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Fundamental Limits in Cellular Networks with Point Process Partial Area Statistics

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Abstract

In this paper, we propose a new approach exploiting the cell capacity of the Spatial Continuum Broadcast Channel (SCBC) recently introduced for an isolated cell. The network capacity is linked to the cells' geometry statistics in a Voronoi tessellation. The fundamental limit is characterized by the minimal average cell power required in a network modelled as a Point Process (PP) to achieve a desired rate distribution. A direct relation is established between this minimum average power and the partial area statistics of the cells geometry, which constitute a sufficient statistic. Our approach is validated through Monte-Carlo simulations.

Problem

Evaluating an analogous result to the Shannon capacity for a cellular network.

Contributions

1. The fundamental Energy Efficiency - Spectral Efficiency limit is established under the assumptions mentioned below.
2. Average minimal cell power (AMCP) extended at the network scale.
3. Identification of the area and complementary partial area as sufficient statistics.
4. Validation of the theoretical results by extensive simulations.
5. Price of randomness computed and analysed by confronting Poisson and Matérn Point Processes.

Transmission model

Notation	Description
λ_{BS}	Density of base stations
λ_T	Density of mobile nodes
η	Spectral efficiency
$l(r)$	Pathloss function evaluated at the distance r
N_0	Noise power density
$\nu(r)$	Equivalent noise at distance r, $\nu(r) = N_0/l(r)$

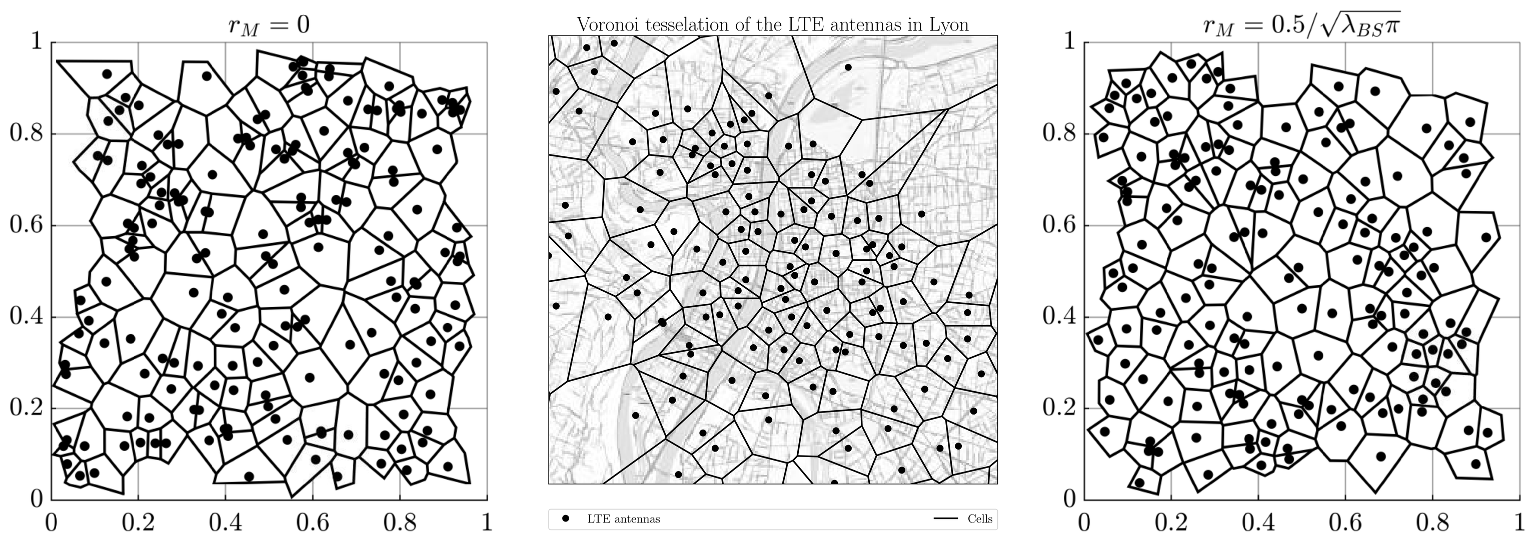
- Downlink transmission.
- Neither fast and static fading nor shadowing are considered.
- Additive White Gaussian Noise (AWGN) channel.
- Single antenna is considered at both the base station (BS) and the nodes.

