Directions: You should work through this worksheet while watching the following video:

Finding the Determinant of a 3×3 matrix

Feel free to pause (and rewind) the video as you work through this handout and take notes.

1. **Definition:** For an arbitrary 3×3 matrix

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

the determinant of a 3×3 matrix is given by the following formula:

$$\det(A) = a_{11} \det\left(\begin{bmatrix} \text{a22} & \text{a23} \\ \text{a32} & \text{a33} \end{bmatrix}\right) - \text{a12} \det\left(\begin{bmatrix} \text{a21} & \text{a23} \\ \text{a31} & \text{a33} \end{bmatrix}\right) + a_{13} \det\left(\begin{bmatrix} \text{a21} & \text{a22} \\ \text{a31} & \text{a32} \end{bmatrix}\right).$$

2. **Notation**The determinant of A, is denoted by $|\text{A}|$ or $\det(\text{A})$.

Example 1: Compute the determinant for A. (This is the same matrix in the video)

$$A = \begin{bmatrix} 1 & 6 & 4 \\ 2 & 7 & 3 \\ 8 & 9 & 5 \end{bmatrix}.$$

$$\det(A) = 1 \det\left(\begin{bmatrix} 7 & 3 \\ 9 & 5 \end{bmatrix}\right) - 6 \det\left(\begin{bmatrix} 2 & 3 \\ 8 & 5 \end{bmatrix}\right) + 4 \det\left(\begin{bmatrix} 2 & 7 \\ 8 & 9 \end{bmatrix}\right)$$

$$= 1\,(35 - 27) - 6\,(10 - 24) + 4\,(18 - 56)$$

$$=$$

Final Answer: $\det(A) = -60$

Example 2: Now let's work through a second example. Compute the determinant for B.

$$B = \begin{bmatrix} 2 & -1 & -3 \\ 0 & 5 & 6 \\ 7 & 1 & -4 \end{bmatrix}.$$

$$\det(B) = 2\det\left(\begin{bmatrix} 5 & 6 \\ 1 & -4 \end{bmatrix}\right) - (-1)\det\left(\begin{bmatrix} 0 & 6 \\ 7 & -4 \end{bmatrix}\right) + (-3)\det\left(\begin{bmatrix} 0 & 5 \\ 7 & 1 \end{bmatrix}\right)$$

$$= 1\,(-20 - 6) + 1\,(0 - 42) - 3\,(0 - 35)$$

$$=$$

Final Answer: $\det(B) = 37$

Practice Problems: Compute the determinant of the following 3×3 matrices.

1. $\begin{bmatrix} 5 & -2 & 2 \\ 0 & 3 & -3 \\ 2 & -4 & 7 \end{bmatrix}$ det(A) = 5(21 - 12) + 2(0 - (-6)) + 2(0 - 6) = 165

2. $\begin{bmatrix} 2 & -2 & 3 \\ 3 & 1 & 2 \\ 1 & 3 & -1 \end{bmatrix}$ det(A) = 2(-1 - 6) + 2(-3 - 2) + 3(9 - 1) = 0

3. $\begin{bmatrix} 2 & 3 & -3 \\ 4 & 0 & 3 \\ 6 & 1 & 5 \end{bmatrix}$ det(A) = 2(0 - 3) - 3(20 - 18) - 3(4 - 0) = -24