By signing below, you attest that you have neither given nor received help of any kind on this exam.

Signature: ______________________ Printed Name: ______________________

Instructions: Show work to get full credit (the correct answer may NOT be enough). Do all your work on the paper provided. Write clearly! Double check your answers!

You will **not** receive full credit for using methods other than those discussed in class.

Exam III

MATH 214 – Linear Algebra

Problem Number	Available Points	Your Points
1	15	
2	15	
3	15	
4	15	
5	15	
6	25	
7	15	
8	25	
Total	100	

1. Determine whether the following matrices are invertible using few calculations as possible. Be sure to justify your answers. [12]

(a) $\begin{bmatrix} 6 & 7 \\ -3 & -5 \end{bmatrix}$ (c) $\begin{bmatrix} -7 & 0 & 4 \\ 3 & 0 & -1 \\ 2 & 0 & 9 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & -5 & -4 \\ 0 & 3 & 4 \\ -3 & 6 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 3 & 7 & 4 \\ 0 & 2 & -6 & 2 \\ 0 & 0 & 7 & 8 \\ 0 & 0 & 0 & -9 \end{bmatrix}$

2. Use co-factor expansion to find the determinant of $A = \begin{bmatrix} 4 & 2 & 0 & 0 \\ 0 & -3 & 1 & 0 \\ 0 & 2 & 0 & 4 \\ -3 & 0 & 5 & 1 \end{bmatrix}$. [15]

3. For this problem, let $H = \left\{ \begin{bmatrix} a+2c \\ 2a+b+3c \\ 3b-3c \\ a+4b-2c \end{bmatrix} : a, b, c \in \mathbb{R} \right\}$. Show that H is a subspace of $\mathbb{R}^4$ and find a basis for H. [15]

4. For this problem, let $A = \begin{bmatrix} 1 & 2 & 1 & 0 & 1 \\ 0 & 3 & 6 & -3 & 0 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & -2 & 4 \end{bmatrix}$ which has echelon form of $\begin{bmatrix} 1 & 2 & 1 & 0 & 1 \\ 0 & 1 & 2 & -1 & 0 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$. Find a **basis** for Col(A). What is the **dimension** of Col(A)? Find a **basis** for Nul(A). What is the **dimension** of Nul(A)? [15]

5. Let $T : M_{2\times 2} \to \mathbb{R}^3$ be the transformation defined by $T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = \begin{bmatrix} a+b \\ c \\ d+1 \end{bmatrix}$. Compute $T(A)$ for $A = \begin{bmatrix} 1 & -8 \\ 6 & -1 \end{bmatrix}$. Find the Kernel of T, $\ker(T)$. [15]

6. Provide an example of the following. [8]
   (a) Give an example of a finite dimensional vector space of dimension 12. Be sure to justify your answer.
   (b) Give an example of an infinite dimensional vector space. Be sure to justify your answer.

7. If A is a 6×8 matrix, what is the smallest possible dimension of Nul(A)? Justify your answer. [10]

8. For each of the following statements, please *circle* T (True) or F (False). You do not need to justify your answer. [10]
   (a) T or F? The matrix $A = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 3 \end{bmatrix}$ is invertible.
   (b) T or F? Let A be an $n \times n$ matrix. If the rows of A span $\mathbb{R}^n$, then the columns of A must be linearly independent.
   (c) T or F? If A is invertible then, $(A^T)^{-1} = A^{-1}$.
   (d) T or F? Let A, B, C be $n \times n$ matrices, then $\det(ABC) = \det(C)\det(B)\det(A)$.