15-210 Assignment ThesaurusLab

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Section E
3/29/2014

5: Written Problems

Task 7.1
First we prove that there is no such tenary tree that has only vertices with degree 3 or more. We will do this by contradiction. We first assume that there is such a tenary tree. This means that the total degree of all the vertices is $3 * n$. This also implies that the number of edges is $\frac{3*n}{2}$. This is a contradiction because in a ternary tree there is only supposed to be $n - 1$ edges. Thus no such tree exists which means that at least one of the vertices needs to have a degree strictly less than three. So we have shown that k can at most be n. Also we will examine a tree where all the vertices have degree strictly less than 3. This is just a straight line of vertices, but it is still considered a a tenary tree. So we also have shown that $k > 1$. Thus we have shown that at least $\frac{n}{k}$ vertices need to have degree strictly less than three, where k goes from 1 to n.
Task 7.2
So our strategy to contract will to select a child and contract it with it's parent's node. So since we know that a fraction of the vertices in the tree would go away, we have it that the expected value of the number of vertices that are have degree less than 3, would be $\frac{n}{2}$. This is because from the previous part we have already shown that $\frac{n}{k}$ of the vertices have degree less than 3, where $1 \leq k \leq n$. So this means that the expected number of vertices of that we lose at each iteration would be $\frac{n}{2}$.
Also since we are essentially removing a leaf node from the parent node, the new graph would still be considered a ternary tree. This is because we are simply contracting the leaf to the parent, and since it is a tree. We are not adding vertices, or edges. We are simply taking it away and a leaf, to be exact, so again we won't have to create an edge since the leaf only had one edge going to it's parent.
So at each iteration we would be remove withe an expected value of half of the vertices.
Task 7.3
Using our method of finding which vertices to contract and contracting them. Since we are only removing leafs, we only remove a single vertex and a single edge. With this logic we would have to remove all the edges to get everything contracted, so the work would be $O(m)$. For each iteration we would be removing half of the vertices, the number of iterations would then be $O(logn)$.