Inference for numerical data

North Carolina births

In 2004, the state of North Carolina released a large data set containing information on births recorded in this state. This data set is useful to researchers studying the relation between habits and practices of expectant mothers and the birth of their children. We will work with a random sample of observations from this data set.

Exploratory analysis

Load the `nc` data set into our workspace.

```
load("more/nc.RData")
```

We have observations on 13 different variables, some categorical and some numerical. The meaning of each variable is as follows.

variable	description
`fage`	father's age in years.
`mage`	mother's age in years.
`mature`	maturity status of mother.
`weeks`	length of pregnancy in weeks.
`premie`	whether the birth was classified as premature (premie) or full-term.
`visits`	number of hospital visits during pregnancy.
`marital`	whether mother is `married` or `not married` at birth.
`gained`	weight gained by mother during pregnancy in pounds.
`weight`	weight of the baby at birth in pounds.

variable	description
`lowbirthweight`	whether baby was classified as low birthweight (`low`) or not (`not low`).
`gender`	gender of the baby, `female` or `male`.
`habit`	status of the mother as a `nonsmoker` or a `smoker`.
`whitemom`	whether mom is `white` or `not white`.

1. What are the cases in this data set? How many cases are there in our sample?

Exercise 1 Answer:

Each case is the information recorded about a birth during 2004 in North Carolina. There are 1,000 cases in the sample.

As a first step in the analysis, we should consider summaries of the data. This can be done using the `summary` command:

```
summary(nc)
```

```
##       fage            mage            mature        weeks
##  Min.   :14.00   Min.   :13   mature mom :133   Min.   :20.00
##  1st Qu.:25.00   1st Qu.:22   younger mom:867   1st Qu.:37.00
##  Median :30.00   Median :27                     Median :39.00
##  Mean   :30.26   Mean   :27                     Mean   :38.33
##  3rd Qu.:35.00   3rd Qu.:32                     3rd Qu.:40.00
##  Max.   :55.00   Max.   :50                     Max.   :45.00
##  NA's   :171                                    NA's   :2
##        premie        visits            marital        gained
##  full term:846   Min.   : 0.0   married    :386   Min.   : 0.00
##  premie   :152   1st Qu.:10.0   not married:613   1st Qu.:20.00
##  NA's     :  2   Median :12.0   NA's       :  1   Median :30.00
##                  Mean   :12.1                     Mean   :30.33
##                  3rd Qu.:15.0                     3rd Qu.:38.00
##                  Max.   :30.0                     Max.   :85.00
##                  NA's   :9                        NA's   :27
##      weight       lowbirthweight    gender          habit
##  Min.   : 1.000   low    :111    female:503   nonsmoker:873
##  1st Qu.: 6.380   not low:889    male  :497   smoker   :126
##  Median : 7.310                                NA's     :  1
##  Mean   : 7.101
##  3rd Qu.: 8.060
##  Max.   :11.750
##
```

```
##       whitemom
##  not white:284
##  white    :714
##  NA's     :  2
##
##
##
##
```

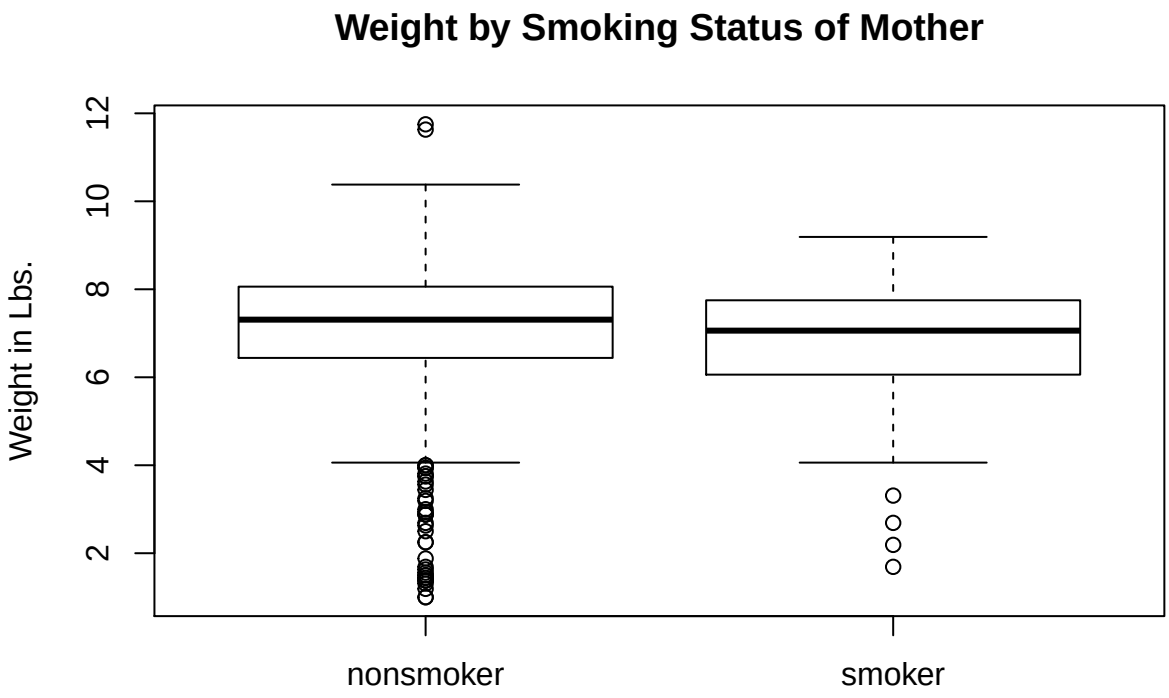
As you review the variable summaries, consider which variables are categorical and which are numerical. For numerical variables, are there outliers? If you aren't sure or want to take a closer look at the data, make a graph.

Consider the possible relationship between a mother's smoking habit and the weight of her baby. Plotting the data is a useful first step because it helps us quickly visualize trends, identify strong associations, and develop research questions.

2. Make a side-by-side boxplot of `habit` and `weight`. What does the plot highlight about the relationship between these two variables?

Exercise 2 Answer:

```
boxplot(weight~habit,data=nc, main = "Weight by Smoking Status of Mother", ylab = "Weight in Lbs.")
```



The medians, min, and interquartile ranges seem fairly similar, but the max is considerably higher for nonsmoking mothers. There are some high-side outliers for nonsmoking mothers. There are also way more data points for nonsmoking mothers (873 observations) than for smoking mothers (126 observations).

The box plots show how the medians of the two distributions compare, but we can also compare the means of the distributions using the following function to split the `weight` variable into the `habit` groups, then take the mean of each using the `mean` function.

```
by(nc$weight, nc$habit, mean)
```

```
## nc$habit: nonsmoker
## [1] 7.144273
## ------------------------------------------------------------
## nc$habit: smoker
## [1] 6.82873
```

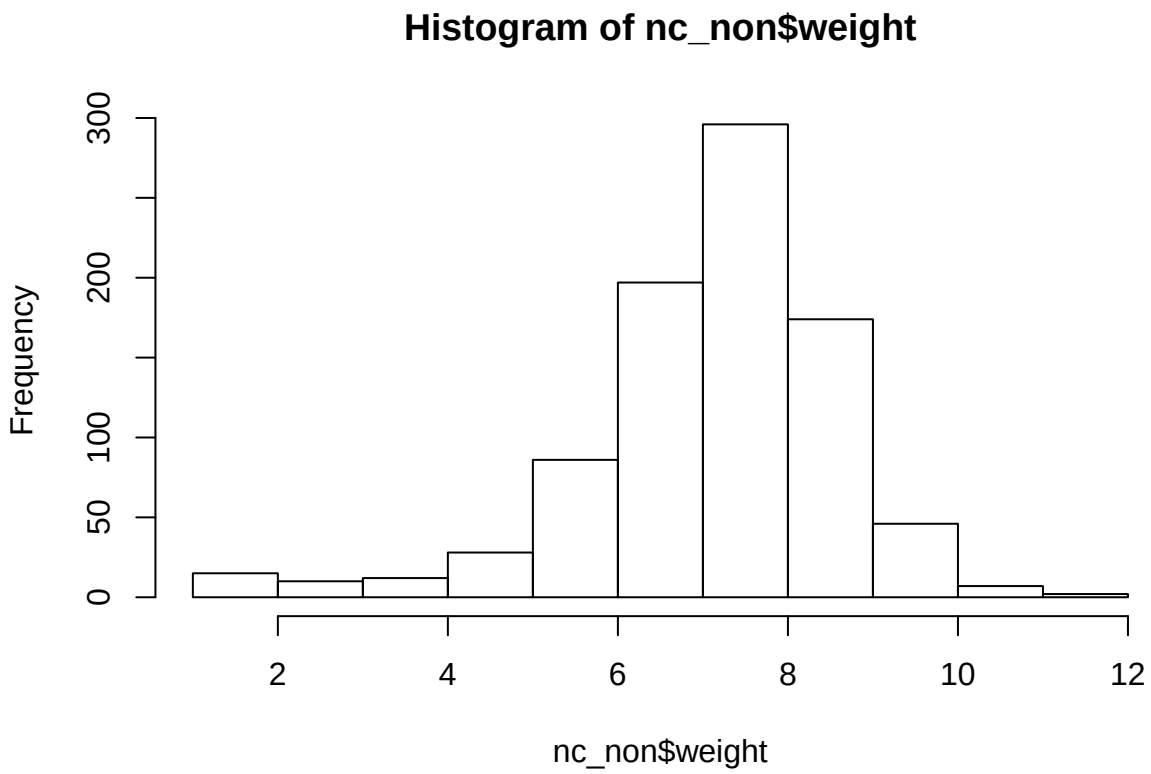
There is an observed difference, but is this difference statistically significant? In order to answer this question we will conduct a hypothesis test .

