Ratio Test

#Calculus_3 #series #year2

Overview

The ratio test is a powerful tool for determining the convergence of series, especially when other convergence tests like the divergence test or the integral test are inconclusive .

Theorem

Theorem 16 (Ratio Test)**.**

Let $\sum a_n$ *be a series with nonzero terms and let*

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = L$$

- *If* $L < 1 \implies$ *convergent*
- *If* $L > 1 \implies$ *diverges*
- *If* $L = 1 \implies$ *inconclusive*

Example

Example 1.

$$\sum_{n=1}^{\infty} \frac{n}{7^n}$$

Solution.

$$\lim_{n\to\infty}\left|\frac{\frac{n+1}{7^{n+1}}}{\frac{n}{7^n}}\right|$$

$$= \lim_{n\to\infty}\frac{n+1}{7^{n+1}}\cdot\frac{7^n}{n}$$

$$= \lim_{n\to\infty}\frac{n+1}{\cancel{7^n}\cdot 7}\cdot\frac{\cancel{7^n}}{n}$$

$$= \lim_{n\to\infty}\left|\frac{n+1}{7n}\right| = \frac{\infty}{\infty}$$

lopital

$$\lim_{x\to\infty}\frac{1}{7} < 1 \therefore \text{convergent by ratio test}$$

Example 2.

$$\sum_{n=1}^{\infty}\frac{2^n}{n!}$$

Solution.

$$\lim_{n\to\infty}\left|\frac{\frac{2^{n+1}}{(n+1)!}}{\frac{2^n}{n!}}\right|$$

$$= \frac{\cancel{2^n}\cdot 2}{(n+1)\,\cancel{n!}}\cdot\frac{\cancel{n!}}{\cancel{2^n}}$$

$$= \frac{2}{n+1} = \frac{2}{\infty} = 0$$

$$0 < 1 \therefore \text{convergent by ratio test}$$

Example 3.

$$\sum_{n=1}^{\infty}\frac{n^2(2^{n+1})}{3^n}$$

Solution.

$$\lim_{n\to\infty}\frac{\frac{(n+1)^2(2^{n+2})}{3^{n+1}}}{\frac{n^2(2^{n+1})}{3^n}}$$

$$= \lim_{n\to\infty} \frac{(n+1)^2(\cancel{2^{n+1}})2}{\cancel{3^n}\cdot 3} \cdot \frac{\cancel{3^n}}{n^2(\cancel{2^{n+1}})}$$

$$= \lim_{n\to\infty} \frac{2(n+1)^2}{3n^2} = \frac{\infty}{\infty}$$

lopital

$$\lim_{x\to\infty} \frac{4(x+1)}{6x} = \frac{\infty}{\infty}$$

lopital again

$$\lim_{x\to\infty} \frac{4}{6} = \frac{4}{6}$$

$$\frac{4}{6} < 1 \therefore \text{convergent by ratio test}$$

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see also

Infinite Series
Root Test