Optimization of travel costs for a transportation company using a prototype system that combines trucks and drones for last-mile delivery services

BY LEILA OUELLAJ

Problem Statement:

A transportation company is evaluating a prototype system that combines trucks and drones for last-mile delivery services. The test run considers a set of orders that must be delivered to known locations. A delivery truck starts from a depot and visits "launch sites" corresponding to the customers locations. From each launch site, the truck deploys a series of drones that deliver the orders and return back to meet the truck at the launch site. Once all the drones are recovered, the truck moves to the next launch site and repeats the process until all orders are delivered.

1. Generate random x and y coordinates between 0 and 100 for each order. Repeat this process 10 times to generate a testbed of problem instances.
2. Assume that the depot is located at the origin (0,0)
3. Build a Euclidean distance matrix between all the orders using the coordinates generated. Round the distances to the nearest integer.
4. The truck must start and end at the depot. The truck can deploy up to K drones at each stop in a launch site. The drones have limited cargo capacity and as a result they can only visit 1 customer (not counting the starting point) before returning to the truck.
5. Each customer should be visited at least once by a drone. The customer locations used as launch sites will be visited by the truck and also will serve as the starting and ending point of a drone tour.
6. For each unit of distance traveled by the truck there is a cost of $10, while for each unit of distance traveled by a drone there is a cost of $3.
7. Consider the objective of minimizing the total travel cost (both by the trucks and the drones)

PROBLEM FORMULATION

Variables-

x – Array of x coordinates

y – Array of y coordinates

Euc – Euclidean Matrix

C_T – Cost per unit of Truck

C_D – Cost per unit of Drones

Primary_nodes: Number of Primary nodes (Integer)

Secondary_nodes: Number of Secondary nodes (Integer)

Positional – Array of positions (Integer)

Truck_travel_bin: Truck binary variable

Drones_travel_bin: Drones binary variable

LIMIT – Customers to be served + 2

Objective function –

Minimize

$\sum^{P}\sum^{P} T\_e\_(,)$ * Euc(i,j)* C_T + $\sum^{P}\sum^{P} De\_e\_(,)$ * Euc(i,j)* C_D

Constraints –

Origin constraints-

x(1):= 0

y(1):= 0

x(LIMIT):= 0

y(LIMIT):= 0

K Drones allowed

$\sum_{=1}^{IIT} T\_e\_(,)$ + $\sum^{IIT} De\_e\_(,)$ <= K+1 $\forall$ i $\in$ (2 to (LIMIT – 1))

Positional Constraints

Positional(1) = 1

Positional(22) = $\sum_{=1}^{IIT}\sum^{IIT} T\_e\_(,)$ + 1

Positional(j) >= Positional(i) + 1 - 200*(1-Truck_travel_bin(i,j)) $\forall$ i $\in$ (1 to LIMIT), j $\in$ (1 to LIMIT)

Balance constraint out - in = 0

$\sum_{=1}^{IIT} T\_e\_(,)$ - $\sum^{IIT} T\_e\_(,)$ = 0 $\forall$ i $\in$ (2 to (LIMIT – 1))

Nodes are greater than 0

Primary_nodes >= 0

Secondary_nodes >= 0

Euc(p,j) = round(sqrt((y(j)-y(p))^2 + (x(j)-x(p))^2)) ∀ p ∈ (1 to LIMIT), j ∈ (1 to LIMIT)

Limit constraints

Primary_nodes + Secondary_nodes =22

$\sum_{=1}^{IIT} \sum^{IIT} De\_e\_(,) = 2$*Secondary_nodes

$\sum_{=1}^{IIT} \sum^{IIT} T\_e\_(,) =$ Primary_nodes -1

Constraint for removing subtours of drones

Drones_travel_bin(i,j) = Drones_travel_bin(j,i) ∀ i ∈ (2 to (LIMIT – 1)) j ∈ (2 to (LIMIT – 1))

Constraint for removing subtour of trucks

Truck_travel_bin(i,j) + Truck_travel_bin(j,i) <=1 ∀ i ∈ (2 to (LIMIT – 1)) j ∈ (2 to (LIMIT – 1))

Drones_travel_bin(i,j) <= $\sum_{a=1}^{IIT} T\_e\_(,) + \sum^{IIT} T\_e\_(,)$ ∀ i ∈ (2 to (LIMIT – 1)) j ∈ (2 to (LIMIT – 1))

Origin constraints

$\sum_{=1}^{IIT} T\_e\_(1,) = 1$

$\sum_{=1}^{IIT} T\_e\_(,1) = 0$

$\sum_{=1}^{IIT} De\_e\_(1,) = 0$

$\sum_{=1}^{IIT} De\_e\_(,1) = 0$

Last node constraints

$\sum_{=1}^{IIT} T\_e\_(IIT,) = 0$

$\sum_{=1}^{IIT} T\_e\_(,IIT) = 1$

$\sum_{=1}^{IIT} De\_e\_(IIT,) = 0$

$\sum_{=1}^{IIT} De\_e\_(,IIT) = 0$

Truck_travel_bin(1,LIMIT) =0

Truck_travel_bin(LIMIT,1) =0

Removing arcs on itself

$\sum_{=1}^{IIT} T\_e\_(,) = 0$

$\sum_{=1}^{IIT} De\_e\_(,) = 0$

All nodes should be having incoming nodes - except origin

$\sum_{=1}^{IIT} T\_e\_(,) - \sum^{IIT} De\_e\_(,)$ >= 1 ∀ i ∈ (2 to LIMIT)

All nodes should be having outgoing nodes - except destination

$\sum_{=1}^{IIT} T\_e\_(,)$ - $\sum^{IIT} De\_e\_(,)$ >= 1 ∀ i ∈ (1 to (LIMIT-1))

Constraint for not being secondary and primary at the same time

Truck_travel_bin(i,j) + Drones_travel_bin(i,j) <=1 ∀ i ∈ (1 to LIMIT) j ∈ (1 to LIMIT)

Constraint for traveling back to j if drone travels from j

Drones_travel_bin(i,j) = Drones_travel_bin(j,i) ∀ i ∈ (2 to (LIMIT – 1)) j ∈ (2 to (LIMIT – 1))

