = \frac{1}{k_t^2}\left[\frac{i\omega\mu}{c\rho}\frac{\partial H_z}{\partial\phi} + ik_z\frac{\partial E_z}{\partial\rho}\right] \qquad E_\phi = \frac{1}{k_t^2}\left[\frac{ik_z}{\rho}\frac{\partial E_z}{\partial\phi} - \frac{i\omega\mu}{c}\frac{\partial H_z}{\partial\rho}\right]$$

$$H_\rho = \frac{1}{k_t^2}\left[-\frac{i\omega\varepsilon}{c}\frac{1}{\rho}\frac{\partial E_z}{\partial\phi} + ik_z\frac{\partial H_z}{\partial\rho}\right] \qquad H_\phi = \frac{1}{k_t^2}\left[\frac{ik_z}{\rho}\frac{\partial H_z}{\partial\phi} + \frac{i\omega\varepsilon}{c}\frac{\partial E_z}{\partial\rho}\right]$$

Para el medio 1, se reemplazan por las formulas $E_z^{(1)}, H_z^{(1)}$ en las formulas anteriores para obtener las formulas de los campos transversales. Para las derivadas respecto de $\phi : \partial_\phi \to in$. En los términos con ∂_ϕ ya hay un factor i entonces se simplifica $i^2 = -1$, es decir que el signo cambia. En las derivadas respecto de la coordenada radial ∂_ρ aparece el factor $k_{t,1}$ ($k_{t,1} = \sqrt{k_1^2 - k_z^2}$):

$$E_\rho^{(1)} = \frac{1}{k_{t,1}^2} \sum_n \left[-\frac{\omega\mu_1 n}{c\rho} \left[C_n^{(1)} J_n(k_{t,1}\rho) + h_{zn}(\rho) \right] + ik_z \left[k_{t,1} A_n^{(1)} J_n'(k_{t,1}\rho) + e_{zn}'(\rho) \right] \right] e^{in\phi} e^{i(k_z z - \omega t)}$$

$$E_\phi^{(1)} = \frac{1}{k_{t,1}^2} \sum_n \left[-\frac{k_z n}{\rho} \left[A_n^{(1)} J_n(k_{t,1}\rho) + e_{zn}(\rho) \right] - \frac{i\omega\mu_1}{c} \left[k_{t,1} C_n^{(1)} J_n'(k_{t,1}\rho) + h_{zn}'(\rho) \right] \right] e^{in\phi} e^{i(k_z z - \omega t)}$$

$$H_\rho^{(1)} = \frac{1}{k_{t,1}^2} \sum_n \left[\frac{\omega\varepsilon_1 n}{c\rho} \left[A_n^{(1)} J_n(k_{t,1}\rho) + e_{zn}(\rho) \right] + ik_z \left[k_{t,1} C_n^{(1)} J_n'(k_{t,1}\rho) + h_{zn}'(\rho) \right] \right] e^{in\phi} e^{i(k_z z - \omega t)}$$

$$H_\phi^{(1)} = \frac{1}{k_{t,1}^2} \sum_n \left[-\frac{k_z n}{\rho} \left[C_n^{(1)} J_n(k_{t,1}\rho) + h_{zn}(\rho) \right] + \frac{i\omega\varepsilon_1}{c} \left[k_{t,1} A_n^{(1)} J_n'(k_{t,1}\rho) + e_{zn}'(\rho) \right] \right] e^{in\phi} e^{i(k_z z - \omega t)}$$

Para el medio 2, se reemplazan por las formulas $E_z^{(2)}, H_z^{(2)}$ y el razonamiento sobre las derivadas que se menciono en el medio 1 se repiten:

$$E_\rho^{(2)} = \frac{1}{k_{t,2}} \sum_n \left[-\frac{\omega\mu_2 n}{c\rho k_{t,2}} (A_o i^n J_n(k_{t,2}\rho) + D_n^{(2)} H_n^{(1)}(k_{t,2}\rho)) + ik_z (B_o i^n J_n'(k_{t,2}\rho) + B_n^{(2)} H_n'^{(1)}(k_{t,2}\rho)) \right] e^{in\phi} e^{i(k_z z - \omega t)}$$

$$E_\phi^{(2)} = \frac{1}{k_{t,2}} \sum_n \left[-\frac{k_z n}{\rho k_{t,2}} (B_o i^n J_n(k_{t,2}\rho) + B_n^{(2)} H_n^{(1)}(k_{t,2}\rho)) - \frac{i\omega\mu_2}{c} (A_o i^n J_n'(k_{t,2}\rho) + D_n^{(2)} H_n'^{(1)}(k_{t,2}\rho)) \right] e^{in\phi} e^{i(k_z z - \omega t)}$$

$$H_\rho^{(2)} = \frac{1}{k_{t,2}} \sum_n \left[\frac{\omega\varepsilon_2 n}{c\rho k_{t,2}} (B_o i^n J_n(k_{t,2}\rho) + B_n^{(2)} H_n^{(1)}(k_{t,2}\rho)) + ik_z (A_o i^n J_n'(k_{t,2}\rho) + D_n^{(2)} H_n'^{(1)}(k_{t,2}\rho)) \right] e^{in\phi} e^{i(k_z z - \omega t)}$$

$$H_\phi^{(2)} = \frac{1}{k_{t,2}} \sum_n \left[-\frac{k_z n}{\rho k_{t,2}} (A_o i^n J_n(k_{t,2}\rho) + D_n^{(2)} H_n^{(1)}(k_{t,2}\rho)) + \frac{i\omega\varepsilon_2}{c} (B_o i^n J_n'(k_{t,2}\rho) + B_n^{(2)} H_n'^{(1)}(k_{t,2}\rho)) \right] e^{in\phi} e^{i(k_z z - \omega t)}$$

Las formulas del medio 2 no cambiaron porque el dipolo esta en el interior del cilindro.

4 Condiciones de contorno

Las condiciones de borde ($\rho = R$) para los campos son:

$$E_z^{(1)}(\rho = R, \phi, z) = E_z^{(2)}(\rho = R, \phi, z), \tag{20}$$

$$E_\phi^{(1)}(\rho = R, \phi, z) = E_\phi^{(2)}(\rho = R, \phi, z), \tag{21}$$

$$H_z^{(1)}(\rho = R, \phi, z) - \frac{4\pi}{c}\sigma(\omega) E_\phi(\rho = R, \phi, z) = H_z^{(2)}(\rho = R, \phi, z), \tag{22}$$

$$H_\phi^{(1)}(\rho = R, \phi, z) + \frac{4\pi}{c}\sigma(\omega) E_z(\rho = R, \phi, z) = H_\phi^{(2)}(\rho = R, \phi, z). \tag{23}$$

Observemos que los campos E_z y E_ϕ son continuos en $\rho = R$, por lo tanto podemos elegir el campo $E_\phi^{(1)}$ o el campo $E_\phi^{(2)}$ en la tercera ecuación. Lo mismo para la cuarta ecuación: podemos elegir el campo $E_z^{(1)}$ o el campo $E_z^{(2)}$. De la Eq. 20:

$$E_z^{(1)}(\rho = R, \phi, z) = E_z^{(2)}(\rho = R, \phi, z) \to A_n^{(1)} J_n(k_{t,1}R) + e_{zn}(R) = B_o i^n J_n(k_{t,2}R) + B_n^{(2)} H_n^{(1)}(k_{t,2}R)$$

La ultima condición de borde Eq. 23: (vamos a ordenar las condiciones segun cómo se arme la matriz)

$$H_\phi^{(1)}(\rho=R,\phi,z)+\frac{4\pi}{c}\sigma(\omega)E_z^{(1)}(\rho=R,\phi,z)=H_\phi^{(2)}(\rho=R,\phi,z)$$
$$\frac{1}{k_{t,1}^2}\left[-\frac{k_z n}{R}\left[C_n^{(1)}J_n(k_{t,1}R)+h_{zn}(R)\right]+\frac{i\omega\varepsilon_1}{c}\left[k_{t,1}A_n^{(1)}J_n'(k_{t,1}R)+e_{zn}'(R)\right]\right]+\frac{4\pi\sigma}{c}\left[A_n^{(1)}J_n(k_{t,1}R)+e_{zn}(R)\right]=$$
$$\frac{1}{k_{t,2}}\left[-\frac{k_z n}{Rk_{t,2}}(A_o i^n J_n(k_{t,2}R)+D_n^{(2)}H_n^{(1)}(k_{t,2}R))+\frac{i\omega\varepsilon_2}{c}(B_o i^n J_n'(k_{t,2}R)+B_n^{(2)}H_n'^{(1)}(k_{t,2}R))\right]$$

$$A_n^{(1)}\Big(\frac{4\pi\sigma}{c}J_{1n}+\frac{i\omega\varepsilon_1}{ck_{t,1}}J_{1n}'\Big)-\frac{i\omega\varepsilon_2}{ck_{t,2}}B_n^{(2)}H_{2n}'-C_n^{(1)}\frac{k_z n}{Rk_{t,1}^2}J_{1n}+\frac{k_z n}{Rk_{t,2}^2}D_n^{(2)}H_{2n}=$$
$$=-\frac{k_z n}{Rk_{t,2}^2}A_o i^n J_{2n}+\frac{i\omega\varepsilon_2}{ck_{t,2}}B_o i^n J_{2n}'-\frac{4\pi\sigma}{c}e_{zn}(R)+\frac{k_z n}{Rk_{t,1}^2}h_{zn}(R)-\frac{i\omega\varepsilon_1}{ck_{t,1}^2}e_{zn}'(R)$$

$$\boxed{\begin{aligned}&A_n^{(1)}\Big(\frac{4\pi\sigma}{c}J_{1n}+\frac{i\omega\varepsilon_1}{ck_{t,1}}J_{1n}'\Big)-\frac{i\omega\varepsilon_2}{ck_{t,2}}B_n^{(2)}H_{2n}'-C_n^{(1)}\frac{k_z n}{Rk_{t,1}^2}J_{1n}+\frac{k_z n}{Rk_{t,2}^2}D_n^{(2)}H_{2n}=\\&=\frac{k_z n}{R}\left[\frac{h_{zn}(R)}{k_{t,1}^2}-\frac{A_o i^n}{k_{t,2}^2}J_2\right]+\frac{i\omega}{c}\left[\frac{\varepsilon_2 B_o i^n}{k_{t,2}}J_2'-\frac{\varepsilon_1}{k_{t,1}^2}e_{zn}'(R)\right]-\frac{4\pi\sigma}{c}e_{zn}(R)\end{aligned}}$$

La tercerca condicion de contorno Eq. 22 es:

$$H_z^{(1)}(\rho=R,\phi,z)-\frac{4\pi}{c}\sigma(\omega)E_\phi^{(1)}(\rho=R,\phi,z)=H_z^{(2)}(\rho=R,\phi,z):$$
$$C_n^{(1)}J_n(k_{t,1}R)+h_{zn}(R)-\frac{4\pi\sigma}{c}\cdot\frac{1}{k_{t,1}^2}\left\{-\frac{k_z n}{R}\left[A_n^{(1)}J_n(k_{t,1}R)+e_{zm}(R)\right]-\frac{i\omega\mu_1}{c}\left[k_{t,1}C_n^{(1)}J_n'(k_{t,1}R)+h_{zn}'(R)\right]\right\}=$$
$$[A_o i^n J_n(k_{t,2}R)+D_n^{(2)}H_n^{(1)}(k_{t,2}R)]$$

$$\frac{4\pi\sigma}{c}\frac{k_z n}{Rk_{t,1}^2}A_n^{(1)}J_{1n}+C_n^{(1)}\Big(J_{1n}+\frac{4\pi\sigma}{c}\frac{i\omega\mu_1}{ck_{t,1}}J_{1n}'\Big)-D_n^{(2)}H_{2n}=$$
$$=A_o i^n J_{2n}-h_{zn}(R)-\frac{4\pi\sigma}{c}\frac{k_z n}{Rk_{t,1}^2}e_{zn}(R)-\frac{4\pi\sigma}{c}\frac{i\omega\mu_1}{ck_{t,1}^2}h_{zn}'(R)$$

$$\frac{4\pi\sigma}{c}\frac{k_z n}{Rk_{t,1}^2}A_n^{(1)}J_{1n}+C_n^{(1)}\Big(J_{1n}+\frac{4\pi\sigma}{c}\frac{i\omega\mu_1}{ck_{t,1}}J_{1n}'\Big)-D_n^{(2)}H_{2n}=A_o i^n J_{2n}-h_{zn}(R)-\frac{4\pi\sigma}{c}\left[\frac{k_z n}{Rk_{t,1}^2}e_{zn}(R)+\frac{i\omega\mu_1}{ck_{t,1}^2}h_{zn}'(R)\right]$$

