# Національний технічний університет України «Київський політехнічний інститут імені Ігоря Сікорського» Факультет інформатики та обчислювальної техніки Кафедра обчислювальної техніки

## Методи наукових досліджень

Лабораторна робота №5

## «ПРОВЕДЕННЯ ТРЬОХФАКТОРНОГО ЕКСПЕРИМЕНТУ ПРИ ВИКОРИСТАННІ РІВНЯННЯ РЕГРЕСІЇ З УРАХУВАННЯМ КВАДРАТИЧНИХ ЧЛЕНІВ(ЦЕНТРАЛЬНІЙ ОРТОГОНАЛЬНИЙ КОМПОЗИЦІЙНИЙ ПЛАН)»

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Мета роботи: Провести трьохфакторний експеримент з урахуванням квадратичних членів ,використовуючи центральний ортогональний композиційний план. Знайти рівняння регресії, яке буде адекватним для опису об'єкту.

### Завдання

- 1. Взяти рівняння з урахуванням квадратичних членів.
- 2. Скласти матрицю планування для ОЦКП
- Провести експеримент у всіх точках факторного простору (знайти значення функції відгуку Y). Значення функції відгуку
  знайти у відповідності з варіантом діапазону, зазначеного далі. Варіанти вибираються по номеру в списку в журналі
  викладача.

$$\begin{aligned} y_{\rm rmax} &= 200 + x_{\rm cpmax} \\ y_{\rm rmin} &= 200 + x_{\rm cpmin} \end{aligned}$$
 Где  $x_{\rm cpmax} = \frac{x_{\rm 1max} + x_{\rm 2max} + x_{\rm 3max}}{3}, \; x_{\rm cpmin} = \frac{x_{\rm 1min} + x_{\rm 2min} + x_{\rm 3min}}{3} \end{aligned}$ 

- 4. Розрахувати коефіцієнти рівняння регресії і записати його.
- Провести 3 статистичні перевірки.

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## Лістинг програми:

```
from random import randint
from numpy.linalg import det
from copy import deepcopy
from scipy.stats import t
def Naturalize(MatrixOfPlan, MinMaxArr, flag):
    result = []
    for i in range(len(MatrixOfPlan)):
            result.append(MinMaxArr[1]) if MatrixOfPlan[i] == 1 else
result.append(MinMaxArr[0])
            x0 = (max(MinMaxArr) + min(MinMaxArr)) / 2
            dx = x0 - min(MinMaxArr)
            value = None
            if flag == 1:
                value = MatrixOfPlan[i] * dx + x0 if i == 8 or 9 else x0
            elif flag == 2:
                value = MatrixOfPlan[i] * dx + x0 if i == 10 or 11 else x0
            elif flag == 3:
                value = MatrixOfPlan[i] * dx + x0 if i == 12 or 13 else x0
            result.append(value)
    return result
def Cocharan(y_arr, y_avg, m, N):
    dispersion = []
    for i in range(len(y_arr[0])):
        current_sum = 0
        for j in range(len(y_arr)):
            current_sum += (y_arr[j][i] - y_avg[j]) ** 2
        dispersion.append(current_sum / len(y_arr))
    print('dispersion:', dispersion)
    gp = max(dispersion) / sum(dispersion)
    print('Gp =', gp)
```

```
# Рівень значимості q = 0.05
    if gp < 0.3346:
        return dispersion
def Students(plan1x0, plan1x1, plan1x2, plan1x3, y_avg_arr, dispersion, m):
    # Оцінка значимості коефіцієнтів регресії згідно критерію Стьюдента
    s2b = sum(dispersion) / 15
    s2bs_avg = s2b / 15 * m
sb = s2bs_avg ** (1 / 2)
    beta arr = [
        sum([y_avg_arr[i] * plan1x0[i] for i in range(15)]) / 15,
