

# Reflecting Intelligent Surfaces-Assisted Multiple-Antenna Coded Caching

Xiaofan Niu\*, Minquan Cheng<sup>†</sup>, Kai Wan\*, Robert Caiming Qiu\*, Giuseppe Caire<sup>§</sup>

\*Huazhong University of Science and Technology, 430074 Wuhan, China, {nxfan,kai\_wan,caiming}@hust.edu.cn

<sup>†</sup>Guangxi Normal University, 541004 Guilin, China, chengqinshi@hotmail.com

<sup>§</sup>Technische Universität Berlin, 10587 Berlin, Germany, caire@tu-berlin.de

**Abstract**—Reconfigurable intelligent surface (RIS) has been treated as a core technique in improving wireless propagation environments for the next generation wireless communication systems. This paper proposes a new coded caching problem, referred to as Reconfigurable Intelligent Surface (RIS)-assisted multiple-antenna coded caching, which is composed of a server with multiple antennas and some single-antenna cache-aided users. Different from the existing multi-antenna coded caching problems, we introduce passive RIS (with limited number of units) into the systems to further increase the multicast gain (i.e., degrees of freedom (DoF)) in the transmission, which is done by using RIS-assisted interference nulling. That is, by using RIS, we can ‘erase’ any path between one transmission antenna and one receive antenna. We first propose a new RIS-assisted interference nulling approach to search for the phase-shift coefficients of RIS for the sake of interference nulling, which converges faster than the state-of-the-art algorithm. After erasing some paths in each time slot, the delivery can be divided into several non-overlapping groups including transmission antennas and users, where in each group the transmission antennas serve the contained users without suffering interference from the transmissions by other groups. The division of groups for the sake of minimizing the DoF could be formulated into a combinatorial optimization problem. We propose a grouping algorithm which can find the optimal solution with low complexity, and the corresponding coded caching scheme achieving this DoF.

**Index Terms**—Coded caching, reconfigurable intelligent surface, zero-forcing

## I. INTRODUCTION

Coded Caching was introduced to address content-related traffic congestion in networks. In addition to the local caching gain (i.e., when a user requires some content which has been already stored, the server does not need to transmit), the strategy of coded caching involves transmitting multicast messages and leveraging cached content to eliminate interference such that a single transmission can serve multiple users, resulting in a preferable multicast gain and showing a great potential to alleviate a significant part of the traffic. The original coded caching work was introduced by Maddah-Ali and Niesen (MN) in [1] for a Single Input Single Output (SISO) shared-link model, where a central server with access to a library containing  $N$  files connects to  $k$  cache-aided users via an error-free shared link. Each user is equipped with a cache of size  $M$  files. The coded caching process contains two phases: placement and delivery. During the placement phase, each user stores parts of content from each file without knowing future demands. During the delivery phase, each user requests a file

from the library. The server broadcasts coded messages based on users’ caches and requests to satisfy all users’ demands. The objective is to minimize the worst-case transmission load among all possible demands (or equivalently maximize the worst-case coded caching gain). If each file is stored totally  $t = KM/N$  times by users, the seminal MN coded caching scheme constructs multicast messages, each of which is useful to  $t + 1$  users, i.e., the coded caching gain is  $t + 1$  (the corresponding load is  $(K - t)/(t + 1)$  where  $K - t$  represents the local caching gain). It was shown in [2], [3] that under the constraint of uncoded cache placement (i.e., each user directly caches a subset of the library bits) and  $N \geq K$ , the achieved coded caching gain was proved to be optimal [2], [3].

Built upon the groundbreaking coded caching strategy, considerable research has been dedicated to wireless coded caching problems, with an emphasis on harnessing the benefits of coded caching gain. Multi-antenna coded caching problem was originally proposed in [4], where  $K_T$  transmitters with memory size  $M_T$  and  $K_R$  receivers with memory size  $M_R$  are connected through a wireless network. By smartly combining spatial multiplexing gain and coded caching gain, a multi-antenna coded caching scheme was proposed in [4] with a total degree-of-freedom (sum-DoF, i.e., overall multiplexing gain)  $\min\{\frac{K_T M_T + K_R M_R}{N}, K_R\}$ , which was proved to be optimal under the constraint of uncoded cache placement and one-shot linear delivery. Following the work in [4], numerous works have been proposed for various multi-antenna coded caching problems to find out coded caching schemes to maximizing the multiplexing gain; just list a few works [4]–[10]. A new combinatorial construction structure on multi-antenna coded caching schemes with uncoded cache placement and one-shot zero-forcing delivery, referred to as Multiple-Antenna Placement Delivery Array (MAPDA), was proposed in [11], [12], by extending the placement delivery array (PDA) structure proposed in [13] for the original shared-link coded caching problem. Under this structure, designing a coded caching scheme could be transformed into designing an array satisfying some constraints, where the latter one is a combinatorial problem and could be solved by using combinatorial tools.

