

By signing below, you attest that you have neither given nor received help of any kind on this exam.

Signature: \_\_\_\_\_ Printed Name: \_\_\_\_\_

**Instructions:** Show work to get full credit (the correct answer may NOT be enough). Do all your work on the paper provided. Write clearly! Double check your answers!

You will **not** receive full credit for using methods other than those discussed in class.

# EXAM I

## MATH 214 – LINEAR ALGEBRA

Problem Number	Available Points	Your Points
1	8	
2	12	
3	12	
4	20	
5	20	
6	8	
7	20	
Total	100	

BLANK PAGE

1. For each of the following statements please **circle** your choice for the best answer.

[8]

(a) T or F? If one row of an echelon form of an **augmented** matrix is  $[0 \ 0 \ 0 \ 3 \ 0]$ , then the associated linear system is inconsistent.

(b) T or F? The vector  $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$  is in the span of  $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 4 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix} \right\}$ .

(c) T or F? Let  $A$  be an  $m \times n$  matrix. If the rows of  $A$  span  $\mathbb{R}^m$ , then the  $A$  has a pivot in every column.

(d) If augmented matrix  $\begin{bmatrix} 2 & 0 & 0 \\ 5 & 1 & 1 \\ 10 & 0 & 2 \end{bmatrix}$  is row equivalent to the matrix  $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$  then the associated linear system has a

(i) no solution

(ii) a unique solution

(iii) infinitely many solutions

2. Consider the system of equations:

[12]

$$\begin{array}{rrrrrrrrcl} 2x_1 & + & x_2 & & & + & 4x_4 & - & x_5 & = & 9 \\ 3x_1 & + & x_2 & + & x_3 & & & + & 2x_5 & = & 3 \\ 4x_1 & & & + & 3x_3 & + & 2x_4 & - & x_5 & = & 7 \end{array}$$

(a) Write the associated augmented matrix for this system.

(b) Write this system as a vector equation.

(c) Write this system as a matrix equation.

3. The following augmented matrix represents a system of equations.

[12]

$$\left[ \begin{array}{ccccc} 1 & -6 & 3 & 0 & 1 \\ 0 & 7 & -7 & 14 & 21 \\ 0 & 0 & 0 & 2 & 6 \\ 0 & 0 & 0 & 6 & 18 \end{array} \right]$$

- (a) Is the augmented matrix in REF, RREF, or neither. Justify your answer.

- (b) Locate the pivot positions.

- (c) Determine if the associated system of linear equations is consistent or inconsistent. If consistent determine how many solution(s) the system has. Be sure to justify your answer.

4. Find all values of  $h$  such that the linear system

[20]

$$\begin{array}{rcccccl} & & 3x_2 & + & 6hx_3 & = & 0 \\ x_1 & + & 2x_2 & + & 6x_3 & = & 2 \\ 3hx_1 & & & + & 6x_3 & = & 3 \end{array}$$

has

- (a) no solutions.
- (b) a unique solution.
- (c) infinitely many solutions.

[*This requires you to put the associated augmented matrix in row echelon form.*]

5. Let  $\vec{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix}$ ,  $\vec{v}_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 2 \end{bmatrix}$ , and  $\vec{v}_3 = \begin{bmatrix} 0 \\ -2 \\ 1 \\ 0 \end{bmatrix}$ . [20]

(a) Is  $\vec{b} = \begin{bmatrix} 1 \\ -2 \\ 3 \\ 0 \end{bmatrix}$  in  $\text{Span}\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ ? If so, write  $\vec{b}$  as a linear combination of  $\vec{v}_1, \vec{v}_2$ , and  $\vec{v}_3$ .

(b) Does  $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$  Span  $\mathbb{R}^3$ ? Justify your answer.

6. Given  $A = \begin{bmatrix} 2 & 4 & 0 & -1 \\ 3 & 0 & 1 & 2 \\ 7 & 0 & -4 & 1 \end{bmatrix}$  and  $\vec{x} = \begin{bmatrix} 1 \\ 5 \\ -2 \\ 0 \end{bmatrix}$  compute  $A\vec{x}$ . [8]

7. For the following questions provide a brief explanation to support your answer.

[20]

- (a) Suppose  $A$  is a  $4 \times 3$  matrix and  $\vec{b}$  is a vector in  $\mathbb{R}^4$  with the property that  $A\vec{x} = \vec{b}$  has a unique solution. What can you say about the reduced echelon form of  $A$ .

- (b) Let  $A$  be a  $3 \times 2$  matrix. Explain why the equation  $A\vec{x} = \vec{b}$  cannot be consistent for all  $\vec{b}$  in  $\mathbb{R}^3$ . Generalize your argument to the case of an arbitrary  $A$  with more rows than columns.

BLANK PAGE