sum([y_avg_arr[i] * plan1x1[i] for i in range(15)]) / 15,
        sum([y_avg_arr[i] * plan1x2[i] for i in range(15)]) / 15,
        sum([y_avg_arr[i] * plan1x3[i] for i in range(15)]) / 15,
        sum([y_avg_arr[i] * plan1x1[i] * plan1x2[i] for i in range(15)]) / 15,
        sum([y_avg_arr[i] * plan1x1[i] * plan1x3[i] for i in range(15)]) / 15,
        sum([y_avg_arr[i] * plan1x2[i] * plan1x3[i] for i in range(15)]) / 15,
        sum([y_avg_arr[i] * plan1x1[i] * plan1x2[i] * plan1x3[i] for i in range(15)])
/ 15,
        sum([y_avg_arr[i] * plan1x1[i] ** 2 for i in range(15)]) / 15,
        sum([y_avg_arr[i] * plan1x2[i] ** 2 for i in range(15)]) / 15,
        sum([y_avg_arr[i] * plan1x3[i] ** 2 for i in range(15)]) / 15
    print('beta:', beta_arr)
    t_arr = [abs(beta_arr[i]) / sb for i in range(11)]
    print('t:', t_arr)
    # f3 = f1*f2 = 2*15 = 30
    f1 = m - 1
    f3 = f1 * f2
    b_arr = []
    for i in range(len(t_arr)):
        if t_arr[i] > t.ppf(q=0.975, df=f3):
            b_arr.append(t_arr[i])
            print(f'Koe\phiiцieht b{i} приймаемо не значним')
            b_arr.append(0)
    return b_arr, s2b
def Fisher(b_arr, s2b, y_avg, y_res, m):
    d = len([i for i in b_arr if i != 0]) # кількість значимих коефіцієнтів
    s2_ad = m * sum([(y_res[i] - y_avg[i]) ** 2 for i in range(15)]) / 15 - d
    fp = s2 ad / s2b
    print(f'Fp = {fp}')
```

```
if fp > 2.1:
        print('Рівняння регресії неадекватно оригіналу при рівні значимості 0.05')
def main(m):
   x2 = [-4, 10]
   x3 = [-10, 10]
    # Величина зоряного плеча
    1 = 1.215
   plan1x0 = [1 for _ in range(N)]
plan1x1 = [-1, -1, 1, 1, -1, 1, 1, -1, 1, 0, 0, 0, 0, 0]
plan1x2 = [-1, 1, -1, 1, -1, 1, -1, 1, 0, 0, -1, 1, 0, 0, 0]
    plan1x3 = [1, -1, -1, 1, -1, 1, -1, 0, 0, 0, 0, -1, 1, 0]
    print('x1:', plan1x1)
print('x2:', plan1x2)
    print('-' * 100)
    plan2x1 = Naturalize(plan1x1, x1, 1)
    plan2x2 = Naturalize(plan1x2, x2, 2)
    plan2x3 = Naturalize(plan1x3, x3, 3)
    plan2x4 = [plan2x1[i] * plan2x2[i] for i in range(len(plan2x1))]
    plan2x5 = [plan2x1[i] * plan2x3[i] for i in range(len(plan2x1))]
    plan2x6 = [plan2x2[i] * plan2x3[i] for i in range(len(plan2x1))]
    plan2x7 = [plan2x1[i] * plan2x2[i] * plan2x3[i] for i in range(len(plan2x1))]
    # Квадратичні значення факторів
    plan2x8 = [plan2x1[i] ** 2 for i in range(len(plan2x1))]
    plan2x9 = [plan2x2[i] ** 2 for i in range(len(plan2x1))]
    plan2x10 = [plan2x3[i] ** 2 for i in range(len(plan2x1))]
    print(f'x1: {plan2x1}')
    print(f'x2: {plan2x2}'
    print(f'x3: {plan2x3}')
    print(f'x4: {plan2x4}')
    print(f'x5: {plan2x5}')
    print(f'x6: {plan2x6}')
    print(f'x7: {plan2x7}')
    print(f'x8: {plan2x8}')
    print(f'x9: {plan2x9}')
    print(f'x10: {plan2x10}')
    x_avg_max = (max(plan2x1) + max(plan2x2) + max(plan2x3)) / 3
    x_avg_min = (min(plan2x1) + min(plan2x2) + min(plan2x3)) / 3
    print()
    print(f'x_avg_max = {x_avg_max}')
    print(f'x avg min = {x avg min}')
    print('-' * 100)
```

```
y_max = int(200 + x_avg_max)
    y_min = int(200 + x_avg_min)
    print(f'y_max = {y_max}'
    print(f'y_min = {y_min}')
    print()
    y_arr = [[randint(y_min, y_max) for _ in range(N)] for _ in range(m)]
    for i in range(len(y_arr)):
        print(f'y{i + 1}: {y_arr[i]}')
    y_avg = []
    for i in range(len(y_arr[0])):
        current_sum = 0
        for j in range(len(y_arr)):
            current_sum += y_arr[j][i]
        y_avg.append(current_sum / len(y_arr))
    print('y average:', y_avg)
print('-' * 100)
    dispersion = Cocharan(y_arr, y_avg, m, N)
    if dispersion:
        mx1 = sum(plan2x1) / len(plan2x1)
        mx2 = sum(plan2x2) / len(plan2x2)
        mx3 = sum(plan2x3) / len(plan2x3)
        mx4 = sum(plan2x4) / len(plan2x4)
        mx5 = sum(plan2x5) / len(plan2x5)
        mx6 = sum(plan2x6) / len(plan2x6)
        mx7 = sum(plan2x7) / len(plan2x7)
        mx8 = sum(plan2x8) / len(plan2x8)
mx9 = sum(plan2x9) / len(plan2x9)
        mx10 = sum(plan2x10) / len(plan2x10)
        my = sum(y_avg) / len(y_avg)
        a1 = sum([y_avg[i] * plan2x1[i] for i in range(len(plan2x1))]) / len(plan2x1)
        a11 = mx8
        a12 = mx4
        a13 = mx5
        a14 = sum([plan2x1[i] * plan2x4[i] for i in range(len(plan2x1))]) /
len(plan2x1)
        a15 = sum([plan2x1[i] * plan2x5[i] for i in range(len(plan2x1))]) /
len(plan2x1)
        a16 = sum([plan2x1[i] * plan2x6[i] for i in range(len(plan2x1))]) /
len(plan2x1)
        a17 = sum([plan2x1[i] * plan2x7[i] for i in range(len(plan2x1))]) /
len(plan2x1)
        a18 = sum([plan2x1[i] * plan2x8[i] for i in range(len(plan2x1))]) /
len(plan2x1)
        a19 = sum([plan2x1[i] * plan2x9[i] for i in range(len(plan2x1))]) /
len(plan2x1)
        a2 = sum([y_avg[i] * plan2x2[i] for i in range(len(plan2x1))]) / len(plan2x2)
        a21 = a12
        a22 = mx9
        a23 = mx6
        a24 = sum([plan2x2[i] * plan2x4[i] for i in range(len(plan2x2))]) /
len(plan2x2)
        a25 = sum([plan2x2[i] * plan2x5[i] for i in range(len(plan2x2))]) /
len(plan2x2)
        a26 = sum([plan2x2[i] * plan2x6[i] for i in range(len(plan2x2))]) /
len(plan2x2)
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```
a27 = sum([plan2x2[i] * plan2x7[i] for i in range(len(plan2x2))]) /
len(plan2x2)
        a28 = sum([plan2x2[i] * plan2x8[i] for i in range(len(plan2x2))]) /
len(plan2x2)
        a29 = sum([plan2x2[i] * plan2x9[i] for i in range(len(plan2x2))]) /
len(plan2x2)
        a3 = sum([y_avg[i] * plan2x3[i] for i in range(len(plan2x3))]) / len(plan2x3)
        a31 = a13
        a32 = a23
        a33 = mx10
        a34 = sum([plan2x3[i] * plan2x4[i] for i in range(len(plan2x3))]) /
len(plan2x3)
        a35 = sum([plan2x3[i] * plan2x5[i] for i in range(len(plan2x3))]) /
len(plan2x3)
        a36 = sum([plan2x3[i] * plan2x6[i] for i in range(len(plan2x3))]) /
len(plan2x3)
        a37 = sum([plan2x3[i] * plan2x7[i] for i in range(len(plan2x3))]) /
len(plan2x3)
        a38 = sum([plan2x3[i] * plan2x8[i] for i in range(len(plan2x3))]) /
