COS30019 - Introduction to Artificial Intelligence
Tutorial Problems Week 8

Task 1: Represent the following sentences in first-order logic, using a consistent vocabulary (which you must define):

Vocabulary:
Constants:
iAI: a *constant* denoting (subject) iAI;
OOP: a *constant* denoting (subject) OOP;
S1Y2001: a *constant* denoting (semester) S1 of year 2001;
UK: a *constant* denoting (the country) UK;

Predicates:
Student**(x): x** is a student;
Take**(u, v, w): u** takes (subject) **v** in (semester) **w**;
Pass**(u, v, w): u** passes (subject) **v** in (semester) **w;**
Semester**(t): t** is a semester;
Person**(x): x** is a person;
Agent**(x): x** is an agent;
Policy**(x): x** is a policy;
Smart**(x): x** is smart;
Buy**(x,y): x** buys **y**;
SellTo**(x,p,y): x** sells (a policy) **p** to **y**;
Barber**(x): x** is a barber;
Man**(x): x** is a man;
InTown**(x): x** is in town;
Shave**(x,y): x** shaves **y**;
BornIn**(x, y): x** was born in (the country) **y**;
Parent**(x, y): x** is a parent of **y**;
Citizen**(x, y): x** is a citizen of (the country) **y**;
Resident**(x, y): x** is a resident of (the country) **y**;
CitizenByBirth**(x, y): x** is a citizen by birth of (the country) **y**;
CitizenByDescent**(x, y): x** is a citizen by descent of (the country) **y**;
Politician**(x): x** is a politician;
Occasion**(x): x** is an occasion;
FoolOnOccasion**(x, y, z): x** fools **y** on the occasion **z**;

Functions:
bestScore(**x**,**y**): a *function* that takes input (subject) **x** and (semester) **y** and returns the best score in subject **x** in semester **y**;

a. Some students took iAI in Semester 1 of 2001.
   $\exists$x Student(x) $\wedge$ Take(x, iAI, S1Y2001)

   $\exists$S

b. Every student who takes OOP passes it.
   $\forall$x (Student(x) $\Rightarrow$ $\forall$t (Semester(t) $\wedge$ Take(x, OOP, t)) $\Rightarrow$ Pass(x, OOP, t)))

   $\forall$x(Student(x) $\wedge$ Takes(x, OOP) => Pass(x, OOP))

c. Only one student took OOP in Semester 1 of 2001.

$\exists x\ (Student(x) \wedge Take(x, OOP, S1Y2001) \wedge \forall y\ (Student(y) \wedge Take(y, OOP, S1Y2001) \Rightarrow x=y))))$

OR

$\exists x\ (Student(x) \wedge Take(x, OOP, S1Y2001) \wedge \neg\exists y\ (Student(y) \wedge Take(y, OOP, S1Y2001) \wedge \neg(x=y)))))$

d. The best score in iAI is always higher than the best score in OOP.

$\forall t\ (Semester(t) \Rightarrow >(bestScore(iAI, t), bestScore(OOP, t)))$

e. Every person who buys a policy is smart.

$\forall x\ (Person(x) \wedge \exists y\ (Policy(y) \wedge Buy(x, y)) \Rightarrow Smart(x))$

ExpensivePolicy(x)

f. No person buys an expensive policy.

$\neg\exists x\ (Person(x) \wedge \exists y\ (Policy(y) \wedge Expensive(y) \wedge Buy(x,y)))$

OR

$\forall x\ (Person(x) \Rightarrow \neg\exists y\ (Policy(y) \wedge Expensive(y) \wedge Buy(x,y)))$

g. There is an agent who sells policies only to people who are not insured.
(anyone who buys policy from this agent is not insured)

$\exists x\ Agent(x) \wedge (\forall y\ Person(y) \wedge (\exists p\ Policy(p) \wedge SellTo(x, p, y)) \Rightarrow \neg Insured(y))$

h. There is a barber who shaves all men in town who do not shave themselves.