In this paper, we introduce Reconfigurable Intelligent Surfaces (RIS), seen as a promising technique for the next generation wireless systems to reconfigure wireless propagation environment [14]–[18], into coded caching systems in order to further increase the sum-DoF. As illustrated in Fig. 1,

RIS serves as a passive configurable relay (by its phase-shift coefficients) in the channel, where the system channel transition matrix is equal to the product of the transition matrix from the transmitter to the RIS, the RIS phase-shift matrix, and the transition matrix from the RIS to the receiver. Several information theoretic works have been proposed on RIS-assisted interference management, in order to increase the sum-DoF. In [19] the authors considered RIS-assisted  $K$ -user interference channel. RIS-assisted interference alignment scheme was proposed for different numbers of RIS units. With the increasing of the RIS units, the sum-DoF can be increased from  $K/2$  to  $K$ . Following the RIS-assisted  $K$ -user interference channel, the authors in [20] proposed a RIS-assisted interference nulling approach with zero-forcing transmission, which achieves the sum-DoF equal to  $K$  when the number of RIS units is over than a threshold approximately equal to  $2K(K-1)$ . This is done by using RIS to ‘erase’ all the interfering paths for the users. To find out the RIS phase-shift coefficients for interference nulling, an alternating projection algorithm was proposed in [20]. Then RIS-assisted zero-forcing and interference alignment were also extended to Multiple-Input-Multiple-Output (MIMO) networks in [21], [22], in order to increase the sum-DoF.

**Main Contribution:** We propose a RIS-assisted multiple-input single-output (MISO) broadcast coded caching system with one-shot linear delivery, for fixed numbers of antennas at the transmitter and of RIS units. Our main contribution on constructing RIS-assisted multi-antenna coded caching schemes is as follows.

- We first propose a new RIS-assisted interference nulling approach to ‘erase’ some paths in the wireless channel, which outputs the needed RIS phase-shift coefficients with a faster convergence rate than the algorithm in [20].
- With the help of RIS-assisted interference nulling, we can divide the transmission in each time slot of the delivery phase into multiple groups including transmission antennas and users, where in each group the transmission antennas serve the contained users without suffering interference from the transmissions by other groups. By formulating the grouping optimization to maximize the sum-DoF as a combinatorial optimization problem, we propose an low-complexity algorithm to find out the optimal grouping method.
- After determining the optimal grouping approach, we design the two-phase coded caching scheme under a new MAPDA struture, referred to as RMAPDA, by adding a new constraint on group-based delivery into MAPDA.

**Notation Convention:** Scalars are denoted by lowercase letters, vectors by bold lowercase letters, and matrices by bold uppercase letters. For a matrix  $\mathbf{A}$ ,  $\mathbf{A}^T$  represents the transpose of the matrix,  $\mathbf{A}^*$  represents the conjugate.  $|v|$  denotes the magnitude of a complex scalar.  $\text{Re}[\cdot]$  denotes taking the real part of a complex number.  $\cdot$  represents the element-wise multiplication of matrices.  $\odot$  indicates the Hadamard product.  $\text{diag}$  denotes creating a diagonal matrix from a given

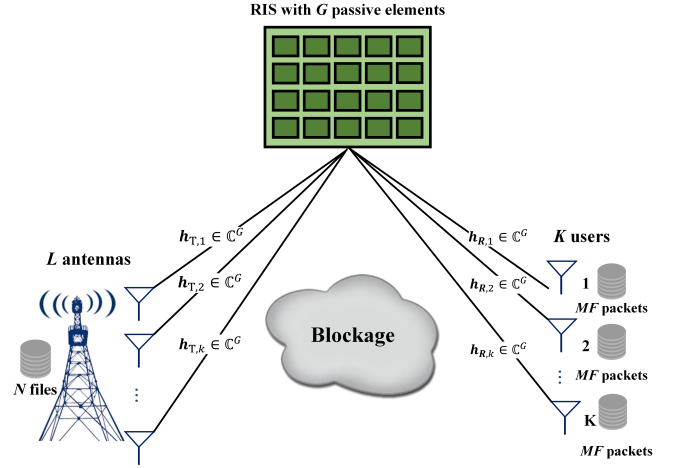


Fig. 1: RIS-assisted MISO broadcast system.

column vector, where the elements of the vector form the main diagonal elements. For a vector  $\mathbf{v}$ ,  $\text{diag}(\mathbf{v})$  represents the diagonal matrix whole diagonal elements are the elements in  $\mathbf{v}$ . Define that  $[a] = \{1, 2, \dots, a\}$ .

## II. SYSTEM MODEL AND PRELIMINARY RESULTS

### A. System model

As illustrated in Fig. 1, we consider a  $(L, G, K, M, N)$  RIS-assisted MISO broadcast coded caching system with one-shot linear delivery. A server has  $L$  transmission antennas and can access to a library containing  $N$  files, denoted by  $\mathcal{W} = \{\mathbf{W}_n \mid n \in [N]\}$ . Each file  $\mathbf{W}_n$  consists of  $F$  packets, defined as  $\mathbf{W}_n \triangleq \{\mathbf{W}_{n,f} \mid f \in [F]\}$ . Each user is equipped with one antenna and a cache capable of storing  $MF$  packets, where  $0 \leq M \leq N$ .

A coded caching scheme consists of two phases:

**Placement Phase:** During this phase,  $F$  data packets of each file are strategically placed into the users’ caches without knowledge of the specific demands.

**Delivery Phase:** Each user  $k \in [K]$  requests an arbitrary file  $\mathbf{W}_{d_k}$  from the library, where  $d_k$  is an index within the range  $[N]$ . Once the user demands are known, denoted by  $\mathbf{d} = (d_0, d_1, \dots, d_K)$ , the server transmits coded packets through  $L$  antennas. We first encode each packet of desired files by Gaussian encoding with rate  $\log P$  to obtain a coded packet, where  $P$  represents the Signal-to-Noise (SNR) ratio. The communication process contains  $S$  time slots, each denoted by  $s \in [S]$ . In time slot  $s$ , the signal transmitted by antenna  $j$  is denoted by  $X_j(s)$ , which is a linear combination of some coded packets desired by the users in  $\mathcal{K}_s \subseteq [K]$ .

In the wireless channel, there exists a passive RIS with  $G$  units. For the ease of description, we assume that the direct paths between the transmitters and receivers are blocked. As shown in [20], the RIS-assisted interference nulling method proposed in this case of no direct path could be directly extended to ‘erase’ the path between a transmission antenna and a user for the systems where direct paths exist. Define  $\mathbf{h}_{T,j}$

as the channel transition vector with dimension  $G$  between the  $j$ -th transmission antenna (or equivalently called the  $j$ -th transmitter) and the RIS, and define  $\mathbf{h}_{R,k}$  as the channel transition vector with dimension  $G$  between the RIS and the  $k$ -th receiver. It is assumed that the coefficients in these vectors are i.i.d over some continuous distributed and that channel state information (CSI) is fully known to the server and users. The reflection coefficients of the RIS are given by  $\mathbf{v} = [e^{j\omega_1}, e^{j\omega_2}, \dots, e^{j\omega_G}]^T \in \mathbb{C}^G$ , where  $\omega_i \in (0, 2\pi)$  indicates the phase-shift coefficient of the  $i$ -th unit of the RIS. Note that the phase-shift vector  $\mathbf{v}$  could be re-configured during each time slot. Each receiver  $k$  receives a reflected signal from the RIS at time slot  $s$ , which can be expressed as

$$Y_k(s) = \sum_{j=1}^L \mathbf{h}_{R,k}^T \text{diag}(\mathbf{v}) \mathbf{h}_{T,j} X_j(s) + n_k(s) \quad (1a)$$

$$= \sum_{j=1}^L \mathbf{a}_{k,j}^T \mathbf{v} X_j(s) + n_k(s), \quad (1b)$$

where  $\mathbf{a}_{k,j} \triangleq \text{diag}(\mathbf{h}_{T,j}) \mathbf{h}_{R,k}$  and  $n_k(s)$  represents the additive Gaussian white noise at receiver  $k$ . Assume that the delivery is one-shot: in time slot  $s$ , each user  $k \in \mathcal{K}_s$  should recover one coded packet from  $Y_k(s)$  and its cache content.

**Objective.** We consider the high SNR regime and sum-DoF as the metric. Under the one-shot linear delivery, the sum-DoF indicates the average number of users served per time slot. Our objective is to maximize the worst-case sum-DoF among all possible demands.

### B. MAPDA

In [11], for the MISO broadcast coded caching systems, the authors proposed a combinatorial construction structure on multi-antenna coded caching schemes, by combining PDA in [13] and zero-forcing transmission.

**Definition 1** (MAPDA, [11]). For any positive integers  $L, K, F, Z$ , and  $S$ , an  $F \times K$  array  $P$  composed of "\*" and  $[S]$  is called an  $(L, K, F, Z, S)$  multiple-antenna placement delivery array (MAPDA) if it satisfies the following conditions:<sup>1</sup>

- C1: The symbol "\*" appears  $Z$  times in each column;
- C2: Each integer from 1 to  $S$  occurs at least once in the array;
- C3: Each integer  $s$  appears at most once in each column;
- C4: For any integer  $s \in [S]$ , define  $P(s)$  to be the subarray of  $P$  including the rows and columns containing  $s$ , and let  $r'_s \times r_s$  denote the dimensions of  $P(s)$ . The number of integer entries in each row of  $P(s)$  is less than or equal to  $\min\{L, K\}$ , i.e.,

$$|\{k_1 \in [r_s] \mid P(s)(f_1, k_1) \in [S]\}| \leq \min\{L, K\}, \quad \forall f_1 \in [r'_s].$$

Note that when  $L = 1$  the MAPDA reduces to the PDA in [13]. From each MAPDA, we can obtain a corresponding multi-antenna coded caching scheme for the system containing

<sup>1</sup>An array is composed of "\*" and integers. Each column represents a user and each row represents a packet of files. A "\*" appears in the position  $(i, j)$  of the array, meaning that user  $j$  stores the  $i$ -th packet of each file. Each integer corresponds to a multicast message transmitted in one time slot.

a server with  $L$  antennas and  $K$  single-antenna users with memory ratio (i.e., the ratio between memory size and library size)  $M/N = Z/F$ , which has sum-DoF  $S$ .

**Lemma 1** (Maximum DoF [11]). *Under the MAPDA structure, the maximum achievable sum-DoF is  $\min\{K, L + KM/N\}$ , when  $KM/N$  is an integer.*

We review some PDA and MAPDA that will be used later.

**Construction 1** (MN PDA [1]). For any integer  $t$  within the set  $[K]$ , we define a MN PDA  $P = (P(\mathcal{T}, k))$ , with dimensions  $\binom{K}{t} \times K$ , where  $\mathcal{T} \subseteq \binom{[K]}{t}$  and  $k \in [K]$ . The elements of  $P$  are defined as follows:

$$P(\mathcal{T}, k) = \begin{cases} * & \text{if } k \in \mathcal{T}, \\ \mathcal{T} \cup \{k\} & \text{otherwise.} \end{cases}$$

Hence, there exists a  $(K, \binom{[K]}{t}, \binom{K-1}{t-1}, \binom{K}{t+1})$  PDA with the sum-DoF  $t + 1$ .

**Construction 2** (MS MAPDA [6]). For any integers  $t, L$  and  $K$  with  $t+L \in [K]$ , we can define the following MS MAPDA  $P = (P(\mathcal{T}, \mathcal{L}, k))$  where  $\mathcal{T} \in \binom{[K]}{t}$ ,  $\mathcal{L} \in \binom{[K-t-1]}{L-1}$ ,  $k \in [K]$ , whose dimension is  $\binom{K}{t} \binom{K-t-1}{L-1} \times K$ :

$$P(\mathcal{T}, \mathcal{L}, k) = \begin{cases} * & \text{if } k \in \mathcal{T}, \\ (\mathcal{S}(\mathcal{T}, \mathcal{L}, k), \text{order}(\mathcal{S}(\mathcal{T}, \mathcal{L}, k))) & \text{otherwise,} \end{cases}$$

where  $\mathcal{S}(\mathcal{T}, \mathcal{L}, k) = \mathcal{T} \cup ([K] \setminus (\mathcal{T} \cup \{k\})) \setminus \mathcal{L}$  and  $\text{order}(\mathcal{S})$  as the order of appearance of the set  $\mathcal{S}$  in each column. Hence, there exists a  $(L, K, \binom{K}{t} \binom{K-t-1}{L-1}, \binom{K-1}{t-1} \binom{K-t-1}{L-1}, \binom{K}{t+L} \binom{t+L-1}{t})$  MAPDA with the sum-DoF  $t + L$ .

### III. IMPROVED RIS-ASSISTED INTERFERENCE NULLING ALGORITHM

In this section, we will propose a new RIS-assisted interference nulling algorithm to find out the phase-shift vector to eliminate some paths in the channel. When we want to eliminate the path from the  $j$ -th transmission antenna and user  $k$ , we need to have  $\mathbf{a}_{k,j}^T \mathbf{v} = 0$  in (1b). It was shown in [20] that to eliminate  $p$  paths, the number of RIS units needs to be very slightly larger than  $2p$ . In this paper, for ease of presentation, we assume that the elimination  $p$  paths requires  $2p$  RIS units.

Assume that for user  $k$ , we want to eliminate the paths between itself and antennas  $j_{k,1}, j_{k,2}, \dots, j_{k,q_k}$  where  $j_q \in [L]$ , and  $q_k$  denotes the total number of paths that user  $k$  needs to eliminate. Considering all users, the total number of paths needed to eliminate is  $\sum_{k \in [K]} q_k$ , which requires slightly more than  $G = 2 \sum_{k \in [K]} q_k$ . Subsequently, we will assume that this is approximately equal to this value.