Inference

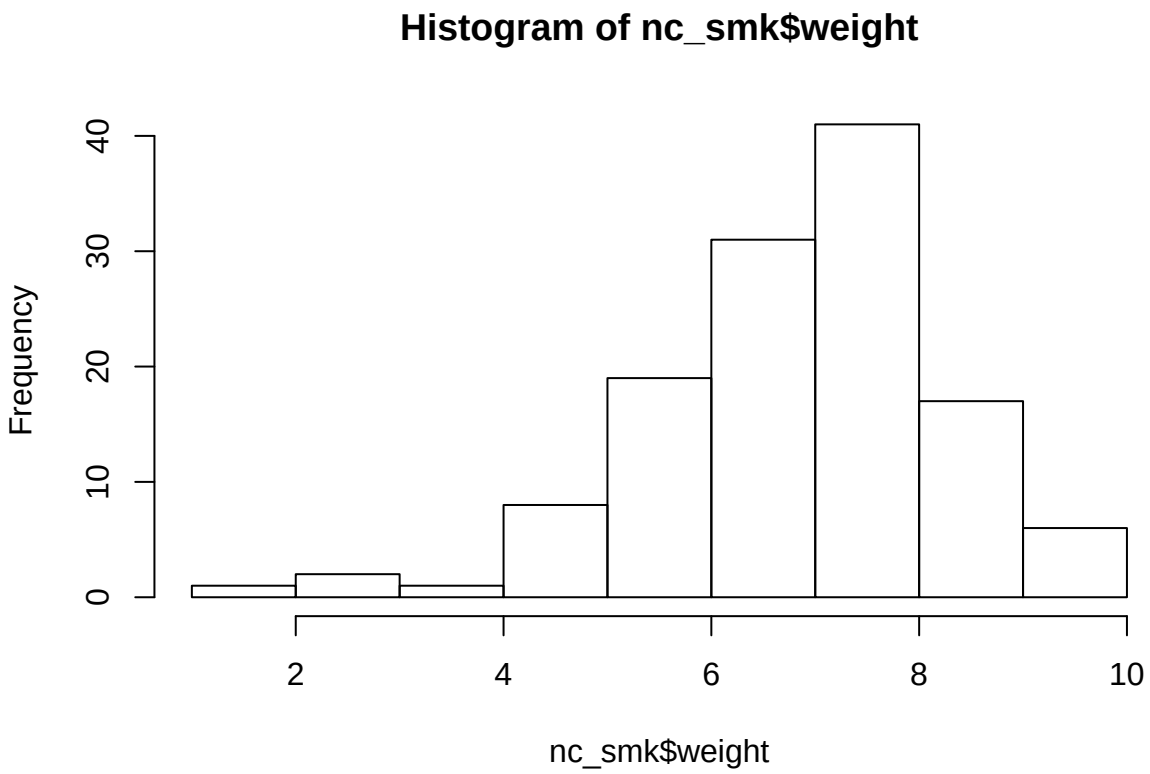
3. Check if the conditions necessary for inference are satisfied. Note that you will need to obtain sample sizes to check the conditions. You can compute the group size using the same `by` command above but replacing `mean` with `length`.

Exercise 3 Answer:
First we check if the observations are independent. The data is from a simple random sample and consists of less than 10% of all cases (presumably there were 10,000 or more births in North Carolina in 2004). Next, we make sure both distributions are normal. See histograms and notes below:

```
# Subset nonsmoker and smoker observations
nc_non <- subset(nc, habit == "nonsmoker")
nc_smk <- subset(nc, habit == "smoker")
# Create histograms
hist(nc_non$weight)
```




```
hist(nc_smk$weight)
```



Though both histograms exhibit some skew, the sample sizes are reasonably large enough that statistical inference analyses can be pursued.

4. Write the hypotheses for testing if the average weights of babies born to smoking and non-smoking mothers are different.

Exercise 4 Answer:

H_0: $\mu_{diff} = 0$ There is no difference in average birth weights for children born from mothers who did or did not smoke (null hypothesis)

H_A: $\mu_{diff} \neq 0$ There is a difference in average birth weights for children born from mothers who did or did not smoke (alternative hypothesis)

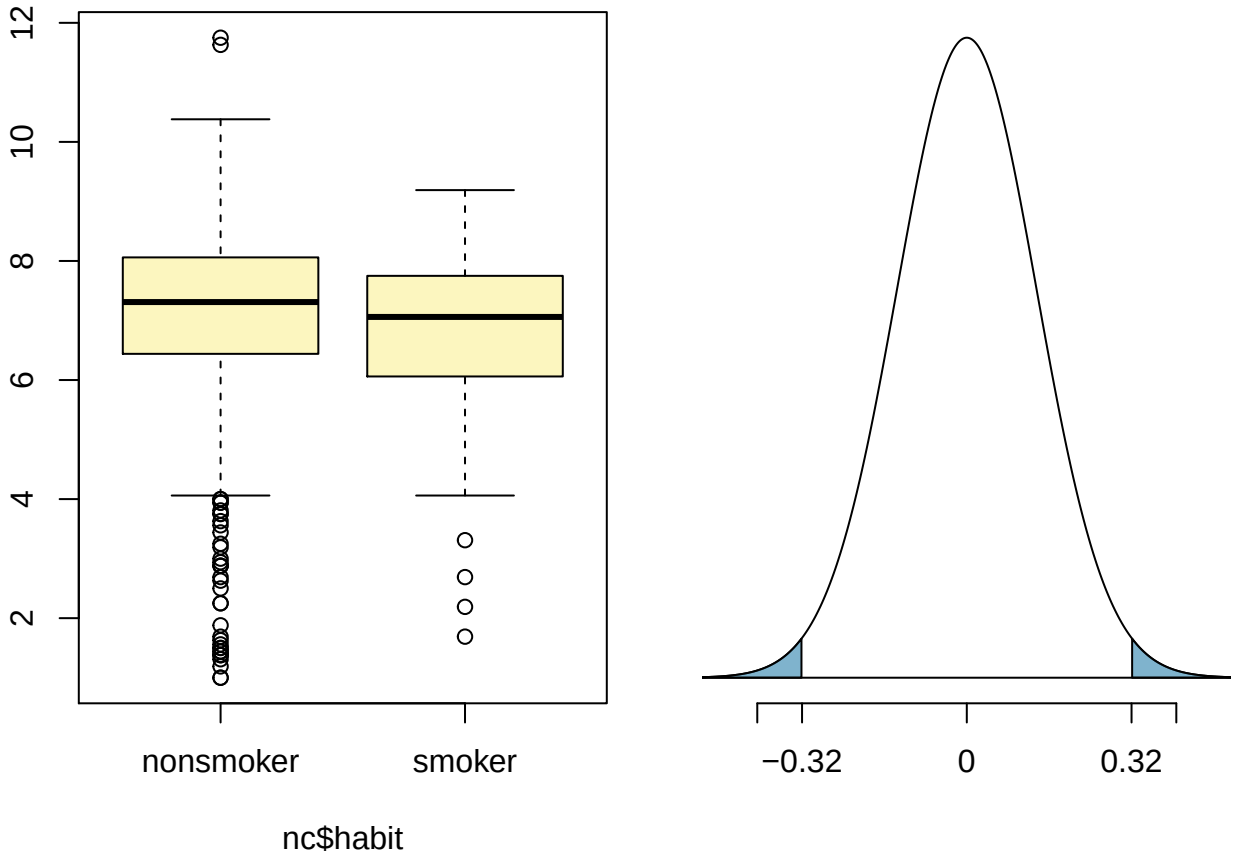
This is s a two-tailed test, because the research question is asking if there is a difference in either direction.

Next, we introduce a new function, `inference`, that we will use for conducting hypothesis tests and constructing confidence intervals.

```
inference(y = nc$weight, x = nc$habit, est = "mean", type = "ht", null = 0,
          alternative = "twosided", method = "theoretical")
```

```
## Response variable: numerical, Explanatory variable: categorical
## Difference between two means
## Summary statistics:
## n_nonsmoker = 873, mean_nonsmoker = 7.1443, sd_nonsmoker = 1.5187
## n_smoker = 126, mean_smoker = 6.8287, sd_smoker = 1.3862
```

```
## Observed difference between means (nonsmoker-smoker) = 0.3155
##
## H0: mu_nonsmoker - mu_smoker = 0
## HA: mu_nonsmoker - mu_smoker != 0
## Standard error = 0.134
## Test statistic: Z =  2.359
## p-value =  0.0184
```



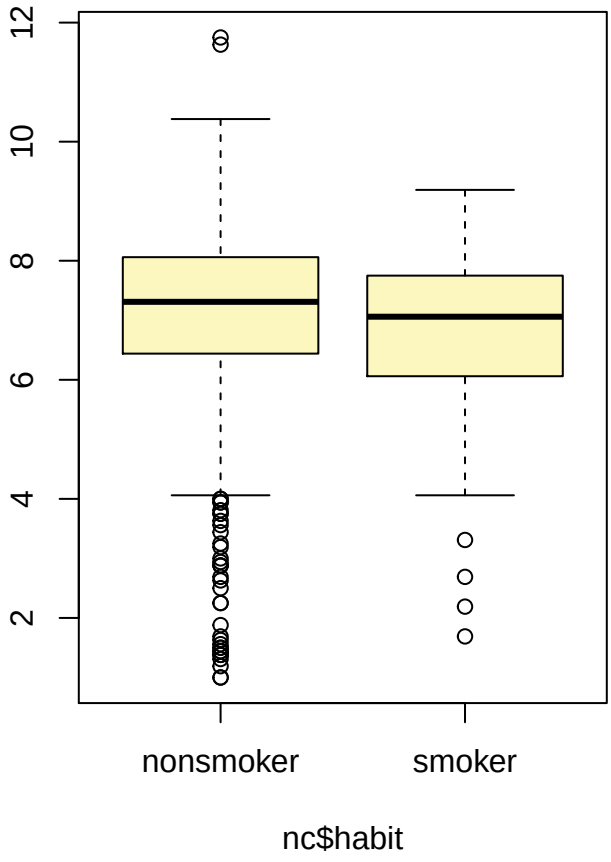
Let's pause for a moment to go through the arguments of this custom function. The first argument is `y`, which is the response variable that we are interested in: `nc$weight`. The second argument is the explanatory variable, `x`, which is the variable that splits the data into two groups, smokers and non-smokers: `nc$habit`. The third argument, `est`, is the parameter we're interested in: `"mean"` (other options are `"median"`, or `"proportion"`.) Next we decide on the `type` of inference we want: a hypothesis test (`"ht"`) or a confidence interval (`"ci"`). When performing a hypothesis test, we also need to supply the `null` value, which in this case is `0`, since the null hypothesis sets the two population means equal to each other. The `alternative` hypothesis can be `"less"`, `"greater"`, or `"twosided"`. Lastly, the `method` of inference can be `"theoretical"` or `"simulation"` based.

5. Change the `type` argument to `"ci"` to construct and record a confidence interval for the difference between the weights of babies born to smoking and non-smoking mothers.

Exercise 5 Answer:

```
inference(y = nc$weight, x = nc$habit, est = "mean", type = "ci", null = 0,
          alternative = "twosided", method = "theoretical")
```

```
## Response variable: numerical, Explanatory variable: categorical
## Difference between two means
## Summary statistics:
## n_nonsmoker = 873, mean_nonsmoker = 7.1443, sd_nonsmoker = 1.5187
## n_smoker = 126, mean_smoker = 6.8287, sd_smoker = 1.3862
```

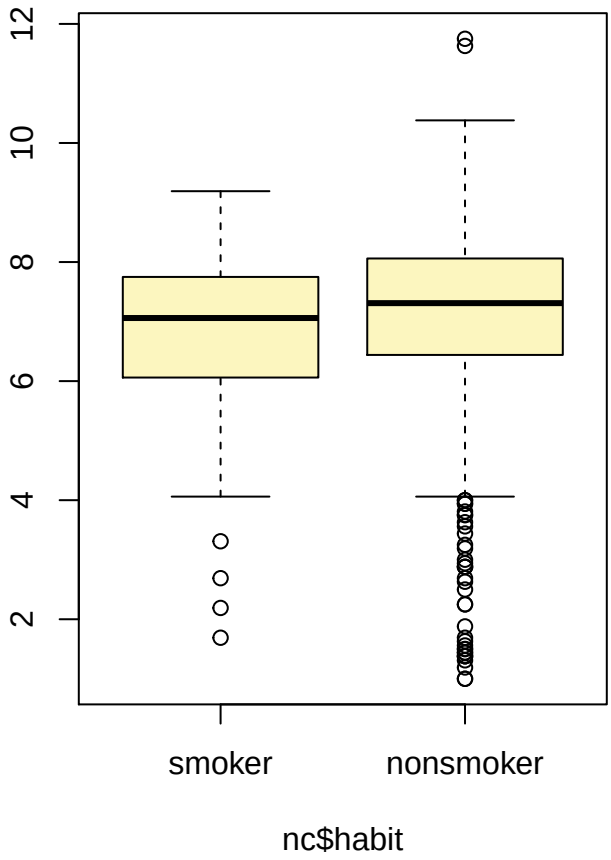



```
## Observed difference between means (nonsmoker-smoker) = 0.3155
##
## Standard error = 0.1338
## 95 % Confidence interval = ( 0.0534 , 0.5777 )
```

By default the function reports an interval for $(\mu_{nonsmoker} - \mu_{smoker})$. We can easily change this order by using the `order` argument:

```
inference(y = nc$weight, x = nc$habit, est = "mean", type = "ci", null = 0,
          alternative = "twosided", method = "theoretical",
          order = c("smoker","nonsmoker"))
```

```
## Response variable: numerical, Explanatory variable: categorical
## Difference between two means
## Summary statistics:
## n_smoker = 126, mean_smoker = 6.8287, sd_smoker = 1.3862
## n_nonsmoker = 873, mean_nonsmoker = 7.1443, sd_nonsmoker = 1.5187
```




```
## Observed difference between means (smoker-nonsmoker) = -0.3155
##
## Standard error = 0.1338
## 95 % Confidence interval = ( -0.5777 , -0.0534 )
```

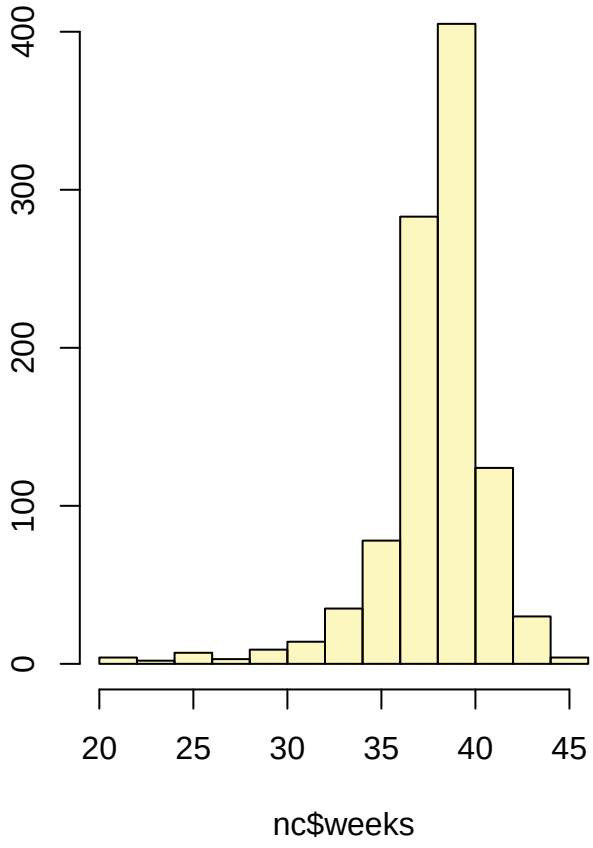
On your own

- Calculate a 95% confidence interval for the average length of pregnancies (`weeks`) and interpret it in context. Note that since you're doing inference on a single population parameter, there is no explanatory variable, so you can omit the `x` variable from the function.
- Calculate a new confidence interval for the same parameter at the 90% confidence level. You can change the confidence level by adding a new argument to the function: `conflevel = 0.90`.
- Conduct a hypothesis test evaluating whether the average weight gained by younger mothers is different than the average weight gained by mature mothers.
- Now, a non-inference task: Determine the age cutoff for younger and mature mothers. Use a method of your choice, and explain how your method works.
- Pick a pair of numerical and categorical variables and come up with a research question evaluating the relationship between these variables. Formulate the question in a way that it can be answered using a hypothesis test and/or a confidence interval. Answer your question using the `inference` function, report the statistical results, and also provide an explanation in plain language.

Number 1 Answer:

```
inference(y = nc$weeks, est = "mean", type = "ci", null = 0,
          alternative = "twosided", method = "theoretical")
```

```
## Single mean
## Summary statistics:
```



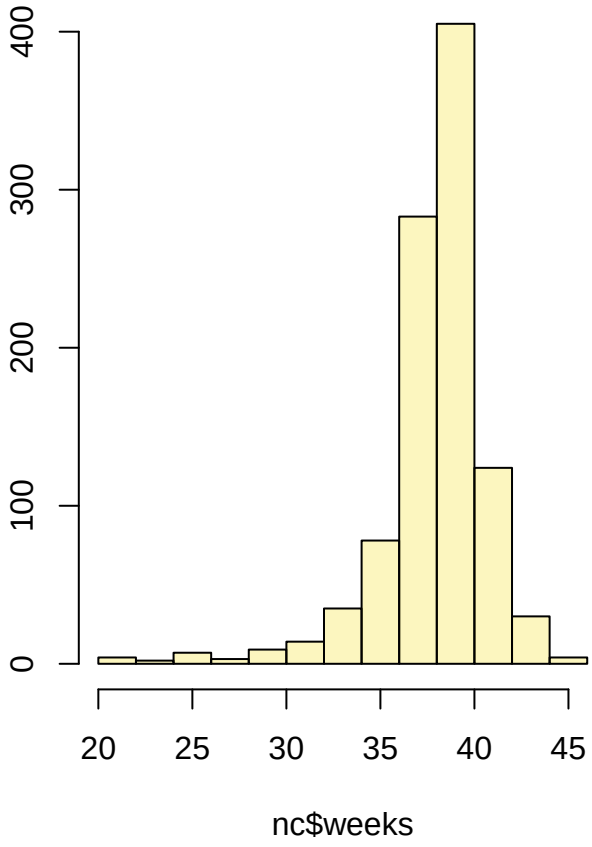

```
## mean = 38.3347 ;  sd = 2.9316 ;  n = 998
## Standard error = 0.0928
## 95 % Confidence interval = ( 38.1528 , 38.5165 )
```

Interpretation: We can be 95% confident that the pregnancy length in weeks for a mother is between 38.1528 and 38.5165.

Number 2 Answer:

```
inference(y = nc$weeks, est = "mean", type = "ci", null = 0,
          alternative = "twosided", method = "theoretical", conflevel = 0.90)
```

```
## Single mean
## Summary statistics:
```



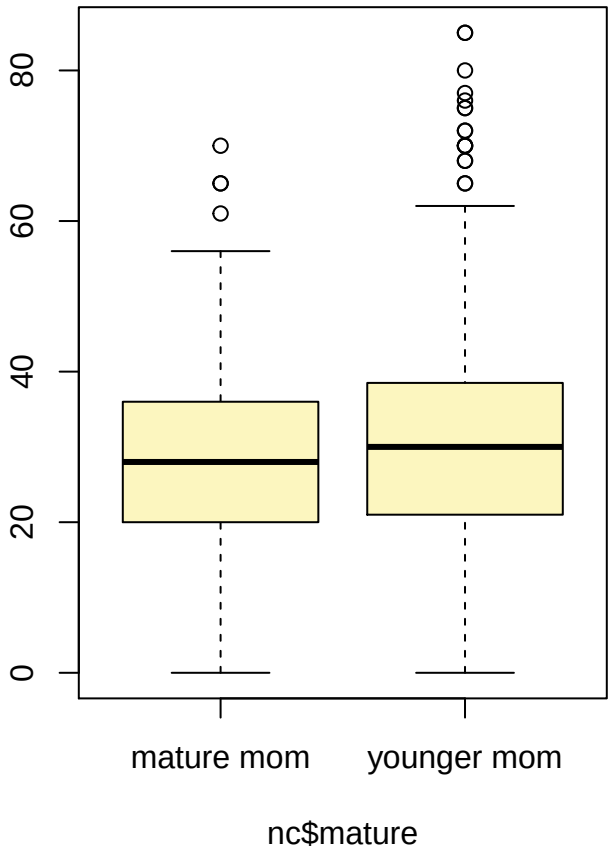

```
## mean = 38.3347 ;  sd = 2.9316 ;  n = 998
## Standard error = 0.0928
## 90 % Confidence interval = ( 38.182 , 38.4873 )
```

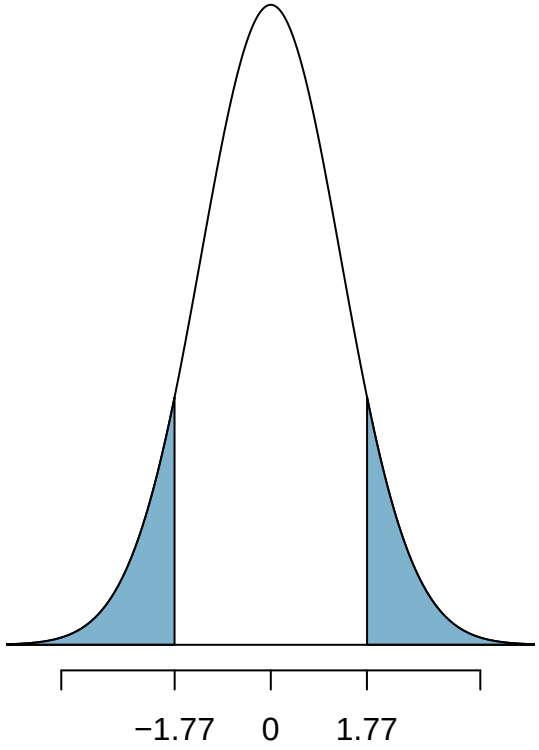
Number 3 Answer:

```
inference(y = nc$gained, x = nc$mature, est = "mean", type = "ht", null = 0,
          alternative = "twosided", method = "theoretical")
```

```
## Response variable: numerical, Explanatory variable: categorical
## Difference between two means
## Summary statistics:
## n_mature mom = 129, mean_mature mom = 28.7907, sd_mature mom = 13.4824
## n_younger mom = 844, mean_younger mom = 30.5604, sd_younger mom = 14.3469

## Observed difference between means (mature mom-younger mom) = -1.7697
##
## H0: mu_mature mom - mu_younger mom = 0
## HA: mu_mature mom - mu_younger mom != 0
## Standard error = 1.286
## Test statistic: Z =  -1.376
## p-value =  0.1686
```





Because the p-value exceeds the α value, we fail to reject the null hypothesis. The data does not provide strong evidence that the weight gain during pregnancy differed for younger mothers vs. mature mothers.

Number 4 Answer:

```
# Create data frame containing observations with younger moms
nc_younger <- subset(nc, mature == "younger mom")
# Create data frame of obsverations with younger moms
nc_mature <- subset(nc, mature == "mature mom")
# Run summary statistics for mage for both data frames
summary(nc_younger$mage)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##   13.00   21.00   25.00   25.44   30.00   34.00
```

```
summary(nc_mature$mage)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##   35.00   35.00   37.00   37.18   38.00   50.00
```

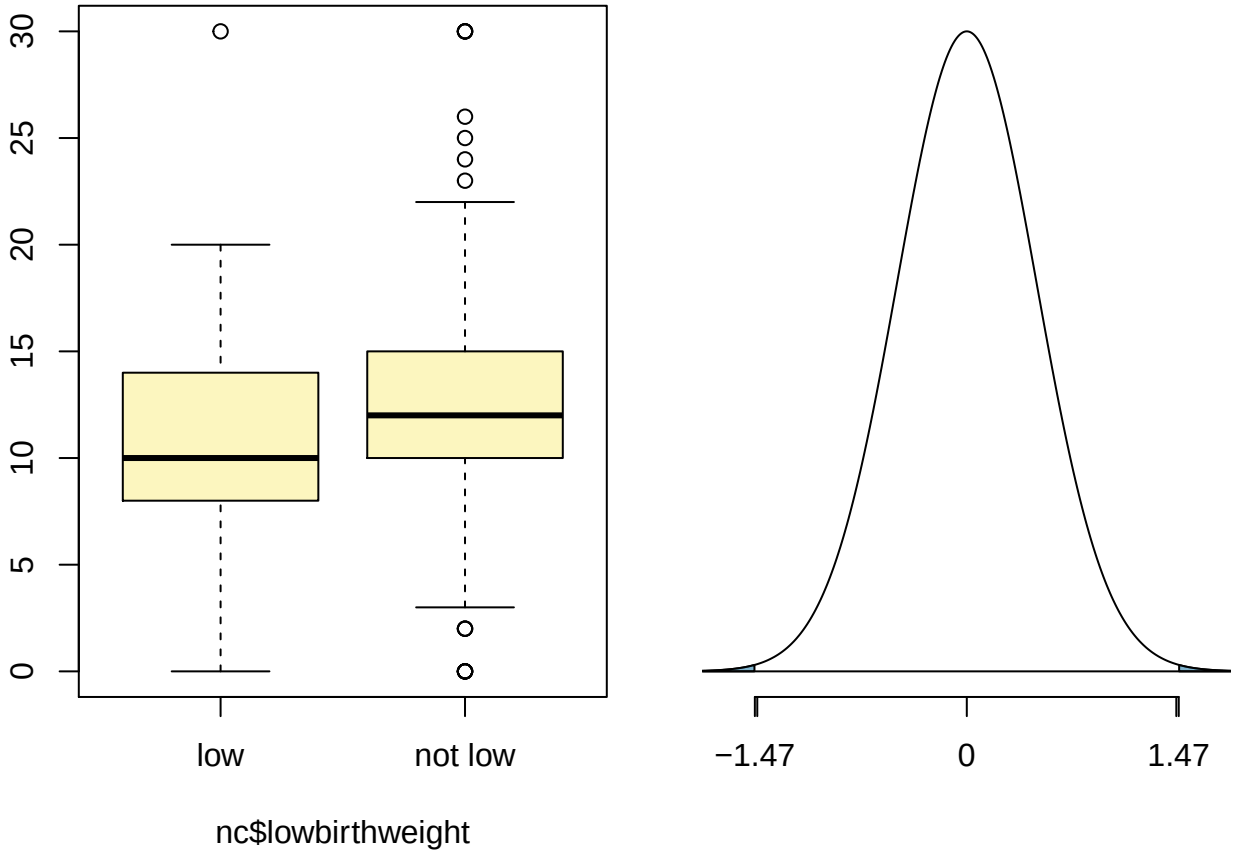
```
# Max age for younger moms is 34, and min age for mature moms is 35, so the
# age cut-off for younger and mature mothers is 35. If the mother is 35 or
# older, she is classified as mature, otherwise she is classified as younger
```

Number 5 Answer:

```r
inference(y = nc$visits, x = nc$lowbirthweight, est = "mean", type = "ht", null = 0, alternative = "two
```

```
## Response variable: numerical, Explanatory variable: categorical
## Difference between two means
## Summary statistics:
## n_low = 108, mean_low = 10.7963, sd_low = 4.8506
## n_not low = 883, mean_not low = 12.265, sd_not low = 3.8036

## Observed difference between means (low-not low) = -1.4687
##
## H0: mu_low - mu_not low = 0
## HA: mu_low - mu_not low != 0
## Standard error = 0.484
## Test statistic: Z =  -3.035
## p-value =  0.0024
```



The inference test indicates that the number of hospital visits during the pregnancy was significantly lower for mothers who gave birth to low weight babies than the number of hospital visits during the pregnancy for mothers who gave birth to babies which were not low weight.