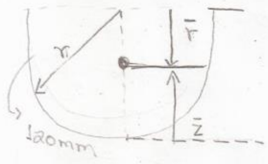
Mecânica I

Cap 5.

5/3 - $\bar{x} = ?$ e $\bar{z} = ?$



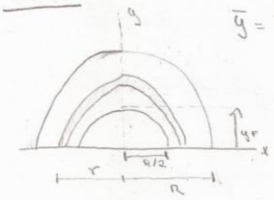
• $\bar{r} = \frac{2r}{\pi} \Rightarrow \frac{2 \cdot 120}{\pi} \Rightarrow \bar{r} = 76{,}4\,mm$

• $\bar{z} = r - \bar{r} \Rightarrow \bar{z} = 120 - 76{,}4$

$\bar{z} = 43{,}6\,mm$

• $\bar{x} = -120\,mm$

5/5 -



$\bar{y} = ?$

• $y_c = \frac{2r}{\pi}$; $dA = \pi r\,dr$; $A = \pi \int_{R/2}^{R} r\,dr \Rightarrow A = \frac{3\pi R^2}{8}$

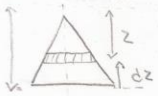
• $\int y_c\,dA \Rightarrow \int_{R/2}^{R} \frac{2r}{\pi} \pi r\,dr \Rightarrow \frac{2r^3}{3}\Big|_{R/2}^{R}$

$= \frac{7}{12} R^3$

• $\bar{y}A = \int y_c\,dA$; $\bar{y} = \frac{7}{12} R^3 \Big| \frac{3\pi R^3}{12}$

$= \frac{14R}{9\pi}$

5/10 - $\bar{z}$ o vértice ao centroide do seu volume.



• $x = \frac{r}{h} \cdot z$ $\quad dv = \pi x^2 dz = \pi \left(\frac{r}{h} \cdot z\right)^2 dz$.

• $v = \frac{\pi r^2}{h^2} \int_0^h z\,dz = \frac{\pi r^2 h}{3}$

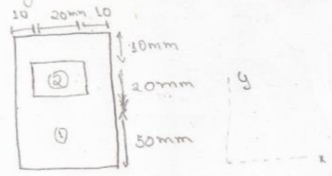
• $\int z\,dv = \pi \frac{r^2}{h} \int_0^h z^3\,dz \Rightarrow \frac{\pi r^2 h^2}{4}$

• $\bar{z} = \int \frac{z\,dv}{V} \Rightarrow \frac{\pi r^2 h^2/4}{\pi r^2 h/3}$

$\bar{z} = \frac{3h}{4}$

5/45.

y do centroide da área sombreada.



① - $A = 80.40 = 3200\ mm^2$

$\bar{y} = 40\ mm$

$\bar{y}A = 3200.40 = 128\,000\ mm^3$

① - $A = -20.20 = -400\ mm^2$

$\bar{y} = 60$

$\bar{y}A = -400.60 = -24000\ mm^3$

$\Sigma A = 2800$ e $\Sigma yA = 104000$

$$\bar{Y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{104000\ mm^3}{2800\ mm^2} = 37{,}1\ mm$$

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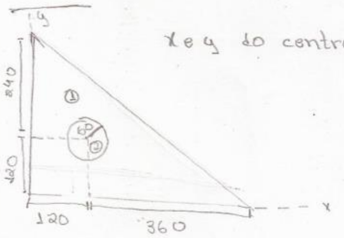

x e y do centroide da área sombreada.

① $A_1 = \frac{1}{2} \cdot 480 \cdot 360 \Rightarrow A_1 = 86400\ mm^2$

$\bar{X}_1 = \frac{480}{3} \Rightarrow \bar{X}_1 = 160mm$

$\bar{y}_1 = \frac{360}{3} \Rightarrow \bar{y}_1 = 120\ mm.$

② - $A_2 = -\pi . 60^2 \Rightarrow A_2 = -11310\ mm^2$

$\bar{X}_2 = \bar{y}_2 = 120mm$

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} \Rightarrow \bar{Y} = \frac{86400 . 120 - 11310 . 120}{86400 - 11310}$$

$\bar{Y} = 120\ mm$

$$\bar{X} = \frac{\Sigma A\bar{x}}{\Sigma A} = \frac{86400 . 160 - 11310 . 120}{86400 - 11310}$$

$\bar{X} = 166{,}03\ mm$

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volume do sólido, gerado por rev = 180° em torno do eixo z.

$$V = \theta \bar{r} A \Rightarrow V = \pi \left(8 + \frac{2}{5} \cdot 12\right) \frac{1}{2} \cdot 12 . 12$$