XPRESS CODE

```
model final_project
uses "mmxprs"; !gain access to the Xpress-Optimizer solver
uses "mmive" !gain access to graphical capabilities

 parameters
 LIMIT=22
 end-parameters
 setparam("XPRS_MAXTIME", 7200)


 declarations
P= 1..22
graph : integer
x: array(P) of real !declaring random X coordinates
y: array(P) of real !declaring random Y coordinates
Euc: array(P,P) of integer !declaring Euclidean matrix
Truck_travel_bin: array(P,P) of mpvar !declaring truck binary variable
Drones_travel_bin: array(P,P) of mpvar !declaring drones binary variable
Primary_nodes: mpvar !primary nodes no
Secondary_nodes: mpvar !secondary nodes no
Positional: array(P) of mpvar
end-declarations

 writeln("RANDOM numbers(", LIMIT," of them) between 1 and 20 :")

setrandseed(4)
cloud:=IVEaddplot("DISTRIBUTION OF GENERATED X and Y", IVE_BLUE)
 forall(p in 2..LIMIT) do
   x(p):= 100*random
   y(p):= 100*random
 end-do


 ! origin constraints
x(1):= 0
y(1):= 0
x(22):= 0
y(22):= 0

!type_constraints
forall( p in 1..LIMIT) do
Positional(p) is_integer
end-do

!K drones allowed constraints
forall(i in 2..21) sum(j in 1..LIMIT) Truck_travel_bin(i,j) + sum(j in 1..LIMIT) Drones_travel_bin(i,j)  <= 6

!Positional Constraints
Positional(1) = 1
Positional(22) = sum(i in 1..LIMIT, j in 1..LIMIT) Truck_travel_bin(i,j)+1

forall( i in 1..LIMIT, j in 1..LIMIT)
Positional(j) >= Positional(i) + 1 - 200*(1-Truck_travel_bin(i,j))

!Balance constraint out - in = 0
forall(i in 2..21) sum(j in 1..LIMIT) Truck_travel_bin(i,j)-sum(j in 1..LIMIT) Truck_travel_bin(j,i)=0

forall(p in 1..LIMIT,j in 1..LIMIT) do
  Truck_travel_bin(p,j) is_binary
  Drones_travel_bin(p,j) is_binary
end-do
```

```
Primary_nodes is_integer
Secondary_nodes is_integer
Primary_nodes >=0
Secondary_nodes >=0

 forall(p in 1..LIMIT,j in 1..LIMIT) do
    Euc(p,j):= round(sqrt((y(j)-y(p))^2 + (x(j)-x(p))^2))
 end-do

!Limit constraints

Primary_nodes + Secondary_nodes =22

!Primary_nodes=(Secondary_nodes/2)+2

!Primary_nodes*2=Secondary_nodes

sum(i in 1..LIMIT, j in 1..LIMIT) Drones_travel_bin(i,j) = 2*Secondary_nodes

sum(i in 1..LIMIT, j in 1..LIMIT) Truck_travel_bin(i,j) = Primary_nodes -1

! constraint for removing subtours of drones
forall(i in 2..21, j in 2..21)
   Drones_travel_bin(i,j) = Drones_travel_bin(j,i)

!Constraint for removing subtour of trucks
forall(i in 2..21, j in 2..21)
   Truck_travel_bin(i,j) + Truck_travel_bin(j,i) <=1
```

```
forall (i in 2..21, j in 2..21) Drones_travel_bin(i,j) <= sum(a in 1..22) Truck_travel_bin(a,i)+ sum(a in 1..22) Truck_travel_bin(a,j)
if S(i,j) exists no S(j,i) should exist
forall(i in 2..11) sum(j in 2..11) Drones_travel_bin(j,i) + sum(j in 2..11) Drones_travel_bin(i,j)>= 2- sum(j in 2..11)Truck_travel_bin(i,j)

 CONSTRAINT FOR Kth


! origin constraints - part 2
sum(i in 1..LIMIT) Truck_travel_bin(1,i)= 1
sum(i in 1..LIMIT) Drones_travel_bin(1,i)= 0

sum(i in 1..LIMIT) Truck_travel_bin(i,1)= 0
sum(i in 1..LIMIT) Drones_travel_bin(i,1)= 0


!last node constraints
sum(i in 1..LIMIT) Truck_travel_bin(22,i)= 0
sum(i in 1..LIMIT) Drones_travel_bin(22,i)= 0

sum(i in 1..LIMIT) Truck_travel_bin(i,22)= 1
sum(i in 1..LIMIT) Drones_travel_bin(i,22)= 0

Truck_travel_bin(1,22) =0
Truck_travel_bin(22,1) =0

!removing_arcs_on_itself
sum(i in 1..LIMIT) Truck_travel_bin(i,i) = 0
sum(i in 1..LIMIT) Drones_travel_bin(i,i) = 0

!all nodes should be have incoming nodes - except origin
forall(i in 2..LIMIT) sum(j in 1..LIMIT) Truck_travel_bin(j,i) + sum(j in 1..LIMIT) Drones_travel_bin(j,i) >=1
```

```
 !all nodes should have outgoing nodes - except destination
  forall(j in 1..21) sum(i in 1..LIMIT) Truck_travel_bin(j,i) + sum(i in 1..LIMIT) Drones_travel_bin(j,i) >=1


 ! constraint for not being secondary and primary at the same time
 forall(i in 1..LIMIT, j in 1..LIMIT)
    Truck_travel_bin(i,j) + Drones_travel_bin(i,j) <=1

! constraint for traveling back to j if drone travels from j
 forall(i in 2..21, j in 2..21)
    Drones_travel_bin(i,j) = Drones_travel_bin(j,i)


obj:=sum(i in P)sum(j in P)(Truck_travel_bin(i,j))*Euc(i,j)*20+sum(i in P)sum(j in P)(Drones_travel_bin(i,j))*Euc(i,j)*1
minimize(obj)


 writeln("Printing random numbers generated below:")
 forall(p in 1..LIMIT) do
 writeln("Number ",p," in Array X is :",x(p)," and the Number ",p," in Array  Y is:", y(p))
 end-do


 IVEzoom(0, -0, 10, 10)
 writeln(" Generating the Plot for co-ordinates below")
 graph:=IVEaddplot("Y(X)",IVE_RED)      !Create a graph
 graph2:=IVEaddplot("Y(X)",IVE_GREEN)

 forall(p in 1..LIMIT) do
 IVEdrawpoint(graph, x(p), y(p))
 IVEdrawlabel(graph, x(p), y(p), ""+p)
 end-do
```

```
forall(i in 1..LIMIT,j in 1..LIMIT |getsol(Drones_travel_bin(i,j)) > 0) do
IVEdrawline(graph2, x(i), y(i),x(j), y(j))
end-do

 forall(i in 1..LIMIT,j in 1..LIMIT |getsol(Truck_travel_bin(i,j)) > 0) do
IVEdrawline(graph, x(i), y(i),x(j), y(j))
end-do

forall(i in 1..LIMIT,j in 1..LIMIT |getsol(Truck_travel_bin(i,j)) > 0 ) do
    writeln("Send Truck from ",i," to ", j," : ",getsol(Truck_travel_bin(i,j)))
end-do

forall(i in 1..LIMIT,j in 1..LIMIT |getsol(Drones_travel_bin(i,j)) > 0 ) do
    writeln("Send Drones from ",i," to ", j," : ",getsol(Drones_travel_bin(i,j)))
    end-do

end-model
```

