I. CONSTRUCTION

Definition 1 (MAPDA, [1])**.** For any positive integers L, K, F, Z, and S, an $F \times K$ array P composed of "*" and $[S]$ is called an (L, K, F, Z, S) multiple-antenna placement delivery array (MAPDA) if it satisfies the following conditions:

C1: The symbol "*" appears Z times in each column;

C2: Each integer from 1 to S occurs at least once in the array;

C3: Each integer s appears at most once in each column;

C4: For any integer $s \in [S]$, define $P(s)$ to be the subarray of P including the rows and columns containing s, and let $r'_s \times r_s$ denote the dimensions of $P(s)$. The number of integer entries in each row of $P(s)$ is less than or equal to $\min\{L, K\}$, i.e.,

$$|\{k_1 \in [r_s] \mid P(s)(f_1, k_1) \in [S]\}| \leq \min\{L, K\}, \quad \forall f_1 \in [r'_s].$$

The array is composed of "*" and integers. Each column represents a user and each row represents a packet of files. A $" * "$ appears in the position (i, j) of the array, meaning that user j stores the i-th packet of each file. Each integer corresponds to a multicast message transmitted in one time slot.

Each MAPDA corresponds to a specific coded caching scheme where the cache ratio $\frac{M}{N}$ equals $\frac{Z}{F}$. This process is summarized in Algorithm 1.

Algorithm 1 Caching Scheme Based on MAPDA in [1]

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procedure PLACEMENT(Q, W)
  Split each file $W_n \in W$ into F packets, i.e., $W_n = \{W_{n,f} | f = 1, 2, \ldots, F\}$.
  for $k \in [K]$ **do**
    $Z_k \leftarrow \{W_{n,f} | Q(f, k) = *, n \in [N], f \in [F]\}$
  end for
end procedure
procedure DELIVERY(Q, W, d)
  for $s = 1, 2, \ldots, S$ **do**
    Server uses L antennas to send $\mathbf{W}_{d_k,j}$ where $P(j, k) = s$ to the users.
  end for
end procedure

Note that when $L = 1$ the the MAPDA reduces to the PDA in [2]. From each MAPDA, we can obtain a corresponding multi-antenna coded caching scheme for the system containing a server with L antenas and K single-antenna users with memory ratio (i.e., the ratio between memory size and library size) $M/N = Z/F$, which has sum-DoF S.

We review some PDA and MAPDA that will be used later.

Lemma 1 (Maximum DoF [1]). *Under the MAPDA structure, the maximum achievable sum-DoF is* $\min\{K, L + KM/N\}$, *when* KM/N *is an integer.*

Lemma 2 (MN PDA [3]). *For any positive integers* K *and* t *with* $t < K$, *there exists a* $(K, \binom{[K]}{t}, \binom{K-1}{t-1}, \binom{K}{t+1})$ *PDA with the maximum DoF* $t + 1$

Construction 1 (MN PDA [4]). For any integer t within the set $[K]$, we difine a MN PDA. $P = (P(\mathcal{T}, k))$, with dimensions $\binom{K}{t} \times K$, where $\mathcal{T} \subseteq \binom{[K]}{t}$ and $k \in [K]$. The elements of P are defined as follows:

$$P(\mathcal{T}, k) = \begin{cases} * & \text{if } k \in \mathcal{T}, \\ \mathcal{T} \cup \{k\} & \text{otherwise.} \end{cases}$$

Example 1. For $K = 7, t = 1$, we have a $(7, 7, 1, 21)$ MN PDA: Just sort and label these groups,

	1	2	3	4	5	6	7
1	$*$	$\{1,2\}$	$\{1,3\}$	$\{1,4\}$	$\{1,5\}$	$\{1,6\}$	$\{1,7\}$
2	$\{1,2\}$	$*$	$\{2,3\}$	$\{2,4\}$	$\{2,5\}$	$\{2,6\}$	$\{2,7\}$
3	$\{1,3\}$	$\{2,3\}$	$*$	$\{3,4\}$	$\{3,5\}$	$\{3,6\}$	$\{3,7\}$
4	$\{1,4\}$	$\{2,4\}$	$\{3,4\}$	$*$	$\{4,5\}$	$\{4,6\}$	$\{4,7\}$
5	$\{1,5\}$	$\{2,5\}$	$\{3,5\}$	$\{4,5\}$	$*$	$\{5,6\}$	$\{5,7\}$
6	$\{1,6\}$	$\{2,6\}$	$\{3,6\}$	$\{4,6\}$	$\{5,6\}$	$*$	$\{6,7\}$
7	$\{1,7\}$	$\{2,7\}$	$\{3,7\}$	$\{4,7\}$	$\{5,7\}$	$\{6,7\}$	$*$