$V = 3620\ mm^3$

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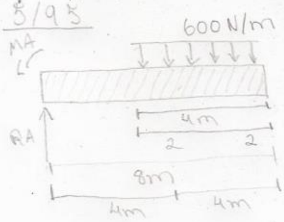

Força RA e o momento MA em. A

$\bullet\ F = 600 \cdot 4 = 2400\,N$

$+\uparrow \sum F = 0 : R_A - 2400 = 0$

$R_A = 2400\,N = \underline{2,4\,kN}$

$\circlearrowleft + \sum M_A = 0 : M_A - 2400 \cdot 6 = 0$

$M_A = 14.400\,N$

$= \underline{14,4\,kN}$

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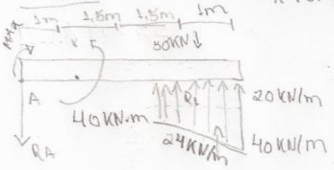

A força e o momento de reação em A.

$\cdot\ R_2 = A_2 = 36 \cdot \frac{25}{2} = 45\,kN$

$\circlearrowleft + \sum M_x = 0 \therefore M_A - 40 + 50 \cdot 4 - 60 \cdot 3,75 - 45 \cdot 4,15$

$M_A = 252,65\,kN.m$

$\uparrow x \sum F = 0$

$-R_A - 50 + 60 + 45 = 0$

$R_A = 55\,kN$

$\bar{X} = \frac{1,5+1}{3} = 0,833$

$R_1 = A_1 = 24 \cdot 2,5 = 60\,kN$

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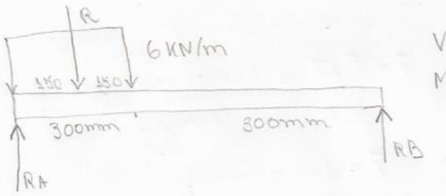


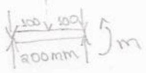
$V = ?$

$M = ?$

$R = 6 \cdot 0,3 = 1,8\,kN$

$\sum M_B = 0 \therefore 1,8(0,3+0,15) - 0,6 R_A = 0$

$R_A = \underline{1,35\,kN}$



1,35 kN

$R_1 = 6 \cdot 0,2 = 1,2\,kN$

$\sum F = 0 \therefore 1,35 - 1,2 - V = 0$

$\underline{V = 0,15\,kN}$

$\sum M_A = 0$

$M - 1,2 \cdot 0,1 - 0,15 \cdot 0,2 = 0$

$\underline{M = 0,15\,kN}$

5/146 Força tratira no meio do vão T_0

$L = 1280m$
$h = 145m$
$W = 310 \, KN \cdot 10^{-3}$
$\omega = 0{,}3108$

$T_0 = \dfrac{\omega L^2}{8h} \therefore T_0 = \dfrac{0{,}3108 \cdot (1280)^2}{8 \cdot 145} \Rightarrow T_0 = 448{,}12 \, KN$

$C = 2 T_{máx} \cdot sen\theta$
$= \cancel{2} \left(\dfrac{\omega L}{\cancel{2}} \right) \Rightarrow \omega L \quad \Rightarrow C = \omega \cdot L$
$C = 0{,}3108 \cdot 1280$
$\underline{C = 397{,}82 \, KN}$

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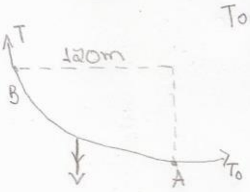
$T_0 = ?$ e $T_{0 \, parabola} = ?$



$x = 60m$
$y = 30m$

$\mu = 0{,}750 \cdot 9{,}81 \cdot 10^{-3}$
$\mu = 0{,}00735$

cabo catenário

$y = \dfrac{T_0}{\mu} \left(\cosh \dfrac{\mu x}{T_0} - 1 \right)$

$30 = \dfrac{T_0}{\mu} \left(\cosh \dfrac{7{,}35 \cdot (120)}{T_0} - 1 \right)$

$T_0 = 1801 \, N$

cabo parabólico

$y = \dfrac{\omega x^2}{2T_0}$

$\dfrac{7{,}35 \cdot (120^2)}{2T_0} = 30$

$7{,}35 \cdot 14.400 = 60 T_0$

$\underline{T_0 = 1764 \, N}$

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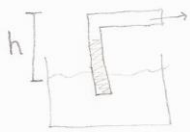
$h_{água} = ?$ e $h_{mercurio} = ?$

$P_{at} = 1{,}0133 \cdot 10^5 \, Pa$
$\rho_{água} = 1000 \, Kg/m^3$
$\rho_{mercurio} = 13570 \, Kg/m^3$