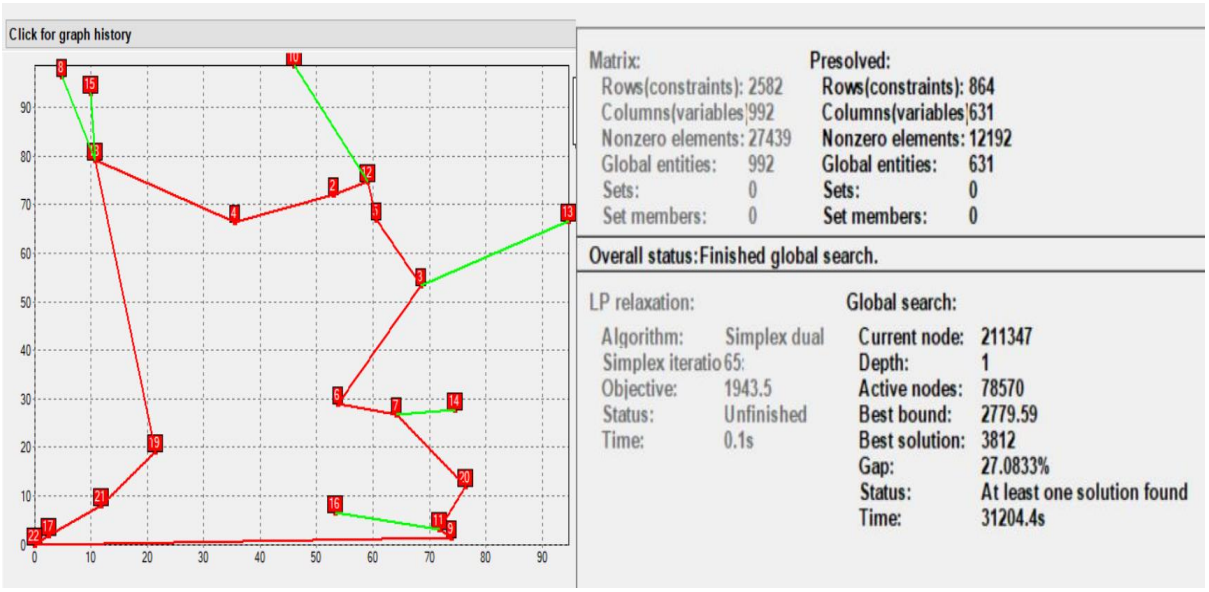
NOTE: We have also used a code with manually generated random points

