COMPENG 3SK3: Computer-Aided Engineering
Assignment #3: SVD for Image Compression

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As a future member of the engineering profession, the student is responsible for performing the required work in an honest manner, without plagiarism and cheating. Submitting this work with my name and student number is a statement and understanding that this work is my own and adheres to the Academic Integrity Policy of McMaster University and the Code of Conduct of the Professional Engineers of Ontario. Submitted by **[Lelna Gwet, gwetl, 400060749]**

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Part I:

See Appendix and attached Matlab files for complete code.

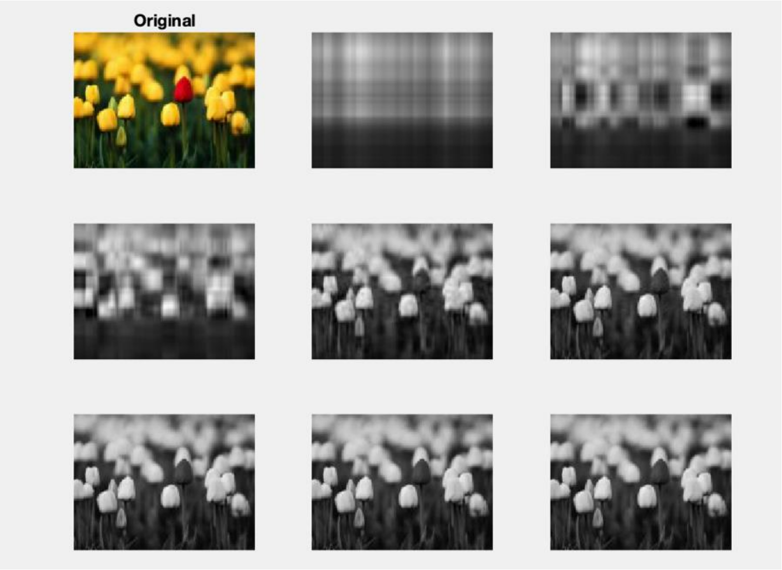

Figure 1a: SVD for lily.jpg

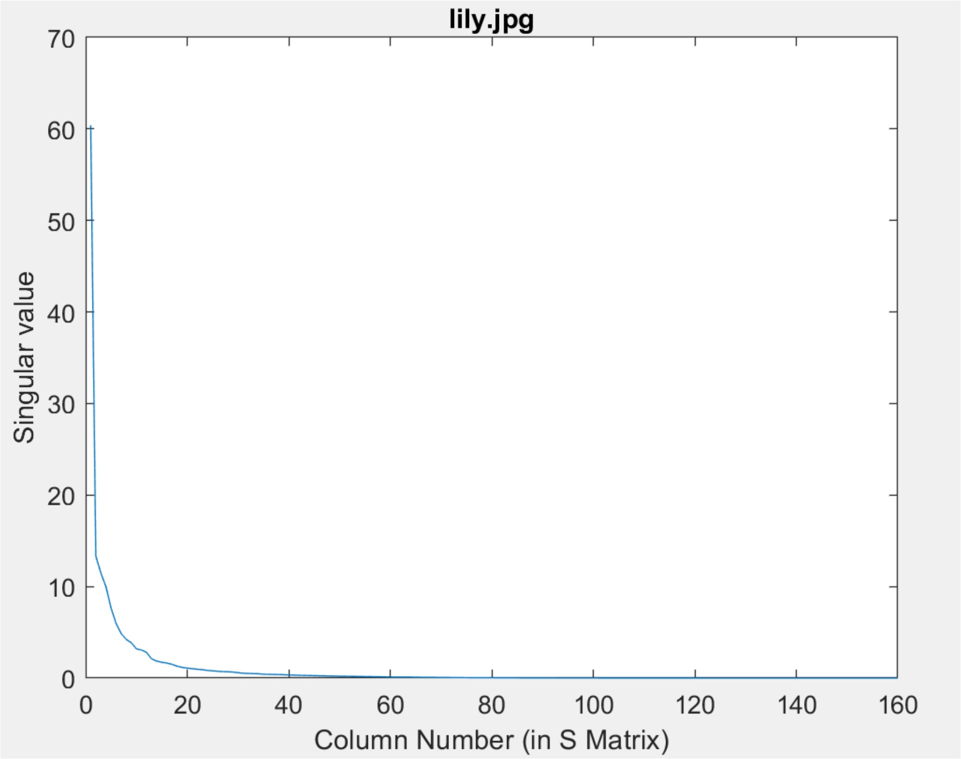

Figure 1b: Singular values for lily.jpg

Table *1*: Residual Error & Compression Rate for Lily.jpg

Rank = 1	$Residual\ error\ = 0.3802$	Compression rate $= 0.0146$
Rank = 2	$Residual\ error\ = 0.3206$	Compression rate $= 0.0293$
Rank = 4	$Residual\ error\ = 0.2204$	Compression rate $= 0.0585$
