

# Reflecting Intelligent Surfaces-Assisted Multiple-Antenna Group Coded Caching Scheme

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**Abstract**—Reconfigurable intelligent surface (RIS) has been treated as a core technique in improving wireless propagation environments for the next generation wireless communication systems. This paper proposes a new coded caching problem, referred to as Reconfigurable Intelligent Surface (RIS)-assisted multiple-antenna coded caching, which is composed of a server with multiple antennas and some single-antenna cache-aided users. Different from the existing multi-antenna coded caching problems, we introduce passive RIS (with limited number of units) into the systems to further increase the multicast gain (i.e., degrees of freedom (DoF)) in the transmission, which is done by using RIS-assisted interference nulling. That is, by using RIS, we can ‘erase’ any path between one transmission antenna and one receive antenna. We first propose a new RIS-assisted interference nulling approach to search for the phase-shift matrix of RIS for the sake of interference nulling, which converges faster than the state-of-the-art algorithm. After erasing some paths in each time slot, the delivery can be divided into several non-overlapping groups including transmission antennas and users, where in each group the transmission antennas serve the contained users without suffering interference from the transmissions by other groups. The division of groups for the sake of minimizing the DoF could be formulated into a combinatorial optimization problem. We propose a grouping algorithm which can find the optimal solution with low complexity, and the corresponding coded caching scheme achieving this DoF.

**Index Terms**—Coded caching, reconfigurable intelligent surface, zero-forcing

## I. INTRODUCTION

Coded Caching was introduced to address content-related traffic congestion in networks. In addition to the local caching gain (i.e., when a user requires some content which has been already stored, the server does not need to transmit), the strategy of coded caching involves transmitting multicast messages and leveraging cached content to eliminate interference such that a single transmission can serve multiple users, resulting in a preferable multicast gain and showing a great potential to alleviate a significant part of the traffic. The original coded caching work was introduced by Maddah-Ali and Niesen (MN) in [1] for a Single Input Single Output (SISO) shared-link model, where a central server with access to a library containing  $N$  files connects to  $k$  cache-aided users via an error-free shared link. Each user is equipped with a cache of size  $M$  files. The coded caching process contains two phases: placement and delivery. During the placement phase, each user stores parts of content from each file without knowing future demands. During the delivery phase, each user requests a file

from the library. The server broadcasts coded messages based on users’ caches and requests to satisfy all users’ demands. The objective is to minimize the worst-case transmission load among all possible demands (or equivalently maximize the worst-cast coded caching gain). If each file is stored totally  $t = KM/N$  times by users, the seminal MN coded caching scheme constructs multicast messages, each of which is useful to  $t + 1$  users, i.e., the coded caching gain is  $t + 1$  (the corresponding load is  $(K - t)/(t + 1)$  where  $K - t$  represents the local caching gain). It was shown in [2], [3] that under the constraint of uncoded cache placement (i.e., each user directly caches a subset of the library bits) and  $N \geq K$ , the achieved coded caching gain was proved to be optimal [2], [3].

Built upon the groundbreaking coded caching strategy, considerable research has been dedicated to wireless coded caching problems, with an emphasis on harnessing the benefits of coded caching gain. Multi-antenna coded caching problem was originally proposed in [4], where  $K_T$  transmitters with memory size  $M_T$  and  $K_R$  receivers with memory size  $M_R$  are connected through a wireless network. By smartly combining spatial multiplexing gain and coded caching gain, a multi-antenna coded caching scheme was proposed in [4] with a total degree-of-freedom (sum-DoF, i.e., overall multiplexing gain)  $\min\{\frac{K_T M_T + K_R M_R}{N}, K_R\}$ , which was proved to be optimal under the constraint of uncoded cache placement and one-shot linear delivery. Following the work in [4], numerous works have been proposed for various multi-antenna coded caching problems to find out coded caching schemes to maximizing the multiplexing gain; just list a few works [4]–[10], a new combinatorial construction structure on multi-antenna coded caching schemes with uncoded cache placement and one-shot zero-forcing delivery, referred to as Multiple-Antenna Placement Delivery Array (MAPDA), was proposed in [11], by extending the placement delivery array (PDA) structure proposed for the original shared-link coded caching problem. Under this structure, designing a coded caching scheme could be transformed into designing an array satisfying some constraints, where the latter one is a combinatorial problem and could be solved by using combinatorial tools.

In this paper, we introduce Reconfigurable Intelligent Surfaces (RIS), seen as a promising technique for the next generation wireless systems to reconfigure wireless propagation environment [12]–[16], into coded caching systems in order to further increase the sum-DoF. As illustrated in Fig. 1,

RIS serves as a passive configurable relay (by its phase-shift matrix) in the channel, where the system channel transition matrix is equal to the product of the transition matrix from the transmitter to the RIS, the RIS phase-shift matrix, and the transition matrix from the RIS to the receiver. Several information theoretic works have been proposed on RIS-assisted interference management, in order to increase the sum-DoF. In [17] the authors considered RIS-assisted  $K$ -user interference channel. RIS-assisted interference alignment scheme was proposed for different numbers of RIS units. With the increasing of the RIS units, the sum-DoF can be increased from  $K/2$  to  $K$ . Following the RIS-assisted  $K$ -user interference channel, the authors in [18] proposed a RIS-assisted interference nulling approach with zero-forcing transmission, which achieves the sum-DoF equal to  $K$  when the number of RIS units is over than a threshold approximately equal to  $2K(K - 1)$ . This is done by using RIS to ‘erase’ all the interfering paths for the users. To find out the RIS phase-shift matrix for interference nulling, an alternating projection algorithm was proposed in [18]. Then RIS-assisted zero-forcing and interference alignment were also extended to Multiple-Input-Multiple-Output (MIMO) networks in [19], [20], in order to increase the sum-DoF.

**Main Contribution:** We propose a RIS-assisted multiple-input single-output (MISO) broadcast coded caching problem, for fixed numbers of antennas at the transmitter and of RIS units. Our main contribution on constructing RIS-assisted multi-antenna coded caching schemes is as follows.

- We first propose a new RIS-assisted interference nulling approach to ‘erase’ some paths in the wireless channel, which outputs the needed RIS phase-shift matrix with a faster convergence rate than the algorithm in [18].
- With the help of RIS-assisted interference nulling, we can divide the transmission in each time slot of the delivery phase into multiple groups including transmission antennas and users, where in each group the transmission antennas serve the contained users without suffering interference from the transmissions by other groups. By formulating the grouping optimization to maximize the sum-DoF as a combinatorial optimization problem, we propose an low-complexity algorithm to find out the optimal grouping method.
- After determining the optimal grouping approach, we design the two-phase coded caching scheme under a new MAPDA struture, referred to as RAPDA, by adding a new constraint on group-based delivery into MAPDA.

**Notation Convention:** Scalars are denoted by lowercase letters, vectors by bold lowercase letters, and matrices by bold uppercase letters. For a matrix  $\mathbf{A}$ ,  $\mathbf{A}^T$  represents the transpose of the matrix,  $\mathbf{A}^*$  represents the conjugate.  $|v|$  denotes the magnitude of a complex scalar.  $\text{Re}[\cdot]$  denotes taking the real part of a complex number.  $\cdot$  represents the element-wise multiplication of matrices.  $\odot$  indicates the Hadamard product.  $\text{diag}$  denotes creating a diagonal matrix from a given column vector, where the elements of the vector form the

main diagonal elements. For a vector  $\mathbf{v}$ ,  $\text{diag}(\mathbf{v})$  represents the diagonal matrix whole diagonal elements are the elements in  $\mathbf{v}$ . Define that  $[a] = \{1, 2, \dots, a\}$ .

## II. SYSTEM MODEL AND PRELIMINARY RESULTS

### A. System model

As illustrated in Fig. 1, we consider a  $(L, G, K, M, N)$  RIS-assisted MISO broadcast coded caching system with one-shot linear delivery. A server has  $L$  transmission antennas and can access to a library containing  $N$  files, denoted by  $\mathcal{W} = \{\mathbf{W}_n \mid n \in [N]\}$ . Each file  $\mathbf{W}_n$  consists of  $F$  packets, defined as  $\mathbf{W}_n \triangleq \{\mathbf{W}_{n,f} \mid f \in [F]\}$ . Each user is equipped with one antenna and a cache capable of storing  $MF$  packets, where  $0 \leq M \leq N$ .

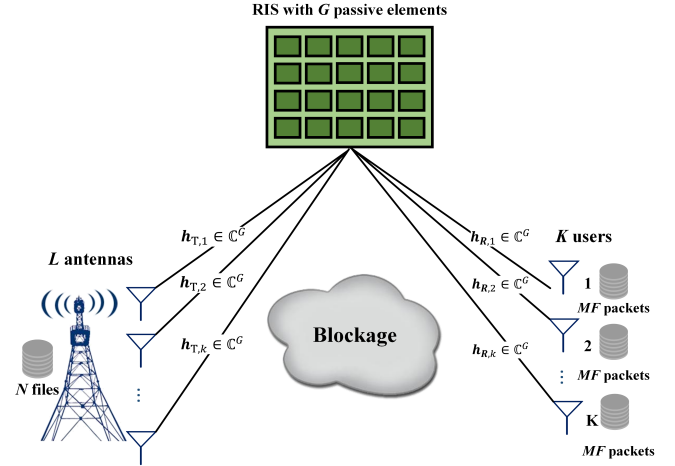


Fig. 1: RIS assisted MISO broadcast system.

A coded caching scheme consists of two phases:

**Placement Phase:** During this phase,  $F$  data packets of each file are strategically placed into the users’ caches without knowledge of the specific demands.

**Delivery Phase:** Each user  $k \in [K]$  requests an arbitrary file  $\mathbf{W}_{d_k}$  from the library, where  $d_k$  is an index within the range  $[N]$ . Once the user demands are known, denoted by  $\mathbf{d} = (d_0, d_1, \dots, d_K)$ , the server transmits coded packets through  $L$  antennas. We first encode each packet of desired files by Gaussian encoding with rate  $\log P$  to obtain a coded packet, where  $P$  represents the Signal-to-Noise (SNR) ratio. The communication process contains  $S$  time slots, each denoted by  $s \in [S]$ . In time slot  $s$ , the signal transmitted by antenna  $j$  is denoted by  $X_j(s)$ , which is a linear combination of some coded packets desired by the users in  $\mathcal{K}_s \subseteq [K]$ .

In the wireless channel, there exists a passive RIS with  $G$  units. For the ease of description, we assume that the direct paths between the transmitters and receivers are blocked. As shown in [18], the RIS-assisted interference nulling method proposed in this case of no direct path could be directly extended to ‘erase’ the path between a transmission antenna and a user for the systems where direct paths exist. Define  $\mathbf{h}_{T,j}$  as the channel transition vector with dimension  $G$  between

the  $j$ -th transmission antenna (or equivalently called the  $j$ -th transmitter) and the RIS, and define  $\mathbf{h}_{R,k}$  as the channel transition vector with dimension  $G$  between the RIS and the  $k$ -th receiver. It is assumed that the coefficients in these vectors are i.i.d over some continuous distributed and that channel state information (CSI) is fully known to the server and users. The reflection coefficients of the RIS are given by  $\mathbf{v} = [e^{j\omega_1}, e^{j\omega_2}, \dots, e^{j\omega_G}]^T \in \mathbb{C}^G$ , where  $\omega_i \in (0, 2\pi)$  indicates the phase-shift coefficient of the  $i$ -th unit of the RIS. Note that the phase-shift vector  $\mathbf{v}$  could be re-configured during each time slot. Each receiver  $k$  receives a reflected signal from the RIS at time slot  $s$ , which can be expressed as

$$Y_k(s) = \sum_{j=1}^L \mathbf{h}_{R,k}^T \text{diag}(\mathbf{v}) \mathbf{h}_{T,j} X_j(s) + n_k(s) \quad (1a)$$

$$= \sum_{j=1}^L \mathbf{a}_{k,j}^T \mathbf{v} X_j(s) + n_k(s), \quad (1b)$$

where  $\mathbf{a}_{k,j} \triangleq \text{diag}(\mathbf{h}_{T,j}) \mathbf{h}_{R,k}$  and  $n_k(s)$  represents the additive Gaussian white noise at receiver  $k$ . Assume that the delivery is one-shot: in time slot  $s$ , each user  $k \in \mathcal{K}_s$  should recover one coded packet from  $Y_k(s)$  and its cache content.

**Objective.** We consider the high SNR regime and sum-DoF as the metric. Under the one-shot linear delivery, the sum-DoF indicates the average number of users served per time slot. Our objective is to maximize the worst-case sum-DoF among all possible demands.

**Definition 1** (MAPDA, [11]). For any positive integers  $L, K, F, Z$ , and  $S$ , an  $F \times K$  array  $P$  composed of "\*" and  $[S]$  is called an  $(L, K, F, Z, S)$  multiple-antenna placement delivery array (MAPDA) if it satisfies the following conditions:

- C1: The symbol "\*" appears  $Z$  times in each column;
- C2: Each integer from 1 to  $S$  occurs at least once in the array;
- C3: Each integer  $s$  appears at most once in each column;
- C4: For any integer  $s \in [S]$ , define  $P(s)$  to be the subarray of  $P$  including the rows and columns containing  $s$ , and let  $r'_s \times r_s$  denote the dimensions of  $P(s)$ . The number of integer entries in each row of  $P(s)$  is less than or equal to  $\min\{L, K\}$ , i.e.,

$$|\{k_1 \in [r_s] \mid P(s)(f_1, k_1) \in [S]\}| \leq \min\{L, K\}, \quad \forall f_1 \in [r'_s].$$

The array is composed of "\*" and integers. Each column represents a user and each row represents a packet of files. A "\*" appears in the position  $(i, j)$  of the array, meaning that user  $j$  stores the  $i$ -th packet of each file. Each integer corresponds to a multicast message transmitted in one time slot.

Each MAPDA corresponds to a specific coded caching scheme where the cache ratio  $\frac{M}{N}$  equals  $\frac{Z}{F}$ . This process is summarized in Algorithm 1.

Note that when  $L = 1$  the MAPDA reduces to the PDA in [21]. From each MAPDA, we can obtain a corresponding multi-antenna coded caching scheme for the system containing a server with  $L$  antennas and  $K$  single-antenna users with

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**Algorithm 1** Caching Scheme Based on MAPDA in [11]

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1: procedure PLACEMENT( $Q, W$ )
2:   Split each file  $W_n \in W$  into  $F$  packets, i.e.,  $W_n = \{W_{n,f} \mid f = 1, 2, \dots, F\}$ .
3:   for  $k \in [K]$  do
4:      $Z_k \leftarrow \{W_{n,f} \mid Q(f, k) = *, n \in [N], f \in [F]\}$ 
5:   end for
6: end procedure
7: procedure DELIVERY( $Q, W, d$ )
8:   for  $s = 1, 2, \dots, S$  do
9:     Server uses  $L$  antennas to send  $\mathbf{W}_{d_k,j}$  where  $P(j, k) = s$  to the users.
10:  end for
11: end procedure

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memory ratio (i.e., the ratio between memory size and library size)  $M/N = Z/F$ , which has sum-DoF  $S$ .

We review some PDA and MAPDA that will be used later.

**Lemma 1** (Maximum DoF [11]). *Under the MAPDA structure, the maximum achievable sum-DoF is  $\min\{K, L + KM/N\}$ , when  $KM/N$  is an integer.*

**Lemma 2** (MN PDA [1]). *For any positive integers  $K$  and  $t$  with  $t < K$ , there exists a  $(K, \binom{K}{t}, \binom{K-1}{t-1}, \binom{K}{t+1})$  PDA with the maximum DoF  $t + 1$ .*

**Construction 1** (MN PDA [22]). For any integer  $t$  within the set  $[K]$ , we define a MN PDA.  $P = (P(\mathcal{T}, k))$ , with dimensions  $\binom{K}{t} \times K$ , where  $\mathcal{T} \subseteq \binom{[K]}{t}$  and  $k \in [K]$ . The elements of  $P$  are defined as follows:

$$P(\mathcal{T}, k) = \begin{cases} * & \text{if } k \in \mathcal{T}, \\ \mathcal{T} \cup \{k\} & \text{otherwise.} \end{cases}$$

**Example 1.** For  $K = 5, t = 1$ , we have a  $(5, 5, 1, 10)$  MN PDA: Just sort and label these groups, and the elements in the

*	{1, 2}	{1, 3}	{1, 4}	{1, 5}
{1, 2}	*	{2, 3}	{2, 4}	{2, 5}
{1, 3}	{2, 3}	*	{3, 4}	{3, 5}
{1, 4}	{2, 4}	{3, 4}	*	{4, 5}
{1, 5}	{2, 5}	{3, 5}	{4, 5}	*

TABLE I: A MN PDA example

array can then be represented in the form of integers.

**Lemma 3** (MS MAPDA [6]). *For any positive integers  $K$  and  $t$ , there exists a  $(L, K, \binom{K}{t} \binom{K-t-1}{L-1}, \binom{K-1}{t-1} \binom{K-t-1}{L-1}, \binom{K}{t+L} \binom{t+L-1}{t})$  MAPDA with the maximum DoF  $t + L$ .*

**Construction 2** (MS MAPDA). For any integers  $t, L$  and  $K$  with  $t + L \in [K]$ , we can define a  $\binom{K}{t} \binom{K-t-1}{L-1} \times K$  array  $P = (P(\mathcal{T}, \mathcal{L}, k))$  where  $\mathcal{T} \in \binom{[K]}{t}$ ,  $\mathcal{L} \in \binom{[K-t-1]}{L-1}$ ,  $k \in [K]$

in the following way.

$$P(\mathcal{T}, \mathcal{L}, k) = \begin{cases} * & \text{if } k \in \mathcal{T}, \\ (\mathcal{S}(\mathcal{T}, \mathcal{L}, k), \text{order}(\mathcal{S}(\mathcal{T}, \mathcal{L}, k))) & \text{otherwise.} \end{cases}$$

where  $\mathcal{S}(\mathcal{T}, \mathcal{L}, k) = \mathcal{T} \cup ([K] \setminus (\mathcal{T} \cup \{k\}))[\mathcal{L}] \cup \{k\}$  and  $\text{order}(\mathcal{S})$  as the order of appearance of the set  $\mathcal{S}$  in each column.

**Example 2.** For  $K = 5, t = 1, L = 2$ , we have a  $(5, 10, 2, 20)$  MS MAPDA.

In this  $15 \times 5$  array is first generated, and "\*" is filled in according to the construction rules. Each remaining position should be filled with a set of size  $|t + L|$ , which includes the user indices corresponding to "\*" in this row, the index of the protector at this position, and the indices of  $L - 1$  other users. Therefore, there are  $\binom{K-1-t}{L-1} = 3$  ways to select this, so based on  $\binom{K}{t} = 5$ , it is vertically replicated 3 times to ensure that all possibilities for each position occur.

*	{1, 2, 3}	{1, 2, 3}	{1, 2, 4}	{1, 2, 5}
{1, 2, 3}	*	{1, 2, 3}	{1, 2, 4}	{1, 2, 5}
{1, 2, 3}	{1, 2, 3}	*	{1, 3, 4}	{1, 3, 5}
{1, 2, 4}	{1, 2, 4}	{1, 3, 4}	*	{1, 4, 5}
{1, 2, 5}	{1, 2, 5}	{1, 3, 5}	{1, 4, 5}	*
*	{1, 2, 4}	{1, 3, 4}	{1, 3, 4}	{1, 3, 5}
{1, 2, 4}	*	{2, 3, 4}	{2, 3, 4}	{2, 3, 5}
{1, 3, 4}	{2, 3, 4}	*	{2, 3, 4}	{2, 3, 5}
{1, 3, 4}	{2, 3, 4}	{2, 3, 4}	*	{2, 4, 5}
{1, 3, 5}	{2, 3, 5}	{2, 3, 5}	{2, 4, 5}	*
*	{1, 2, 5}	{1, 3, 5}	{1, 4, 5}	{1, 4, 5}
{1, 2, 5}	*	{2, 3, 5}	{2, 4, 5}	{2, 4, 5}
{1, 3, 5}	{2, 3, 5}	*	{3, 4, 5}	{3, 4, 5}
{1, 4, 5}	{2, 4, 5}	{3, 4, 5}	*	{3, 4, 5}
{1, 4, 5}	{2, 4, 5}	{3, 4, 5}	{3, 4, 5}	*

TABLE II: Example of MS MAPDA with  $K = 5, t = 1$ (sets)

After filling in, it is observed that the same sets occur 2 times in each column. Therefore, afterwards, it is only necessary to label each set according to the order of its occurrence in each column to obtain an array in integer form, for example, the set  $\{1, 2, 3\}$  at  $P(2, 1), P(1, 2), P(3, 1)$  are denoted by 1, and the set at  $P(3, 1), P(3, 2), P(2, 3)$  will be denoted by 2. And the set  $\{1, 2, 4\}$  at  $P(4, 1), P(4, 2), P(1, 4)$  will be denoted by 2, and so on. i.e.

### III. IMPROVED RIS-ASSISTED INTERFERENCE NULLING ALGORITHM

In this section, we will propose a new RIS-assisted interference nulling algorithm to find out the phase-shift vector to eliminate some paths in the channel. When we want to eliminate the path from the  $j$ -th transmission antenna and user  $k$ , we need to have  $\mathbf{a}_{k,j}^T \mathbf{v} = 0$  in (1b). It was shown in [18] that to eliminate  $p$  paths, the number of RIS units needs to be very slightly larger than  $2p$ . In this paper, for ease of presentation, we assume that the elimination  $p$  paths requires  $2p$  RIS units.

*	1	1	3	5
1	*	2	4	6
2	2	*	7	9
3	3	7	*	11
5	5	9	11	*
*	4	8	8	10
4	*	13	13	15
7	13	*	14	16
8	14	14	*	17
9	15	15	17	*
*	6	10	12	12
6	*	16	18	18
10	16	*	19	19
11	17	19	*	20
12	18	20	20	*

TABLE III: Example of MS MAPDA with  $K = 5, t = 1$ (integers)

Assume that for user  $k$ , we want to eliminate the paths between itself and antennas  $j_{k,1}, j_{k,2}, \dots, j_{k,q_k}$  where  $j_q \in [L]$ , and  $q_k$  denotes the total number of paths that user  $k$  needs to eliminate. Considering all users, the total number of paths needed to eliminate is  $\sum_{k \in [K]} q_k$ , which requires slightly more than  $G = 2 \sum_{k \in [K]} q_k$ . Subsequently, we will assume that this is approximately equal to this value.

Define that

$$\mathbf{A} = [\mathbf{a}_{1,j_{1,1}}, \dots, \mathbf{a}_{1,j_{1,q_1}}, \mathbf{a}_{2,j_{2,1}}, \dots, \mathbf{a}_{K,j_{K,q_K}}]. \quad (2)$$

Then our aim can be rewritten as

$$\mathcal{S}_1 = \{\mathbf{A}^T \mathbf{v} = 0\}. \quad (3)$$

Since the RIS is passive, it is essential to also ensure that

$$\mathcal{S}_2 = \{|v_i| = 1\}. \quad (4)$$

The algorithm presented in [18] operates by performing alternating projections between these two sets, i.e.

$$\Pi_{\mathcal{S}_1}(\mathbf{v}) = \mathbf{v} - \mathbf{A}^*(\mathbf{A}^T \mathbf{A}^*)^{-1} \mathbf{A}^T \mathbf{v} \quad (5)$$

$$\Pi_{\mathcal{S}_2}(\mathbf{v}) = \frac{\mathbf{v}}{|\mathbf{v}|} \quad (6)$$

thereby gradually converging to the intersection of these sets.

Different from the alternative projection algorithm in [18], we adopt the approach of projecting onto the tangent space to accelerate convergence rate which increases the step size of each projection to speeds up the convergence. The pseudo code of the improved algorithm is given in Algorithm 2.

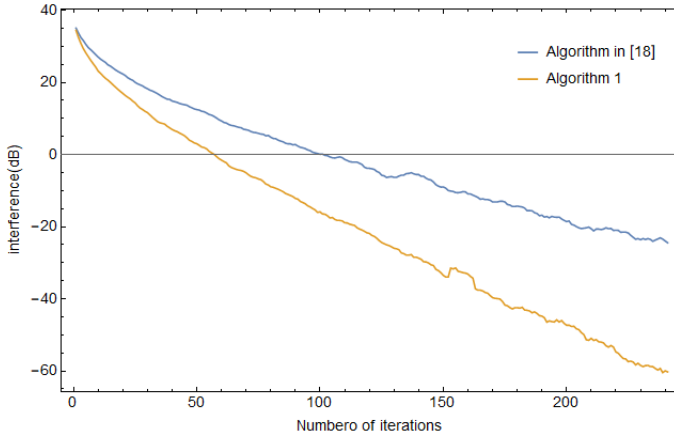
In Algorithm 2,  $\mathbf{y} = \mathbf{A}^*(\mathbf{A}^T \mathbf{A}^*)^{-1} \mathbf{A}^T \mathbf{v}$  represents the projection of  $\mathbf{v}$  onto the orthogonal complement space of set  $\mathcal{S}_1$ . Then we project vector  $\mathbf{y}$  onto  $\mathbf{v}$ . i.e.  $\mathbf{m} = \mathbf{y} - \text{Re}(\mathbf{v}^* \odot \mathbf{y}) \cdot \mathbf{v}$ . Subtract  $2\mathbf{m}$  from  $\mathbf{v}$ , where the coefficient 2 is to achieve a larger step size. In essence, the operation still entails

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**Algorithm 2** Improved Alternating Projection Algorithm

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- 1: Initialize reflection coefficients  $\mathbf{v}$ , channel matrix  $\mathbf{A}_k$ , and number of iterations  $m$
  - 2: Start with initial value  $\mathbf{v} = \mathbf{v}^0 \in \mathcal{S}_2$
  - 3: **for**  $t = 0, 1, 2, \dots, m$  **and** interference not nullified **do**  
 $\mathbf{y}^t = \mathbf{v}^t - \Pi_{\mathcal{S}_1}(\mathbf{v}^t)$   
 $\mathbf{m}^t = \mathbf{y}^t - \Pi_{\mathbf{v}^t}(\mathbf{y}^t)$   
 $\tilde{\mathbf{v}}^t = \mathbf{v}^t - 2\mathbf{m}^t$   
 $\mathbf{v}^{t+1} = \Pi_{\mathcal{S}_2}(\tilde{\mathbf{v}}^t)$
  - 4: **end for**
  - 5: **if** iteration limit reached or interference nullified **then**  
Stop iteration
  - 6: **end if**
  - 7: Output the resulting reflection coefficients  $\mathbf{v}^{t+1}$
- 



**Fig. 2:** Comparison of two algorithms in interference

subtracting the direction that is orthogonal to the set  $\mathcal{S}_1$ .<sup>1</sup> After that,  $\mathbf{v}$  is projected onto  $\mathcal{S}_2$ , resulting in a larger step size compared to the projection onto  $\mathcal{S}_2$  as described in (6).

*Simulation:* Next we compare Algorithm 2 with the RIS-assisted interference nulling algorithm in [18], in the  $K$ -transmitter and  $K$ -user interference channel. In this channel, we need to use RIS eliminate  $K - 1$  interference paths for each user; thus there are totally  $K(K - 1)$  paths to eliminate. Assume that  $K = 25$  and the number of RIS units  $G$  is 1600. The channel coefficients are i.i.d. over a standard normal distribution. The number of algorithm's iterations is set to be 250. For each iteration, we compute the interference power at users in dB, defined as  $\sum_{j \neq k} |\mathbf{a}_{k,j} \mathbf{v}|^2$ .

It can be seen from Fig. 2 that Algorithm 2 performs significantly better than the algorithm in [18] in terms of interference power with the same number of iterations.

#### IV. OPTIMAL GROUPING APPROACH

By RIS-assisted interference nulling method proposed in the last section, we can divide the transmission in each time

<sup>1</sup>This step is the primary difference from the algorithm presented in [18], namely, projecting onto the tangent plane of the current  $\mathbf{v}$  prior to projecting onto the set  $\mathcal{S}_2$ .

slot of the delivery phase into multiple groups; inside each group the contained antennas serve the contained users without suffering interference from the transmissions by other groups. In this section, we aim to find out the optimal grouping method leading to the maximum sum-DoF.

Given the total number of antennas  $L$ , memory ration  $M/N$  and  $K$  users, assume that the number of active antennas is  $L_0 \leq L$ , i.e., we can keep some antenna 'silent'.  $L_0$  antennas are divided into  $r$  groups each of which contains  $L_i$  antennas,  $\sum_{i=1}^r L_i = L_0$ . Consider an integer  $t = KM/N$ . By Lemma 1 and the grouping method, we have the following sum-DoF:<sup>2</sup>

$$g = \sum_{i=1}^r (L_i + t) = L_0 + tr. \quad (7)$$

Since the number of RIS elements can be expressed as twice the number of interference paths and the maximum of users served by each group is  $L_i + t$ , the number of RIS units  $G$  needed is at least

$$G = 2 \sum_{i=1}^r (L_i + t)(L_0 - L_i). \quad (8)$$

We consider an alternative optimization task, which is more feasible: given a task sum DoF  $g$ , we want to minimize the needed number of RIS units; i.e.,

$$\begin{aligned} & \underset{L_i \in \mathbb{Z}^+}{\text{minimize}} && G \\ & \text{subject to} && L_0 \leq L, r \leq L, g = L_0 + tr. \end{aligned} \quad (9)$$

**Theorem 1.** *Given the sum-DoF  $g$ , the number of groups  $r$ , and the number of active antennas is  $L_0$ , the grouping scheme with minimum number of RIS elements is that  $L_1 = L_0 - (r - 1)$  and  $L_2 = L_3 = \dots = L_r = 1$ . The minimum of RIS elements is*

$$G = 2((r - 1)[(t + 2)L_0 - r]). \quad (10)$$

The proof of Theorem 1 could be found in Appendix A. By Theorem 1, we only need to find out  $r$  and  $L_0$  that minimizes (10), which could be done by exhaustive search with complexity  $O(L^2)$ .

For the case that there exists a scheme where the sum-DoF exactly equal to the given  $g$ , Algorithm 3 can find the optimal grouping scheme with complexity  $O(L)$ . Detailed analysis and design process of it can be find in Appendix B.

#### V. RIS-ASSISTED GROUPING CODED CACHING SCHEME

After finding the solution for the grouping optimization problem, we then propose how to construct a coded caching scheme. Recall that by grouping, the transmission for each time slot could be divided into multiple separate transmissions, each of which is for one group. Once the groupings are established. Once grouped, the users and antennas within each group are independent of those in other groups. This allows us to treat each group's transmission as an isolated problem.

<sup>2</sup>For the ease of description, we assume that  $g$  in (7) is no larger than  $K$ .

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**Algorithm 3** Optimal Grouping Scheme

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1: Input: Initialize total antennas  $L$ ,  $t$ , and sum-DoF  $g$ 
2: Compute initial  $L_{\min} = \lceil \frac{g}{t+1} \rceil$  and  $L_{\max} = \min(L_0, g - t)$ 
3: for  $L_0 = L_{\max}$  down to  $L_{\min}$  do
4:   Compute  $r = \frac{g-L_0}{t}$ 
5:   if  $r$  is an integer and  $r > 0$  then
6:     break
7:   end if
8: end for
9: Calculate  $\lambda_0 = \text{solve for } f(\lambda) = 0 \text{ and } \lambda_0 \neq 0$ 
10: Compute maximum  $\lambda_{\max} = \lfloor \frac{L_{\max}-r}{t+1} \rfloor$ 
11: Determine the optimal values:
12: if  $\lambda_{\max} \geq \lambda_0$  then
13:    $L_{\text{opt}} = L_{\max} - \lambda_{\max} \cdot t$ 
14:    $r_{\text{opt}} = r + \lambda_{\max}$ 
15:    $G_{\text{opt}} = G + f(\lambda_{\max})$ 
16: else
17:    $L_{\text{opt}} = L_{\max}$ ,  $r_{\text{opt}} = r$ ,  $G_{\text{opt}} = G$ 
18: end if
19: Output: Optimal values  $L_{\text{opt}}, r_{\text{opt}}, G_{\text{opt}}$ 

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Inspired by MAPDA, our scheme can also be represented in the form of an array. Similar to MAPDA, the symbol "\*" is used to denote cached content at the users. For transmission, to enable simultaneous broadcasting across  $r$  groups.

From our previous analyses, the antenna groups are limited to two configurations: one consisting of a single antenna with  $t + 1$  users, and the other consisting of  $L - r + 1$  antennas with  $t + L - r + 1$  users. So our objective is to serve  $g = (t + 1)(r - 1) + (t + L - r + 1)$  users per time slot, which necessitates the construction of a MAPDA. Besides meeting requirements in definition 1, it must also satisfy the following condition.

C5: Each integer  $s \in [S]$ , distributed across  $g$  columns, can be divided into  $r$  groups, where the first  $r - 1$  groups satisfy PDA's properties and the last group satisfy MAPDA's properties.

The new MAPDA structure proposed for the RIS-assisted MISO coded caching problem is called RMAPDA.

We are now equipped to begin the construction of a MAPDA that fulfills the specified condition based on the MN PDA in lemma 2 and MS MAPDA in lemma 3.

Given that under the one-shot transmission strategy, the DoF  $g \leq K$ , it is sufficient to study cases where  $g \leq K$ .

In each MN PDA, groups of  $t + 1$  users require only one time slot each for data handling. In addition, in the MAPDA, each group of  $t + L'$  users necessitates  $\binom{t+L'-1}{t}$  time slots. To ensure that the number of time slots across  $r - 1$  MN PDAs and a MAPDA matches, we replicate them vertically until the total number of slots aligns.

Let  $L' = L - r + 1$  and the parameter  $L$  in lemma 3 is  $L'$ . To ensure that the total time slots transmitted are the same. Let  $S_1 = \binom{K}{t+1}$  and  $S_2 = \binom{K}{t+L'} \binom{t+L'-1}{t}$ . Assume each MN PDA is replicated vertically  $x$  times, and the MS MAPDA is

replicated vertically  $y$  times. To align the total time slots of MN PDA and MS MAPDA when vertically concatenated, it is sufficient to satisfy the equation  $x(r - 1)S_1 = yS_2$ .

We will provide a specific example to illustrate the following points:

- 1) The construction method for the RMAPDA mentioned earlier.
- 2) For any set of  $g$  elements, this set can always be partitioned into  $r - 1$  subsets, each containing  $t + 1$  elements, denoted as  $s_1, \dots, s_{r-1}$ , and one subset containing  $t + L - r + 1$  elements, denoted as  $s_r$ . Thus, it is always possible to find  $g$  columns that can accommodate a integer  $s$  and meet the condition C5.
- 3) A RMAPDA can represent a specific coded caching scheme.

**Example:** Consider a system with  $K = 5$ ,  $M = 1$ , and  $L = 3$ ,  $t = KM/N = 1$  dividing antennas into  $r = 2$  groups, one group with a single antenna and another group with  $L' = 2$  antennas. Then we have the MN PDA  $\mathbf{P}'$  from example 1 and the MS MAPDA  $\mathbf{P}''$  from example 2. In order to make the same time slot for a group with DoF  $t + 1 = 2$  based on  $\mathbf{P}'$  and a group with DoF  $t + L' = 3$  based on  $\mathbf{P}''$ , we can obtain our desired array vertically by replicating the  $\mathbf{P}'$  twice below the  $\mathbf{P}''$

Then we should adjust the integers of the placed  $\mathbf{P}'$  such that each integer  $s$  appears in different columns from the  $s$  in  $\mathbf{P}''$ , i.e. ensure  $s$  are divided into different groups, aiming to satisfy C5. For instance, the set  $\{1, 2, 3\}$  in table II represents the integer  $s = 1$ , and for table I,  $\{4, 5\}$  can be a another group which is independent from  $\{1, 2, 3\}$ , so we can replace  $\{4, 5\}$  by  $s = 1$  which means these two groups can be served at the same time. Since there has two  $\{1, 2, 3\}$  each column in table III so the integer  $s = 2$  is also the set  $\{1, 2, 3\}$ . We have replicate table I twice, so just replace the set  $\{4, 5\}$  by  $s = 1$  and  $s = 2$  respectively in the two MN PDAs, their time slot will match perfectly. All other integers are similarly placed. Finally we can get our desired array  $\mathbf{P}$  in Fig. 3.

*	1	1	3	5
1	*	2	4	6
15	9	5	*	1
13	7	3	1	*

TABLE IV: Subarray of time slot 1

Based on the array provided in Fig. 3, we can obtain the corresponding coded caching scheme:

- 1) **Placement Phase:** Denote the array in Fig. 3 by  $\mathbf{P}$ , each row of  $\mathbf{P}$  represents a subfile, and each column represents a user. If  $P(j, k)$  is "\*", it means user  $k$  stores subfile  $\mathbf{W}_{n,j}$ , where  $n \in [N]$ .
- 2) **Delivery Phase:** For instance, assume that the request vector is  $\mathbf{d} = (1, 2, 3, 4, 5)$ . We consider the first time slot, the subarray of  $s = 1$  is in table IV, since the delivery for other time slots are similar.

$P''$	*	1	1	3	5
	1	*	2	4	6
	2	2	*	7	9
	3	3	7	*	11
	5	5	9	11	*
	*	4	8	8	10
	4	*	13	13	15
	7	13	*	14	16
	8	14	14	*	17
	9	15	15	17	*
	*	6	10	12	12
	6	*	16	18	18
$P'$	10	16	*	19	19
	11	17	19	*	20
	12	18	20	20	*
	*	19	17	15	13
	19	*	11	9	7
	17	11	*	5	3
	15	9	5	*	1
	13	7	3	1	*
$P'$	*	20	18	16	14
	20	*	12	10	8
	18	12	*	6	4
	16	10	6	*	2
	14	8	4	2	*

Fig. 3: Example of RMAPDA with  $K = g = 5, t = 1, L = 3, r = 2$

Since in Fig. 3,  $P(1,2) = P(1,3) = P(2,1) = P(19,4) = P(20,5) = 1$ , the served users' request packets are  $\mathbf{W}_{1,2}, \mathbf{W}_{2,1}, \mathbf{W}_{3,1}, \mathbf{W}_{4,19}, \mathbf{W}_{5,2}$

From the subarray, it can be seen that in time slot 1, users are divided into two groups: users 1, 2, and 3 form one group, while users 4 and 5 make up another group. Users 4 and 5 are part of a MN PDA. They do not receive signals for files 1, 2, and 3. For the files  $\mathbf{W}_{4,19}$  and  $\mathbf{W}_{5,2}$ , each user stores content required by the other user. so the antenna only needs to send the sum of these two files then users 4 and 5 can accurately retrieve the files they need.

While users 1, 2, and 3 are situated in a MS MAPDA which means they only need to handle the subfiles of files 1, 2, and 3. Consider, for instance, user 2 (similar to user 3), who neither has  $\mathbf{W}_{3,1}$  cached nor requires it. In this case, antenna zero-forcing can be implemented which involves using signal precoding to adjust the transmitted signals such that, after passing through the channel, the signal coefficients at unintended receivers are reduced to zero. While for  $\mathbf{W}_{1,2}$ , it can be cancelled using the contents already present in user 2's cache. Then user 2 can correctly get the subfile  $\mathbf{W}_{2,1}$ .

As for user 1, it has cached  $\mathbf{W}_{3,1}$  and  $\mathbf{W}_{2,1}$ , so it can just subtract them from received signal to correctly get subfile  $\mathbf{W}_{1,2}$ .

## VI. CONCLUSIONS

This paper presents a novel approach to coded caching assisted by RIS with multi-antenna systems. By integrating

RIS, we enhance multicast gains by enabling interference nulling, effectively "erasing" unwanted transmission paths. We introduce a fast-converging algorithm for RIS phase adjustment. We then use the strategy that dividing antennas and users into several independent groups with RIS, reducing interference and maximizing DoF, which advances RIS-assisted coded caching technology.

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## APPENDIX A PROOF OF THEOREM 1

Given the number of active antennas  $L_0$ , the total number  $G$  of RIS elements used must satisfy the following inequality.

$$\begin{aligned}
\frac{G}{2} &\geq \sum_{i=1}^r (L_i + t)(L_0 - L_i) \\
&= \sum_{i=1}^r (L_0 L_i + t L_0 - L_i^2 - t L_i) \\
&= (L_0 - t) \sum_{i=1}^r L_i + t r L_0 - \sum_{i=1}^r L_i^2 \\
&= L_0(L_0 - t) + t r L_0 - \sum_{i=1}^r L_i^2 \quad (11)
\end{aligned}$$

We prefer to design a scheme such that the value  $G$  is as small as possible. This implies that given the parameters  $L$ ,  $r$  and  $t$ , the value of  $\sum_{i=1}^r L_i^2$  is as large as possible under the condition  $\sum_{i=1}^r L_i = L_0$ . Without loss of generality, we assume that  $L_1 \geq L_2 \geq \dots \geq L_r \geq 1$ . When we move  $a_2, a_3, \dots, a_r$  antennas from the second, third,  $\dots$ ,  $r$ th group respectively to the first group, then the first group has  $L'_1 = L_1 + a_1 + a_2 + \dots + a_{r-1}$  and the  $i$ th group has  $L'_i = L_i - a_i$  antennas for each  $2 \leq i \leq r$  where  $a_i < L_i$ . Now let us consider the value of  $\sum_{i=1}^r L_i'^2$  and obtain the following equality

$$\begin{aligned}
&L_1'^2 + L_2'^2 + \dots + L_r'^2 \\
&= (L_1 + a_2 + a_3 + \dots + a_r)^2 + (L_2 - a_2)^2 + (L_3 - a_3)^2 \\
&\quad + \dots + (L_r - a_r)^2 \\
&= L_1^2 + L_2^2 + L_3^2 + \dots + L_r^2 + (a_2 + a_3 + \dots + a_r)^2 \\
&\quad + 2a_2(L_1 - L_2) + 2a_3(L_1 - L_3) + \dots + 2a_r(L_1 - L_r).
\end{aligned}$$

Clearly  $L_1 - L_2, L_1 - L_3, \dots, L_1 - L_r$  are all non-negative. Therefore, we have  $\sum_{i=1}^r L_i'^2 \geq \sum_{i=1}^r L_i^2$ . This implies that the more antennas  $a_1, a_2, \dots, a_r$  are moved to the group with the highest number of antennas, the greater increase of the sum of squares  $\sum_{i=1}^r L_i'^2$ . Clearly the extreme cases is that the first group has exactly  $L_0 - r + 1$  antennas and each of the last  $r - 1$  groups has exactly one antenna.

## APPENDIX B DETAILED ANALYSIS AND DESIGN PROCESS OF THE ALGORITHM 3

Based on the analysis of scenarios where the number of used antennas is known, it can be deduced that even if some antennas is allowed to be silenced, dividing antennas into  $r$  groups, with  $r - 1$  groups containing one antenna each, and the remaining group consisting of  $L_0 - (r - 1)$  antennas is still is considered optimal only under the conditions specified for grouping.

Assuming two schemes have the same sum-DoF, denoted as  $g$ , and the number of antennas for these schemes are  $L$  and  $L'$  respectively. Let us assume that  $L > L'$  and the  $L$  antennas and  $L'$  antennas are divided into  $r$  and  $r'$  groups respectively.

By our hypothesis we have  $g = L + tr = L' + tr'$  for each  $t$ , i.e.,  $L - L' = t(r' - r) > 0$ . So we have  $r < r'$  and let  $\lambda = r' - r$ .

From (11), the required number of RIS elements  $G$  for  $L$  antennas is

$$\begin{aligned}
\frac{G}{2} &= (t+1)(L-1) + (L-(r-1)+t)(r-1) \\
&= (r-1)[(t+2)L-r] \quad (12)
\end{aligned}$$

and the number of RIS elements  $G'$  for  $L'$  antennas is

$$\begin{aligned}
\frac{G'}{2} &= (t+1)(L'-1) + (L'-(r'-1)+t)(r'-1) \\
&= (\lambda+r-1)[(t+2)(L-\lambda t) - (\lambda+r)] \\
&= \lambda[(t+2)(L-\lambda t) - (\lambda+r)] + (r-1)((t+2)L-r) \\
&\quad + (r-1)[-(t+2)\lambda t - \lambda] \quad (13)
\end{aligned}$$

Then we have

$$\begin{aligned}
f(\lambda) &= G' - G \\
&= 2(\lambda((t+2)(L-\lambda t) - (\lambda+r)) + \\
&\quad (r-1)(-(t+2)\lambda t - \lambda)) \\
&= 2(\lambda((-t^2 - 2t - 1)\lambda + (t+2)L - \\
&\quad 2r + (1-r)t^2 - 2tr + 2t + 1)) \quad (14)
\end{aligned}$$

The value of (14) represents the difference in the number of RIS elements between the original scheme and the scheme with fewer antennas.

For a given sum-DoF  $g$ , assume  $L$  represents the maximum number of antennas used to achieve this sum-DoF. Consider this scheme as the original scheme referred to in (14).

Starting from this maximum value, consider other schemes with fewer antennas (i.e. consider the value of  $\lambda$ ). It is not difficult to check that  $f(\lambda)$  in (14) is a downward-opening parabola that passes through the origin, with  $\lambda$  as the independent variable. This implies that if  $f(\lambda) \geq 0$ , the reducing the number of antennas  $L - L'$  leads to an increase or no change in RIS elements; if  $f(\lambda) < 0$ , the scheme with  $L'$  antennas utilizes fewer RIS elements than the original scheme, making it more optimal. Additionally, the larger the value of  $\lambda$ , the greater the reduction in the number of RIS elements.

Since  $\lambda$  is positive, it is only necessary to determine whether  $\lambda$  can exceed another zero point. If it can, then by reducing the number of antennas, a more optimal scheme can be found. Otherwise, the scheme using the maximum number of antennas is the best solution.

- To find the scheme that uses the maximum number of antennas under a given sum-DoF  $g$ , it is necessary first to determine the range of  $L$ .

First, when the used antennas is  $L$ , the maximum achievable sum-DoF is  $L(t+1)$ . Therefore,  $g \leq L(t+1)$ , which implies  $L \geq \frac{g}{t+1}$ . Since  $L$  must be a positive integer, we have  $L \geq \lceil \frac{g}{t+1} \rceil$ . From (7), we have the number of groups  $r = \frac{(g-L)}{t} \geq 1$ , i.e.,  $L \leq g - t$ . So we have the range for

$L$  as follows,

$$\lceil \frac{g}{t+1} \rceil \leq L \leq \min\{L_0, g-t\} \quad (15)$$

where  $L_0$  represents the total number of existing antennas.

In addition, the value of  $L$  must also ensure that  $r = \frac{(g-L)}{t}$  is a positive integer.

After determining the range and constraints for  $L$ , it's possible to verify from the upper bound of  $L$  downwards, checking each value whether it meets the condition that  $r$  is a positive integer. This process will help identify the scheme that uses the maximum number of antennas, along with the corresponding number of groups and RIS elements. Then substitute these values into the function  $f(\lambda)$  and compute the non-zero roots  $\lambda_0$  of  $f(\lambda)$ .

$$\lambda_0 = \frac{(t+2)L - 2r + (1-r)t^2 - 2tr + 2t + 1}{(t+1)^2} \quad (16)$$

- After determining  $\lambda_0$ , it is important to consider the range of possible values for  $\lambda$ .

Given that reducing the number of antennas by  $t$  results in an increase of one group, the reduced number of antennas  $L'$  must be greater than or equal to the new number of groups  $r'$ , that is,  $L' \geq r'$ . Substituting in the expressions for  $L'$  and  $r'$ , we get  $L - \lambda t \geq \lambda + r$ . Considering also that  $\lambda$  must be a positive integer, the range for  $\lambda$  can be determined as follows,

$$0 \leq \lambda \leq \lfloor \frac{L-r}{t+1} \rfloor \quad (17)$$

Therefore, we only need to compare the maximum value of  $\lambda$  with  $\lambda_0$ .