Se repite lo mismo para la segunda condición de borde (Eq. 21):

$$E_\phi^{(1)}(\rho=R,\phi,z)=E_\phi^{(2)}(\rho=R,\phi,z):$$
$$\frac{1}{k_{t,1}^2}\left[-\frac{k_z n}{R}\left[A_n^{(1)}J_n(k_{t,1}R)+e_{zn}(R)\right]-\frac{i\omega\mu_1}{c}\left[k_{t,1}C_n^{(1)}J_n'(k_{t,1}R)+h_{zn}'(R)\right]\right]=$$
$$\frac{1}{k_{t,2}}\left[-\frac{k_z n}{Rk_{t,2}}[B_o i^n J_n(k_{t,2}R)+B_n^{(2)}H_n^{(1)}(k_{t,2}R)]-\frac{i\omega\mu_2}{c}[A_o i^n J_n'(k_{t,2}R)+D_n^{(2)}H_n'^{(1)}(k_{t,2}R)]\right]$$

$$-\frac{k_z n}{Rk_{t,1}^2}A_n^{(1)}J_{1n}-\frac{i\omega\mu_1}{ck_{t,1}}C_n^{(1)}J_{1n}'+\frac{k_z n}{Rk_{t,2}^2}B_n^{(2)}H_{2n}+\frac{i\omega\mu_2}{ck_{t,2}}D_n^{(2)}H_{2n}'=$$
$$=-\frac{i\omega\mu_2}{ck_{t,2}}A_o i^n J_{2n}'-\frac{k_z n}{Rk_{t,2}^2}B_o i^n J_{2n}+\frac{k_z n}{Rk_{t,1}^2}e_{zn}(R)+\frac{i\omega\mu_1}{ck_{t,1}^2}h_{zn}'(R)=$$

Con $J_{1n} \equiv J_n(k_{t,1}R)$, $H^{(1)}_{2n} \equiv H_n(k_{t,2}R)$ y los terminos que van con A_o, B_o tienen $J_{2n} \equiv J_n(k_{t,2}R)$. La segunda condicion de contorno es:

$$-\frac{k_z n}{Rk_{t,1}^2}A_n^{(1)}J_{1n} - \frac{i\omega\mu_1}{ck_{t,1}}C_n^{(1)}J'_{1n} + \frac{k_z n}{Rk_{t,2}^2}B_n^{(2)}H_{2n} + \frac{i\omega\mu_2}{ck_{t,2}}D_n^{(2)}H'_{2n} = \frac{i\omega}{c}\left[\frac{\mu_1}{k_{t,1}^2}h'_{zn}(R) - \frac{\mu_2}{k_{t,2}}A_o i^n J'_{2n}\right] + \frac{k_z n}{R}\left[\frac{e_{zn}(R)}{k_{t,1}^2} - \frac{B_o i^n}{k_{t,2}^2}J_{2n}\right]$$

Se simplifica la notación de la siguiente manera: $J_i = J_n(k_{t,i}R), H_2 = H_n^{(1)}(k_{t,2}R))$ sobreentendiendo el modo n-esimo. Con las 4 ecuaciones recuadradas, se construye la matriz de 4×4 del caso inhomogeneo:

$$\begin{pmatrix} J_1 & -H_2 & 0 & 0 \\ \frac{4\pi\sigma}{c}J_1 + \frac{i\omega\varepsilon_1}{ck_{t,1}}J'_1 & -\frac{i\omega\varepsilon_2}{k_{t,2}c}H'_2 & -\frac{k_z\nu}{k_{t,1}^2 R}J_1 & \frac{k_z\nu}{k_{t,2}^2 R}H_2 \\ \frac{4\pi\sigma k_z\nu}{cRk_{t,1}^2}J_1 & 0 & J_1 + \frac{4\pi i\sigma\omega\mu_1}{c^2k_{t,1}}J'_1 & -H_2 \\ -\frac{k_z\nu}{Rk_{t,1}^2}J_1 & \frac{k_z\nu}{Rk_{t,2}^2}H_2 & -\frac{i\omega\mu_1}{ck_{t,1}}J'_1 & \frac{i\omega\mu_2}{ck_{t,2}}H'_2 \end{pmatrix}\begin{bmatrix} A_\nu^{(1)} \\ B_\nu^{(2)} \\ C_\nu^{(1)} \\ D_\nu^{(2)} \end{bmatrix} =$$