Define that

$$\mathbf{A} = [\mathbf{a}_{1,j_{1,1}}, \dots, \mathbf{a}_{1,j_{1,q_1}}, \mathbf{a}_{2,j_{2,1}}, \dots, \mathbf{a}_{K,j_{K,q_K}}]. \quad (2)$$

Then our aim can be rewritten as

$$\mathcal{S}_1 = \{\mathbf{A}^T \mathbf{v} = 0\}. \quad (3)$$

Since the RIS is passive, it is essential to also ensure that

$$\mathcal{S}_2 = \{|v_i| = 1\}. \quad (4)$$

The algorithm presented in [20] operates by performing alternating projections between these two sets, i.e.

$$\Pi_{\mathcal{S}_1}(\mathbf{v}) = \mathbf{v} - \mathbf{A}^*(\mathbf{A}^T \mathbf{A}^*)^{-1} \mathbf{A}^T \mathbf{v} \quad (5)$$

$$\Pi_{\mathcal{S}_2}(\mathbf{v}) = \frac{\mathbf{v}}{|\mathbf{v}|} \quad (6)$$

thereby gradually converging to the intersection of these sets.

Different from the alternative projection algorithm in [20], we adopt the approach of projecting onto the tangent space to accelerate convergence rate which increases the step size of each projection to speeds up the convergence. The pseudo code of the improved algorithm is given in Algorithm 1.

---

**Algorithm 1** Improved Alternating Projection Algorithm

---

- 1: Initialize reflection coefficients  $\mathbf{v}$ , channel matrix  $\mathbf{A}_k$ , and number of iterations  $m$
  - 2: Start with initial value  $\mathbf{v} = \mathbf{v}^0 \in \mathcal{S}_2$
  - 3: **for**  $t = 0, 1, 2, \dots, m$  **and** interference not nullified **do**  
 $\mathbf{y}^t = \mathbf{v}^t - \Pi_{\mathcal{S}_1}(\mathbf{v}^t)$   
 $\mathbf{m}^t = \mathbf{y}^t - \Pi_{\mathbf{v}^t}(\mathbf{y}^t)$   
 $\tilde{\mathbf{v}}^t = \mathbf{v}^t - 2\mathbf{m}^t$   
 $\mathbf{v}^{t+1} = \Pi_{\mathcal{S}_2}(\tilde{\mathbf{v}}^t)$
  - 4: **end for**
  - 5: **if** iteration limit reached or interference nullified **then**  
Stop iteration
  - 6: **end if**
  - 7: Output the resulting reflection coefficients  $\mathbf{v}^{t+1}$
- 

In Algorithm 1,  $\mathbf{y} = \mathbf{A}^*(\mathbf{A}^T \mathbf{A}^*)^{-1} \mathbf{A}^T \mathbf{v}$  represents the projection of  $\mathbf{v}$  onto the orthogonal complement space of set  $\mathcal{S}_1$ . Then we project vector  $\mathbf{y}$  onto  $\mathbf{v}$ , i.e.  $\mathbf{m} = \mathbf{y} - \text{Re}(\mathbf{v}^* \odot \mathbf{y}) \cdot \mathbf{v}$ . Subtract  $2\mathbf{m}$  from  $\mathbf{v}$ , where the coefficient 2 is to achieve a larger step size. In essence, the operation still entails subtracting the direction that is orthogonal to the set  $\mathcal{S}_1$ .<sup>2</sup> After that,  $\mathbf{v}$  is projected onto  $\mathcal{S}_2$ , resulting in a larger step size compared to the projection onto  $\mathcal{S}_2$  as described in (6).

*Simulation:* Next we compare Algorithm 1 with the RIS-assisted interference nulling algorithm in [20], in the  $K$ -transmitter and  $K$ -user interference channel. In this channel, we need to use RIS eliminate  $K - 1$  interference paths for each user; thus there are totally  $K(K - 1)$  paths to eliminate. Assume that  $K = 25$  and the number of RIS units  $G$  is 1600. The channel coefficients are i.i.d. over a standard normal distribution. The number of algorithm's iterations is set to be 250. For each iteration, we compute the interference power at users in dB, defined as  $\sum_{j \neq k} |\mathbf{a}_{k,j} \mathbf{v}|^2$ .

It can be seen from Fig. 2 that Algorithm 1 performs significantly better than the algorithm in [20] in terms of interference power with the same number of iterations.

<sup>2</sup>This step is the primary difference from the algorithm presented in [20], namely, projecting onto the tangent plane of the current  $\mathbf{v}$  prior to projecting onto the set  $\mathcal{S}_2$ .

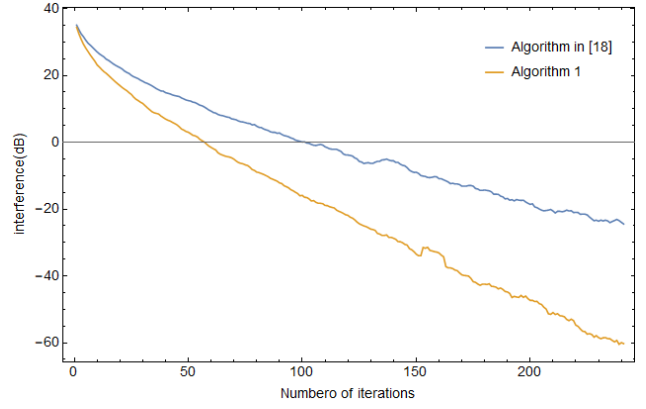


Fig. 2: Comparison of two algorithms in interference

#### IV. OPTIMAL GROUPING APPROACH

By RIS-assisted interference nulling method proposed in the last section, we can divide the transmission in each time slot of the delivery phase into multiple groups; inside each group the contained antennas serve the contained users without suffering interference from the transmissions by other groups. In this section, we aim to find out the optimal grouping method leading to the maximum sum-DoF.

Given the total number of antennas  $L$ , memory ration  $M/N$  and  $K$  users, assume that the number of active antennas is  $L_0 \leq L$ , i.e., we can keep some antenna 'silent'.  $L_0$  antennas are divided into  $r$  groups each of which contains  $L_i$  antennas,  $\sum_{i=1}^r L_i = L_0$ . Consider an integer  $t = KM/N$ . By Lemma 1 and the grouping method, we have the following sum-DoF:<sup>3</sup>

$$g = \sum_{i=1}^r (L_i + t) = L_0 + tr. \quad (7)$$

Since the number of RIS elements can be expressed as twice the number of interference paths and the maximum of users served by each group is  $L_i + t$ , the number of RIS units  $G$  needed is at least

$$G = 2 \sum_{i=1}^r (L_i + t)(L_0 - L_i). \quad (8)$$

We consider an alternative optimization task, which is more feasible: given a task sum DoF  $g$ , we want to minimize the needed number of RIS units; i.e.,

$$\begin{aligned} & \underset{L_i \in \mathbb{Z}^+}{\text{minimize}} && G \\ & \text{subject to} && L_0 \leq L, r \leq L, g = L_0 + tr. \end{aligned} \quad (9)$$

**Theorem 1.** *Given the sum-DoF  $g$ , the number of groups  $r$ , and the number of active antennas is  $L_0$ , the grouping scheme with minimum number of RIS elements is that  $L_1 = L_0 - (r - 1)$  and  $L_2 = L_3 = \dots = L_r = 1$ . The minimum of RIS elements is*

$$G = 2((r - 1)[(t + 2)L_0 - r]). \quad (10)$$

<sup>3</sup>For the ease of description, we assume that  $g$  in (7) is no larger than  $K$ .

The proof of Theorem 1 could be found in the extended version of this paper [23]. By Theorem 1, we only need to find out  $r$  and  $L_0$  that minimizes (10), which could be done by exhaustive search with complexity  $O(L^2)$ .

## V. RIS-ASSISTED GROUPING CODED CACHING SCHEME

After finding the solution for the grouping optimization problem, we then propose how to construct a coded caching scheme. Recall that by grouping, the transmission for each time slot could be divided into multiple separate transmissions, each of which is for one group. Inspired by MAPDA, our scheme can also be represented in the form of an array. Our objective is to serve  $g$  users with  $r$  groups in each time slot. Besides meeting requirements in Definition 1, it must also satisfy the following condition.

C5: Each integer  $s \in [S]$ , distributed across  $g$  columns, can be divided into  $r$  groups, where the first  $r-1$  groups satisfy PDA's properties and the last group satisfy MAPDA's properties.

The new MAPDA structure proposed for the RIS-assisted MISO coded caching problem is called RMAPDA.

Our main strategy is to construct a RMAPDA based on the MN PDA in Construction 1 and MS MAPDA in Construction 2. After obtaining the RMAPDA, we can use the similar method as described in [11] which constructs the multicast messages based on zero-forcing from MAPDA.

Due to the limitation of space, we provide an example to illustrate the construction, while the general description could be found in [23].

**Example:** Consider a system with  $K = 5$ ,  $M = 1$ , and  $L = 3$ ,  $t = KM/N = 1$  dividing antennas into  $r = 2$  groups, one group with a single antenna and another group with  $L' = 2$  antennas. Then we have a  $(5, 5, 1, 10)$  MN PDA  $\mathbf{P}'$  in Construction 1 and a  $(5, 10, 2, 20)$  MS MAPDA  $\mathbf{P}''$  in Construction 2. In order to make the same time slot for a group with DoF  $t+1 = 2$  based on  $\mathbf{P}'$  and a group with DoF  $t+L' = 3$  based on  $\mathbf{P}''$ , we can obtain our desired array  $\mathbf{P}$  in Fig. 3 vertically by replicating the  $\mathbf{P}'$  twice below the  $\mathbf{P}''$  and adjust the integers of the placed  $\mathbf{P}'$  such that each integer  $s$  appears in different columns from the  $s$  in  $\mathbf{P}''$ , i.e. ensure  $s$  are divided into different groups, aiming to satisfy C5.

