CS1026 Hilary Term Lab 2

Aim: Design a circuit to implement the function F using only NOR gates.

$$F = (X + Z)(Y' + Z)(X' + Y + Z)$$

The general form of a NOR gate is:

$$F = (A + B)'$$

Or for n inputs:

$$F = (E_1 + E_2 + \cdots + E_{n-1} + E_n)'$$

Where E_x is an element of the NOR function which can itself be a function (eg: $E_x = (Y' + Z)$) in which case we would further manipulate the element to the NOR form.

So to use the minimum number of gates we want to manipulate the above equation to be of that form. For simplicity, let:

(X+Z) = A,

(Y'+Z)=B,

(X'+Y+Z)=C.

Then:

$$F = A \cdot B \cdot C$$

First we invert the function and use De Morgan's theorem:

$$F' = A' + B' + C'$$

$$\Rightarrow F = (A' + B' + C')'$$

The above is in NOR form, so we can construct it simply using a NOR gate.

Undoing the substitutions gives:

$$F = ((X + Z)' + (Y' + Z)' + (X' + Y + Z)')'$$

and so the elements, happily, are also in NOR form. Since we can make a NOT gate by connecting the same input into a NOR gate, this manipulated function can be used to easily make a circuit of the function with minimal NOR gates.

We can use a truth table to ensure the algebra was applied correctly, and that the original and modified functions are equivalent.

XYZ	$(X + Z)(Y' + Z)(X' + Y + Z)$	$((X + Z)' + (Y' + Z)' + (X' + Y + Z)')'$
000	0	0
001	0	0
010	0	0
011	1	1
100	0	0
101	1	1
110	0	0
111	1	1

Finally, a diagram of the circuit: