

$$4) \begin{bmatrix} 1 & 0 & 6 & 5 \\ 0 & 2 & 5 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 6 & 5 \\ 0 & 1 & \frac{5}{2} & \frac{3}{2} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{aligned} x_1 &= 6x_2 + 5x_3 \\ x_2 &= \frac{5}{2}x_3 - \frac{3}{2}x_4 \end{aligned} \quad (a) \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -6x_3 - 5x_4 \\ -\frac{5}{2}x_3 - \frac{3}{2}x_4 \\ x_3 \\ x_4 \end{bmatrix} = x_3 \begin{bmatrix} -6 \\ -\frac{5}{2} \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -5 \\ -\frac{3}{2} \\ 0 \\ 1 \end{bmatrix}$$

$$\text{Nul}(A) = \text{Span} \left\{ \begin{bmatrix} -6 \\ -\frac{5}{2} \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -5 \\ -\frac{3}{2} \\ 0 \\ 1 \end{bmatrix} \right\}$$

$$\text{Col}(A) = \text{Span} \left\{ \begin{bmatrix} -2 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ -6 \\ 8 \end{bmatrix} \right\}$$

$$5) \begin{bmatrix} 1 & 1 & 0 \\ 3 & 0 & 1 \\ 0 & 3 & 5 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 0 \\ 0 & -3 & 1 \\ 0 & 3 & 5 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 0 \\ 0 & -3 & 1 \\ 0 & 0 & 6 \end{bmatrix} \sim$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & -3 & 1 \\ 0 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

pivot in every column  $\Rightarrow$  linearly independent  
 pivot in every row  $\Rightarrow$  span  $\mathbb{R}^3$   
 All of the set form a standard basis  
 for  $\mathbb{R}^3$