# Melt pond parameters

Enrico Calzavarini & Silvia C. Hirata

## Physical parameters

$$
\begin{aligned}
\beta_T &= -7.5264 \cdot 10^{-5}\ (^{\circ}C)^{-1} \text{ water thermal expansion coefficient between -1.7 and -0.17 °C} && (1)\\
\beta_S &= 8.076 \cdot 10^{-4}\ (psu)^{-1} \quad \text{salinity contraction coefficient} && (2)\\
\nu &= 1.8 \cdot 10^{-6} \quad \text{water kinematic viscosity} && (3)\\
\kappa_T &= 1.39 \cdot 10^{-7}\ m^2 s^{-1} \quad \text{thermal diffusivity of water} && (4)\\
\kappa_S &= 6.8 \cdot 10^{-10}\ m^2 s^{-1} \quad \text{molecular diffusivity of salt} && (5)\\
c_p &= 4.185\ kJ/(Kg^{\circ}K) \quad \text{specific heat of water} && (6)\\
L &= 333.5\ kJ/Kg \quad \text{latent heat of water} && (7)\\
g &= 9.81\ ms^{-2} \quad \text{gravity acceleration} && (8)\\
T_{ice} &= -\alpha\ S = -0.054\ \frac{^{\circ}C}{psu}\ S \quad \text{Slope of the liquid curve as a function of salt concentration} && (9)\\
& && (10)
\end{aligned}
$$

## Boundary conditions

$$
\begin{aligned}
T_{top} &= -0.17\ ^{\circ}C \quad \text{top temperature} && (11)\\
T_{bot} &= -1.7\ ^{\circ}C \quad \text{bottom temperature} && (12)\\
S_{top} &= 3.2\ psu \quad \text{top salt concentration} && (13)\\
S_{bot} &= 32\ psu \quad \text{bottom salt concentration} && (14)\\
H &= 0.5\ m \quad \text{height of the liquid layer} && (15)\\
& && (16)
\end{aligned}
$$

## Dimensionless parameters

$$
\begin{aligned}
Ra_T &= \frac{\beta_T\ g\ \Delta T\ H^3}{\nu\ \kappa_T} = 5.6 \cdot 10^8 \quad \text{thermal Rayleigh number} && (17)\\
Ra_S &= \frac{\beta_S\ g\ \Delta S\ H^3}{\nu\ \kappa_S} = 2.3 \cdot 10^{13} \quad \text{solutal Rayleigh number} && (18)\\
Pr &= \frac{\nu}{\kappa_T} = 13. \quad \text{Prandtl number} && (19)\\
Sc &= \frac{\nu}{\kappa_S} = 2647. \quad \text{Schmidt number} && (20)\\
Le &= \frac{\kappa_T}{\kappa_S} = 204. \quad \text{Lewis number} && (21)\\
Ste &= \frac{c_p\ \Delta T}{L} = 0.02 \quad \text{Stefan number} && (22)\\
\frac{\alpha\ \Delta S}{\Delta T} &= 1.016 && (23)
\end{aligned}
$$

## Equations for the pure fluid case without solidification

$$
\begin{aligned}
\partial_t \boldsymbol{u} + (\boldsymbol{u} \cdot \nabla)\boldsymbol{u} &= -\nabla p/\rho + \nu\ \Delta \boldsymbol{u} + \beta_T (T - T_0)\hat{\boldsymbol{z}} - \beta_S (S - S_0)\hat{\boldsymbol{z}} && (24)\\
\partial_t T + (\boldsymbol{u} \cdot \nabla) T &= \kappa_T\ \Delta T && (25)\\
\partial_t S + (\boldsymbol{u} \cdot \nabla) S &= \kappa_S\ \Delta S && (26)
\end{aligned}
$$

When nondimensionalizing lengths with $H$, velocities with $\kappa/H$, temperatures with $\Delta T$, salt concentration with $\Delta S$

$$\partial_t \boldsymbol{u} + (\boldsymbol{u}\cdot\nabla)\boldsymbol{u} = -\nabla p + Pr\left(\Delta\boldsymbol{u} + Ra_T\,\theta\,\hat{\boldsymbol{z}} - \frac{Ra_S}{Le}\,\sigma\,\hat{\boldsymbol{z}}\right) \tag{27}$$

$$\partial_t \theta + (\boldsymbol{u}\cdot\nabla)\theta = \Delta\theta + u_z \tag{28}$$

$$\partial_t \sigma + (\boldsymbol{u}\cdot\nabla)\sigma = \frac{1}{Le}\Delta\sigma + u_z \tag{29}$$

**Non-dimensional equations for the three-layer case without solidification**

$$\partial_t\left(\frac{\boldsymbol{u}}{\phi}\right) + \frac{1}{\phi}(\boldsymbol{u}\cdot\nabla)\frac{\boldsymbol{u}}{\phi} = \nabla\cdot\left(\frac{1}{\phi}\nabla\boldsymbol{u} - P\boldsymbol{I}\right) - \frac{1}{Da}\boldsymbol{u} + Gr_T T\hat{\boldsymbol{z}} + Gr_S S\hat{\boldsymbol{z}} \tag{30}$$

$$\partial_t T + (\boldsymbol{u}\cdot\nabla)T = \frac{1}{Pr}\nabla\cdot\left(\frac{\kappa_T}{\kappa_{Tf}}\nabla T\right) \tag{31}$$

$$\phi\partial_t S + (\boldsymbol{u}\cdot\nabla)S = \frac{1}{Sc}\nabla\cdot(\phi\nabla S) \tag{32}$$

**STEPS**

1) Validation of the stability analysis for RB problem with salinity field and no melting (Nield,1967)

$$\rho = 1 - \beta_T(T_{top} - T_{bot}) - \beta_S(S_{top} - S_{bot}) \tag{33}$$

$$Ra_T + Ra_S = 1707.765 \tag{34}$$

2) Validation of the melting: conduction (Hubert et al., 2008)
3) Validation of the melting: convection (benchmark problem DG and PQ)