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Quiz 7

1) (a) H contains the zero vector because when $a=0$ and $b=0$, then

$$\begin{bmatrix} 2(0)-3(0) \\ 3(0)+4(0) \\ 2(0)+0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2a-3b \\ 3a+4b \\ 2a+b \\ a \end{bmatrix} + \begin{bmatrix} 2c-3d \\ 3c+4d \\ 2c+d \\ c \end{bmatrix} = \begin{bmatrix} 2(a+c)-3(b+d) \\ 3(a+c)+4(b+d) \\ 2(a+c)+(b+d) \\ (a+c) \end{bmatrix}$$

Since $(a+c)$ and $(b+d)$ are real numbers, this condition is satisfied.

$$c\begin{bmatrix} 2a-3b \\ 3a+4b \\ 2a+b \\ a \end{bmatrix} = \begin{bmatrix} c(2a-3b) \\ c(3a+4b) \\ c(2a+b) \\ c(a) \end{bmatrix}$$

Since the entries are still all real numbers, this condition is satisfied.

(b) Given the nonzero entry at row 1, col 1, this matrix can never be all zeros, so H is not a subspace of V.

(2) $\mathbb{R}^n$

(3) $\mathbb{R}^m$

4) $\begin{bmatrix} 1 & 0 & 6 & 5 \\ 0 & 2 & 5 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 6 & 5 \\ 0 & 1 & \frac{5}{2} & \frac{3}{2} \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$x_1 + 6x_3 + 5x_4$

$x_2 + \frac{5}{2}x_3 + \frac{3}{2}x_4$

(a) $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -6x_3 - 5x_4 \\ -\frac{5}{2}x_3 - \frac{3}{2}x_4 \\ x_3 \\ x_4 \end{bmatrix} = x_3 \begin{bmatrix} -6 \\ -5/2 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -5 \\ -3/2 \\ 0 \\ 1 \end{bmatrix}$

$Nul(A) = Span\left\{ \begin{bmatrix} -6 \\ -5/2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -5 \\ -3/2 \\ 0 \\ 1 \end{bmatrix} \right\}$

$Col(A) = Span\left\{ \begin{bmatrix} -2 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ -6 \\ 8 \end{bmatrix} \right\}$

5) $\begin{bmatrix} 1 & 1 & 0 \\ 3 & 0 & 1 \\ 0 & 3 & 5 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 0 \\ 0 & -3 & 1 \\ 0 & 3 & 5 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 0 \\ 0 & -3 & 1 \\ 0 & 0 & 6 \end{bmatrix} \sim$

$\begin{bmatrix} 1 & 1 & 0 \\ 0 & -3 & 1 \\ 0 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

pivot in every column $\Rightarrow$ linearly independent

pivot in every row $\Rightarrow$ span $\mathbb{R}^3$

ALL of the set form a standard basis for $\mathbb{R}^3$