15-210 Assignment BabbleLab

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4: Fun With Sets

Task 7.1
The work given by the union of two sets on the course website is $O(mlog(1+n))$ and the span is $O(log(n+m))$. m is the size of the smaller of the two sets and n is the the size of the larger of the two sets. So in this specific case $n = n$ and $m = \sqrt(n)$. So if we input this into our equation we would get that the work would be $O(\sqrt{n}log(1+n))$ and the span would be $O(log(n^{3/2}))$.

Task 7.2
The $\Omega(n)$ is not the case because of the implementation of the sets. We can make is so that when we create X, X refers(points) to the set A and B. That way X would be the union of the two sets as it has access to both elements in A and B and we did not create an entire new set containing all the elements of A and B. This can be done in the functional setting as functions are seen as values thus will the functions will not be automatically be evaluated, Thus we can maintain our set A, B while creating X.

Task 7.3
If we assume that each set has roughly the same cardinality, we then have the recurrence of $W(n) = 2W(\frac{n}{2}) + W_{union}(m/2, m/2)$. We can this recurrence from saying that the reduce is just divide and conquer algorithm where the division of the set is in half. So we know the work for the union which is $W_{reduce}(n, m) = mlog(1 + \frac{n}{m})$ where $m > n$. If we say that all the sets have roughly equal size this would reduce down to $\frac{m}{2}$. So now we have $W(n) = 2W(\frac{n}{2}) + \frac{m}{2}$. So if we go down the tree level at each level there would be $2^i \frac{m}{2^{i+1}}$ work being done saying that the i is the current level. This reduces down to $\frac{m}{2}$ work being done at each level. So then we can say that there are $logn$ levels and at each level there is $\frac{m}{2}$ work being done so we have that in total the amount of work being done is $\frac{m}{2}log(n)$.
For the span we take out the 2^i at each level so then at each level there is $\frac{m}{2^{i+1}}$ being done. There are still $logn$ level and if we reduce the summation we would get that the span $O(n)$.