len(plan2x3)
        a39 = sum([plan2x3[i] * plan2x9[i] for i in range(len(plan2x3))]) /
len(plan2x3)
        a4 = sum([y_avg[i] * plan2x4[i] for i in range(len(plan2x4))]) / len(plan2x4)
        a41 = a14
        a42 = a24
        a43 = a34
        a44 = sum([plan2x4[i] ** 2 for i in range(len(plan2x4))]) / len(plan2x4)
        a45 = sum([plan2x4[i] * plan2x5[i] for i in range(len(plan2x4))]) /
len(plan2x4)
        a46 = sum([plan2x4[i] * plan2x6[i] for i in range(len(plan2x4))]) /
len(plan2x4)
        a47 = sum([plan2x4[i] * plan2x7[i] for i in range(len(plan2x4))]) /
len(plan2x4)
        a48 = sum([plan2x4[i] * plan2x8[i] for i in range(len(plan2x4))]) /
len(plan2x4)
        a49 = sum([plan2x4[i] * plan2x9[i] for i in range(len(plan2x4))]) /
len(plan2x4)
        a5 = sum([y_avg[i] * plan2x5[i] for i in range(len(plan2x5))]) / len(plan2x5)
        a51 = a15
        a52 = a25
        a53 = a35
        a54 = a45
        a55 = sum([plan2x5[i] ** 2 for i in range(len(plan2x5))]) / len(plan2x5)
        a56 = sum([plan2x5[i] * plan2x6[i] for i in range(len(plan2x5))]) /
len(plan2x5)
        a57 = sum([plan2x5[i] * plan2x7[i] for i in range(len(plan2x5))]) /
len(plan2x5)
        a58 = sum([plan2x5[i] * plan2x8[i] for i in range(len(plan2x5))]) /
len(plan2x5)
        a59 = sum([plan2x5[i] * plan2x9[i] for i in range(len(plan2x5))]) /
len(plan2x5)
        a6 = sum([y_avg[i] * plan2x6[i] for i in range(len(plan2x6))]) / len(plan2x6)
        a61 = a16
        a62 = a26
        a63 = a36
        a64 = a46
        a65 = a56
        a66 = sum([plan2x6[i] ** 2 for i in range(len(plan2x6))]) / len(plan2x6)
```

```
a67 = sum([plan2x6[i] * plan2x7[i] for i in range(len(plan2x6))]) /
len(plan2x6)
        a68 = sum([plan2x6[i] * plan2x8[i] for i in range(len(plan2x6))]) /
len(plan2x6)
        a69 = sum([plan2x6[i] * plan2x9[i] for i in range(len(plan2x6))]) /
len(plan2x6)
        a7 = sum([y_avg[i] * plan2x7[i] for i in range(len(plan2x7))]) / len(plan2x7)
        a71 = a17
        a72 = a27
        a73 = a37
        a74 = a47
        a75 = a57
        a76 = a67
        a77 = sum([plan2x7[i] ** 2 for i in range(len(plan2x7))]) / len(plan2x7)
        a78 = sum([plan2x7[i] * plan2x8[i] for i in range(len(plan2x7))]) /
len(plan2x7)
        a79 = sum([plan2x7[i] * plan2x9[i] for i in range(len(plan2x7))]) /
len(plan2x7)
        a8 = sum([y_avg[i] * plan2x8[i] for i in range(len(plan2x8))]) / len(plan2x8)
        a81 = a18
        a82 = a28
        a83 = a38
        a84 = a48
        a85 = a58
        a86 = a68
        a87 = a78
        a88 = sum([plan2x8[i] ** 2 for i in range(len(plan2x8))]) / len(plan2x8)
        a89 = sum([plan2x8[i] * plan2x9[i] for i in range(len(plan2x8))]) /
len(plan2x8)
        a9 = sum([y_avg[i] * plan2x9[i] for i in range(len(plan2x9))]) / len(plan2x9)
        a91 = a19
        a92 = a29
        a93 = a39
        a94 = a49
        a95 = a59
        a96 = a69
        a97 = a79
        a98 = a89
        a99 = sum([plan2x9[i] ** 2 for i in range(len(plan2x9))]) / len(plan2x9)
        a10 = sum([y_avg[i] * plan2x10[i] for i in range(len(plan2x10))]) /
len(plan2x10)
        a101 = sum([plan2x10[i] * plan2x1[i] for i in range(len(plan2x10))]) /
len(plan2x10)
        a102 = sum([plan2x10[i] * plan2x2[i] for i in range(len(plan2x10))]) /
len(plan2x10)
        a103 = sum([plan2x10[i] * plan2x3[i] for i in range(len(plan2x10))]) /
len(plan2x10)
        a104 = sum([plan2x10[i] * plan2x4[i] for i in range(len(plan2x10))]) /
len(plan2x10)
        a105 = sum([plan2x10[i] * plan2x5[i] for i in range(len(plan2x10))]) /
len(plan2x10)
        a106 = sum([plan2x10[i] * plan2x6[i] for i in range(len(plan2x10))]) /
len(plan2x10)
        a107 = sum([plan2x10[i] * plan2x7[i] for i in range(len(plan2x10))]) /
len(plan2x10)
        a108 = sum([plan2x10[i] * plan2x8[i] for i in range(len(plan2x10))]) /
len(plan2x10)
        a109 = sum([plan2x10[i] * plan2x9[i] for i in range(len(plan2x10))])
```

```
len(plan2x10)
        a1010 = sum([plan2x10[i] ** 2 for i in range(len(plan2x10))]) / len(plan2x10)
        main_determinant = [[1, mx1, mx2, mx3, mx4, mx5, mx6, mx7, mx8, mx9, mx10],
                             [mx1, a11, a21, a31, a41, a51, a61, a71, a81, a91, a101]
                             [mx2, a12, a22, a32, a42, a52, a62, a72, a82, a92, a102]
                             [mx3, a13, a23, a33, a43, a53, a63, a73, a83, a93, a103]
                             [mx4, a14, a24, a34, a44, a54, a64, a74, a84, a94, a104]
                             [mx5, a15, a25, a35, a45, a55, a65, a75, a85, a95, a105]
                             [mx6, a16, a26, a36, a46, a56, a66, a76, a86, a96, a106]
                             [mx7, a17, a27, a37, a47, a57, a67, a77, a87, a97, a107]
                             [mx8, a18, a28, a38, a48, a58, a68, a78, a88, a98, a108]
                             [mx9, a19, a29, a39, a49, a59, a69, a79, a89, a99, a109]
                             [mx10, a101, a102, a103, a104, a105, a106, a107, a108,
a109, a1010]]
        column_to_change = [my, a1, a2, a3, a4, a5, a6, a7, a8, a9, a10]
        main_determinant_value = det(main_determinant)
        matrices = []
        for i in range(len(main_determinant[0])):
            new_matrix = deepcopy(main_determinant)
            for j in range(len(main_determinant)):
                new_matrix[j][i] = column_to_change[j]
            matrices.append(new_matrix)
        b list = []
        for i in range(len(matrices)):
            b_list.append(det(matrices[i]) / main_determinant_value)
        print('-' * 100)
        print(f'b: {b_list}')
        y_list = []
        for i in range(len(plan2x1)):
            y = b_list[0] + b_list[1] * plan2x1[i] + b_list[2] * plan2x2[i] +
b_list[3] * plan2x3[i] +\
                b_list[4] * plan2x4[i] + b_list[5] * plan2x5[i] + b_list[6] *
plan2x6[i] + b_list[7] * plan2x7[i] +\
                b_list[8] * plan2x8[i] + b_list[9] * plan2x9[i] + b_list[10] *
plan2x10[i]
            y_list.append(y)
        print(f'y = {y}; y avg = {y_avg[i]}')
print('-' * 100)
        t arr, s2b = Students(plan1x0, plan1x1, plan1x2, plan1x3, y avg, dispersion,
m)
        b_arr = []
        for i in range(len(b_list)):
            b = b_list[i] if t_arr[i] != 0 else 0
            b_arr.append(b)
        print('-' * 100)
        y_res = []
        for i in range(N):
            y = b_arr[0] + b_arr[1] * plan1x1[i] + b_arr[2] * plan1x2[i] + b_arr[3] *
plan1x3[i] + 
                b_arr[4] * plan1x1[i] * plan1x2[i] + b_arr[5] * plan1x1[i] *
plan1x3[i] + 
                b_arr[6] * plan1x2[i] * plan1x3[i] + b_arr[7] * plan1x1[i] *
plan1x2[i] * plan1x3[i] +\
                b arr[8] * plan1x1[i] ** 2 + b arr[9] * plan1x2[i] ** 2 + b arr[10] *
```

Результат виконання програми:

```
x1: [-6, -6, 1, 1, -6, -6, 1, 1, -6.7525, 1.752500000000000004, -2.5, -2.5, -2.5, -2.5, -2.5]
x2: [-4, 10, -4, 10, -4, 10, -4, 10, 3.0, 3.0, -5.50500000000000, 11.505, 3.0, 3.0, 3.0, 3.0]
x3: [10, -10, -10, 10, -10, 10, 10, -10, 0.0, 0.0, 0.0, 0.0, -12.15, 12.15, 0.0]
x4: [24, -60, -4, 10, 24, -60, -4, 10, -20.2575, 5.25750000000000, 13.7625000000000, -28.76250
x5: [-60, 60, -10, 10, 60, -60, 10, -10, -0.0, 0.0, -0.0, -0.0, -0.3, 0.375, -0.0]
x6: [-40, -100, 40, 100, 40, 100, -40, -100, 0.0, -0.0, -0.0, -0.0, -36.45, 36.45, 0.0]
x7: [240, 600, 40, 100, -240, -600, -40, -100, -0.0, 0.0, -0.0, -0.125, -91.125, -9.1]
x8: [36, 36, 1, 1, 36, 36, 1, 1, 45.59025625, 3.0712562500000000, 6.25, 6.25, 6.25, 6.25, 6.25]
x9: [16, 100, 16, 100, 16, 100, 16, 100, 9.0, 9.0, 9.0, 0.0, 147.6225, 147.6225, 0.0]
  b: [202.32831513871685, 0.4152661752827493, 0.11717/
y = 203.03514440882708; y avg = 193.0
y = 195.973497887362; y avg = 196.66666666666666
y = 197.71477697201328; y avg = 196.6666666666666666
y = 197.71477697201328; y avg = 197.3333333333333
y = 195.71098672416996; y avg = 195.3333333333333
y = 199.96416928019838; y avg = 201.0
y = 201.3726799080462; y avg = 203.0
y = 201.8928086662064; y avg = 203.0
y = 201.8928086662064; y avg = 203.0
y = 203.2136631140807; y avg = 202.33333333333333
y = 199.79031256730977; y avg = 202.333333333333334
y = 199.79031256730977; y avg = 202.6666666666666
y = 17.98282657102573; y avg = 195.3333333333334
y = 199.7903125673983; y avg = 198.0
y = 201.45037036259794; y avg = 204.3333333333333334
        beta: [199.8666666666667, 0.39688888888893]3, -0.54311111111111117, 0.939777777777776, -0.022222222222255, -0.8222222222222, -1.044444444444, -0.600000000000000, 146.116272222222222, 145 t: [79.79313869181006, 0.15845068457146297, 0.2168772525714743, 0.375189218954486, 0.008871818844986917, 0.32825729726450686, 0.4169754857143715, 0.2395391088146422, 58.334269385714066, 58.0461397
        Коефіцієнт b1 приймаємо не значним
Коефіцієнт b2 приймаємо не значним
Коефіцієнт b3 приймаємо не значним
Коефіцієнт b4 приймаємо не значним
           \hat{\mathbf{v}} = 202.3295202665002
           Рівняння регресії адекватно оригіналу при рівні значимості 0.05
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