CS1026 Hilary Term Lab 3

Aim: To find the minimal version of the function F, representing the input states of sensors of an engine which will cause the engine to enter an “eco-friendly” mode to comply with regulations. The function will be used to construct the most cost-effective circuit. F can be is defined as:

$$F = \Sigma m(0, 2, 5, 6, 9, 11)$$

The first step in reducing the function is to find its prime implicants. The function expressed as prime implicants is the function containing only terms which cannot be further reduced by algebraic methods (ie: Adjacency theorem).

In order to find the prime implicants we can arrange the true minterms (the combinations of inputs which output true) by the amount of 1’s in their binary representation. This allows us to easily see terms which can be reduced by adjacency theorem (those only differing by 1 bit).

	ABCD	Prime Implicants
m_0	0000	$\Sigma(m_0, m_2)$= A’B’D’
m_2	0010	---
m_5	0101	m_5 = A’BC’D
m_6	0110	$\Sigma(m_6, m_2)$= A’CD’
m_9	1001	---
m_{11}	1011	$\Sigma(m_{11}, m_9)$ = AB’D

Now we have reduced the function to:

$$F = A'B'D' + A'BC'D + A'CD' + AB'D$$

The prime implicants themselves cannot be reduced any further, however it may be possible that some of them are not necessary for the function to operate correctly, ie: the truth table may be unchanged by removing one or more of the prime implicants. If this is possible the function can be implemented even more efficiently.

We could find this out through trial and error, however this would be lengthly. Instead we will construct a table showing the outcome of each minterm on the equation, for each row marking which implicants output true.

For the equation to remain the same, each minterm must have at least one prime implicant which outputs true. (Note: the prime implicants are arranged as columns because it seems more intuitive in picturing the equation and its minterm inputs).

Minterms / **Prime Implicant**	**A'B'D'**	**A'BC'D**	**A'CD'**	**AB'D**
m_0				
m_2				
m_5				
m_6				
m_9				
m_{11}				

It is clear from the table that each prime implicant is necessary, because **for each prime implicant in *F* there is one minterm which only that implicant serves.**

Thus the function cannot be reduced further than its prime implicants, so the minimal-cost function is:

$$F = A'B'D' + A'BC'D + A'CD' + AB'D$$

Below is a diagram of the logic circuit to be used for the engine: