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Quiz 9

1) a) $\begin{bmatrix} 2-\lambda & 0 & 0 \\ 1 & 2-\lambda & 1 \\ -1 & 0 & 1-\lambda \end{bmatrix}$

$(2-\lambda)\cdot\det\begin{bmatrix} 2-\lambda & 1 \\ 0 & 1-\lambda \end{bmatrix} = (2-\lambda)[(2-\lambda)(1-\lambda)-(0)(1)]$

$= (2-\lambda)(2-2\lambda-\lambda+\lambda^2)$

$= (2-\lambda)(\lambda^2-3\lambda+2)$

$= (2-\lambda)(\lambda-2)(\lambda-1)$

eigenvalues: 1 with a mult. of 1 and 2 with a multiplicity of 2

$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ -1 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$ $\beta = \left\{ \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\}$

$\begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 1 \\ -1 & 0 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ $\beta = \left\{ \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \right\}$

Since the sum of the dimensions of the eigenspaces corresponding to the eigenvalues is equal to 3 and therefore a basis for $\mathbb{R}^3$, the matrix A is diagonalizable.

$P = \begin{bmatrix} 0 & 0 & -1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ $D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

with P being invertible and $A = PDP^{-1}$