Directions: You should work through this worksheet while watching the following video:

Finding the Determinant of a 3×3 matrix

Feel free to pause (and rewind) the video as you work through this handout and take notes.

1. **Definition:** For an arbitrary 3×3 matrix

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

the determinant of a 3×3 matrix is given by the following formula:

$$\det(A) = a_{11} \det\left(\begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}\right) - \_\_\_ \det\left(\begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}\right) + a_{13} \det\left(\begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}\right).$$

2. **Notation**The determinant of A, is denoted by __________ or __________.

Example 1: Compute the determinant for A. (This is the same matrix in the video)

$$A = \begin{bmatrix} 1 & 6 & 4 \\ 2 & 7 & 3 \\ 8 & 9 & 5 \end{bmatrix}.$$

$$\det(A) = 1 \det\left(\begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}\right) - 6 \det\left(\begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}\right) + 4 \det\left(\begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}\right)$$

$$= 1\,(\_\_\_ - \_\_\_) - 6\,(\_\_\_ - \_\_\_) + 4\,(\_\_\_ - \_\_\_)$$

$$=$$

Final Answer: $\det(A) = \_\_\_$

Example 2: Now let's work through a second example. Compute the determinant for B.

$$B = \begin{bmatrix} 2 & -1 & -3 \\ 0 & 5 & 6 \\ 7 & 1 & -4 \end{bmatrix}.$$

$$\det(B) = 2\det\left(\begin{bmatrix} 5 & \underline{\quad} \\ \underline{\quad} & -4 \end{bmatrix}\right) - (-1)\det\left(\begin{bmatrix} \underline{\quad} & \underline{\quad} \\ 7 & \underline{\quad} \end{bmatrix}\right) + (-3)\det\left(\begin{bmatrix} \underline{\quad} & 5 \\ \underline{\quad} & \underline{\quad} \end{bmatrix}\right)$$

$$= 1\,(-20 \; - \; \underline{\quad\quad}) + 1\,(\underline{\quad\quad} \; - \; \underline{\quad\quad}) - 3\,(\underline{\quad\quad} \; - \; 35)$$

$$=$$

Final Answer: $\det(B) = \underline{\quad\quad}$

Practice Problems: Compute the determinant of the following 3×3 matrices.

1. $\begin{bmatrix} 5 & -2 & 2 \\ 0 & 3 & -3 \\ 2 & -4 & 7 \end{bmatrix}$

2. $\begin{bmatrix} 2 & -2 & 3 \\ 3 & 1 & 2 \\ 1 & 3 & -1 \end{bmatrix}$

3. $\begin{bmatrix} 2 & 3 & -3 \\ 4 & 0 & 3 \\ 6 & 1 & 5 \end{bmatrix}$