By signing below, you attest that you have neither given nor received help of any kind on this exam.

Signature: ____________________ Printed Name: ____________________

Instructions: Show work to get full credit (the correct answer may NOT be enough). Do all your work on the paper provided. Write clearly! Double check your answers!

You will **not** receive full credit for using methods other than those discussed in class.

Exam II

MATH 214 – Linear Algebra

Problem Number	Available Points	Your Points
1	16	
2	12	
3	10	
4	4	
5	20	
6	15	
7	15	
8	8	
Total	100	

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1. Determine if the following sets of vectors are linearly independent or linearly dependent. Justify your answer.

   (a) $\left\{ \begin{bmatrix} 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 5 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 6 \end{bmatrix} \right\}$ [4]

   (b) $\left\{ \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 6 \\ 3 \\ 3 \end{bmatrix} \right\}$ [4]

   (c) $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\}$ [4]

   (d) $\left\{ \begin{bmatrix} 2 \\ 5 \\ 4 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 10 \\ 8 \end{bmatrix} \right\}$ [4]

2. Let $A = \begin{bmatrix} 1 & 4 & 8 & 1 \\ 0 & 2 & 1 & 3 \\ 0 & 0 & 0 & 5 \end{bmatrix}$ and let $T : \mathbb{R}^4 \to \mathbb{R}^3$ be the linear transformation give by $T(x) = Ax$.

   (a) Find the image of $u = \begin{bmatrix} 1 \\ 0 \\ -3 \\ 2 \end{bmatrix}$ under the transformation T. [4]

   (b) Is T one-to-one? Be sure to justify your answer. [4]

   (c) Is T onto? Be sure to justify your answer. [4]

3. Let $A = \begin{bmatrix} 1 & -4 & -2 & 0 & 3 & -5 \\ 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ which has reduced echelon form of $\begin{bmatrix} 1 & -4 & 0 & 0 & 0 & 5 \\ 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

(a) Do the columns of A span $\mathbb{R}^4$? Explain your answer. [2]

(b) Write the solution set of $A\mathbf{x} = \mathbf{0}$ in parametric vector form. [5]

(c) Given that $\begin{bmatrix} 7 \\ -1 \\ 0 \\ 0 \\ 2 \\ 3 \end{bmatrix}$ is a solution to the matrix equation $A\mathbf{x} = \mathbf{b}$ where $\mathbf{b} = \begin{bmatrix} 2 \\ -3 \\ -10 \\ 0 \end{bmatrix}$ write the entire solution set of $A\mathbf{x} = \mathbf{b}$. [3]

4. Carefully write out the definition of a **linear transformation**. [4]

5. Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be the transformation defined by

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x - 2y \\ y \\ -2x + y \end{bmatrix}.$$

(a) Show T is a linear transformation. [15]

(b) Find the standard matrix for T. [5]

6. Given the following matrices, perform the indicated operation if possible. If it is not possible to perform the operation, explain why it could not be performed.

$$A = \begin{bmatrix} 2 & 4 & 0 \\ 3 & 0 & 8 \end{bmatrix},\ B = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 3 & 4 & 0 \end{bmatrix},\ C = \begin{bmatrix} 3 & 2 \\ 1 & 5 \end{bmatrix}, D = \begin{bmatrix} 7 & -1 & 4 \\ 0 & 9 & -2 \end{bmatrix}.$$

(a) $-2A + D$ [5]

(b) CB [5]

(c) AB [5]

7. Let $A = \begin{bmatrix} 1 & 0 & -2 \\ -3 & 1 & 4 \\ 2 & -3 & 4 \end{bmatrix}$. Find A^{-1}. [15]

8. Explain why the columns of an $n \times n$ matrix A are linearly independent when A is invertible. [8]

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