Exam 4 Study Guide
Math 214

Exam 4 will be **Wednesday April 29, 2020**. The exam will cover sections 4.6, 5.1, 5.2, 5.3, and 6.1 of the textbook.

The exam consist of 100 points and will have around 7-8 problems, some of which have multiples parts. Many of the problems will be similar to those appearing on the problem sets and in class examples. Some problems will be straightforward computations (e.g. using row reductions to solve a system of equations) while others will be conceptual in nature (i.e. assessing your understanding of the major definitions and theorems). I may ask you to provide definitions of a few of the major concepts we've discussed. There will also be some true/false questions which will require explanations.

- Determining the rank of a matrix (4.6)
- Using the Rank Theorem to determine information about a matrix or system (4.6)
- Determining if a matrix is invertible using the rank of a matrix (4.6)
- Given a vector $\vec{u}$ determine if it is an eigenvector for a given matrix. (5.1)
- Given a number, show that it is an eigenvalue for a given matrix, and find the corresponding eigenvectors (5.1)
- Given an eigenvalue, find a basis for the corresponding eigenspace (5.1)
- Be able to find the characteristic polynomial of a given matrix (5.2)
- Be able to use the characteristic polynomial to find eigenvalues of a given matrix (5.2)
- Determining if a given matrix is diagonalizable (5.3)
- Finding a diagonalization of a matrix (5.3)
- Computing $\vec{u} \cdot \vec{v}$, $||\vec{v}||$, normal vectors, $\text{dist}(\vec{u}, \vec{v})$ (6.1)
- Computing a basis for $W^{\perp}$ (6.1)

Practice problems: These are meant to illustrate for you the varying types of problems which may appear on the exam. They are **NOT** meant to indicate in any way what specific topics will be addressed on the exam, nor the length of the exam. Any similarity between these practice problems and your actual exam problems is completely coincidental.

1. Let $B = \begin{bmatrix} 1 & 6 & 2 & -4 \\ -3 & 2 & -2 & -8 \\ 4 & -1 & 3 & 9 \end{bmatrix}$. Find a basis for each of $\text{Nul}(B)$, $\text{Col}(B)$ and $\text{Row}(B)$. State the dimension of each subspace.

2. Can a 6×15 matrix B have a 5 dimensional Null Space? Why or why not?

3. Let A be a $m \times n$ matrix. Justify the following:
   (a) $\dim(\text{Row}(A)) + \dim(\text{Nul}(A)) = n$
   (b) $\dim(\text{Col}(A)) + \dim(\text{Nul}(A^T)) = m$

4. Suppose a 4×7 matrix A has four pivot columns. Is $(A) = \mathbb{R}^4$? Is $\text{Nul}(A) = \mathbb{R}^3$? Explain your answers.

5. Is $\vec{v} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ an eigenvector of $\begin{bmatrix} 1 & -1 \\ 6 & -4 \end{bmatrix}$? If so, find the eigenvalue.

6. Find all eigenvalues for the following matrices:

(a) $A = \begin{bmatrix} 1 & 0 & 2 \\ 3 & 1 & 3 \\ 2 & 0 & 1 \end{bmatrix}$ (b) $B = \begin{bmatrix} 2 & 0 & 1 \\ 4 & 1 & 3 \\ 2 & 1 & 2 \end{bmatrix}$ (c) $C = \begin{bmatrix} 5 & -2 \\ 1 & 3 \end{bmatrix}$.

7. Find a basis for the eigenspace of $A = \begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$ corresponding to the eigenvalue $\lambda = 3$.

8. Find the characteristic polynomial of the matrix $A = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 1 & -2 & 4 & 1 \\ 0 & 1 & 0 & 2 \\ 2 & 0 & 0 & -4 \end{bmatrix}$.

9. Let A have the diagonalization $A = \begin{bmatrix} 3 & -2 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} -3 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} -1 & 2 \\ -2 & 3 \end{bmatrix}$. Use this to compute A^4.

10. Diagonalize the following matrices. If the matrix is not diagonalizable, explain why.

(a) $\begin{bmatrix} 1 & 0 & 2 \\ 3 & 1 & 3 \\ 2 & 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 0 & 0 \\ 1 & 2 & 1 \\ -1 & 0 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 2 & 4 & 6 \\ 0 & 2 & 2 \\ 0 & 0 & 4 \end{bmatrix}$

11. Let $\vec{u}_1 = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}$, $\vec{u}_2 = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$, $\vec{u}_3 = \begin{bmatrix} -1 \\ -4 \\ 7 \end{bmatrix}$, and $S = \{\vec{u}_1, \vec{u}_2, \vec{u}_3\}$.

(a) Is S an orthogonal set? Explain.

(b) Is S an orthonormal set? Explain.

(c) Is S a basis for $\mathbb{R}^3$? Explain.

(d) Use inner products to write $\vec{b} = \begin{bmatrix} 6 \\ 1 \\ -8 \end{bmatrix}$ as a linear combination of the vectors in S.

12. Let $W = \text{Span}\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \right\}$. Find a basis for $W^{\perp}$.

13. Let A be a 4×4 matrix with only 3 distinct eigenvalues.

(a) Is it possible to determine if A is diagonalizable with the given information? Explain.

(b) Is it possible to determine if A is invertible with the given information? Explain.