Rank = 16	$Residual\ error\ = 0.0619$	Compression rate $= 0.2342$
Rank = 32	$Residual\ error\ = 0.0238$	Compression rate $= 0.4683$
Rank = 64	$Residual\ error\ = 0.0043$	Compression rate $= 0.9367$
Rank = 100	$Residual\ error\ = 0.0005$	Compression rate $= 1.4635$
Rank = 120	$Residual\ error\ = 0.0000$	Compression rate $= 1.7563$

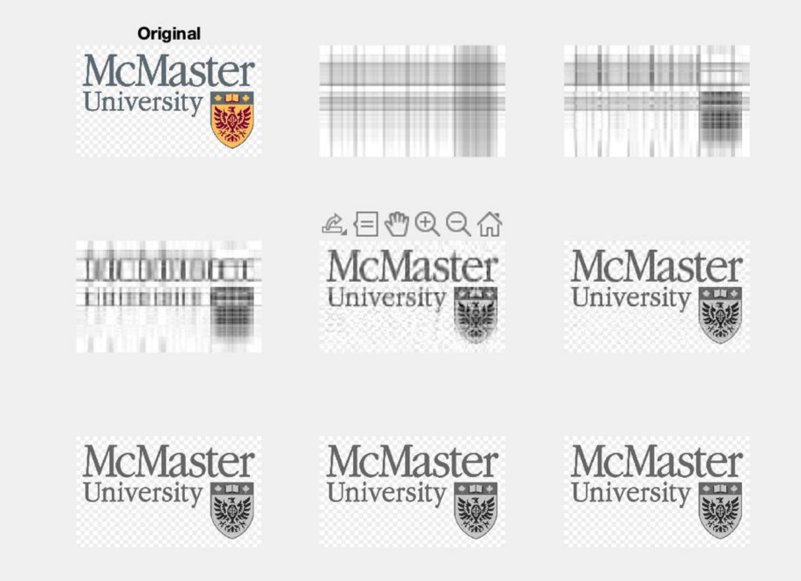

Figure *1*a: SVD for McmasterLogo.png

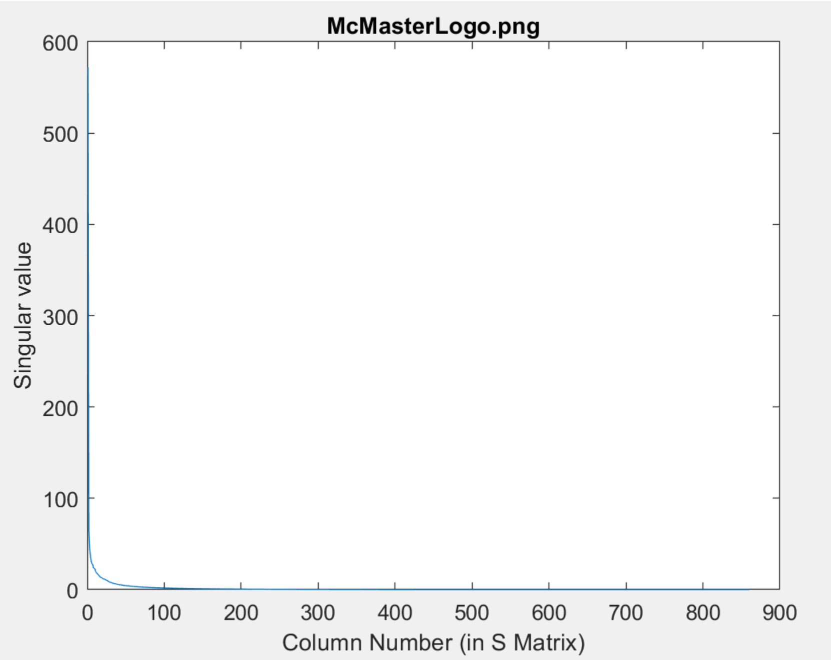

Figure 2b: Singular values for McMasterLogo.png

Table 2: Residual Error & Compression Rate for McmasterLogo.png

Rank = 1	$Residual\ error\ = 0.2196$	Compression rate $= 0.0031$
Rank = 2	$Residual\ error\ = 0.1891$	Compression rate $= 0.0062$
Rank = 4	$Residual\ error\ = 0.1605$	Compression rate $= 0.0124$
Rank = 16	$Residual\ error\ = 0.0922$	Compression rate $= 0.0496$
Rank = 32	$Residual\ error\ = 0.0568$	Compression rate $= 0.0992$
Rank = 64	$Residual\ error\ = 0.0318$	Compression rate $= 0.1984$
Rank = 128	$Residual\ error\ = 0.0149$	Compression rate $= 0.3967$
Rank = 430	$Residual\ error\ = 0.0000$	Compression rate $= 1.3327$

