Melt pond parameters

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Physical parameters

$$\beta_T = -7.5264 \cdot 10^{-5}\ (°C)^{-1} \text{ water thermal expansion coefficient between -1.7 and -0.17 °C} \quad (1)$$
$$\beta_S = 8.076 \cdot 10^{-4}\ (psu)^{-1} \quad \text{salinity contraction coefficient} \quad (2)$$
$$\nu = 1.8 \cdot 10^{-6} \quad \text{water kinematic viscosity} \quad (3)$$
$$\kappa_T = 1.39 \cdot 10^{-7}\ m^2 s^{-1} \quad \text{thermal diffusivity of water} \quad (4)$$
$$\kappa_S = 6.8 \cdot 10^{-10}\ m^2 s^{-1} \quad \text{molecular diffusivity of salt} \quad (5)$$
$$c_p = 4.185\ kJ/(Kg°K) \quad \text{specific heat of water} \quad (6)$$
$$L = 333.5\ kJ/Kg \quad \text{latent heat of water} \quad (7)$$
$$g = 9.81\ ms^{-2} \quad \text{gravity acceleration} \quad (8)$$
$$T_{ice} = -\alpha\ S = -0.054\ \frac{°C}{psu}\ S \quad \text{Slope of the liquid curve as a function of salt concentration} \quad (9)$$
$$(10)$$

Boundary conditions

$$T_{top} = -0.17\ °C \quad \text{top temperature} \quad (11)$$
$$T_{bot} = -1.7\ °C \quad \text{bottom temperature} \quad (12)$$
$$S_{top} = 3.2\ psu \quad \text{top salt concentration} \quad (13)$$
$$S_{bot} = 32\ psu \quad \text{bottom salt concentration} \quad (14)$$
$$H = 0.5\ m \quad \text{height of the liquid layer} \quad (15)$$
$$(16)$$

Dimensionless parameters

$$Ra_T = \frac{\beta_T\ g\ \Delta T\ H^3}{\nu\ \kappa_T} = 5.6 \cdot 10^8 \quad \text{thermal Rayleigh number} \quad (17)$$
$$Ra_S = \frac{\beta_S\ g\ \Delta S\ H^3}{\nu\ \kappa_S} = 2.3 \cdot 10^{13} \quad \text{solutal Rayleigh number} \quad (18)$$
$$Pr = \frac{\nu}{\kappa_T} = 13. \quad \text{Prandtl number} \quad (19)$$
$$Sc = \frac{\nu}{\kappa_S} = 2647. \quad \text{Schmidt number} \quad (20)$$
$$Le = \frac{\kappa_T}{\kappa_S} = 204. \quad \text{Lewis number} \quad (21)$$
$$Ste = \frac{c_p\ \Delta T}{L} = 0.02 \quad \text{Stefan number} \quad (22)$$
$$\frac{\alpha\ \Delta S}{\Delta T} = 1.016 \quad (23)$$

Equations for the pure fluid case without solidification

$$\partial_t \boldsymbol{u} + (\boldsymbol{u} \cdot \nabla)\boldsymbol{u} = -\nabla p/\rho + \nu\ \Delta \boldsymbol{u} + \beta_T (T - T_0)\hat{\boldsymbol{z}} - \beta_S (S - S_0)\hat{\boldsymbol{z}} \quad (24)$$
$$\partial_t T + (\boldsymbol{u} \cdot \nabla)T = \kappa_T\ \Delta T \quad (25)$$
$$\partial_t S + (\boldsymbol{u} \cdot \nabla)S = \kappa_S\ \Delta S \quad (26)$$

When nondimensionalizing lengths with H, velocities with κ/H, temperatures with ΔT, salt concentration with ΔS

$$\partial_t \boldsymbol{u} + (\boldsymbol{u}\cdot\nabla)\boldsymbol{u} = -\nabla p + Pr \left(\Delta \boldsymbol{u} + \ Ra_T\ \theta\ \hat{\boldsymbol{z}} - \frac{Ra_S}{Le}\ \sigma\ \hat{\boldsymbol{z}}\right) \tag{27}$$

$$\partial_t \theta + (\boldsymbol{u}\cdot\nabla)\theta = \Delta\theta + u_z \tag{28}$$

$$\partial_t \sigma + (\boldsymbol{u}\cdot\nabla)\sigma = \frac{1}{Le}\Delta\sigma + u_z \tag{29}$$

Non-dimensional equations for the three-layer case without solidification

$$\partial_t\left(\frac{\boldsymbol{u}}{\phi}\right) + \frac{1}{\phi}(\boldsymbol{u}\cdot\nabla)\frac{\boldsymbol{u}}{\phi} = \nabla\cdot\left(\frac{1}{\phi}\nabla\boldsymbol{u} - P\boldsymbol{I}\right) - \frac{1}{Da}\boldsymbol{u} + Gr_T T\hat{\boldsymbol{z}} + Gr_S S\hat{\boldsymbol{z}} \tag{30}$$

$$\partial_t T + (\boldsymbol{u}\cdot\nabla)T = \frac{1}{Pr}\nabla\cdot\left(\frac{\kappa_T}{\kappa_{Tf}}\nabla T\right) \tag{31}$$

$$\phi\partial_t S + (\boldsymbol{u}\cdot\nabla)S = \frac{1}{Sc}\nabla\cdot(\phi\nabla S) \tag{32}$$

STEPS

1) Validation of the stability analysis for RB problem with salinity field and no melting (Nield,1967)

$$\rho = 1 - \beta_T(T_{top} - T_{bot}) - \beta_S(S_{top} - S_{bot}) \tag{33}$$

$$Ra_T + Ra_S = 1707.765 \tag{34}$$

2) Validation of the melting: conduction (Hubert et al., 2008)
3) Validation of the melting: convection (benchmark problem DG and PQ)