*	1	1	3	5
1	*	2	4	6
15	9	5	*	1
13	7	3	1	*

TABLE I: Subarray of time slot 1

Based on the array provided in Fig. 3, we can obtain the corresponding coded caching scheme:

- 1) **Placement Phase:** Denote the array in Fig. 3 by  $\mathbf{P}$ , each row of  $\mathbf{P}$  represents a subfile, and each column represents a user. If  $P(j, k)$  is "\*", it means user  $k$  stores subfile  $\mathbf{W}_{n,j}$ , where  $n \in [N]$ .

$P''$	*	1	1	3	5
	1	*	2	4	6
	2	2	*	7	9
	3	3	7	*	11
	5	5	9	11	*
	*	4	8	8	10
	4	*	13	13	15
	7	13	*	14	16
	8	14	14	*	17
	9	15	15	17	*
	*	6	10	12	12
	6	*	16	18	18
$P'$	10	16	*	19	19
	11	17	19	*	20
	12	18	20	20	*
	*	19	17	15	13
	19	*	11	9	7
	17	11	*	5	3
$P'$	15	9	5	*	1
	13	7	3	1	*
	*	20	18	16	14
	20	*	12	10	8
	18	12	*	6	4
	16	10	6	*	2
	14	8	4	2	*

Fig. 3: Example of RMAPDA with  $K = g = 5, t = 1, L = 3, r = 2$

- 2) **Delivery Phase:** For instance, assume that the request vector is  $\mathbf{d} = (1, 2, 3, 4, 5)$ . We consider the first time slot, the subarray of  $s = 1$  is in table I, since the delivery for other time slots are similar.

Since in Fig. 3,  $P(1, 2) = P(1, 3) = P(2, 1) = P(19, 4) = P(20, 5) = 1$ , the served users' request packets are  $\mathbf{W}_{1,2}, \mathbf{W}_{2,1}, \mathbf{W}_{3,1}, \mathbf{W}_{4,19}, \mathbf{W}_{5,2}$ .

From the subarray, it can be seen that in time slot 1, users are divided into two groups: users 1, 2, and 3 form one group, while users 4 and 5 make up another group. Users 4 and 5 are part of a MN PDA. They do not receive signals for files 1, 2, and 3. For the files  $\mathbf{W}_{4,19}$  and  $\mathbf{W}_{5,2}$ , each user stores content required by the other user. so the antenna only needs to send the sum of these two files then users 4 and 5 can accurately retrieve the files they need.

While users 1, 2, and 3 are situated in a MS MAPDA which means they only need to handle the subfiles of files 1, 2, and 3. Consider, for instance, user 2 (similar to user 3), who neither has  $\mathbf{W}_{3,1}$  cached nor requires it. In this case, antenna zero-forcing can be implemented which involves using signal precoding to adjust the transmitted signals such that, after passing through the channel, the signal coefficients at unintended receivers are reduced to zero. While for  $\mathbf{W}_{1,2}$ , it can be cancelled using the contents already present in user 2's cache. Then user 2 can correctly get the subfile  $\mathbf{W}_{2,1}$ .

As for user 1, it has cached  $\mathbf{W}_{3,1}$  and  $\mathbf{W}_{2,1}$ , so it can just subtract them from received signal to correctly get subfile  $\mathbf{W}_{1,2}$ .