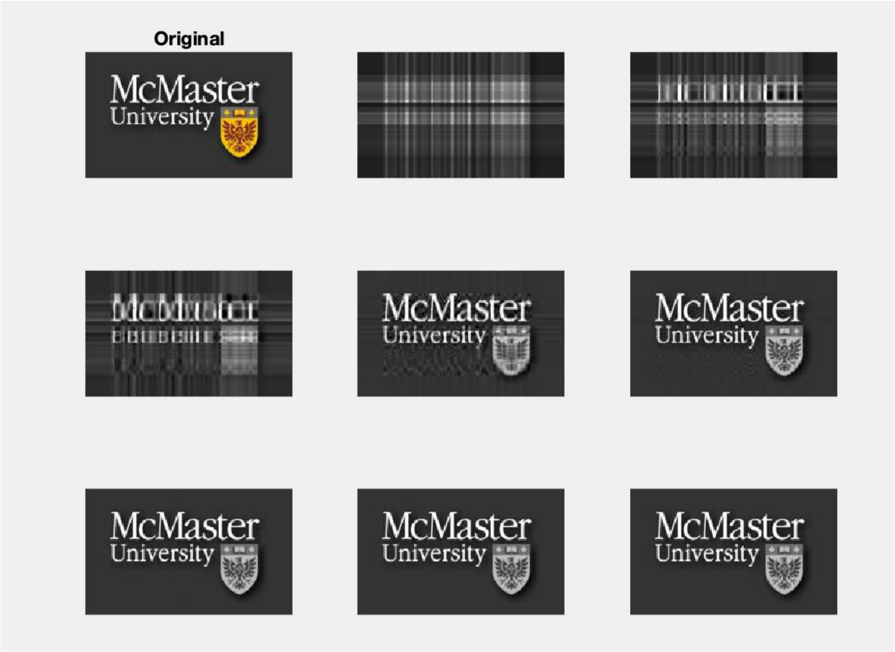

Figure 2a: SVD for Mcmaster_Logo.jpg

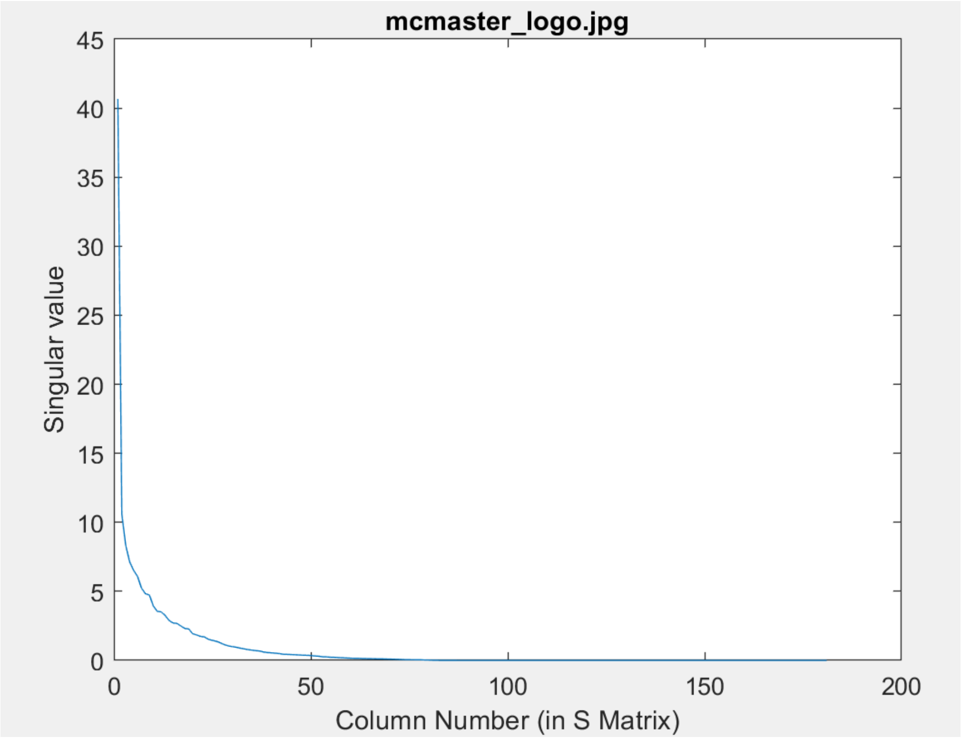

Figure 3b: Singular values for McMaster_logo.jpg

Table 3: Residual Error & Compression Rate for Mcmaster_Logo.jpg

Rank = 1	*Residual error* = 0.4851	Compression rate = 0.0147
Rank = 2	*Residual error* = 0.4281	Compression rate = 0.0293
Rank = 4	*Residual error* = 0.3573	Compression rate = 0.0587
Rank = 16	*Residual error* = 0.1510	Compression rate = 0.2347

Rank = 32	R$esidual\ error$ $= 0.0544$	Compression rate $= 0.4693$
Rank = 64	R$esidual\ error$ $= 0.0070$	Compression rate $= 0.9386$
Rank = 100	R$esidual\ error$ $= 0.0000$	Compression rate $= 1.4666$
Rank = 89	R$esidual\ error$ $= 0.0000$	Compression rate $= 1.3053$

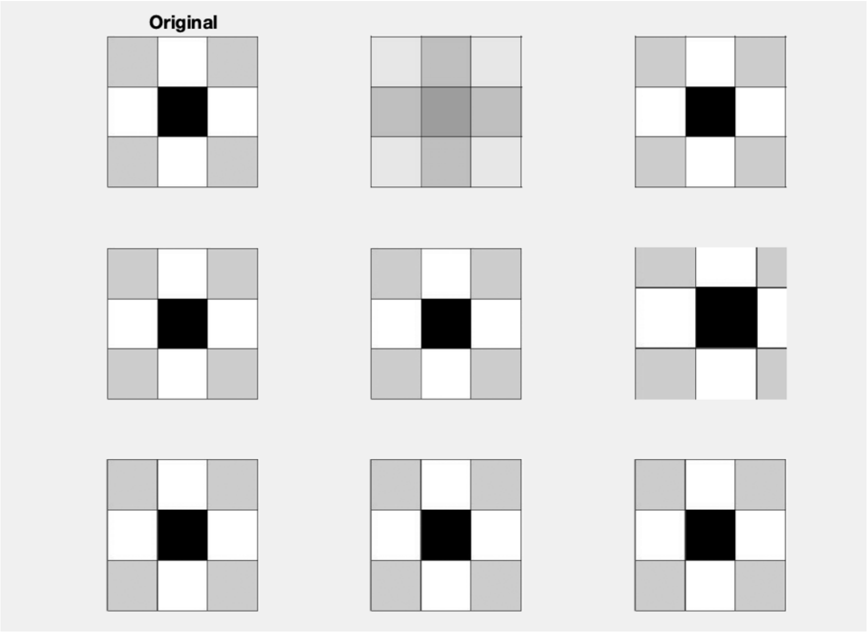

Figure 3a: SVD for Squares.bmp

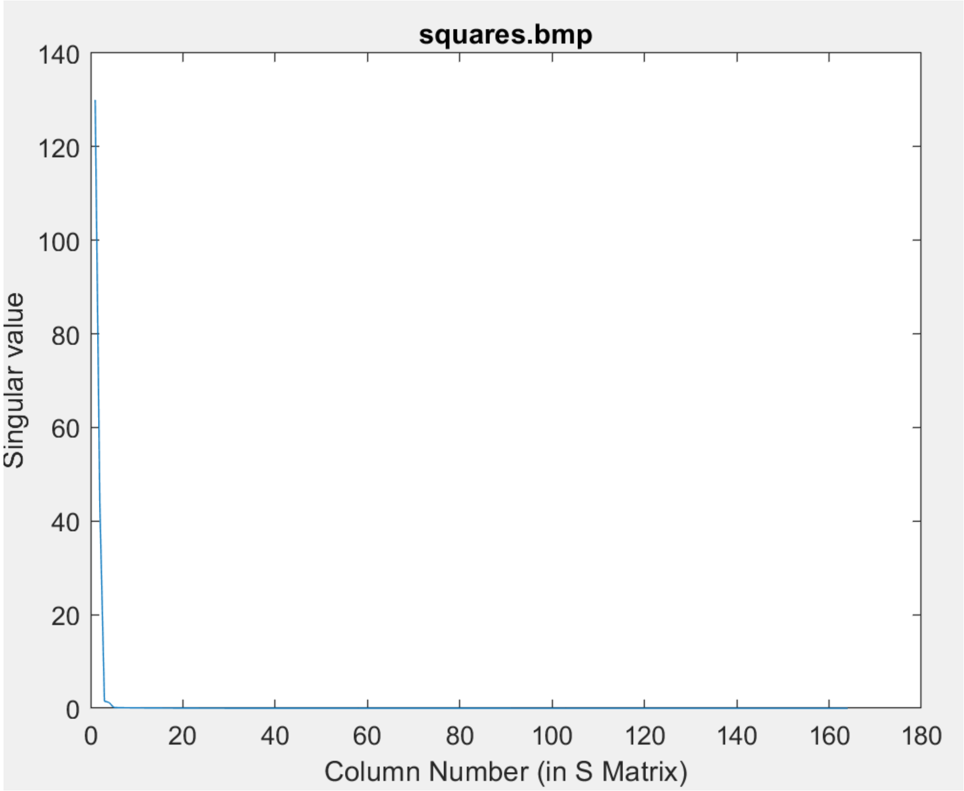

Figure 4b: Singular values for Squares.bmp

Table 4: Residual Error & Compression Rate for Squares.bmp

Rank = 1	$Residual\ error\ = 0.3188$	Compression rate = 0.0122
Rank = 2	$Residual\ error\ = 0.0145$	Compression rate = 0.0245
Rank = 4	$Residual\ error\ = 0.0026$	Compression rate = 0.0489
Rank = 16	$Residual\ error\ = 0.0011$	Compression rate = 0.1957
Rank = 32	$Residual\ error\ = 0.0006$	Compression rate = 0.3914
Rank = 64	$Residual\ error\ = 0.0003$	Compression rate = 0.7829
Rank = 100	$Residual\ error\ = 0.0001$	Compression rate = 1.2232
Rank = 137	$Residual\ error\ = 0.0000$	Compression rate = 1.6758

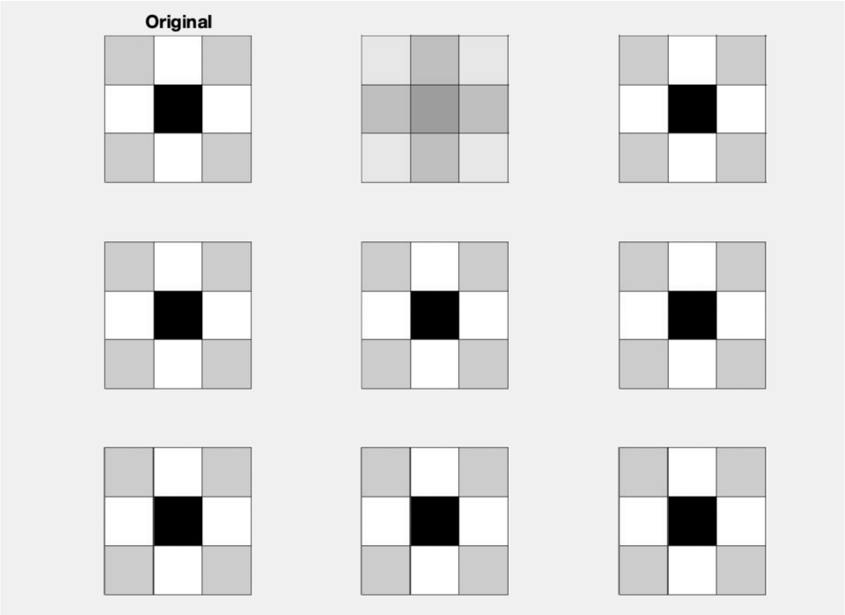

Figure 4a: SVD for Sqaures.jpg

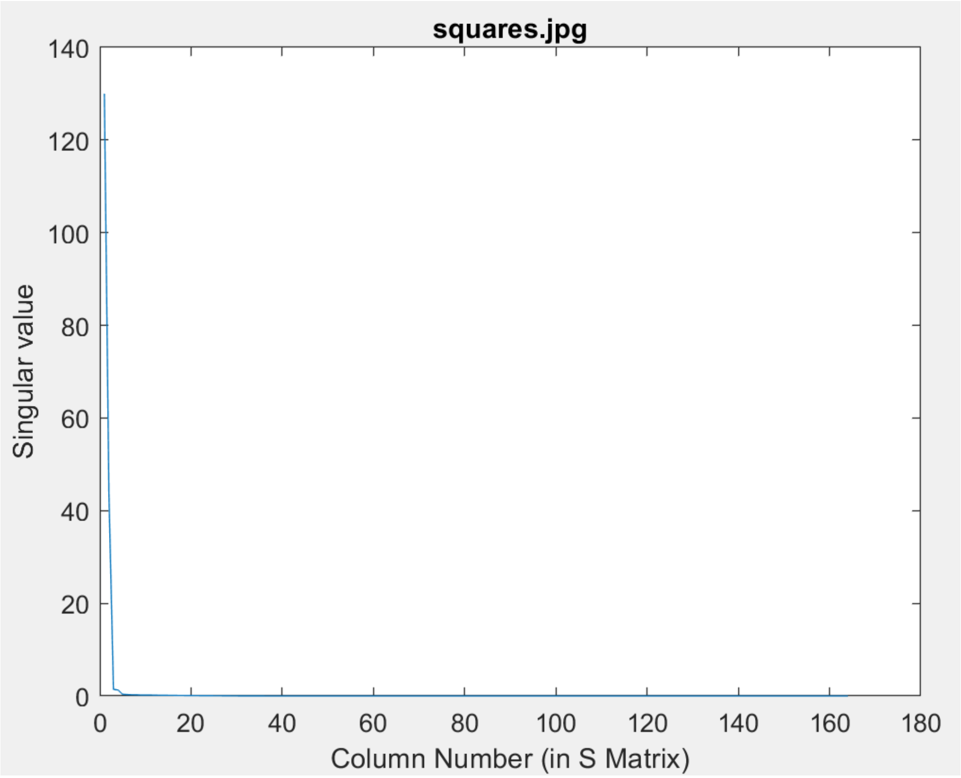

Figure 5b: Singular values for Squares.jpg

Table 5: Residual Error & Compression Rate for Squares.jpg

Rank = 1	$Residual\ error\ = 0.3187$	Compression rate $= 0.0122$
Rank = 2	$Residual\ error\ = 0.0146$	Compression rate $= 0.0245$

Rank = 4	$Residual\ error\ = 0.0029$	Compression rate $= 0.0489$
Rank = 16	$Residual\ error\ = 0.0010$	Compression rate $= 0.1957$
Rank = 32	$Residual\ error\ = 0.0002$	Compression rate $= 0.3914$
Rank = 64	$Residual\ error\ = 0.0000$	Compression rate $= 0.7829$
Rank = 100	$Residual\ error\ = 0.0000$	Compression rate $= 1.2232$
Rank = 54	$Residual\ error\ = 0.0000$	Compression rate $= 0.6605$

