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Quiz 8
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1) $$\begin{bmatrix} 2 & -3 & 6 & 2 & 5 \\ -2 & 3 & -3 & -3 & -4 \\ 4 & -6 & 9 & 5 & 9 \\ -2 & 3 & 3 & -4 & 1 \end{bmatrix} \sim \begin{bmatrix} 2 & -3 & 6 & 2 & 5 \\ 0 & 0 & 3 & -1 & 1 \\ 0 & 0 & -3 & 1 & -1 \\ 0 & 0 & 9 & -2 & 6 \end{bmatrix} \sim$$

$$\begin{bmatrix} 2 & -3 & 6 & 2 & 5 \\ 0 & 0 & 3 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 9 & -2 & 6 \end{bmatrix} \sim \begin{bmatrix} 2 & -3 & 6 & 2 & 5 \\ 0 & 0 & 3 & -1 & 1 \\ 0 & 0 & 9 & -2 & 6 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \sim$$

$$\begin{bmatrix} 2 & -3 & 0 & 4 & 3 \\ 0 & 0 & 3 & -1 & 1 \\ 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 2 & -3 & 0 & 0 & -9 \\ 0 & 0 & 3 & 0 & 4 \\ 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -3/2 & 0 & 0 & -9/2 \\ 0 & 0 & 1 & 0 & 4/3 \\ 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Find a basis for Col(A).

$$\text{Col } A = \text{span}\left\{ \begin{bmatrix} 2 \\ -2 \\ 4 \\ -2 \end{bmatrix}, \begin{bmatrix} 6 \\ -3 \\ 9 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ -3 \\ 5 \\ -4 \end{bmatrix} \right\}$$

Find a basis for Nul(A).

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 3/2 x_2 + 9/2 x_5 \\ x_2 \\ -4/3 x_5 \\ -3x_5 \\ x_5 \end{bmatrix} = x_2 \begin{bmatrix} 3/2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} 9/2 \\ 0 \\ -4/3 \\ -3 \\ 1 \end{bmatrix}$$

$$\text{Nul}A = \text{span} \left\{ \begin{bmatrix} 3/2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 9/2 \\ 0 \\ -4/3 \\ -3 \\ 1 \end{bmatrix} \right\}$$

Find a basis for Row A.

$$\text{Row } A = \text{Span} \{ (1, -3/2, 0, 0, -9/2), (0, 0, 1, 0, 4/3), (0, 0, 0, 1, 3) \}$$

List the rank(A) and dim(Nul A).

rank(A) = 3

dim(Nul A) = 2

2) subspaces of $\mathbb{R}^n$: Row A, Nul A, Col A^T

subspaces of $\mathbb{R}^m$: Row A^T, Nul A^T, Col A

4 distinct subspaces since Col A = Row A^T and Row A = Col A^T

3) $\begin{bmatrix} 2 & 1 \\ 1 & 4 \end{bmatrix}\begin{bmatrix} -1+\sqrt{2} \\ 1 \end{bmatrix} = \begin{bmatrix} 2(-1+\sqrt{2}) + 1(1) \\ 1(-1+\sqrt{2}) + 4(1) \end{bmatrix} = \begin{bmatrix} -2+2\sqrt{2}+1 \\ -1+\sqrt{2}+4 \end{bmatrix}$

$= \begin{bmatrix} -1+2\sqrt{2} \\ 3+\sqrt{2} \end{bmatrix}$

$$\lambda \begin{bmatrix} -1+\sqrt{2} \\ 1 \end{bmatrix} = \begin{bmatrix} -1+2\sqrt{2} \\ 3+\sqrt{2} \end{bmatrix}$$

$\lambda = 3+\sqrt{2}$

4) $Ax = 2x$
$(A-2I)x = 0$

$$\begin{bmatrix} 4 & -1 & 6 \\ 2 & 1 & 6 \\ 2 & -1 & 8 \end{bmatrix} - \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 2 & -1 & 6 \\ 2 & -1 & 6 \\ 2 & -1 & 6 \end{bmatrix}$$

Since $3\begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 6 \\ 6 \\ 6 \end{bmatrix}$, the columns of $A-2I$ are linearly dependent, so $(A-2I)x=0$ has nontrivial solutions. Thus 2 is an eigenvalue of A.

$$\begin{bmatrix} 2 & -1 & 6 \\ 2 & -1 & 6 \\ 2 & -1 & 6 \end{bmatrix} \sim \begin{bmatrix} 2 & -1 & 6 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & -1/2 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2}x_2 - 3x_3 \\ x_2 \\ x_3 \end{bmatrix} = x_2 \begin{bmatrix} 1/2 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix}$$

a basis for the eigenspace:

$$\left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix} \right\}$$

$Ax = 9x$

$(A - 9I)x = 0$

$$\begin{bmatrix} 4 & -1 & 6 \\ 2 & 1 & 6 \\ 2 & -1 & 8 \end{bmatrix} - \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix} = \begin{bmatrix} -5 & -1 & 6 \\ 2 & -8 & 6 \\ 2 & -1 & -1 \end{bmatrix}$$

$$\begin{bmatrix} -5 & -1 & 6 \\ 2 & -8 & 6 \\ 2 & -1 & -1 \end{bmatrix} \sim \begin{bmatrix} -5 & -1 & 6 \\ 1 & -4 & 3 \\ 2 & -1 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & -4 & 3 \\ -5 & -1 & 6 \\ 2 & -1 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & -4 & 3 \\ 0 & -21 & 21 \\ 0 & -7 & 7 \end{bmatrix} \sim$$

$$\begin{bmatrix} 1 & -4 & 3 \\ 0 & -21 & 21 \\ 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & -4 & 3 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

Since there is not a pivot in every column, the columns of $A-9I$ are linearly dependent, so $(A-2I)x=0$ has nontrivial solutions. Thus, 9 is an eigenvalue of A.

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_3 \\ x_3 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

a basis for the eigenspace:

$$\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$$