## REFERENCES

- [1] M. A. Maddah-Ali and U. Niesen, "Fundamental limits of caching," *IEEE Transactions on information theory*, vol. 60, no. 5, pp. 2856–2867, 2014.
- [2] K. Wan, D. Tuninetti, and P. Piantanida, "An index coding approach to caching with uncoded cache placement," *IEEE Transactions on Information Theory*, vol. 66, no. 3, pp. 1318–1332, Mar. 2020.
- [3] Q. Yu, M. A. Maddah-Ali, and S. Avestimehr, "The exact rate-memory tradeoff for caching with uncoded prefetching," *IEEE Trans. Infor. Theory*, vol. 64, no. 2, pp. 1281–1296, Feb. 2018.
- [4] N. Naderializadeh, M. A. Maddah-Ali, and A. S. Avestimehr, "Fundamental limits of cache-aided interference management," *IEEE Transactions on Information Theory*, vol. 63, no. 5, pp. 3092–3107, 2017.
- [5] E. Lampiridis and P. Elia, "Adding transmitters dramatically boosts coded-caching gains for finite file sizes," *IEEE Journal on Selected Areas in Communications*, vol. 36, no. 6, pp. 1176–1188, 2018.
- [6] S. P. Shariatpanahi, S. A. Motahari, and B. H. Khalaj, "Multi-server coded caching," *IEEE Transactions on Information Theory*, vol. 62, no. 12, pp. 7253–7271, 2016.
- [7] J. Hachem, U. Niesen, and S. N. Diggavi, "Degrees of freedom of cache-aided wireless interference networks," *IEEE Transactions on Information Theory*, vol. 64, no. 7, pp. 5359–5380, 2018.
- [8] S. P. Shariatpanahi, G. Caire, and B. H. Khalaj, "Physical-layer schemes for wireless coded caching," *IEEE Transactions on Information Theory*, vol. 65, no. 5, pp. 2792–2807, 2018.
- [9] S. Mohajer and I. Bergel, "Miso cache-aided communication with reduced subpacketization," in *ICC 2020-2020 IEEE International Conference on Communications (ICC)*. IEEE, 2020, pp. 1–6.
- [10] E. Piovano, H. Joudeh, and B. Clerckx, "Generalized degrees of freedom of the symmetric cache-aided miso broadcast channel with partial csit," *IEEE Transactions on Information Theory*, vol. 65, no. 9, pp. 5799–5815, 2019.
- [11] T. Yang, K. Wan, M. Cheng, R. C. Qiu, and G. Caire, "Multiple-antenna placement delivery array for cache-aided miso systems," *IEEE Transactions on Information Theory*, 2023.
- [12] K. K. Nambodiri, E. Peter, and B. S. Rajan, "Extended placement delivery arrays for multi-antenna coded caching scheme," *IEEE Transactions on Communications*, 2023.
- [13] Q. Yan, M. Cheng, X. Tang, and Q. Chen, "On the placement delivery array design for centralized coded caching scheme," *IEEE Transactions on Information Theory*, vol. 63, no. 9, pp. 5821–5833, 2017.
- [14] T. J. Cui, M. Q. Qi, X. Wan, J. Zhao, and Q. Cheng, "Coding metamaterials, digital metamaterials and programmable metamaterials," *Light: science & applications*, vol. 3, no. 10, pp. e218–e218, 2014.
- [15] Q. Wu and R. Zhang, "Towards smart and reconfigurable environment: Intelligent reflecting surface aided wireless network," *IEEE Communications Magazine*, vol. 58, no. 1, pp. 106–112, 2020.
- [16] M. Di Renzo, A. Zappone, M. Debbah, M.-S. Alouini, C. Yuen, J. De Rosny, and S. Tretjakov, "Smart radio environments empowered by reconfigurable intelligent surfaces: How it works, state of research, and the road ahead," *IEEE journal on selected areas in communications*, vol. 38, no. 11, pp. 2450–2525, 2020.
- [17] C. Pan, H. Ren, K. Wang, J. F. Kolb, M. ElKashlan, M. Chen, M. Di Renzo, Y. Hao, J. Wang, A. L. Swindlehurst, X. You, and L. Hanzo, "Reconfigurable intelligent surfaces for 6g systems: Principles, applications, and research directions," *IEEE Communications Magazine*, vol. 59, no. 6, pp. 14–20, 2021.
- [18] Y. Liu, X. Liu, X. Mu, T. Hou, J. Xu, M. Di Renzo, and N. Al-Dhahir, "Reconfigurable intelligent surfaces: Principles and opportunities," *IEEE communications surveys & tutorials*, vol. 23, no. 3, pp. 1546–1577, 2021.
- [19] A. H. A. Bafghi, V. Jamali, M. Nasiri-Kenari, and R. Schober, "Degrees of freedom of the  $k$ -user interference channel in the presence of intelligent reflecting surfaces," *arXiv preprint arXiv:2012.13787*, 2020.
- [20] T. Jiang and W. Yu, "Interference nulling using reconfigurable intelligent surface," *IEEE Journal on Selected Areas in Communications*, vol. 40, no. 5, pp. 1392–1406, 2022.
- [21] M. Fu, Y. Zhou, and Y. Shi, "Reconfigurable intelligent surface for interference alignment in mimo device-to-device networks," in *2021 IEEE International Conference on Communications Workshops (ICC Workshops)*. IEEE, 2021, pp. 1–6.
- [22] S. Zheng, B. Lv, T. Zhang, Y. Xu, G. Chen, R. Wang, and P. Ching, "On dof of active ris-assisted mimo interference channel with arbitrary antenna configurations: When will ris help?" *IEEE transactions on vehicular technology*, 2023.
- [23] X. Niu, M. Cheng, K. Wan, R. C. Qiu, and G. Caire, "Reflecting intelligent surfaces-assisted multiple-antenna coded caching," <https://github.com/Lelouch125/RIS-Assisted-Multiple-Antenna-Group-Coded-Caching-Scheme>, 2024.