SOLUTION

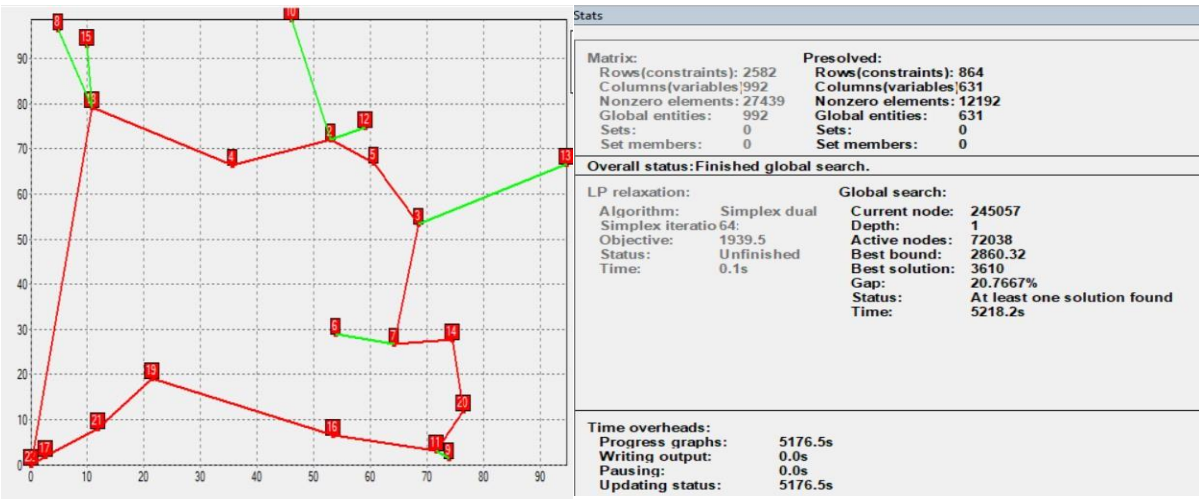
SEED = 4

COST PER UNIT DISTANCE OF TRUCK – 10 COST PER UNIT DISTANCE OF DRONES - 3

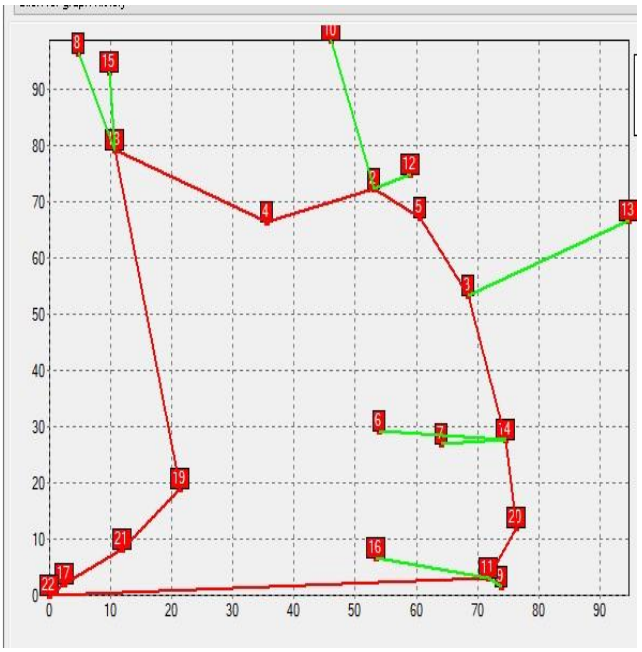
K =2 OBJECTIVE FUNCTION = 3812



K =3 OBJECTIVE FUNCTION = 3610



K =4 OBJECTIVE FUNCTION = 3734



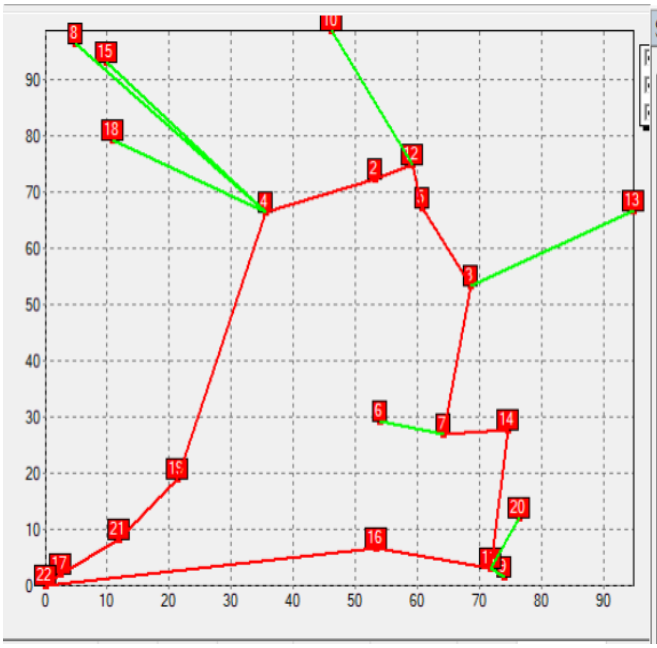
Stats

Matrix:		Presolved:	
Rows(constraints):	2582	Rows(constraints):	864
Columns(variables):	992	Columns(variables):	631
Nonzero elements:	27439	Nonzero elements:	12192
Global entities:	992	Global entities:	631
Sets:	0	Sets:	0
Set members:	0	Set members:	0

Overall status: **Finished global search.**

LP relaxation:		Global search:	
Algorithm:	Simplex dual	Current node:	465819
Simplex iterations:	63	Depth:	1
Objective:	1939.5	Active nodes:	162356
Status:	Unfinished	Best bound:	2905.67
Time:	0.1s	Best solution:	3734
		Gap:	22.1835%
		Status:	At least one solution found
		Time:	7199.5s

K =5 OBJECTIVE FUNCTION = 3738



Stats

Matrix:		Presolved:	
Rows(constraints):	2582	Rows(constraints):	864
Columns(variables):	992	Columns(variables):	631
Nonzero elements:	27439	Nonzero elements:	12192
Global entities:	992	Global entities:	631
Sets:	0	Sets:	0
Set members:	0	Set members:	0

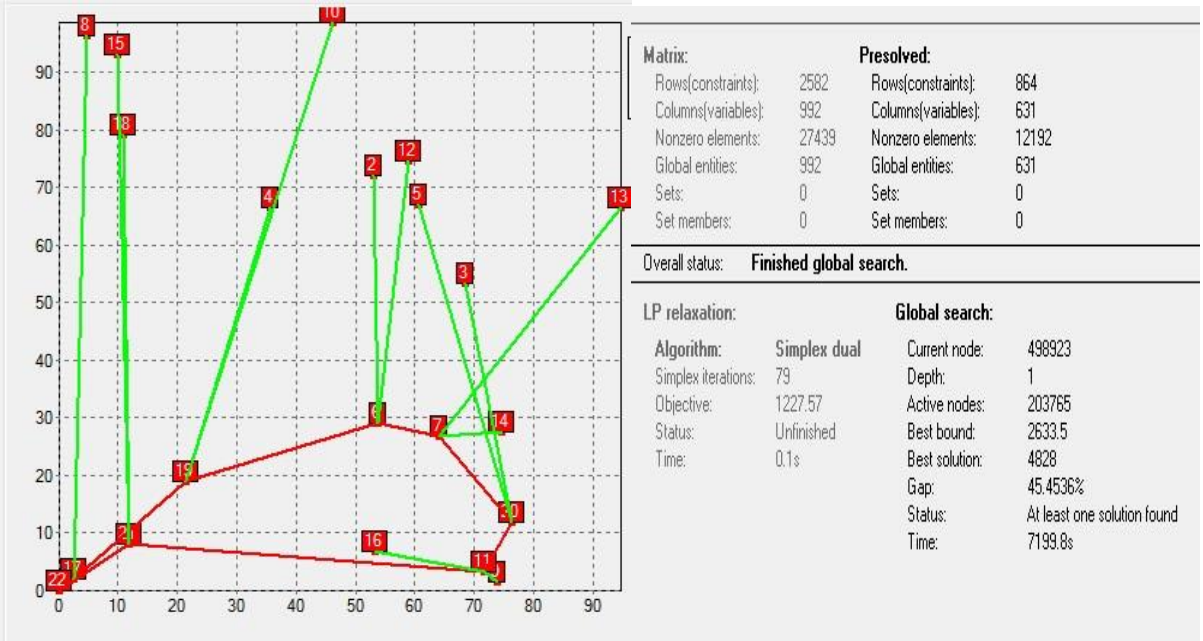
Overall status: **Finished global search.**

LP relaxation:		Global search:	
Algorithm:	Simplex dual	Current node:	242426
Simplex iterations:	61	Depth:	1
Objective:	1939.5	Active nodes:	80457
Status:	Unfinished	Best bound:	2846.32
Time:	0.1s	Best solution:	3738
		Gap:	23.8546%
		Status:	At least one solution found
		Time:	28475.4s