Under these assumptions, the minimum transmission power at the BS for a cell n is [1, 2]:

$$P_{min,n} = 2\eta_n \int_0^{\nu_{max}} \nu f(\nu) e^{2\nu_n} G(\nu) d\nu \qquad (1)$$

Point processes

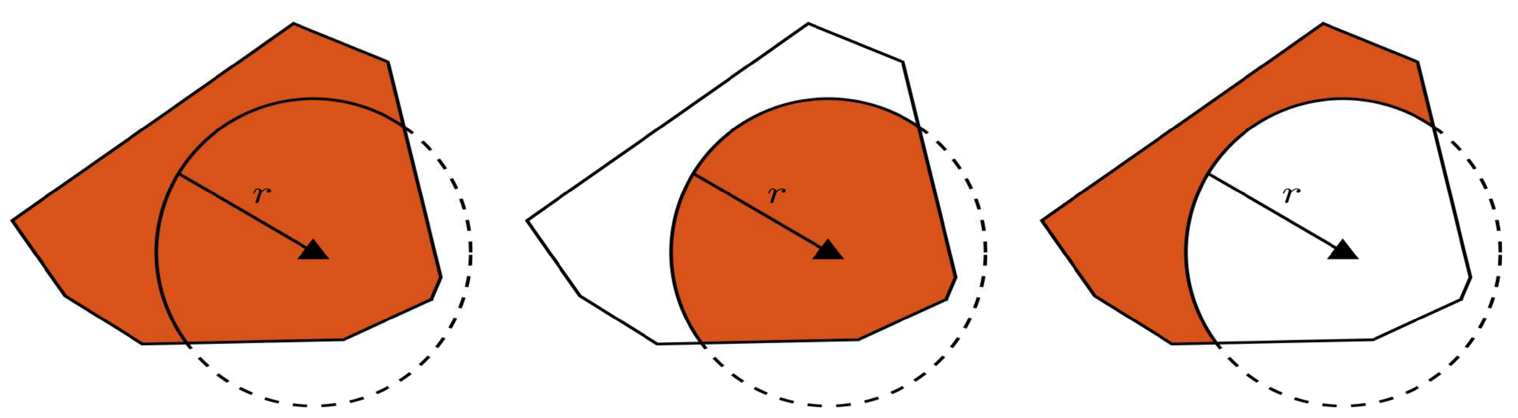
- 1 point process for the mobile nodes λ_T.
- 1 point process for BSs with density λ_{BS}.
- Point process for the BSs is either Poisson [3] or Matérn [4]



(a) Uniform PPP **(b)** LTE antennas **(c)** Matérn

Figure 1 – Three Voronoi tessellations corresponding to a PPP **(a)**, the LTE antennas of the city of Lyon **(b)** and a MPP **(c)** (Matérn hard-core type II point process). Both PPP and MPP have the same density $\lambda_P = \lambda_M = 200$. The cells in periphery of the Voronoi tessellation of the LTE antennas may be much larger than the average since we did not take into account the antennas of the adjacent cities.

Cell areas



(a) Total area **(b)** Partial area **(c)** Compl. partial area

Figure 2 – The total area of a cell associated to a BS (black triangle) is represented in (a). For a given distance r, the partial and complementary partial areas are represented in (b) and (c) respectively.

The complementary partial area (CPA) random variable is denoted by $A_{c,\lambda_{BS}}(r)$. Its moment generating function (MGF) is given by:

$$\mathcal{M}_{A_{c,\lambda(r)}}(t) := \sum_{k=0}^{+\infty} \frac{t^k \mathbb{E}\left[A_{c,\lambda}(r)^k\right]}{k!}. \qquad (2)$$

Thanks to the scaling properties of the MGF, the following relation between the CPA's MGFs for a non normalised and normalised PP holds:

$$\mathcal{M}_{A_{\lambda,c(r)}}(t) = \mathcal{M}_{A_c(u)}\left(\frac{t}{\lambda}\right), \qquad (3)$$

where $u := \sqrt{\lambda} \cdot r$ is a normalised distance.

Towards Average Minimum Cell Power (AMCP)

Average Minimum Cell Power – AMCP

Theorem 1

$$\bar{P}_{min} = \int_0^{\nu_{max}} \left(\mathbb{E}^0\left[e^{R_c(\nu)}\right] - 1\right) d\nu \qquad (4)$$

where $\mathbb{E}^0$ denotes the Palm expectation and $R_c(\nu)$ is the complementary sum-rate for equivalent noise ν.

AMCP with Complementary Partial Areas

Theorem 2

$$\bar{P}_{min} = \int_0^{u_{max}} \nu'\left(\frac{u}{\sqrt{\lambda}}\right) \beta(u) \left(\mathcal{M}_{\tilde{A}_c(u)}(2\eta) - 1\right) du \qquad (5)$$

where u is a normalised distance, $\nu'(u/\sqrt{\lambda}) = N_0 \frac{d}{du} l\left(u/\sqrt{\lambda}\right)^{-1}$ and $\beta(u)$ is the proportion of non-zero CPAs.

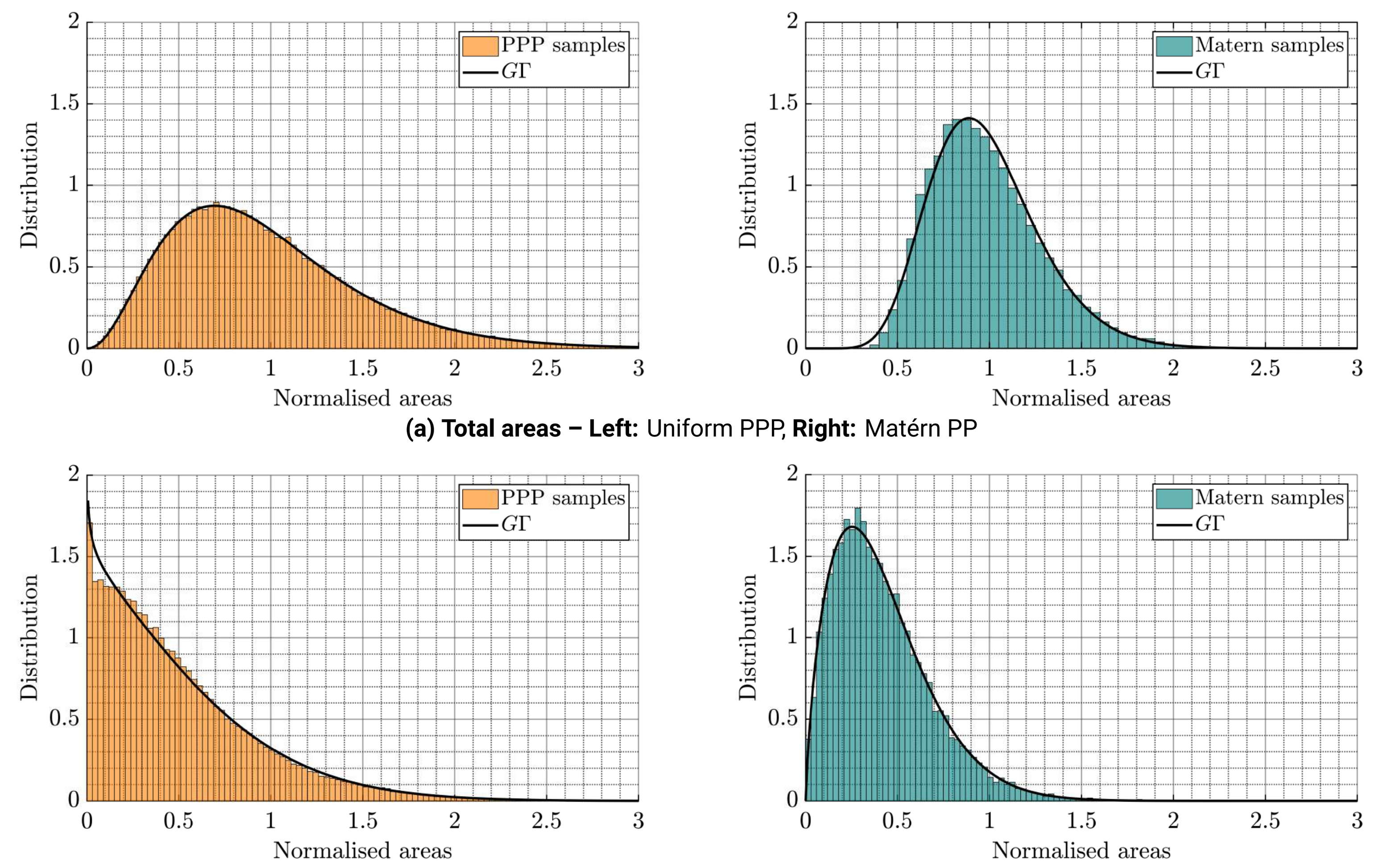
Convergence radius of the MGF

Theorem 3

Assuming that the total area follow a Generalised Gamma Distribution [5] of parameters (a, p, d), the convergence radius of its MGF is bounded by:

$$\varrho_c^* = \begin{cases} 0 & \text{if } p < 1 \\ \frac{1}{a} & \text{if } p = 1 \\ +\infty & \text{if } p > 1 \end{cases} \qquad (6)$$

Fits of area distributions



(a) Total areas – Left: Uniform PPP, **Right:** Matérn PP

(b) CPA – Left: Uniform PPP, **Right:** Matérn PP

Figure 3 – Histograms and generalized gamma fits for cell total area distributions (a) and non-zero CPA distributions with $u = 0.8/\sqrt{\pi}$ (b).

Validation by simulations

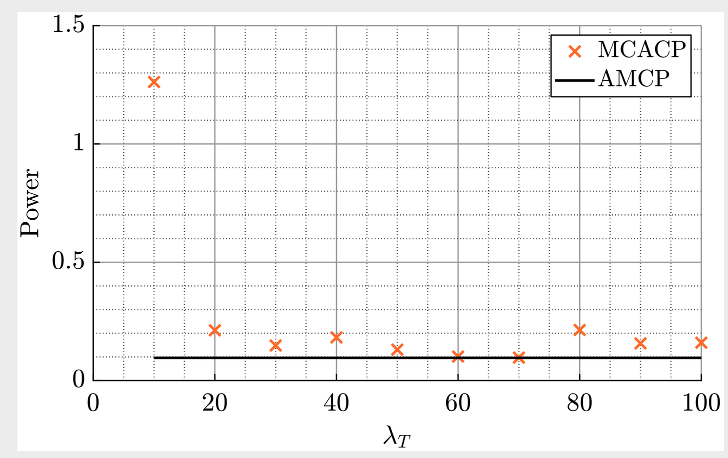


Figure 4 – Comparison between the MCACP and the AMCP. When the node density increases, the MCACP tends to the AMCP.

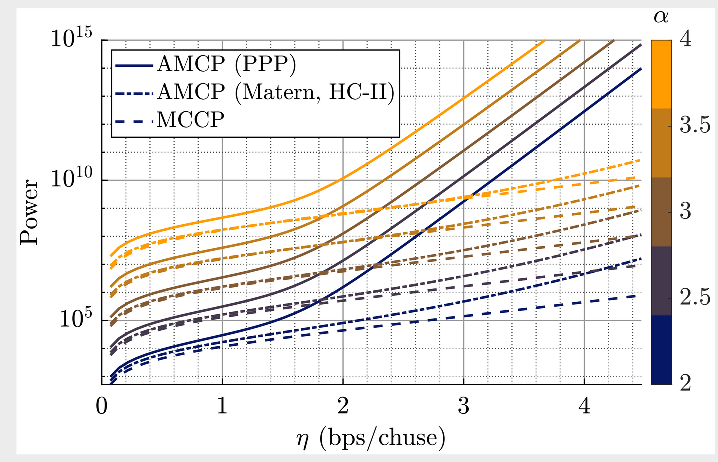


Figure 5 – Confrontation of the PPP's AMCP, the Matérn's AMCP and the MCCP for different values of η and different pathlosses.

Conclusion

- We exploited the **high order** statistics of the radio links, through CPA distribution, from the newly established analytic relation between AMCP and CPA statistics.
- We **validated** the AMCP with Monte-Carlo simulations.
- The cell geometry randomness in a **PPP generates a huge power overconsumption** in the network, which can be significantly **reduced in a more regular network**, as modelled with a MPP.
- At the best of our knowledge, this paper is the **first contribution dealing with the fundamental limit of a cellular network**, considering high order statistics in PPs.

References

[1] J.-M. Gorce et al. "Fundamental limits of a dense iot cell in the uplink". In: *IEEE WiOpt*. 2017.

[2] J.-M. Gorce, H. V. Poor, and J.-M. Kelif. "Spatial Continuum Model: Toward the Fundamental Limits of Dense Wireless Networks". In: *IEEE Globecom*. 2016.

[3] J. G. Andrews, F. Baccelli, and R. K. Ganti. "A tractable approach to coverage and rate in cellular networks". In: *IEEE Trans. Commun.* 59.11 (2011), pp. 3122–3134.

[4] M. Haenggi. *Stochastic geometry for wireless networks*. Cambridge University Press, 2012.

[5] M. Tanemura. "Statistical distributions of Poisson Voronoi cells in two and three dimensions". In: *Forma* 18 (2003), pp. 221–247. ISSN: 0911-6036.

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