$$= \begin{bmatrix} B_o i^\nu J_2 - e_{z\nu}(R) \\ \frac{k_z\nu}{R}\left[\frac{h_{z\nu}(R)}{k_{t,1}^2} - \frac{A_o i^\nu}{k_{t,2}^2}J_2\right] + \frac{i\omega}{c}\left[\frac{\varepsilon_2 B_o i^\nu}{k_{t,2}}J'_2 - \frac{\varepsilon_1}{k_{t,1}^2}e'_{z\nu}(R)\right] - \frac{4\pi\sigma}{c}e_{z\nu}(R) \\ A_o i^\nu J_{2\nu} - h_{z\nu}(R) - \frac{4\pi\sigma}{c}\left[\frac{k_z\nu}{Rk_{t,1}^2}e_{z\nu}(R) + \frac{i\omega\mu_1}{ck_{t,1}^2}h'_{z\nu}(R)\right] \\ \frac{i\omega}{c}\left[\frac{\mu_1}{k_{t,1}^2}h'_{z\nu}(R) - \frac{\mu_2}{k_{t,2}}A_o i^\nu J'_2\right] + \frac{k_z\nu}{R}\left[\frac{e_{z\nu}(R)}{k_{t,1}^2} - \frac{B_o i^\nu}{k_{t,2}^2}J_2\right] \end{bmatrix}$$

Observar que la matriz y los campos quedaron escritos en funcion de $h_{zm}(R)$, $e_{zm}(R)$, con lo cual sólo hay que hallar dichas funciones para $\rho < \rho_D$ y $\rho > \rho_D$.

$$h_{zm}(\rho) = \frac{i\omega}{4c}k_{t,1}e^{-im\theta_D - ik_z z_D}J_m(k_{t,1}\rho)\Big\{-p_+e^{-i\theta_D}H_{m+1}(k_{t,1}\rho_D) - p_-e^{i\theta_D}H_{m-1}(k_{t,1}\rho_D)\Big\} \qquad \rho < \rho_D,$$

$$h_{zm}(\rho) = \frac{i\omega}{4c}k_{t,1}e^{-im\theta_D - ik_z z_D}H_m^{(1)}(k_{t,1}\rho)\Big\{-p_+e^{-i\theta_D}J_{m+1}(k_{t,1}\rho_D) - p_-e^{i\theta_D}J_{m-1}(k_{t,1}\rho_D)\Big\} \qquad \rho > \rho_D$$

Derivando respecto de ρ:

$$h'_{zm}(\rho) = \frac{\partial h_{zm}(\rho)}{\partial(k_{t,1}\rho)}\frac{\partial(k_{t,1}\rho)}{\partial\rho} = k_{t,1}\frac{\partial h_{zm}(\rho)}{\partial(k_{t,1}\rho)} =$$

$$h'_{zm}(\rho) = \frac{i\omega}{4c}k_{t,1}^2 e^{-im\theta_D - ik_z z_D} J'_m(k_{t,1}\rho)\Big\{-p_+ e^{-i\theta_D} H_{m+1}(k_{t,1}\rho_D) - p_- e^{i\theta_D} H_{m-1}(k_{t,1}\rho_D)\Big\} \qquad \rho < \rho_D,$$

$$h'_{zm}(\rho) = \frac{i\omega}{4c}k_{t,1}^2 e^{-im\theta_D - ik_z z_D} H'^{(1)}_m(k_{t,1}\rho)\Big\{-p_+ e^{-i\theta_D} J_{m+1}(k_{t,1}\rho_D) - p_- e^{i\theta_D} J_{m-1}(k_{t,1}\rho_D)\Big\} \qquad \rho > \rho_D$$