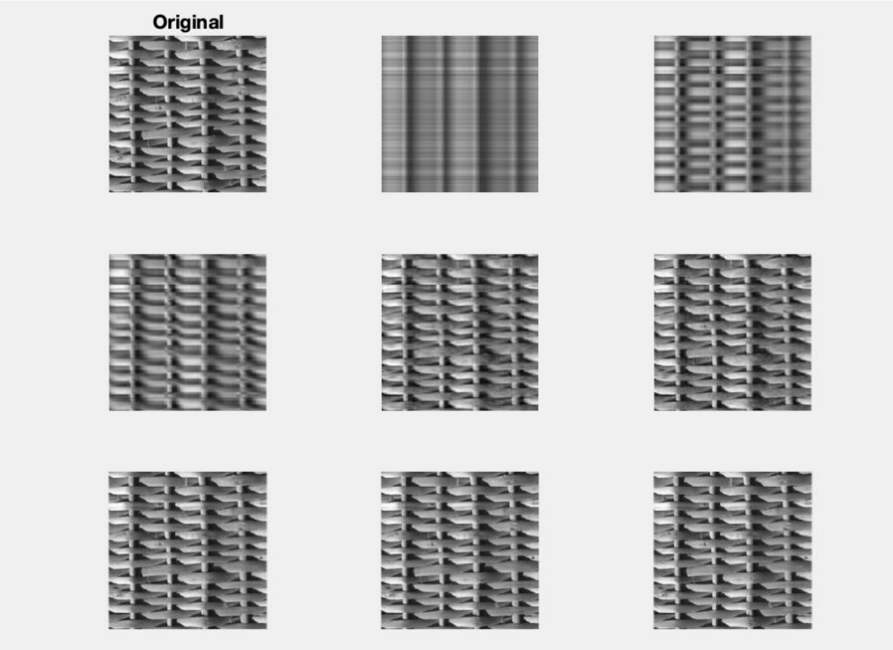

Figure 5a: SVD for Weave.bmp

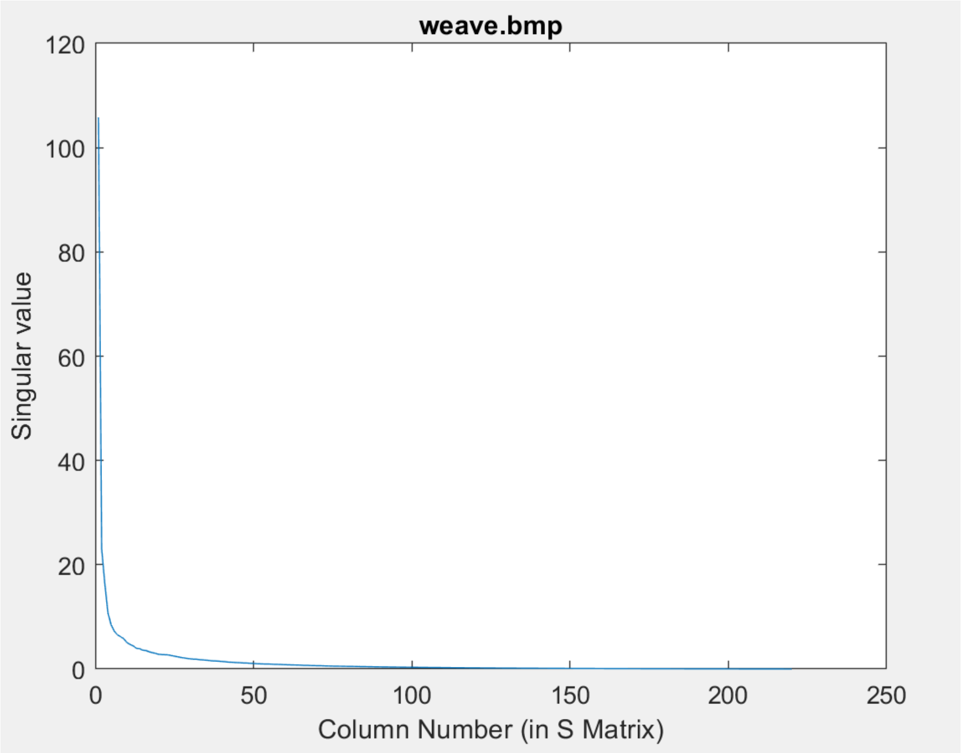

Figure 6b: Singular values for Weave.bmp

Table 6: Residual Error & Compression Rate for Weave.bmp

Rank = 1	$Residual\ error\ = 0.3373$	Compression rate = 0.0091
Rank = 2	$Residual\ error\ = 0.2678$	Compression rate = 0.0182
Rank = 4	$Residual\ error\ = 0.2033$	Compression rate = 0.0364
Rank = 16	$Residual\ error\ = 0.1126$	Compression rate = 0.1458
Rank = 32	$Residual\ error\ = 0.0675$	Compression rate = 0.2916
Rank = 64	$Residual\ error\ = 0.0298$	Compression rate = 0.5831
Rank = 128	$Residual\ error\ = 0.0071$	Compression rate = 1.1663
Rank = 220	$Residual\ error\ = 0.0000$	Compression rate = 2.0045

Part II (Observations):

1. **For the png-format and jpg-format McMaster logo,**
   a. At which rank approximation can you make out what University?
   For the png-format, the university name was made out at rank 16.
   For the jpg-format, the university name was made out at rank 16.
   b. How quickly does the wording come out as opposed to the crest?
   For the png-format, the wording comes 16 ranks before the crest.
   Similarly, for the jpg-format, the wording comes out 16 ranks before the crest.

2. **For bmp-format and jpg-format square picture,**
   a. At which rank approximation can you see the squares?
   For the bmp-format, the squares can be seen in rank 2.
   Similarly, for the jpg-format, the squares can be seen in rank 2.

3. **For the weave,**
   a. At which rank approximation can you see the weave?
   The weave can be seen at rank 16.

4. **Generate a random picture with a resolution of 220x220. Generate the elements from the uniform distribution between 0 and 100. Do the SVD approximations with different rank values {1, 2, 4, 16, 32, 64, 128, and 220}. What can you see from the approximated pictures as you increase the rank?**
   As the rank is increased, more dots appear in the image. The dots disappear at the final rank (220) where the image appears identical to the original image.