$\exists x\ Barber(x) \wedge (\forall y\ (Man(y) \wedge InTown(y) \wedge \neg Shave(y, y)) \Rightarrow Shave(x, y))$

i. A person born in the UK, each of whose parents is a UK citizen or a UK resident, is a UK citizen by birth.

$\forall x\ (Person(x) \wedge BornIn(x, UK) \wedge (\forall y\ Parent(y, x) \Rightarrow (Citizen(y, UK) \vee Resident(y, UK))) \Rightarrow CitizenByBirth(x, UK))$

j. A person born outside the UK, one of whose parents is a UK citizen by birth, is a UK citizen by descent.

$\forall x\ (Person(x) \wedge \neg BornIn(x, UK) \wedge (\exists y\ Parent(y, x) \wedge CitizenByBirth(y, UK))) \Rightarrow CitizenByDescent(x, UK)$

k. Politicians can fool some of the people all of the time, and they can fool all of the people some of the time, but they can't fool all of the people all of the time.

This is a very important example. It shows that natural language (English in this case) is ambiguous. With this sentence, there are multiple ways to interpret it. FOL is unambiguous. It gives you a single interpretation only:

Interpretation 1:

$$\begin{aligned}&\forall x\ \text{Politician}(x) \Rightarrow\\&[(\exists y\ \text{Person}(y) \wedge (\forall t\ \text{Occasion}(t) \Rightarrow \text{FoolOnOccasion}(x, y, t))) \wedge\\&(\forall y\ \text{Person}(y) \Rightarrow (\exists t\ \text{Occasion}(t) \wedge \text{FoolOnOccasion}(x, y, t))) \wedge\\&\neg(\forall y.\forall t\ \text{Person}(y) \wedge \text{Occasion}(t) \Rightarrow \text{FoolOnOccasion}(x, y, t))]\end{aligned}$$

Interpretation 2:

$$\begin{aligned}&\forall x\ \text{Politician}(x) \Rightarrow\\&[(\forall t\ \text{Occasion}(t) \Rightarrow (\exists y\ \text{Person}(y) \wedge \text{FoolOnOccasion}(x, y, t))) \wedge\\&(\forall y\ \text{Person}(y) \Rightarrow (\exists t\ \text{Occasion}(t) \wedge \text{FoolOnOccasion}(x, y, t))) \wedge\\&\neg(\forall y.\forall t\ \text{Person}(y) \wedge \text{Occasion}(t) \Rightarrow \text{FoolOnOccasion}(x, y, t))]\end{aligned}$$

Interpretation 3:

$$\begin{aligned}&\forall x\ \text{Politician}(x) \Rightarrow\\&[(\exists y\ \text{Person}(y) \wedge (\forall t\ \text{Occasion}(t) \Rightarrow \text{FoolOnOccasion}(x, y, t))) \wedge\\&(\exists t\ \text{Occasion}(t) \wedge (\forall y\ \text{Person}(y) \Rightarrow \text{FoolOnOccasion}(x, y, t))) \wedge\\&\neg(\forall y.\forall t\ \text{Person}(y) \wedge \text{Occasion}(t) \Rightarrow \text{FoolOnOccasion}(x, y, t))]\end{aligned}$$

Interpretation 4:

$$\begin{aligned}&\forall x\ \text{Politician}(x) \Rightarrow\\&[(\forall t\ \text{Occasion}(t) \Rightarrow (\exists y\ \text{Person}(y) \wedge \text{FoolOnOccasion}(x, y, t))) \wedge\\&(\exists t\ \text{Occasion}(t) \wedge (\forall y\ \text{Person}(y) \Rightarrow \text{FoolOnOccasion}(x, y, t))) \wedge\\&\neg(\forall y.\forall t\ \text{Person}(y) \wedge \text{Occasion}(t) \Rightarrow \text{FoolOnOccasion}(x, y, t))]\end{aligned}$$

Can you see the differences between the above? Essentially, each of the sentences: "Politicians can fool some of the people all of the time" AND "they can fool all of the people some of the time" could have two possible interpretations.

