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Quiz 8

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$$1) \begin{bmatrix} 2 & -3 & 6 & 2 & 5 \\ -2 & 3 & -3 & -3 & -4 \\ 4 & -6 & 9 & 5 & 9 \\ -2 & 3 & 3 & -4 & 1 \end{bmatrix} \sim \begin{bmatrix} 2 & -3 & 6 & 2 & 5 \\ 0 & 0 & 3 & -1 & 1 \\ 0 & 0 & -3 & 1 & -1 \\ 0 & 0 & 9 & -2 & 6 \end{bmatrix} \sim$$

$$\begin{bmatrix} 2 & -3 & 6 & 2 & 5 \\ 0 & 0 & 3 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 9 & -2 & 6 \end{bmatrix} \sim \begin{bmatrix} 2 & -3 & 6 & 2 & 5 \\ 0 & 0 & 3 & -1 & 1 \\ 0 & 0 & 9 & -2 & 6 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \sim$$

$$\begin{bmatrix} 2 & -3 & 0 & 4 & 3 \\ 0 & 0 & 3 & -1 & 1 \\ 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 2 & -3 & 0 & 0 & -9 \\ 0 & 0 & 3 & 0 & 4 \\ 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -3/2 & 0 & 0 & -9/2 \\ 0 & 0 & 1 & 0 & 4/3 \\ 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Find a basis for  $\text{Col}(A)$ .

$$\text{Col } A = \text{span} \left\{ \begin{bmatrix} 2 \\ -2 \\ 4 \\ -2 \end{bmatrix}, \begin{bmatrix} 6 \\ -3 \\ 9 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ -3 \\ 5 \\ -4 \end{bmatrix} \right\}$$



Find a basis for  $\text{Nul}(A)$ .

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 3/2 x_2 + 9/2 x_5 \\ x_2 \\ -4/3 x_5 \\ -3 x_5 \\ x_5 \end{bmatrix} = x_2 \begin{bmatrix} 3/2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} 9/2 \\ 0 \\ -4/3 \\ -3 \\ 1 \end{bmatrix}$$

left.

$$\text{Nul } A = \text{span} \left\{ \begin{bmatrix} 3/2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 9/2 \\ 0 \\ -4/3 \\ -3 \\ 1 \end{bmatrix} \right\}$$

Find a basis for  $\text{Row } A$ .

$$\text{Row } A = \text{Span} \{ (1, -3/2, 0, 0, -9/2), (0, 0, 1, 0, 4/3), (0, 0, 0, 1, 3) \}$$

List the  $\text{rank}(A)$  and  $\dim(\text{Nul } A)$ .

$$\text{rank}(A) = 3$$

$$\dim(\text{Nul } A) = 2$$

left.

2) subspaces of  $\mathbb{R}^n$ :  $\text{Row } A$ ,  $\text{Nul } A$ ,  $\text{Col } A^T$

subspaces of  $\mathbb{R}^m$ :  $\text{Row } A^T$ ,  $\text{Nul } A^T$ ,  $\text{Col } A$

4 distinct subspaces since  $\text{Col } A = \text{Row } A^T$  and  $\text{Row } A = \text{Col } A^T$

$$3) \begin{bmatrix} 2 & 1 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} -1+\sqrt{2} \\ 1 \end{bmatrix} = \begin{bmatrix} 2(-1+\sqrt{2}) + 1(1) \\ 1(-1+\sqrt{2}) + 4(1) \end{bmatrix} = \begin{bmatrix} -2+2\sqrt{2}+1 \\ -1+\sqrt{2}+4 \end{bmatrix} \\ = \begin{bmatrix} -1+2\sqrt{2} \\ 3+\sqrt{2} \end{bmatrix}$$

$$\lambda \begin{bmatrix} -1+\sqrt{2} \\ 1 \end{bmatrix} = \begin{bmatrix} -1+2\sqrt{2} \\ 3+\sqrt{2} \end{bmatrix}$$

$$\lambda = 3+\sqrt{2} \quad \checkmark$$

$$4) Ax = 2x \\ (A-2I)x = 0$$

$$\begin{bmatrix} 4 & -1 & 6 \\ 2 & 1 & 6 \\ 2 & -1 & 8 \end{bmatrix} - \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 2 & -1 & 6 \\ 2 & -1 & 6 \\ 2 & -1 & 6 \end{bmatrix}$$

Since  $3 \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 6 \\ 6 \\ 6 \end{bmatrix}$ , the columns of  $A-2I$  are linearly dependent, so  $(A-2I)x=0$  has nontrivial solutions.  
Thus 2 is an eigenvalue of  $A$ .

$$\begin{bmatrix} 2 & -1 & 6 \\ 2 & -1 & 6 \\ 2 & -1 & 6 \end{bmatrix} \sim \begin{bmatrix} 2 & -1 & 6 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & -1/2 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

4 pt.

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1/2 x_2 - 3x_3 \\ x_2 \\ x_3 \end{bmatrix} = x_2 \begin{bmatrix} 1/2 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix}$$

a basis for the eigenspace:

$$\left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix} \right\} \quad \checkmark$$



$$Ax = 9x$$

$$(A - 9I)x = 0$$

$$\begin{bmatrix} 4 & -1 & 6 \\ 2 & 1 & 6 \\ 2 & -1 & 8 \end{bmatrix} - \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix} = \begin{bmatrix} -5 & -1 & 6 \\ 2 & -8 & 6 \\ 2 & -1 & -1 \end{bmatrix}$$

Left.

$$\begin{bmatrix} -5 & -1 & 6 \\ 2 & -8 & 6 \\ 2 & -1 & -1 \end{bmatrix} \sim \begin{bmatrix} -5 & -1 & 6 \\ 1 & -4 & 3 \\ 2 & -1 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & -4 & 3 \\ -5 & -1 & 6 \\ 2 & -1 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & -4 & 3 \\ 0 & -21 & 21 \\ 0 & -7 & 7 \end{bmatrix} \sim$$

$$\begin{bmatrix} 1 & -4 & 3 \\ 0 & -21 & 21 \\ 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & -4 & 3 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

Since there is not a pivot in every column, the columns of  $A - 9I$  are linearly dependent, so  $(A - 9I)x = 0$  has nontrivial solutions. Thus, 9 is an eigenvalue of  $A$ .

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_3 \\ x_3 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

a basis for the eigenspace:

$$\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\} \quad \checkmark$$