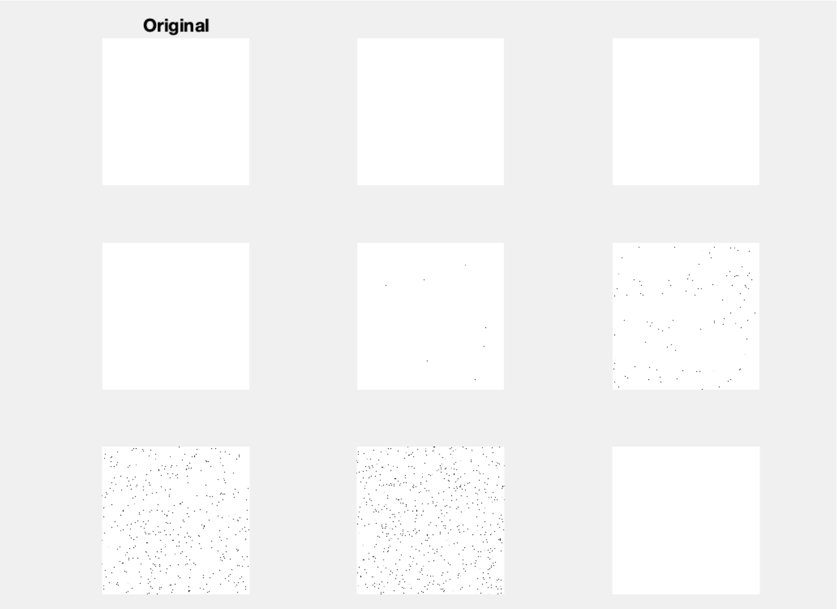

Figure 6: SVD for random image.

Part III (Discussions):

1. **The singular values drop much faster in some pictures than in other ones. What features of the picture or image file may contribute to this fact?**

   Each singular value in the S matrix represents a specific subset of details pertaining to the image. Since the PNG format is a lossless compression format, all the details of the original image are retained. Therefore, not only are there more singular values available to represent all the details but, there is a smoother transition between the singular values (i.e., smaller increments). In contrast, the JPG format is a lossy compressed file format. Since information is removed in order to compress the image, there will be "jumps" in information, resulting in the singular values of jpg image files dropping much faster with a more jagged (i.e., less smooth) curve.

2. **For each picture, what is the minimum number of ranks needed to recover a reasonably good quality of the picture after SVD approximation? How is this number related to the percentage of energy left over in the approximations?**

   With the exception of the 'squares' images, the minimum number of ranks needed to recover a reasonably good quality picture after SVD is 32. For each of the 'squares' images, the minimum number of ranks is 2.

3. **How efficient is the SVD compression for different types of images? Answer the question by comparing jpeg, bmp, and png format images, natural vs. random pictures, and simple (e.g., the square picture) vs. more complicated pictures (e.g., the weave picture).**
   The SVD compression is more efficient for jpeg images when comparing it with bmp and png formats. It is also more efficient with simple pictures than complicated ones. The compression rate for the randomly generated picture was very similar to that of the complicated weave picture.

Appendix A: SVD For Image Compression

```
clc; clear;
% read in image & display
A = imread('weave.bmp');
subplot(3, 3, 1)
imshow(A)
title('Original')

%convert image into grayscale
X_pre = im2gray(A);
X = im2double(X_pre);
size(X)
rank(X)

if width(X) >= 220
   range = [1 2 4 16 32 64 128 rank(X)];
else
   range = [1 2 4 16 32 64 100 rank(X)];
end

%find S V D
[U, S, V] = svd(X);

%initialize vector to store percentage energy error w/ Frobenius norm
residual_error = zeros(8,1);
compression_rate = zeros(8, 1);

plotind = 2;
i = 1;
for r=range
   Xapprox = U(:, 1:r)*S(1:r, 1:r)*V(:, 1:r)';
   subplot(3, 3, plotind), plotind = plotind+1;
   imshow(Xapprox), axis off
   original_norm = norm(X, 'fro');
   residual_error(i) = norm((X-Xapprox), 'fro')/norm(X, 'fro');
   residual_error
   compression_rate(i) = (r + (r*width(X))+ (r*height(X)))/(height(X)*width(X));
   compression_rate
   i = i+1;
end
```

Appendix B: SVD For Random Image

```
X = randi(100, 220, 220);
subplot(3, 3, 1)
imshow(X)
title('Original')
%find S V D
[U, S, V] = svd(X);


%initialize vector to store percentage energy error w/ Frobenius norm
residual_error = zeros(8,1);
compression_rate = zeros(8, 1);

plotind = 2;
i = 1;
for r=[1 2 4 16 32 64 128 220]
    Xapprox = U(:, 1:r)*S(1:r, 1:r)*V(:, 1:r)';
    subplot(3, 3, plotind), plotind = plotind+1;
    imshow(Xapprox), axis off
    original_norm = norm(X, 'fro');
    residual_error(i) = norm((X-Xapprox), 'fro')/norm(X, 'fro');
    residual_error
    compression_rate(i) = (r + (r*width(X))+ (r*height(X)))/(height(X)*width(X));
    compression_rate
    i = i+1;
end
```