Double integral

Overview

Double integration is integration of a function with two variables. It is an integration inside another. You can use integration techniques on the inside integral, then use integration techniques for the outside integral.

Definition

Definition 1 (Definite integrals of functions of two variables).

$$\int_{h_1(y)}^{h_2(y)} f_x(x,y)\,dx = f(x,y)\Big|_{h_1(y)}^{h_2(y)} = f(h_2(y),y) - f(h_1(y),y)$$

$$\int_{g_1(x)}^{g_2(x)} f_y(x,y)\,dy = f(x,y)\Big|_{g_1(x)}^{g_2(x)} = f(x,g_2(x)) - f(x,g_1(x))$$

> ✏️ **Note**
>
> When integrating with respect to x, the limit of integration are from y. Similarly, when integrating with respect to y, the limit of integration are from x.

Examples

Example 1.

Evaluate the integral

$$\int_{1}^{2y} 4xy\,dx$$

Solution.

$$\int_{1}^{2y} 4xy\,dx = 2x^2 y\Big|_{1}^{2y}$$

$$= 2(2y)^2 y - 2(1)^2 y$$

$$= 8y^3 - 2y$$

Example 2.

$$\int_1^x 3x^2 y^2 + 2xy \, dy$$

Solution.

$$\int_1^x 3x^2 y^2 + 2xy \, dy = x^2 y^3 + xy^2 \Big|_1^x$$

$$= x^2 (x)^3 + x(x)^2 - (x^2 (1)^3 + x(1)^2)$$

$$= x^5 + x^3 - x^2 - x$$