15-210 Assignment BignumLab 1

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2/12/14

4: Addition

Task 4.1

GEN	GEN	STOP
GEN	PROP	STOP
GEN	STOP	STOP

5: Recurrences

Task 5.1

1. So we just use the tree method. We begin by saying that $T(n) \leq T(n-1) + k_1 log(n)$. By definition of Big theta. Then So if we go down the the tree it only expands into a single branch and at each level we get that $log(n-i)$ where i is the level of the tree. So then we if take the summation of all of the above we get that the final summation is $\sum_{i=0}^{n-1} log(n-i)$ This summation reduces down $log(n!)$. Thus we get that $T(n) \in O(log(n!)$. To get big omega we just flip the inequality and go the other direction. With this we get that $T(n) \in \Omega(log(n!))$ Thus we have that $T(n) \in \Theta(log(n!)$.

2. So we use the tree method again. We know that there are $log(log(n))$ levels. (We get this from recitation). At each level we have n work. To see this, we look to the first split. At the split there are $\sqrt{n}$ nodes and each node doing $\sqrt{n}$ work. Thus we have that at this level there is $\sqrt{n} * \sqrt{n} = n$. So each level we have n work so the summation is $\sum_{i=1}^{loglog(n)} n$, which will simplify down to $nloglog(n)$. Thus we since we have calculated the exact work we can say that $T(n) \in \Omega(nloglog(n))$.

3. So we start with the base case and this which from the definition of Big Theta we have that $T(1) \leq c$ where c is some constant. Thus the base case holds. We assume that for any n' that is $1 \leq n' \leq n$ that $T(n) \leq k_1 n^{frac32}$. So we start off with the recurrence that $T(n) = 4T(\frac{n}{4}) + c\sqrt{n}$. We apply the induction hypothesis on $T(\frac{n}{4}$ so that we get $T(n) \leq 4k_1 \frac{n}{4}^{\frac{3}{2}} + c\sqrt{n}$. We can simplify this down (all the constants will just make another bigger constant k_3 to $k_3 n^{\frac{3}{2}} + c\sqrt{n} = k_3\sqrt{n}(n+c)$ We can then lose c and and we get that $k_3 n^{\frac{3}{2}}$. Thus the induction step holds. Since the induction step and the base case holds the we have that $T(n) \leq kn^{\frac{3}{2}}$. Thus we have that $T(n) \in O(n^{\frac{3}{2}}$. We can then follow the same steps but just switch around the inequality sign to get the proof for the Big omega. Thus we have that $T(n) \in \Theta(n^{\frac{3}{2}})$.