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Quiz 6

1) a) $ad - bc$
$= -30 - (-21) = -30 + 21 = -9$
The matrix is invertible since $ad - bc \neq 0$.

b) The matrix is not invertible since there is a column of zeroes, meaning there are not 3 pivots, so the Invertible Matrix Theorem is not satisfied.

c) $\begin{bmatrix} 1 & -5 & -4 \\ 0 & 3 & 4 \\ -3 & 6 & 0 \end{bmatrix} \sim \begin{bmatrix} -3 & 6 & 0 \\ 0 & 3 & 4 \\ 1 & -5 & -4 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 & 0 \\ 0 & 3 & 4 \\ 1 & -5 & -4 \end{bmatrix} \sim$

$R1 \leftrightarrow R3$ $\quad -\frac{1}{3}R_1 \to R_1$

$\begin{bmatrix} 1 & -2 & 0 \\ 0 & 3 & 4 \\ 0 & -3 & -4 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 & 0 \\ 0 & 3 & 4 \\ 0 & 0 & 0 \end{bmatrix}$

NOT invertible; there are not 3 pivot positions

d) Invertible because there are 4 pivot positions

2) $\det(A) = a_{11}\det(A_{11}) - a_{12}\det(A_{12}) + a_{13}\det(A_{13}) - a_{14}\det(A_{14})$
$+ a_{15}\det(A_{15})$

$= 1\cdot\det(A_{11}) - 0\cdot\det(A_{12}) + 0\cdot\det(A_{13}) - 1\cdot\det(A_{14}) +$
$0\cdot\det(A_{15})$

$= 1\cdot\det(A_{11}) - 1\cdot\det(A_{14})$

$$= 1\cdot\det\left(\begin{bmatrix} 0 & -3 & 4 & 0 \\ 4 & 0 & -1 & 3 \\ -3 & 1 & 0 & 1 \\ -5 & 0 & 0 & 0 \end{bmatrix}\right) - 1\cdot\det\left(\begin{bmatrix} 5 & 0 & -3 & 0 \\ 0 & 4 & 0 & 3 \\ 0 & -3 & 1 & 1 \\ 0 & -5 & 0 & 0 \end{bmatrix}\right)$$

$\downarrow$

$-5\cdot\det(A_{41})$ $\qquad$ $-5\cdot\det(A_{42})$

$$-5\cdot\det\left(\begin{bmatrix} -3 & 4 & 0 \\ 0 & -1 & 3 \\ 1 & 0 & 1 \end{bmatrix}\right) \qquad -5\cdot\det\left(\begin{bmatrix} 5 & -3 & 0 \\ 0 & 0 & 3 \\ 0 & 0 & 0 \end{bmatrix}\right)$$

$\downarrow$ $\qquad$ $\downarrow$

$-3\cdot\det(A_{11}) - 4\det(A_{12})$ $\qquad$ 0

$$-3\cdot\det\left(\begin{bmatrix} -1 & 3 \\ 0 & 1 \end{bmatrix}\right) - 4\cdot\det\left(\begin{bmatrix} 0 & 3 \\ 1 & 1 \end{bmatrix}\right)$$

$-3\cdot(-1-0) - 4\cdot(0-3)$

$-3(-1) - 4(-3)$

$3 + 12 = 15$

$$\begin{array}{r} 15 \\ \times 5 \\ \hline 75 \end{array}$$

$-5(15)$

-75

$\det(A) = -75$