Let's take the first sentence: "Politicians can fool some of the people all of the time" – let's consider a politician SM, in the first interpretation, we can find some people, say BV, who believes in SM so much that ALL the time this naïve person BV is always fooled by SM (every time)! In another interpretation, no such naïve person exists, but on EVERY occasion (i.e. ALL THE TIME), our politician SM is so convincing that he always manages to fool someone; that is, today, he manages to fool BV, but tomorrow even though BV is no longer fooled by him but he manages to fool another person, say CH, and then the day after tomorrow, another person MA, etc.

We have a similar problem with the English sentence: “they can fool all of the people some of the time.”

Task 2: Represent the sentence “All Germans speak the same languages” in predicate calculus. Use *Speaks(x, l)*, meaning that person *x* speaks language *l*.

Possible correct answers:

$\forall x,y,l$
$(German(x) \wedge German(y) \wedge Speaks(x,l) \Rightarrow Speaks(y,l))$

If X and Y are Germans and X speaks language L, Y will also speak language L (it is true for all languages)

$\forall x,y\ (German(x) \wedge German(y) \Rightarrow (\forall l\ (Speaks(x,l) \Leftrightarrow Speaks(y,l)))$

If X and Y are Germans, any language L spoken by X will be spoken by Y and vice versa.

$\forall x,l\ (German(x) \wedge Speaks(x,l) \Rightarrow (\forall y\ (German(y) \Rightarrow Speaks(y,l))))$

If X is German and speaks language L, any German Y will speak language L.

$\forall l\ ((\forall x(German(x) \Rightarrow \neg Speaks(x,l))) \vee (\forall y\ (German(y) \Rightarrow Speaks(y,l))))$

Wrong answer:

$\forall x \exists l\ (German(x) \Rightarrow Speaks(x, l))$

There exists a language that all Germans speak. This is wrong because it leaves open the possibility that there are languages spoken by some Germans that are not spoken by others.

Task 3: A popular children's riddle is "Brothers and sisters have I none, but that man's father is my father's son." Can you use predicate logic to informally show who that man is?

Constants:

I – for the person “I” in the above riddle;
Man – for the person “that man” in the above riddle;
MF– for the person “my father” in the above riddle;

The predicates Sibling(x, y), Father(x,y) are defined as in the lecture;

Represent in First-order logic:

1. $\forall x. \neg Sibling(x, I)$ – Brothers and sisters have I none
(i.e. no one is my sibling)

2. $\exists p\ Father(p, Man) \wedge Father(MF, p)$ – That man's father is my father's son
(i.e. there is a person p who is that man’s father and is also my father’s son as my father

3. MF is father of p)
Father(MF, I) – My father is father of I.

Definition of sibling:

4. $\forall x,y$ Sibling(x,y) $\Leftrightarrow$ $(x \neq y) \wedge (\exists f$ Father(f,x) $\wedge$ Father(f,y))

Substitution: [y\I] to 4.

5. $\forall x$ Sibling(x,I) $\Leftrightarrow$ $(x \neq I) \wedge (\exists f$ Father(f,x) $\wedge$ Father(f,I))

From 1. And 5: $\forall x \neg ((x \neq I) \wedge (\exists f$ Father(f,x) $\wedge$ Father(f,I)))

6. $\forall x \neg ((x \neq I) \wedge (\exists f$ Father(f,x) $\wedge$ Father(f,I)))

Substitution: [x\p] to 6.

$\neg ((p \neq I) \wedge (\exists f$ Father(f,p) $\wedge$ Father(f,I)))

7. $\neg (p \neq I) \vee \neg (\exists f$ Father(f,p) $\wedge$ Father(f,I))

8. $(\exists f$ Father(f,p) $\wedge$ Father(f,I)) $\Rightarrow \neg (p \neq I)$

Substitution: [f\MF] to 8.

Father(MF,p) $\wedge$ Father(MF,I) $\Rightarrow \neg (p \neq I)$

Use 2 and 3:

$\neg (p \neq I)$

$p = I$.

Thus, Father(I, Man)

I.e. I am that man's father.