AMCS 312. Course project

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Introduction

The aim of this study is to explore how efficient could be Krylov-iterative methods implemented in PETSc solving hyperbolic equations with implicit time stepping, such as wave equations.

As a particular example we consider a simplified case, because it is sufficient for the performance comparison purposes. The framework is as follows:

Equation: Strong form of equation of motion

Discretization: Finite differences in time domain (FDTD)

Time stepping: Implicit

Accuracy: 2nd order schemes in space and time

Model: Homogeneous isotropic media with constant velocity

Programming: C/C++ with PETSc 3.7.3

3D acoustic wave equation

Acoustic wave equation for homogeneous media in strong formulation could be expressed as follows

$$\rho \frac{\partial^2 u}{\partial t^2} - c^2 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) = f \tag{1}$$

We discretize with 2nd order of accuracy both in space and time. Implicit time stepping requires backward time derivative approximation, whereas spacial derivatives are approximated with centered scheme.

$$\begin{aligned}
&\rho_{i,j,k} \frac{2U^{n+1}_{i,j,k} - 5U^{n}_{i,j,k} + 4U^{n-1}_{i,j,k} - U^{n-2}_{i,j,k}}{\Delta t^2} = \\
&c^2_{i,j,k} \frac{U^{n+1}_{i+1,j,k} - 2U^{n+1}_{i,j,k} + U^{n+1}_{i-1,j,k}}{\Delta x^2} + \\
&c^2_{i,j,k} \frac{U^{n+1}_{i,j+1,k} - 2U^{n+1}_{i,j,k} + U^{n+1}_{i,j-1,k}}{\Delta y^2} + \\
&c^2_{i,j,k} \frac{U^{n+1}_{i,j,k+1} - 2U^{n+1}_{i,j,k} + U^{n+1}_{i,j,k-1}}{\Delta z^2} + f_{i,j,k}
\end{aligned} \tag{2}$$

Hence,

$$
\begin{aligned}
&U^{n+1}_{i,j,k}\left(2\Delta x\Delta y\Delta z + 2\frac{c^2_{i,j,k}\Delta t^2}{\rho_{i,j,k}}\left(\frac{\Delta y\Delta z}{\Delta x} + \frac{\Delta x\Delta z}{\Delta y} + \frac{\Delta x\Delta y}{\Delta z}\right)\right) + \\
&U^{n+1}_{i+1,j,k}\left(-\frac{c^2_{i+1,j,k}\Delta t^2}{\rho_{i+1,j,k}}\ \frac{\Delta y\Delta z}{\Delta x}\right) + \\
&U^{n+1}_{i-1,j,k}\left(-\frac{c^2_{i-1,j,k}\Delta t^2}{\rho_{i-1,j,k}}\ \frac{\Delta y\Delta z}{\Delta x}\right) + \\
&U^{n+1}_{i,j+1,k}\left(-\frac{c^2_{i,j+1,k}\Delta t^2}{\rho_{i,j+1,k}}\ \frac{\Delta x\Delta z}{\Delta y}\right) + \\
&U^{n+1}_{i,j-1,k}\left(-\frac{c^2_{i,j-1,k}\Delta t^2}{\rho_{i,j-1,k}}\ \frac{\Delta x\Delta z}{\Delta y}\right) + \\
&U^{n+1}_{i,j,k+1}\left(-\frac{c^2_{i,j,k+1}\Delta t^2}{\rho_{i,j,k+1}}\ \frac{\Delta x\Delta y}{\Delta z}\right) + \\
&U^{n+1}_{i,j,k-1}\left(-\frac{c^2_{i,j,k-1}\Delta t^2}{\rho_{i,j,k-1}}\ \frac{\Delta x\Delta y}{\Delta z}\right) = \Delta x\Delta y\Delta z\left(5U^n_{i,j,k} - 4U^{n-1}_{i,j,k} + U^{n-2}_{i,j,k} + \frac{\Delta t^2}{\rho_{i,j,k}}f_{i,j,k}\right)
\end{aligned}
\tag{3}
$$