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Exam 3

1) a) $\begin{bmatrix} 6 & 7 \\ -3 & -5 \end{bmatrix}$ det $= -30-(-21) = -30+21 = -9 \neq 0$

Thus, the matrix is invertible.

b) The matrix is not invertible because there is a column of 0's, meaning there cannot be 3 pivots, and therefore all the statements of IMT are false.

c) $\begin{bmatrix} 1 & -5 & -4 \\ 0 & 3 & 4 \\ -3 & 6 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & -5 & -4 \\ 0 & 3 & 4 \\ 0 & -9 & -12 \end{bmatrix} \sim \begin{bmatrix} 1 & -5 & -4 \\ 0 & 3 & 4 \\ 0 & 0 & 0 \end{bmatrix}$

The matrix is not invertible because there are not 3 pivots, so all the statements of IMT are false.

d) There are 4 pivots, so the matrix is invertible since all the statements of IMT are true.

2) $A = \begin{bmatrix} 4 & 2 & 0 & 0 \\ 0 & -3 & 1 & 0 \\ 0 & 2 & 0 & 4 \\ -3 & 0 & 5 & 1 \end{bmatrix}$

$\det(A) = a_{11}(-1)^{1+1}A_{11} + a_{12}(-1)^{1+2}A_{12} + a_{13}(-1)^{1+3}A_{13} + a_{14}(-1)^{1+4}A_{14}$

$\det(A) = 4\det\begin{bmatrix} -3 & 1 & 0 \\ 2 & 0 & 4 \\ 0 & 5 & 1 \end{bmatrix} - 2\det\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 4 \\ 3 & 5 & 1 \end{bmatrix} + 0\,A_{13} - 0\,A_{14}$

$\det(A) = 4\left(\overset{a_{11}}{-3}\,\det\overset{A_{11}}{\begin{bmatrix} 0 & 4 \\ 5 & 1 \end{bmatrix}} - 1\det\begin{bmatrix} 2 & 4 \\ 0 & 1 \end{bmatrix}\right) - 2\left(\overset{a_{12}}{-1}\,\det\begin{bmatrix} 0 & 4 \\ 3 & 1 \end{bmatrix}\right)$

$\det(A) = 4(-3(0-20) - (2-0)) + 2(0-12)$

$\det(A) = 4(-3(-20)-2) - 24$

$= 4(58) - 24$

$= 208$

$58 \times 4 = 232$

$232 - 24 = 208$

3) $\begin{bmatrix} a+2c \\ 2a+b+3c \\ 3b-3c \\ a+4b-2c \end{bmatrix}$ subspace of $\mathbb{R}^4$ and find a basis

Subspace:

When $a=0$, $b=0$, and $c=0$, the vector $= \vec{0}$, so the $\vec{0}$ is in H.

$$\begin{bmatrix} a+2c \\ 2a+b+3c \\ 3b-3c \\ a+4b-2c \end{bmatrix} + \begin{bmatrix} d+2f \\ 2d+e+3f \\ 3e-3f \\ d+4e-2f \end{bmatrix} = \begin{bmatrix} (a+d)+2(c+f) \\ 2(a+d)+(b+e)+3(c+f) \\ 3(b+e)-3(c+f) \\ (a+d)+4(b+e)-2(c+f) \end{bmatrix}$$

$$k\begin{bmatrix} a+2c \\ 2a+b+3c \\ 3b-3c \\ a+4b-2c \end{bmatrix} = \begin{bmatrix} k(a+2c) \\ k(2a+b+3c) \\ k(3b-3c) \\ k(a+4b-2c \end{bmatrix}$$

$\text{basis}(H) = \left\{ \begin{bmatrix} 0 \\ 1 \\ 3 \\ 4 \end{bmatrix} \right\}$ by the Spanning Set Theorem

4) $\text{basis}(\text{Col} A) = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -3 \\ 1 \\ -2 \end{bmatrix} \right\}$

$\dim(\text{Col} A) = 3$

$$\begin{bmatrix} 1 & 2 & 1 & 0 & 1 \\ 0 & 1 & 2 & -1 & 0 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 1 & 0 & 1 \\ 0 & 1 & 2 & 0 & 2 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -3 & 0 & -3 \\ 0 & 1 & 2 & 0 & 2 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 3x_3 + 3x_5 \\ -2x_3 - 2x_5 \\ x_3 \\ -2x_5 \\ x_5 \end{bmatrix} = x_3\begin{bmatrix} 3 \\ -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_5\begin{bmatrix} 3 \\ -2 \\ 0 \\ -2 \\ 1 \end{bmatrix}$$

$\text{basis}(\text{Nul}(A)) = \left\{ \begin{bmatrix} 3 \\ -2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ -2 \\ 0 \\ -2 \\ 1 \end{bmatrix} \right.$

$\dim(\text{Nul} A) = 2$

5) $T(A) = T\left(\begin{bmatrix} 1 & -8 \\ 6 & -1 \end{bmatrix}\right) = \begin{bmatrix} 1+(-8) \\ 6 \\ -1+1 \end{bmatrix} = \begin{bmatrix} -7 \\ 6 \\ 0 \end{bmatrix}$

$\ker(T) \equiv T(\vec{v}) = \vec{0}$

$T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = \begin{bmatrix} a+b \\ c \\ d+1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$\ker(T) =$ the set of all vectors such that $a+b=0$, $c=0$, and $d=-1$.

6) a) $\mathbb{R}^{12}$ is an example because for $\mathbb{R}^n$, $\dim(\mathbb{R}^n) = n$.

b) $\mathbb{P}$, the set $\{1, x, x^2, x^3, x^4, \dots\}$ is infinite dimensional since the set is not constrained by some number n.

7) If a matrix is 6×8, the smallest possible number is 2 since 2 columns will be nonpivot columns no matter what since there are only 6 rows.

8) a) FALSE

$A^{-1} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \frac{1}{-2}\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 3/2 & -\frac{1}{2} \end{bmatrix}$

b) FALSE

c) FALSE

$A^T = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} \Rightarrow (A^T)^{-1} = \frac{1}{-2}\begin{bmatrix} 4 & -3 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 3/2 \\ 1 & -\frac{1}{2} \end{bmatrix}$

d) TRUE