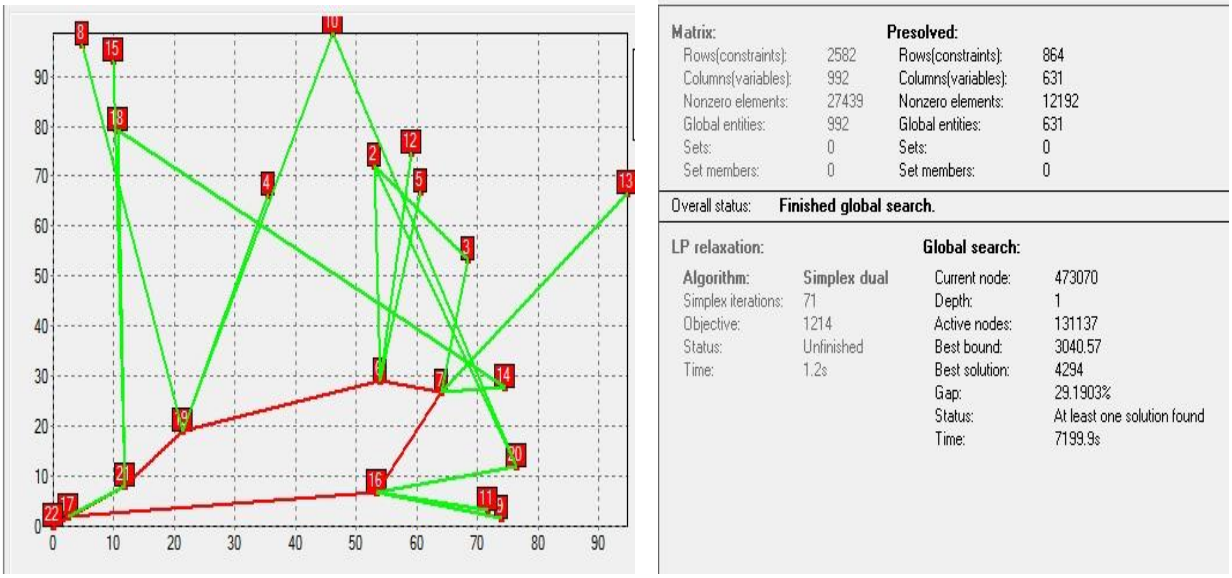
SENSITIVITY ANALYSIS

COST PER UNIT DISTANCE OF TRUCK – 20 COST PER UNIT DISTANCE OF DRONES – 1

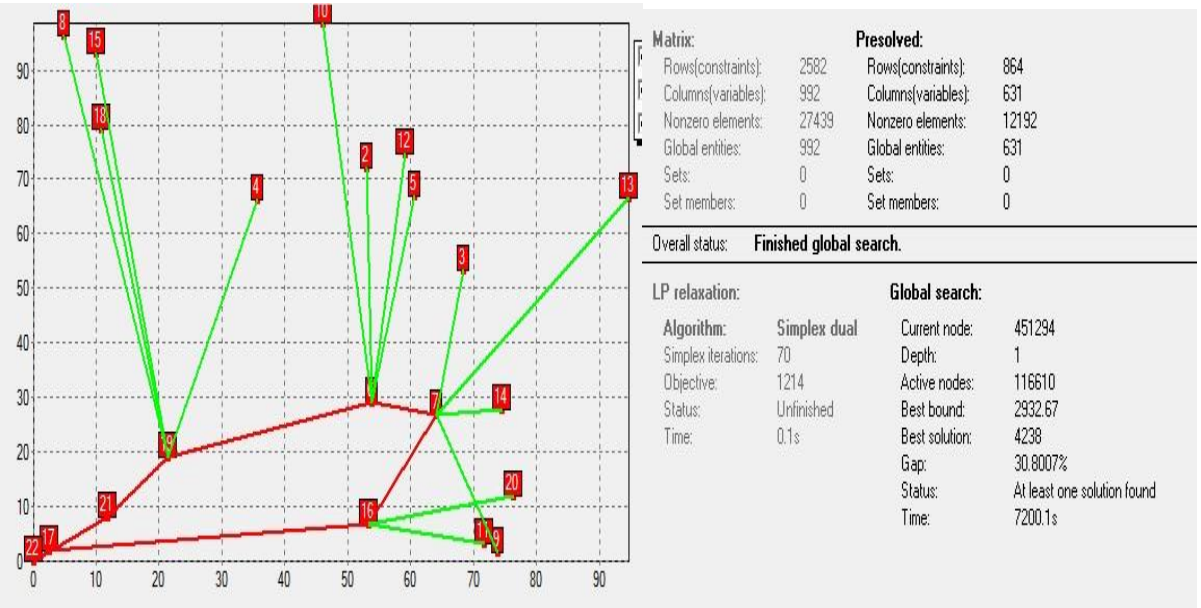
K =2 OBJECTIVE FUNCTION = 4828



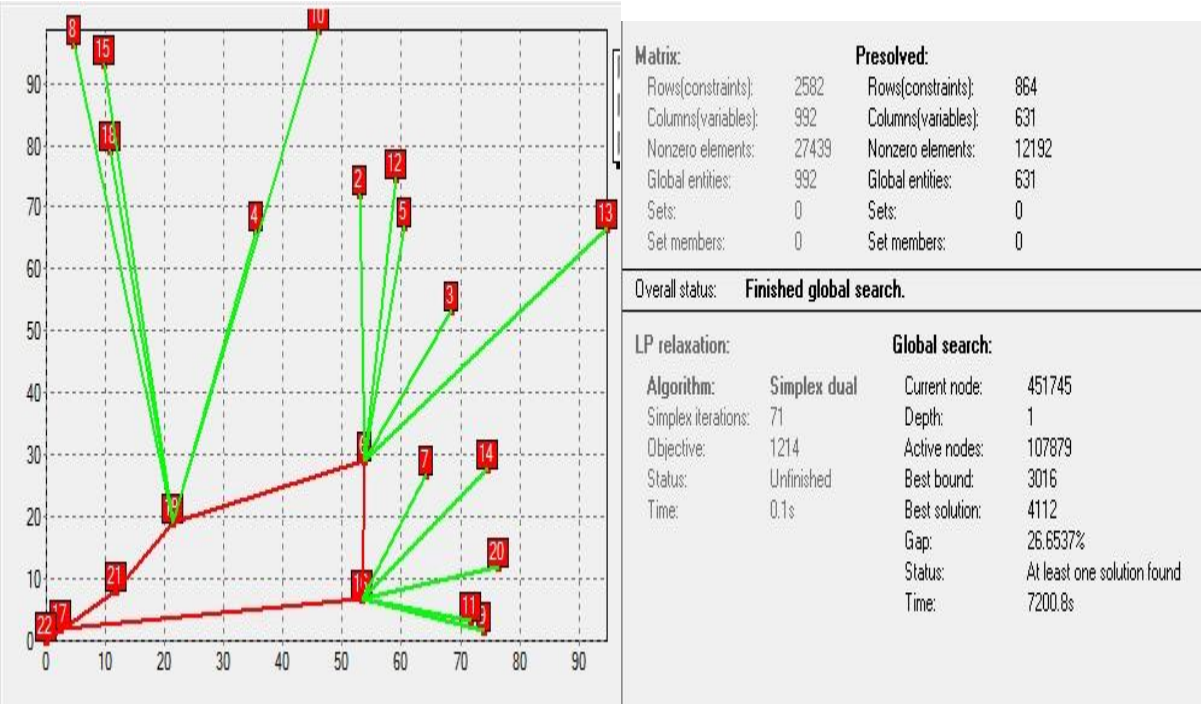
K =3 OBJECTIVE FUNCTION = 4294



K =4 OBJECTIVE FUNCTION = 4238



K =5 OBJECTIVE FUNCTION = 4112

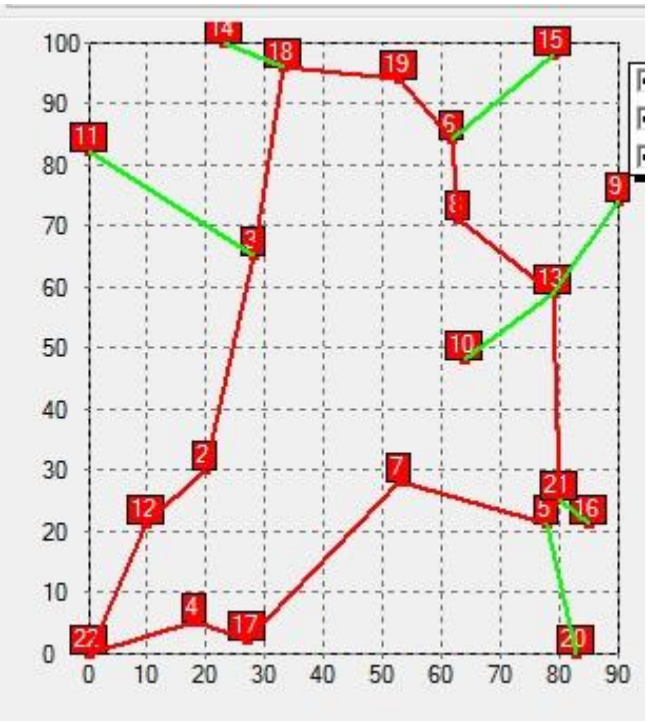


CONCLUSION: We observe by increasing the ratio of Unit cost per distance of Truck to that of Drones, the solution optimizes the route by using more Drones for every value of K

