By signing below, you attest that you have neither given nor received help of any kind on this exam.

Signature: ____________________ Printed Name: ____________________

Instructions: Show work to get full credit (the correct answer may NOT be enough). Do all your work on the paper provided. Write clearly! Double check your answers!

You will **not** receive full credit for using methods other than those discussed in class.

Exam IV

MATH 214 – Linear Algebra

Problem Number	Available Points	Your Points
1	8	
2	22	
3	10	
4	20	
5	16	
6	8	
7	8	
8	8	
Total	100	

1. If C is a 4×5 matrix, what is the largest possible rank of C? What is the smallest possible dimension of the null space of C? Explain your answer. [8]

2. Consider $A = \begin{bmatrix} 1 & -4 & 0 & 5 \\ 0 & 2 & -1 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}$. [22]

   (a) Find a basis for Nul(A). What is the dimension of Nul(A)?

   (b) Find a basis for Col(A). What is the dimension of Col(A)?

   (c) Find a basis for Row(A). What is the dimension of Row(A)?

   (d) Verify the Rank Theorem.

3. Is $\begin{bmatrix} -\sqrt{2} - 1 \\ 1 \end{bmatrix}$ an eigenvector of $\begin{bmatrix} 2 & 1 \\ 1 & 4 \end{bmatrix}$? If so, find the eigenvalue. [10]

4. Consider $A = \begin{bmatrix} 4 & 0 & 1 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 2 \end{bmatrix}$. [20]

   (a) Find the characteristic polynomial of A.

   (b) Find all eigenvalues of A, including the multiplicity.

   (c) Is A invertible? Justify your answer.

5. Consider $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 4 \end{bmatrix}$. Observe, since A is upper triangular the eigenvalues of A are $\lambda = 1$ with multiplicity 2 and $\lambda = 4$. Find a basis for each eigenspace. Determine if A is diagonalizable. Justify your answer. [16]

6. Let A have the diagonalization $A = \begin{bmatrix} 3 & -2 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} -3 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} -1 & 2 \\ -2 & 3 \end{bmatrix}$. Use this to find a formula for A^k. [8]

7. Consider vectors $v = \begin{bmatrix} 2 \\ 3 \\ -6 \end{bmatrix}$ and $u = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$. [8]

   (a) Find $u \cdot v$, that is the inner product of u and v.

   (b) Find a unit vector, u, in the direction of the vector v. In other words, normalize the vector v.

8. Determine whether the pairs of vectors are orthogonal. Justify your answer. [8]

   (a) $\begin{bmatrix} 8 \\ -5 \end{bmatrix}, \begin{bmatrix} -2 \\ -3 \end{bmatrix}$

   (b) $\begin{bmatrix} 12 \\ 3 \\ -5 \end{bmatrix}, \begin{bmatrix} 2 \\ -3 \\ 3 \end{bmatrix}$