• Água:

$\rho g h = P_{at}$
$1000 \cdot 9{,}81 \cdot h = 1{,}0133$
$\underline{h = 10{,}33 \, m}$

• mercúrio

$\rho g h = P_{at}$
$13570 \cdot 9{,}81 \cdot h = 1{,}0133$
$\underline{h = 0{,}761 \, m}$

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$R = ?$ e $x = ?$

$h_1 = 500 + 300$

$h_1 = 800mm$

$h_2 = 500mm$

$\rho = 900 kg/m^3$

$P_2 = \rho g h_2$

$P_2 = 900 \cdot 9,8 \cdot 0,500$

$P_2 = \underline{4,415 \cdot 10^3 Pa}$ //

$P_1 = \rho g h_1$

$P_1 = 900 \cdot 9,81 \cdot 0,800$

$P_1 = 7,063 \cdot 10^3 Pa$

$R_1 = P_2 \cdot A$

$R_1 = (4,415 \cdot 10^3) \cdot 0,600 \cdot 0,400$

$R_1 = 1059 N$

$R_2 = \frac{P_1 - P_2}{2} \cdot A \Rightarrow R_2 = \frac{(7063 - 4.415)}{2} 10^3 \cdot 0,600 \cdot 0,400$

$R_2 = 318 N$ //

$R = R_1 + R_2$

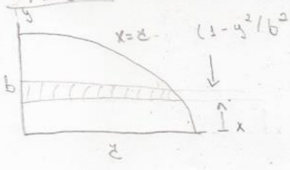
$= 1059 + 318 \Rightarrow \underline{R = 1377 N}$ //

$R_x = \Sigma M_A$; $1377x = 1060 \cdot 300 + 318 \cdot 400$

$X = \frac{445200}{1377}$

$\underline{X = 323,3mm}$ //

5/13 - centroide da área sombreada.



• $dA = x\,dy$; $A = \int_0^b a\left(1 - \frac{y^2}{b^2}\right) dy$

$A = a\left[y - \frac{y^3}{b^2}\right]\Big|_0^b$

$A = \frac{2}{3} ab$,,

• $\bar{x}A = \int x_c\,dA \Rightarrow \int_0^b \frac{x}{2}\,x\,dy \Rightarrow \frac{1}{2}\int_0^b a^2\left(1 - \frac{2y^2}{3b^2} + \frac{y^4}{b^4}\right) dy$

$= \frac{a^2}{2}\left[y - \frac{2y^3}{3b^2} + \frac{y^5}{5b^4}\right]\Big|_0^b$

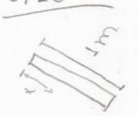
$= \frac{4}{15} a^2 b$,,

• $\bar{x} = \frac{4a^2b/15}{2ab/3} \Rightarrow \bar{x} = \frac{2}{5} a$,,

• $\bar{y}A = \int y\,dA \Rightarrow \int_0^b a\left(y - \frac{y^3}{b^2}\right) dy \Rightarrow a\left[\frac{y^2}{2} - \frac{y^4}{4b^2}\right]\Big|_0^b \Rightarrow \frac{1}{4} ab^2$,,

$\bar{y} = \frac{ab^2/4}{2ab/3} \Rightarrow \bar{y} = \frac{3}{8} b$,,

5/15 -



$\rho = \rho_0\left(1 - \frac{x}{2}\right)$

• $\frac{dm}{dx} = \rho_0\left(1 - \frac{x}{2}\right)$

$m = \int_0^1 \rho\left(1 - \frac{x}{2}\right) dx \Rightarrow m = \rho_0\left(x - \frac{x^2}{4}\right)\Big|_0^1$

$m = \rho_0\left(1 - \frac{1}{4}\right) \cdot \frac{3}{4}\rho_0$

$\bar{x} = \frac{\int x\,dm}{m} = \frac{\int_0^1 x\,\rho_0\left(1 - \frac{x}{2}\right) dx}{3/4 \cdot \rho_0} \Rightarrow \bar{x} = \frac{3}{3\rho_0}\rho_0 \int\left(x - \frac{x^2}{2}\right) dx$

$\bar{x} = \frac{4}{3}\left[\frac{x^2}{2} - \frac{x^3}{6}\right]\Big|_0^1 \Rightarrow \bar{x} = \frac{4}{3}\left(\frac{1}{2} - \frac{1}{6}\right)$

$\bar{x} = \frac{4}{3} \cdot \frac{2}{6} \Rightarrow \bar{x} = \frac{8}{18}$

$\bar{x} = \frac{4}{9}$ m ,,