TABLE I: A MN PDA example

and the elements in the array can then be represented in the form of integers.

Lemma 3 (MS MAPDA [5]). *For any positive integers* K *and* t, *there exists a* $(L, K, \binom{K}{t}\binom{K-t-1}{L-1}, \binom{K-1}{t-1}\binom{K-t-1}{L-1}, \binom{K}{t+L}$ *MAPDA with the maximum DoF* $t + L$

Construction 2 (MS MAPDA). For any integers t, L and K with $t + L \in [K]$, we can define a $\binom{K}{t}\binom{K-t-1}{L-1} \times K$ array $P = (P(\mathcal{T}, \mathcal{L}), k))$ where $\mathcal{T} \in \binom{[K]}{t}, \mathcal{L} \in \binom{[K-t-1]}{L-1}, k \in [K]$ in the following way.

$$P(\mathcal{T},\mathcal{L}),k) = \begin{cases} * & \text{if } k \in \mathcal{T}, \\ (\mathcal{S}(\mathcal{T},\mathcal{L},k), order(\mathcal{S}(\mathcal{T},\mathcal{L},k)) & \text{otherwise.} \end{cases}$$

where $\mathcal{S}(\mathcal{T},\mathcal{L},k) = \mathcal{T} \cup ([K] \setminus (\mathcal{T} \cup \{k\})[\mathcal{L}] \cup \{k\}$ and order($\mathcal{S}$) as the order of appearance of the set $\mathcal{S}$ in each column.

Example 2. For $K = 7, t = 1, L = 2$,we have a $(2, 7, 35, 70)$ MS MAPDA.

	1	2	3	4	5	6	7
1	*	{1,2,3}	{1,2,3}	{1,2,4}	{1,2,5}	{1,2,6}	{1,2,7}
2	{1,2,3}	*	{1,2,3}	{1,2,4}	{1,2,5}	{1,2,6}	{1,2,7}
3	{1,2,3}	{1,2,3}	*	{1,3,4}	{1,3,5}	{1,3,6}	{1,3,7}
4	{1,2,4}	{1,2,4}	{1,3,4}	*	{1,4,5}	{1,4,6}	{1,4,7}
5	{1,2,5}	{1,2,5}	{1,3,5}	{1,4,5}	*	{1,5,6}	{1,5,7}
6	{1,2,6}	{1,2,6}	{1,3,6}	{1,4,6}	{1,5,6}	*	{1,6,7}
7	{1,2,7}	{1,2,7}	{1,3,7}	{1,4,7}	{1,5,7}	{1,6,7}	*
1	*	{1,2,4}	{1,3,4}	{1,3,4}	{1,3,5}	{1,3,6}	{1,3,7}
2	{1,2,4}	*	{2,3,4}	{2,3,4}	{2,3,5}	{2,3,6}	{2,3,7}
3	{1,3,4}	{2,3,4}	*	{2,3,4}	{2,3,5}	{2,3,6}	{2,3,7}
4	{1,3,4}	{2,3,4}	{2,3,4}	*	{2,4,5}	{2,4,6}	{2,4,7}
5	{1,3,5}	{2,3,5}	{2,3,5}	{2,4,5}	*	{2,5,6}	{2,5,7}
6	{1,3,6}	{2,3,6}	{2,3,6}	{2,4,6}	{2,5,6}	*	{2,6,7}
7	{1,3,7}	{2,3,7}	{2,3,7}	{2,4,7}	{2,5,7}	{2,6,7}	*
1	*	{1,2,5}	{1,3,5}	{1,4,5}	{1,4,5}	{1,4,6}	{1,4,7}
2	{1,2,5}	*	{2,3,5}	{2,4,5}	{2,4,5}	{2,4,6}	{2,4,7}
3	{1,3,5}	{2,3,5}	*	{3,4,5}	{3,4,5}	{3,4,6}	{3,4,7}
4	{1,4,5}	{2,4,5}	{3,4,5}	*	{3,4,5}	{3,4,6}	{3,4,7}
5	{1,4,5}	{2,4,5}	{3,4,5}	{3,4,5}	*	{3,5,6}	{3,5,7}
6	{1,4,6}	{2,4,6}	{3,4,6}	{3,4,6}	{3,5,6}	*	{3,6,7}
7	{1,4,7}	{2,4,7}	{3,4,7}	{3,4,7}	{3,5,7}	{3,6,7}	*
1	*	{1,2,6}	{1,3,6}	{1,4,6}	{1,5,6}	{1,5,6}	{1,5,7}
2	{1,2,6}	*	{2,3,6}	{2,4,6}	{2,5,6}	{2,5,6}	{2,5,7}
3	{1,3,6}	{2,3,6}	*	{3,4,6}	{3,5,6}	{3,5,6}	{3,5,7}
4	{1,4,6}	{2,4,6}	{3,4,6}	*	{4,5,6}	{4,5,6}	{4,5,7}
5	{1,5,6}	{2,5,6}	{3,5,6}	{4,5,6}	*	{4,5,6}	{4,5,7}
6	{1,5,6}	{2,5,6}	{3,5,6}	{4,5,6}	{4,5,6}	*	{4,6,7}
7	{1,5,7}	{2,5,7}	{3,5,7}	{4,5,7}	{4,5,7}	{4,6,7}	*
1	*	{1,2,7}	{1,3,7}	{1,4,7}	{1,5,7}	{1,6,7}	{1,6,7}
2	{1,2,7}	*	{2,3,7}	{2,4,7}	{2,5,7}	{2,6,7}	{2,6,7}
3	{1,3,7}	{2,3,7}	*	{3,4,7}	{3,5,7}	{3,6,7}	{3,6,7}
4	{1,4,7}	{2,4,7}	{3,4,7}	*	{4,5,7}	{4,6,7}	{4,6,7}
5	{1,5,7}	{2,5,7}	{3,5,7}	{4,5,7}	*	{5,6,7}	{5,6,7}
6	{1,6,7}	{2,6,7}	{3,6,7}	{4,6,7}	{5,6,7}	*	{5,6,7}
7	{1,6,7}	{2,6,7}	{3,6,7}	{4,6,7}	{5,6,7}	{5,6,7}	*

TABLE II: A MS MAPDA example

In this 35×7 array is first generated, and "$*$" is filled in according to the construction rules. Each remaining position should be filled with a set of size $|t + L|$, which includes the user indices corresponding to " $*$ " in this row, the index of the protector at this position, and the indices of $L-1$ other users. Therefore, there are $\binom{K-1-t}{L-1}=5$ ways to select this, so based on $\binom{K}{t}=7$, it is vertically replicated 3 times to ensure that all possibilities for each position occur.

After filling in, it is observed that the same sets occur 2 times in each column. Therefore, afterwards, it is only necessary to label each set according to the order of its occurrence in each column to obtain an array in integer form, for example, the set $\{1,2,3\}$ at $P(2,1), P(1,2), P(1,3)$ are denoted by 1, and the set at $P(3,1), P(3,2), P(2,3)$ will be denoted by 2. And the set 1,2,4 at $P(4,1), P(4,2), P(1,4)$ will be denoted by 3, and so on.

Inspired by MAPDA, our scheme can also be represented in the form of an array. Similar to MAPDA, the symbol "*" is used to denote cached content at the users. For transmission, to enable simultaneous broadcasting across r groups.

From our previous analyses, the antenna groups are limited to two configurations: one consisting of a single antenna with $t+1$ users, and the other consisting of $L-r+1$ antennas with $t+L-r+1$ users. So our objective is to serve $g=(t+1)(r-1)+(t+L-r+1)$ users per time slot, which necessitates the construction of a MAPDA. Besides meeting requirements C1, C2, C3 in definition 1, it must also satisfy the following condition.

C4: For any integer $s \in [S]$, define $P_1^{(s)}, P_2^{(s)}, \ldots P_r^{(s)}$, to be the subarrays of P including the rows and columns containing s. These subarrays are mutually disjoint in columns, and one of the subarrays is an MAPDA with $L-r+1$ antennas, while the remaining $r-1$ subarrays are each MAPDAs with 1 antenna.

The new MAPDA structure proposed for the RIS-assisted MISO coded caching problem is called RMAPDA.

Definition 2 (RMAPDA)**.** For any positive integers L, K, F, Z, r and S, an $F \times K$ array P composed of "*" and $[S]$ is called an (L,K,F,Z,r,S) multiple-antenna placement delivery array (MAPDA) if it satisfies the following conditions:

C1: The symbol "*" appears Z times in each column;

C2: Each integer from 1 to S occurs at least once in the array;

C3: Each integer s appears at most once in each column;

C4: For any integer $s \in [S]$, define $P_1^{(s)}, P_2^{(s)}, \ldots P_r^{(s)}$, to be the subarrays of P including the rows and columns containing s. These subarrays are mutually disjoint in columns, and one of the subarrays is an MAPDA with $L-r+1$ antennas, while the remaining $r-1$ subarrays are each MAPDAs with 1 antenna.

The array is composed of "*" and integers. Each column represents a user and each row represents a packet of files. A " $*$ " appears in the position (i, j) of the array, meaning that user j stores the i-th packet of each file. Each integer corresponds to a multicast message transmitted in one time slot.

Each RMAPDA corresponds to a specific RIS-assisted coded caching scheme where the cache ratio $\frac{M}{N}$ equals $\frac{Z}{F}$.

We are now equipped to begin the construction of a RMAPDA that fulfills the specified condition based on the MN PDA in lemma 2 and MS MAPDA in lemma 3.

Given that under the one-shot transmission strategy, the DoF $g \leq K$, it is sufficient to study cases where $g \leq K$.

In each MN PDA, groups of $t+1$ users require only one time slot each for data handling. In addition, in the MAPDA, each group of $t+L'$ users necessitates $\binom{t+L'-1}{t}$ time slots. To ensure that the number of time slots across $r-1$ MN PDAs and a MAPDA matches, we replicate them vertically until the total number of slots aligns.

Let $L' = L - r + 1$ and the parameter L in lemma 3 is L'.

First, assume each MS MAPDA is replicated vertically x times, and the MN PDA is replicated vertically y times. To confirm that the time slots of a $t+L'$ users group is the same of the $r-1$ groups with $t+1$ users, x and y should satisfied:

$$x\binom{t+L'-1}{t} = y'\binom{K-(t+L')}{(r-1)(t+1)} \times \frac{[(r-1)(t+1)]!}{((t+1)!)^{(r-1)}(r-1)!} \tag{1}$$

A PDA copied y' times only satisfies the matching of a single multi-user group and the corresponding single-user group. To ensure that each multi-user group can be matched, the copied PDA should be copied n more times, where

$$n = \binom{K-t-1}{t+L'}\binom{K-(t+1)-(t+L')}{(r-2)(t+1)} \times \frac{[(r-2)(t+1)]!}{((t+1)!)^{(r-2)}(r-2)!} \tag{2}$$

	1	2	3	4	5	6	7
1	*	{1,2,3}	{1,2,3}	{1,2,4}	{1,2,5}	{1,2,6}	{1,2,7}
2	{1,2,3}	*	{1,2,3}	{1,2,4}	{1,2,5}	{1,2,6}	{1,2,7}
3	{1,2,3}	{1,2,3}	*	{1,3,4}	{1,3,5}	{1,3,6}	{1,3,7}

The first three rows of Q_0

	1	2	3	4	5	6	7
1	*	{1,2,3}	{1,2,3}	{1,2,4}	{1,2,5}	{1,2,6}	{1,2,7}
2	{1,2,3}	*	{1,2,3}	{1,2,4}	{1,2,5}	{1,2,6}	{1,2,7}
3	{1,2,3}	{1,2,3}	*	{1,3,4}	{1,3,5}	{1,3,6}	{1,3,7}

The first three rows of Q_0

	1	2	3	4	5	6	7
1	*	{1,2,3}	{1,2,3}	{1,2,4}	{1,2,5}	{1,2,6}	{1,2,7}
2	{1,2,3}	*	{1,2,3}	{1,2,4}	{1,2,5}	{1,2,6}	{1,2,7}
3	{1,2,3}	{1,2,3}	*	{1,3,4}	{1,3,5}	{1,3,6}	{1,3,7}

The first three rows of Q_0

	1	2	3	4	5	6	7
1	*	{1,2}	{1,3}	{1,4}	{1,5}	{1,6}	{1,7}
2	{1,2}	*	{2,3}	{2,4}	{2,5}	{2,6}	{2,7}
3	{1,3}	{2,3}	*	{3,4}	{3,5}	{3,6}	{3,7}
4	{1,4}	{2,4}	{3,4}	*	{4,5}	{4,6}	{4,7}
5	{1,5}	{2,5}	{3,5}	{4,5}	*	{5,6}	{5,7}
6	{1,6}	{2,6}	{3,6}	{4,6}	{5,6}	*	{6,7}
7	{1,7}	{2,7}	{3,7}	{4,7}	{5,7}	{6,7}	*

	1	2	3	4	5	6	7
1	*	{1,2}	{1,3}	{1,4}	{1,5}	{1,6}	{1,7}
2	{1,2}	*	{2,3}	{2,4}	{2,5}	{2,6}	{2,7}
3	{1,3}	{2,3}	*	{3,4}	{3,5}	{3,6}	{3,7}
4	{1,4}	{2,4}	{3,4}	*	{4,5}	{4,6}	{4,7}
5	{1,5}	{2,5}	{3,5}	{4,5}	*	{5,6}	{5,7}
6	{1,6}	{2,6}	{3,6}	{4,6}	{5,6}	*	{6,7}
7	{1,7}	{2,7}	{3,7}	{4,7}	{5,7}	{6,7}	*

The first P_1

Fig. 1: Example of $t+L'$-usres $\{1,2,3\}$

Therefore, the total number y of times the PDA is ultimately copied is

$$y = ny' \tag{3}$$

The subpacketization F is

$$F = F_1 + F_2 = x\binom{K}{t}\binom{K-t-1}{L-1} + y\binom{K}{t} \tag{4}$$

The construction process is summarized as Algorithm 2. Then we will propose an example to illustrate the construction process in Algorithm 2 in detail.

Example 3. Consider a system with $K=7$, $L=4$, $M=1$, $t=KM/N=1$, dividing antennas into $r=3$ groups, one group with $L'=2$ antennas and the other two groups both have only

Algorithm 2 General Construction of RMAPDA

procedure CONSTRUCTRMAPDA(K, L, M, N, r)

  $t \leftarrow \frac{KM}{N}$

  $L' \leftarrow L - r + 1$ $\triangleright$ Divide antennas into r groups, one with L' antennas, others with one antenna

  Step 1: Construct MS MAPDA Q_0 and MN PDA P_0

  $P_0 \leftarrow$ CONSTRUCTION OF MN_PDA(K, t)

  $Q_0 \leftarrow$ CONSTRUCTION OF MS_MAPDA(K, t, L')

  Step 2: Construct Q and P

  Determine x and y' such that $x\binom{t+L'-1}{t} = y'\binom{K-(t+L')}{(r-1)(t+1)} \frac{[(r-1)(t+1)]!}{((t+1)!)^{r-1}(r-1)!}$

$$Q \leftarrow \left.\begin{bmatrix} Q_0 \\ \hline Q_0 \\ \hline \vdots \\ \hline Q_0 \end{bmatrix}\right\} x \quad times$$

$$P_1 \leftarrow \left.\begin{bmatrix} P_0 \\ \hline P_0 \\ \hline \vdots \\ \hline P_0 \end{bmatrix}\right\} y' \quad times$$

  Compute n as: $n \leftarrow \binom{K-t-1}{t+L'}\binom{K-(t+1)-(t+L')}{(r-2)(t+1)}\left(\frac{[(r-2)(t+1)]!}{((t+1)!)^{r-2}(r-2)!}\right)$

$$P \leftarrow \left.\begin{bmatrix} P_1 \\ \hline P_1 \\ \hline \vdots \\ \hline P_1 \end{bmatrix}\right\} n \quad times$$

  Step 3: Combine P and Q

$$T \leftarrow \begin{bmatrix} P \\ \hline Q \end{bmatrix}$$

  return T

end procedure

one antenna. Assume each MS MAPDA is replicated vertically x times, and the MN PDA is replicated vertically y times.

- **Step 1.** Construction of MS MAPDA $\mathbf{Q}_0$ and MN PDA $\mathbf{P}_0$.
  - **Step 1.1.** We first get a 7×7 table $\mathbf{P}_0$ from construction 1. In this example, the table I is the $\mathbf{P}_0$ we want.
  - **Step 1.2.** Then we get a 35×7 table $\mathbf{Q}_0$ which can be constructed by construction 2. In fact, tabel II is the $\mathbf{Q}_0$ that we need.
- **Step 2.** Construction of $\mathbf{Q}$ and $\mathbf{P}$.

- **Step 2.1.** The $\mathbf{Q}$ represent the table after replicating $\mathbf{Q}_0$ x times, so it's necessary to decide x. The time slots required to transmit for each $(t+L')$-user group in $\mathbf{Q}$ is $S_1 = \binom{t+L'-1}{t} = \binom{2}{1} = 2$, while each $(t+1)$-user group only need $S_2 = 1$ time slot. To ensure time slot matching, we can consider selecting a group of $(t+L')$ users first, and then traversing all possible combinations of the remaining users.In this example, after excluding $t+L' = 3$ users, there are $\binom{K-(t+L')}{(r-1)(t+1)} \frac{[(r-1)(t+1)]!}{((t+1)!)^{(r-1)}(r-1)!} = \binom{4}{4} \frac{4!}{(2!)^2 2!} = 3$ cases in which $(r-1)(t+1)$ users are selected from the remaining users and divided into $(r-1)$ groups of $(t+1)$ users each. Therefore, select x and y' to satisfy the equation 1, i.e.

$$2x = 3y' \tag{5}$$

  Let $x = 3$, $y' = 2$, which ensures that the time slots of a multi-user group can match those of the single-user groups during transmission.

- **Step 2.2.** Then we replicate $\mathbf{Q}_0$ x times vertically to get a new table $\mathbf{Q}$ and replicate $\mathbf{P}_0$ y' times vertically to get $\mathbf{P}_1$.
- **Step 2.3.** The P_0 replicated vertically y' times obtain P_1 can only meet the time slot requirements of one $(t+L')$ group. For example, for the $(t+L')$ group composed of users $\{1, 2, 3\}$, after replication, it should occupy a total of 6 time slots. There are three ways to select two two-user groups from the remaining users. Each grouping occupies two time slots after being replicated y' times. Take the user group $\{4, 5\}$ as an example. Assuming that the three-user group becomes $\{1, 2, 6\}$, in its corresponding two-user group, $\{4, 5\}$ still needs to occupy two time slots. This means that replicating y' times can only meet the time slot requirements of one three-user group for the two-user groups, and one $(t+1)$ group may be needed n times in equation 2. In this example, n is $\binom{5}{3} = 10$ Therefore, $\mathbf{P}_1$ needs to be replicated 10 times vertically to get $\mathbf{P}$.

- **Step 3.** Combine $\mathbf{P}$ and $\mathbf{Q}$
  - **Step 3.1** Combine the generated $\mathbf{P}$ and $\mathbf{Q}$ vertically to form a new table $\mathbf{T}$, which corresponds to the array of the entire cache placement scheme.
  - **Step 3.2** Rewrite the sets in the table as numbers, which determine the time slot in which each file is sent. First, select a $(t+L')$-users group from $\mathbf{Q}$, for example, choose the set $\{1, 2, 3\}$. This set occupies two time slots in $\mathbf{Q}_0$. Each $\mathbf{Q}$ contains 3 $\mathbf{Q}_0$, thus occupying a total of 6 time slots. One time slot's $\{1, 2, 3\}$ corresponds to two $(t+1)$-users group,

	1	2	3	4	5	6	7
1	*	1	1	{1,2,4}	{1,2,5}	{1,2,6}	{1,2,7}
2	1	*	2	{1,2,4}	{1,2,5}	{1,2,6}	{1,2,7}
3	2	2	*	{1,3,4}	{1,3,5}	{1,3,6}	{1,3,7}
	1	2	3	4	5	6	7
1	*	3	3	{1,2,4}	{1,2,5}	{1,2,6}	{1,2,7}
2	3	*	4	{1,2,4}	{1,2,5}	{1,2,6}	{1,2,7}
3	4	4	*	{1,3,4}	{1,3,5}	{1,3,6}	{1,3,7}
	1	2	3	4	5	6	7
1	*	5	5	{1,2,4}	{1,2,5}	{1,2,6}	{1,2,7}
2	5	*	6	{1,2,4}	{1,2,5}	{1,2,6}	{1,2,7}
3	6	6	*	{1,3,4}	{1,3,5}	{1,3,6}	{1,3,7}
	1	2	3	4	5	6	7
1	*	{1,2}	{1,3}	{1,4}	{1,5}	{1,6}	{1,7}
2	{1,2}	*	{2,3}	{2,4}	{2,5}	{2,6}	{2,7}
3	{1,3}	{2,3}	*	{3,4}	{3,5}	{3,6}	{3,7}
4	{1,4}	{2,4}	{3,4}	*	1	2	3
5	{1,5}	{2,5}	{3,5}	1	*	3	2
6	{1,6}	{2,6}	{3,6}	2	3	*	1
7	{1,7}	{2,7}	{3,7}	3	2	1	*
	1	2	3	4	5	6	7
1	*	{1,2}	{1,3}	{1,4}	{1,5}	{1,6}	{1,7}
2	{1,2}	*	{2,3}	{2,4}	{2,5}	{2,6}	{2,7}
3	{1,3}	{2,3}	*	{3,4}	{3,5}	{3,6}	{3,7}
4	{1,4}	{2,4}	{3,4}	*	4	5	5
5	{1,5}	{2,5}	{3,5}	4	*	6	6
6	{1,6}	{2,6}	{3,6}	5	6	*	4
7	{1,7}	{2,7}	{3,7}	6	5	4	*

Fig. 2: Example of time slots 1-6

i.e., two sets of two users. The remaining four users are divided into two groups of two users with three possible combinations: $(\{4,5\},\{6,7\}),(\{4,6\},\{5,7\}),(\{4,7\},\{5,6\})$. In each time slot of $\{1,2,3\}$, select one of these three combinations in $\mathbf{P}$ to correspond to it, choosing each combination twice, which exactly matches the time slots. Similarly, other $(t+L')$-users in $\mathbf{Q}$ also select the corresponding combinations in $\mathbf{P}$ to match them, writing the same number for the same time slot.

For example, when the $(t+L')$-users group is selected as $\{1,2,3\}$, we extract the first three rows of each $\mathbf{Q}_0$ containing $\{1,2,3\}$ and the first $\mathbf{P}_1$. In Fig. 1, each color represents a time slot

So the final time slots corresponding to $\{1,2,3\}$ can be filled as shown in Fig. 2. The filling of time slots occupied by other $(t+L')$-users groups is similar. When finding the corresponding $(t+1)$-users group in $\mathbf{P}$, you can follow the principle of left to right and top to bottom.

If the remaining users after excluding the $(t+L')$ group exceed $(r-1)(t+1)$, then each possible combination should be considered. Therefore, first select $(r-1)(t+1)$ users from the remaining users, resulting in $\binom{K-(t+L')}{(r-1)(t+1)}$ combinations. For each combination, assign the user groups for each time slot using the method described in the previous example. After completing one combination, move to the next $(r-1)(t+1)$ combination and repeat the above steps. Once all combinations are exhausted, move to the next $(t+L')$ group and repeat the allocation process.

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