El campo electrico del dipolo:

$$e_{zm}(\rho) = \frac{k_{t,1}}{4\varepsilon_1} e^{-im\theta_D - ik_z z_D} J_m(k_{t,1}\rho)\Big[2ip_z H_m(k_{t,1}\rho_D)\left\{\frac{m(m+1)}{\rho^2 k_{t,1}} - k_{t,1}\right\} - k_z p_+ e^{-i\theta_D} H_{m+1}(k_{t,1}\rho_D) +$$

$$+ k_z p_- e^{i\theta_D} H_{m-1}(k_{t,1}\rho_D)\Big] \qquad \rho < \rho_D$$

$$e_{zm}(\rho) = \frac{k_{t,1}}{4\varepsilon_1} e^{-im\theta_D - ik_z z_D} H_m(k_{t,1}\rho)\Big[2ip_z J_m(k_{t,1}\rho_D)\left\{\frac{m(m+1)}{\rho^2 k_{t,1}} - k_{t,1}\right\} - k_z p_+ e^{-i\theta_D} J_{m+1}(k_{t,1}\rho_D) +$$

$$+ k_z p_- e^{i\theta_D} J_{m-1}(k_{t,1}\rho_D)\Big] \qquad \rho > \rho_D$$

Su derivada respecto de ρ:

$$e'_{zm}(\rho) = \frac{k_{t,1}^2}{4\varepsilon_1} e^{-im\theta_D - ik_z z_D} J'_m(k_{t,1}\rho)\Big[2ip_z H_m(k_{t,1}\rho_D)\left\{\frac{m(m+1)}{\rho^2 k_{t,1}} - k_{t,1}\right\} - k_z p_+ e^{-i\theta_D} H_{m+1}(k_{t,1}\rho_D) +$$

$$+ k_z p_- e^{i\theta_D} H_{m-1}(k_{t,1}\rho_D)\Big] \qquad \rho < \rho_D$$

$$e'_{zm}(\rho) = \frac{k_{t,1}^2}{4\varepsilon_1} e^{-im\theta_D - ik_z z_D} H'_m(k_{t,1}\rho)\Big[2ip_z J_m(k_{t,1}\rho_D)\left\{\frac{m(m+1)}{\rho^2 k_{t,1}} - k_{t,1}\right\} - k_z p_+ e^{-i\theta_D} J_{m+1}(k_{t,1}\rho_D) +$$

$$+ k_z p_- e^{i\theta_D} J_{m-1}(k_{t,1}\rho_D)\Big] \qquad \rho > \rho_D$$

El último paso es la adimensionalización de las cantidades para simplificar el trabajo numérico. Para ello, se usan las variables adimensionales: $x_{t,j} \equiv k_{t,j}/k_0$ y $\bar{R} \equiv Rk_0$ en el sistema matricial anterior:

$$\begin{pmatrix} J_1 & -H_2 & 0 & 0 \\ \frac{4\pi\sigma J_1}{c} + \frac{i\varepsilon_1 J'_1}{x_{t,1}} & -\frac{i\varepsilon_2 H'_2}{x_{t,2}} & -\frac{x_z \nu J_1}{x_{t,1}^2 \bar{R}} & \frac{x_z \nu H_2}{x_{t,2}^2 \bar{R}} \\ \frac{4\pi\sigma}{c}\frac{x_z \nu J_1}{\bar{R} x_{t,1}^2} & 0 & J_1 + \frac{4\pi\sigma}{c}\frac{i\mu_1 J'_1}{x_{t,1}} & -H_2 \\ -\frac{x_z \nu J_1}{\bar{R} x_{t,1}^2} & \frac{x_z \nu H_2}{\bar{R} x_{t,2}^2} & -\frac{i\mu_1 J'_1}{x_{t,1}} & \frac{i\mu_2 H'_2}{x_{t,2}} \end{pmatrix} \begin{bmatrix} A_\nu^{(1)} \\ B_\nu^{(2)} \\ C_\nu^{(1)} \\ D_\nu^{(2)} \end{bmatrix} =$$

$=$

$$
= \begin{bmatrix}
B_o i^\nu J_2 - e_{z\nu}(R) \\
\\
\frac{x_z \nu}{\bar{R}} \left[\frac{h_{z\nu}(R)}{x_{t,1}^2} - \frac{i^\nu A_o}{x_{t,2}^2} J_2 \right] + i \left[\frac{\varepsilon_2 i^\nu B_o}{x_{t,2}} J_2' - \frac{\varepsilon_1 (e'_{z\nu}(R)/k_0)}{x_{t,1}^2} \right] - \frac{4\pi\sigma}{c} e_{z\nu}(R) \\
\\
A_o i^\nu J_2 - h_{z\nu}(R) - \frac{4\pi\sigma}{c} \frac{1}{x_{t,1}^2} \left[\frac{x_z \nu}{\bar{R}} e_{z\nu}(R) + i\mu_1 (h'_{z\nu}(R)/k_0) \right] \\
\\
\\
i \left[\mu_1 (h'_{z\nu}(R)/k_0) - \frac{\mu_2}{x_{t,2}} i^\nu A_o J_2' \right] + \frac{x_z \nu}{\bar{R}} \left[\frac{e_{z\nu}(R)}{x_{t,1}^2} - \frac{i^\nu B_o}{x_{t,2}^2} J_2 \right]
\end{bmatrix}
$$

