Ratio Test

#Calculus_3 #series #year2

Overview

The ratio test is a powerful tool for determining the convergence of series, especially when other convergence tests like the divergence test or the integral test are inconclusive.

Theorem

Theorem 16 (Ratio Test).

Let $\sum a_n$ be a series with nonzero terms and let

$$\lim_{n o\infty}\left|rac{a_{n+1}}{a_n}
ight|=L$$

- If $L < 1 \implies convergent$
- If $L>1 \implies$ diverges
- ullet If $L=1 \implies$ inconclusive

Example

Example 1.

$$\sum_{n=1}^{\infty} \frac{n}{7^n}$$

Solution.

$$\begin{split} &\lim_{n \to \infty} \left| \frac{\frac{n+1}{7^{n+1}}}{\frac{n}{7^n}} \right| \\ &= \lim_{n \to \infty} \frac{n+1}{7^{n+1}} \cdot \frac{7^n}{n} \\ &= \lim_{n \to \infty} \frac{n+1}{7^n \cdot 7} \cdot \frac{7^n}{n} \\ &= \lim_{n \to \infty} \left| \frac{n+1}{7n} \right| = \frac{\infty}{\infty} \end{split}$$

lopital

$$\lim_{x \to \infty} \frac{1}{7} < 1$$
 . Convergent by ratio test

Example 2.

$$\sum_{n=1}^{\infty} \frac{2^n}{n!}$$

Solution.

$$egin{aligned} &\lim_{n o\infty}\left|rac{rac{2^{n+1}}{(n+1)!}}{rac{2^n}{n!}}
ight| \ &=rac{2^{arkappa'}\cdot 2}{(n+1)arkappa'}\cdotrac{arkappa'}{2^{arkappa'}} \ &=rac{2}{n+1}=rac{2}{\infty}=0 \end{aligned}$$

0 < 1: convergent by ratio test

Example 3.

$$\sum_{n=1}^{\infty}\frac{n^2(2^{n+1})}{3^n}$$

Solution.

$$\lim_{n o\infty}rac{rac{(n+1)^2(2^{n+2})}{3^{n+1}}}{rac{n^2(2^{n+1})}{3^n}}$$

$$egin{align} &=\lim_{n o\infty}rac{(n+1)^2(2^{n+1})2}{3^n\cdot 3}\cdotrac{3^n}{n^2(2^{n+1})} \ &=\lim_{n o\infty}rac{2(n+1)^2}{3n^2}=rac{\infty}{\infty} \ &= rac{1}{2} \$$

lopital

$$\lim_{x o\infty}rac{4(x+1)}{6x}=rac{\infty}{\infty}$$

lopital again

$$\lim_{x o\infty}rac{4}{6}=rac{4}{6}$$

 $\frac{4}{6} < 1$: convergent by ratio test

see also

Infinite Series Root Test