To better visualize the working of the model we have manually generated customer coordinates which are clustered near a single node (Customer location). Our result for these set of customer location is as follows-

COST PER UNIT DISTANCE OF TRUCK – 10 COST PER UNIT DISTANCE OF DRONES - 3

K =2 OBJECTIVE FUNCTION = 3772

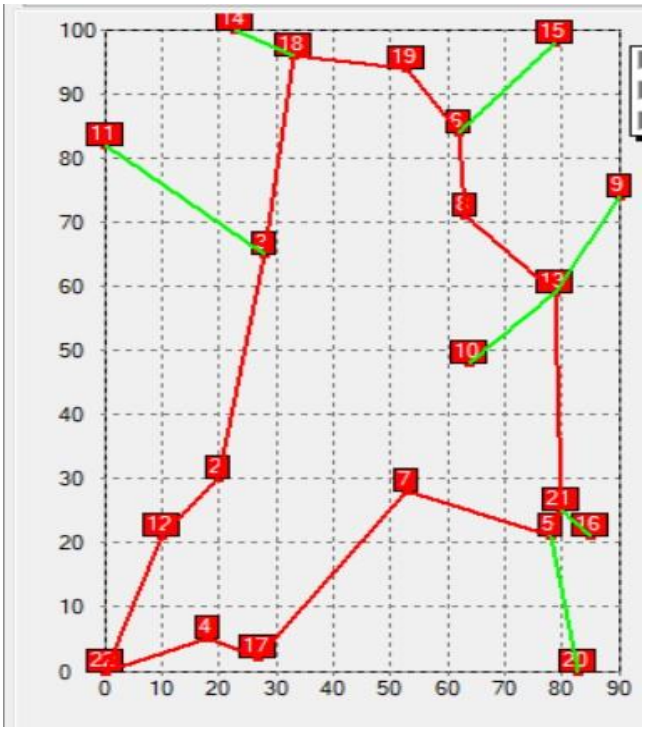


Matrix:		Presolved:	
Rows(constraints):	2582	Rows(constraints):	864
Columns(variables):	992	Columns(variables):	631
Nonzero elements:	27439	Nonzero elements:	12192
Global entities:	992	Global entities:	631
Sets:	0	Sets:	0
Set members:	0	Set members:	0

Overall status: **Finished global search.**

LP relaxation:		Global search:	
Algorithm:	Simplex dual	Current node:	342371
Simplex iterations:	72	Depth:	1
Objective:	2673.67	Active nodes:	38240
Status:	Unfinished	Best bound:	3564.67
Time:	0.1s	Best solution:	3772
		Gap:	5.49664%
		Status:	At least one solution found
		Time:	4799.2s

K =3 OBJECTIVE FUNCTION = 3772



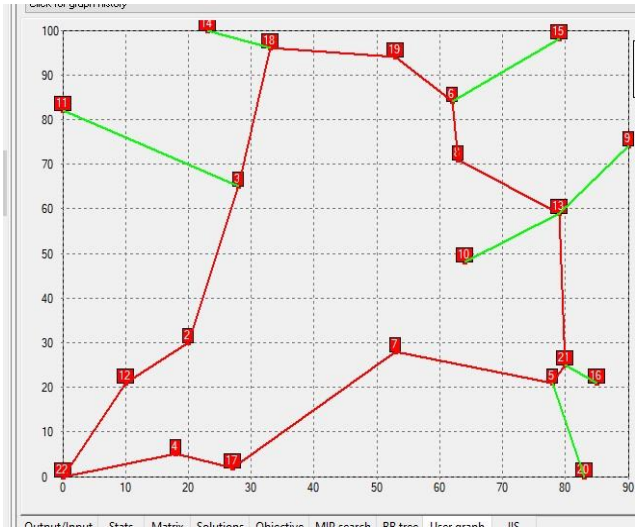
Stats

Matrix:		Presolved:	
Rows(constraints):	2582	Rows(constraints):	864
Columns(variables	992	Columns(variables	631
Nonzero elements:	27439	Nonzero elements:	12192
Global entities:	992	Global entities:	631
Sets:	0	Sets:	0
Set members:	0	Set members:	0

Overall status: Finished global search.

LP relaxation:		Global search:	
Algorithm:	Simplex dual	Current node:	63686
Simplex iteratio	66:	Depth:	1
Objective:	2670.67	Active nodes:	0
Status:	Unfinished	Best bound:	3772
Time:	0.1s	Best solution:	3772
		Gap:	0%
		Status:	Solution is optimal.
		Time:	2431.9s

K=4 OBJECTIVE FUNCTION = 3772



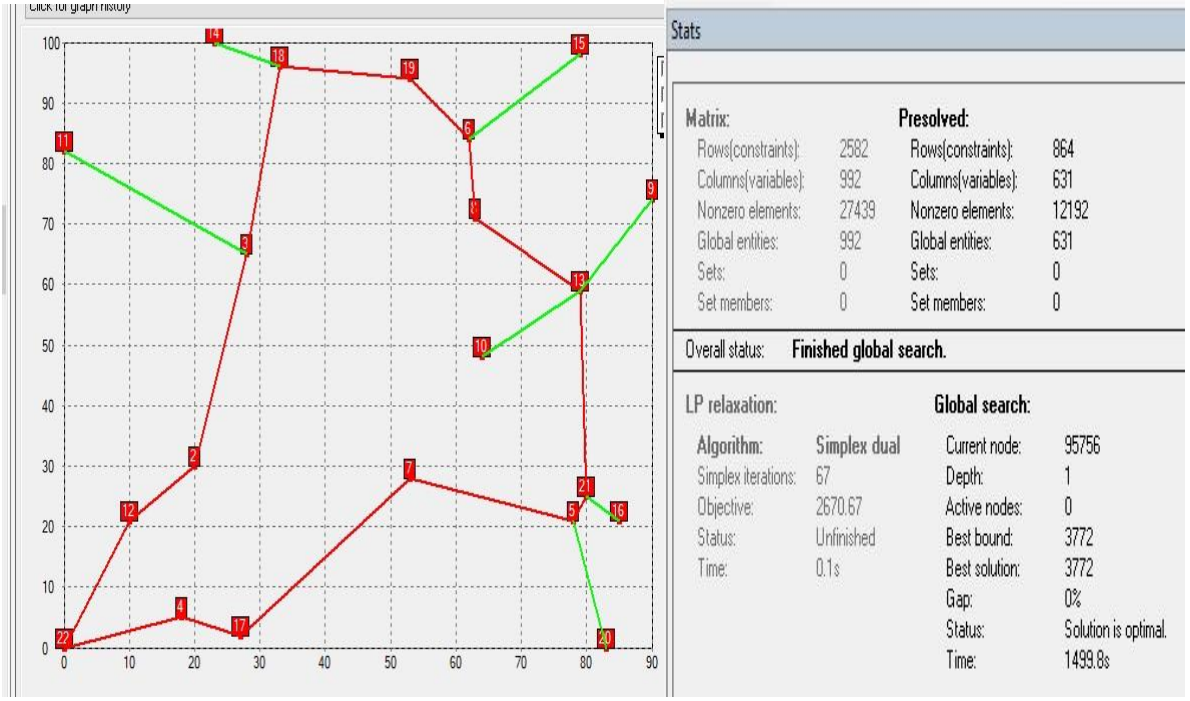
Stats

Matrix:		Presolved:	
Rows(constraints):	2582	Rows(constraints):	864
Columns(variables):	992	Columns(variables):	631
Nonzero elements:	27439	Nonzero elements:	12192
Global entities:	992	Global entities:	631
Sets:	0	Sets:	0
Set members:	0	Set members:	0

Overall status: **Finished global search.**

LP relaxation:		Global search:	
Algorithm:	Simplex dual	Current node:	40286
Simplex iterations:	67	Depth:	1
Objective:	2670.67	Active nodes:	0
Status:	Unfinished	Best bound:	3772
Time:	0.1s	Best solution:	3772
		Gap:	0%
		Status:	Solution is optimal.
		Time:	1106.7s

K=5 OBJECTIVE FUNCTION = 3772



Stats

Matrix:		Presolved:	
Rows(constraints):	2582	Rows(constraints):	864
Columns(variables):	992	Columns(variables):	631
Nonzero elements:	27439	Nonzero elements:	12192
Global entities:	992	Global entities:	631
Sets:	0	Sets:	0
Set members:	0	Set members:	0

Overall status: **Finished global search.**

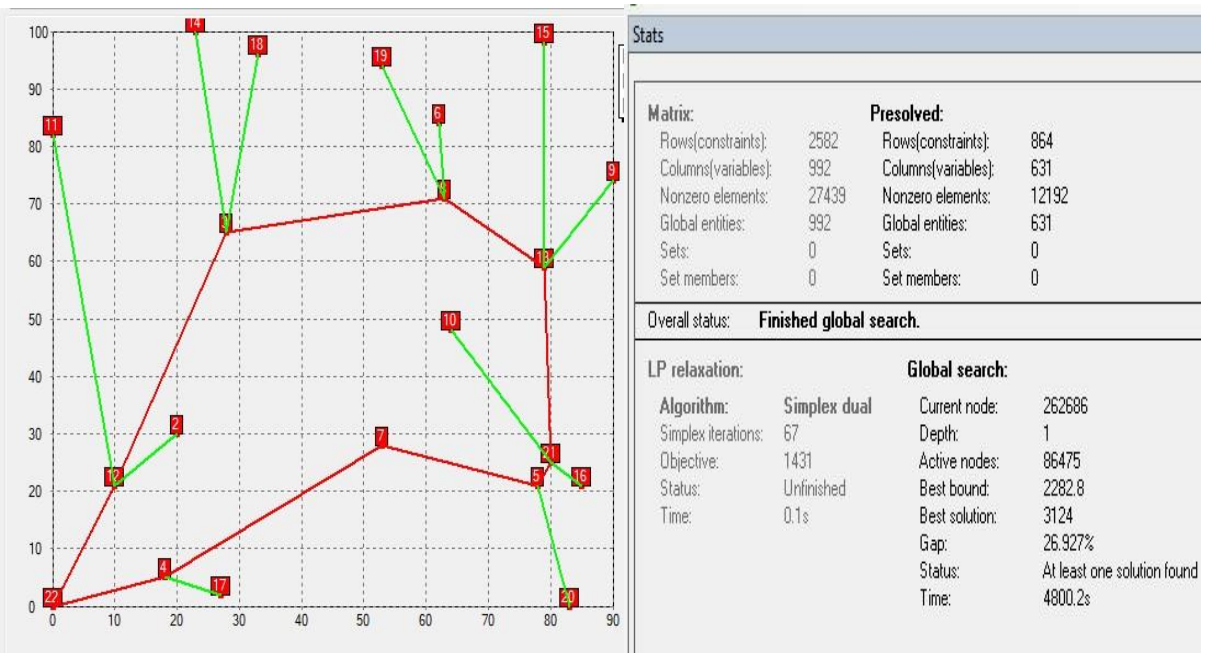
LP relaxation:		Global search:	
Algorithm:	Simplex dual	Current node:	95756
Simplex iterations:	67	Depth:	1
Objective:	2670.67	Active nodes:	0
Status:	Unfinished	Best bound:	3772
Time:	0.1s	Best solution:	3772
		Gap:	0%
		Status:	Solution is optimal.
		Time:	1499.8s

CONCLUSION: Here we observe increasing K does not decrease the objective function.

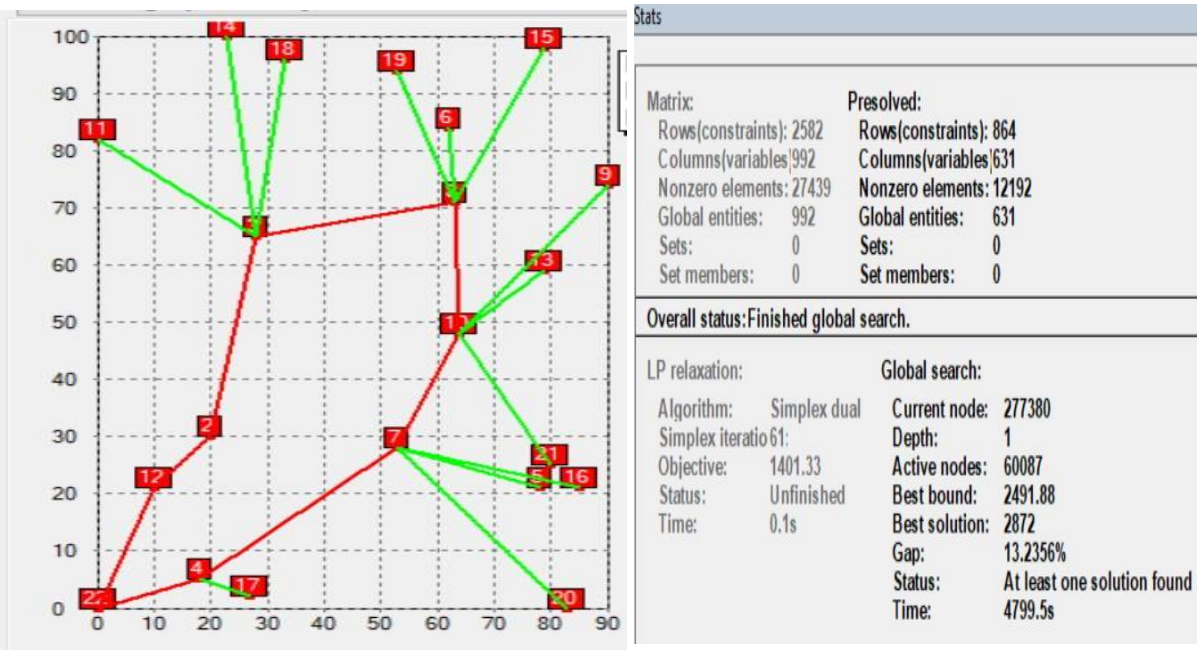
SENSITIVITY ANALYSIS

COST PER UNIT DISTANCE OF TRUCK – 10 COST PER UNIT DISTANCE OF DRONES – 1

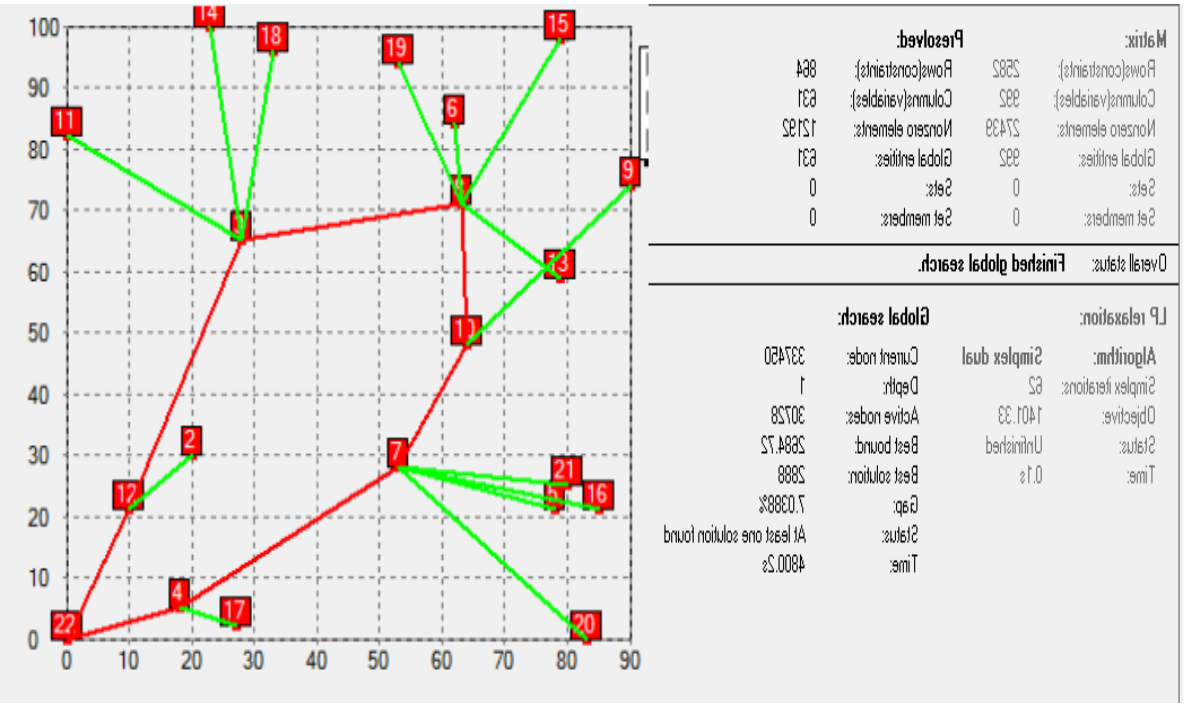
K=2 OBJECTIVE FUNCTION = 3124



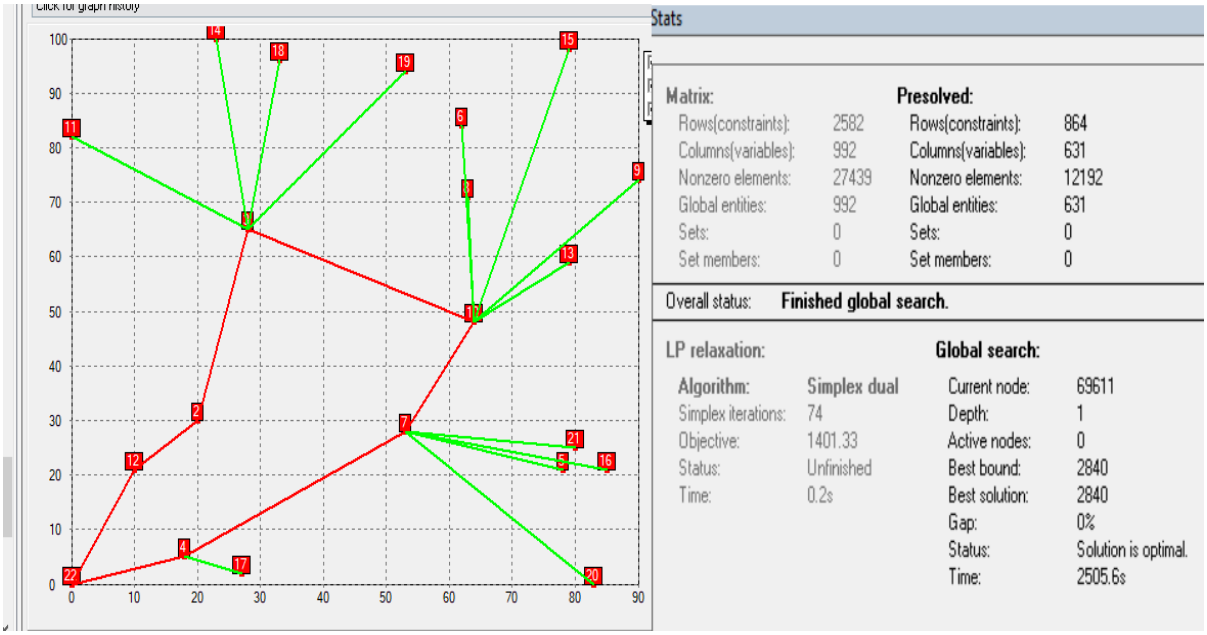
K=3 OBJECTIVE FUNCTION = 2872



K=4 OBJECTIVE FUNCTION = 2888



K=5 OBJECTIVE FUNCTION = 2840



CONCLUSION: Here we observe increasing K decreases the objective function, but not by a huge margin.