Las unidades de los campos del dipolo son las mismas que la de los coeficientes $A_o, B_o, A_\nu^{(1)}, B_\nu^{(2)}, C_\nu^{(1)}, D_\nu^{(2)}$ para que la formula de los campos longitudinales tenga sentido. Las unidades dan bien porque cuando aparece un e', h' siempre hay un k_0 dividiendo.

A Apendice

$$\vec{p} = p_x\hat{x} + p_y\hat{y}$$
$$\vec{p} = p_\rho\hat{\rho} + p_\theta\hat{\theta}$$

Los versores se relacionan de la siguiente manera:

$$\hat{\rho} = \cos(\theta)\hat{x} + \sin(\theta)\hat{y}$$
$$\hat{\theta} = -\sin(\theta)\hat{x} + \cos(\theta)\hat{y}$$

$$\hat{x} = \cos(\theta)\hat{\rho} - \sin(\theta)\hat{\theta}$$
$$\hat{y} = \sin(\theta)\hat{\rho} + \cos(\theta)\hat{\theta}$$

Se reemplazan los versores y se junta todo lo que tiene $\hat{x}$ y todo lo que tiene $\hat{y}$ para obtener p_x y p_y en función de las coordenadas cilíndricas:

$$\vec{p} = p_\rho\hat{\rho} + p_\theta\hat{\theta}$$
$$\vec{p} = p_\rho\Big(\underbrace{\cos(\theta)\hat{x} + \sin(\theta)\hat{y}}_{\hat{\rho}}\Big) + p_\theta\Big(\underbrace{-\sin(\theta)\hat{x} + \cos(\theta)\hat{y}}_{\hat{\theta}}\Big)$$
$$\vec{p} = \underbrace{(p_\rho\cos(\theta) - p_\theta\sin(\theta))}_{p_x}\hat{x} + \underbrace{(p_\rho\sin(\theta) + p_\theta\cos(\theta))}_{p_y}\hat{y} = p_x\hat{x} + p_y\hat{y}$$

$$\boxed{\begin{aligned} p_x &= p_\rho\cos(\theta) - p_\theta\sin(\theta) \\ p_y &= p_\rho\sin(\theta) + p_\theta\cos(\theta) \end{aligned}}$$

Análogo para obtener p_ρ y p_ϕ en función de las coordenadas cartesianas:

$$\vec{p} = p_x\hat{x} + p_y\hat{y}$$
$$\vec{p} = p_x\Big(\underbrace{\cos(\theta)\hat{\rho} - \sin(\theta)\hat{\theta}}_{\hat{x}}\Big) + p_y\Big(\underbrace{\sin(\theta)\hat{\rho} + \cos(\theta)\hat{\theta}}_{\hat{y}}\Big)$$
$$\vec{p} = \underbrace{(p_x\cos(\theta) + p_y\sin(\theta))}_{p_\rho}\hat{\rho} + \underbrace{(-p_x\sin(\theta) + p_y\cos(\theta))}_{p_\theta}\hat{\theta} = p_\rho\hat{\rho} + p_\theta\hat{\theta}$$

$$\boxed{\begin{aligned} p_\rho &= p_x\cos(\theta) + p_y\sin(\theta) \\ p_\theta &= -p_x\sin(\theta) + p_y\cos(\theta) \end{aligned}}$$

Propiedades de las funciones $C = J, Y, H^{(1)}, H^{(2)}$:

$$C_{n-1}(z) + C_{n+1}(z) = \frac{2n}{z}C_n(z), \tag{24a}$$

$$C_{n-1}(z) - C_{n+1}(z) = 2C'_n(z), \tag{24b}$$

$$C'_n(z) = C_{n-1}(z) - \frac{n}{z}C_n(z), \tag{24c}$$

$$C'_n(z) = -C_{n+1}(z) + \frac{n}{z}C_n(z). \tag{24d}$$

References

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