PHYS379: Group Project - Cellular Automata Modelling Topic

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Keywords: cellular automata; statistical physics; traffic models; Conway's game of life; sandpiles and earthquakes; self-organised criticality; forest fires; fluid flow (lattice gas models); galaxy formation.

Introduction

In PHYS379, you should undertake an open-ended group project and submit a group report describing the project. The group project is worth twenty credits. Please see the document "PHYS379-group-project-notes.pdf" for details about organisation and assessment.

Open-ended project

Your task in the project is to model the properties of physical systems using cellular automata. Precisely what you do is your choice - the project is open-ended. You will need to formulate a research question (or questions) that is more specific than simply "model the properties of physical systems using cellular automata". What are you trying to discover? You should do some research in the library or on the internet to find out what the interesting questions are. It may be that you will not finalise the research question until the latter stages of the project, once you have found what is (or is not) interesting. Remember that this is group work - it will make sense to plan for parallel tasks in order to use your personnel effectively. For example, you might want parallel numerical and analytic investigations, investigations looking at different aspects of the same problem, or you might deliberately duplicate effort in order to check important results (in the "real" world, results should always be checked before publication, getting different people to work independently is the best way to do this). There are some ideas below, you can follow some or all of them, or you can follow your own ideas.

Getting started

Cellular automata

A cellular automaton [1,2,3] can be viewed as a lattice of sites (or "cells") such that the state of each site changes with time according to a rule depending on the local neighbourhood around the site. In particular [3]:

1. Space is discrete, consisting of an array of sites. Each site has the value of a variable associated with it.
2. The rule for a new value of a site depends on the values of the sites in the local neighbourhood.
3. Time is discrete. Site values are updated simultaneously based on the site values at the previous time step: the state of the whole lattice advances in discrete time steps.

Example of an elementary cellular automata

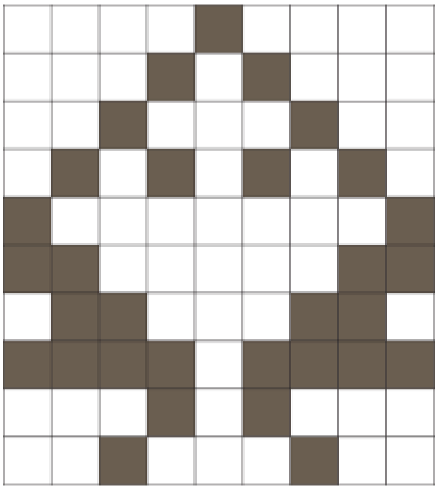

Figure 1 Elementary cellular automaton based on rule 90, with 9 sites and periodic boundary conditions. A site value of 0 is shown as a white cell, 1 is a shaded cell. The initial state is given by the top row, time increases in the downwards direction.

Elementary cellular automata [3,4,5] are one-dimensional cellular automata which have only two possible site values, usually denoted 0 and 1 (also called "one-dimensional Boolean cellular automata"). An example is shown in Fig.1, which consists of nine sites. The initial state of the system is shown in the top row where the central site has value 1 (shaded), the other sites have value 0. Time then evolves, as shown by each subsequent row in the downwards direction. This cellular automaton follows so-called "rule 90" or the "modulo-two" rule. The value of a given site is dictated by the values of its two nearest neighbours at the previous time step: their values are added and if the result is even (i.e. 0 or 2) then the site has value 0, but if the result is odd (i.e. 1) then the site has value 1. In Fig.1, we use periodic boundary conditions.

The following table shows the eight possible states of a site and its two nearest neighbours (e.g. 111), and how the state of the central site at the next time step is subsequently determined (e.g. 0) by rule 90:

current pattern	111	110	101	100	011	010	001	000
new state of central site	0	1	0	1	1	0	1	0

In binary notation, this rule is denoted 01011010: the name "rule 90" arises because 01011010 in binary is the number 90 in base 10. In fact, there are $2^8 = 256$ possible rules [4,5], each corresponding to a different eight-digit binary number.

It is very straightforward to implement the rule 90 cellular automaton (an efficient implementation in Java is given in Section 14.1 of Ref.[3]):

1. For a system with L sites, define an array "slist" to store the integer site values. It can be a one-dimensional array, each consecutive L elements can store the state of the system at a particular time.
2. Define an initial state of the system, by occupying the first L elements in the array "slist".

3. For the next time step, consider each site in turn. For each site, retrieve the state of its two nearest neighbours at the previous time step from “slist” and determine whether they sum to an even or odd number (remember to consider whether to use periodic boundary conditions or not). Use the result to determine the site value at the present step. Add it in the appropriate place to the array “slist”.
4. Repeat step 3. for a (large) number of time steps.

For a larger system, the rule 90 elementary cellular automaton produces the Sierpinski triangle fractal structure [6]. Can you characterise this pattern with a fractal dimension? Try to implement elementary cellular automata that follow different rules, e.g. rules 18 (00010010), 73 (01001001), 110 (01101110), 136 (10001000) [4]. Can you implement random initial states? How sensitive are the patterns to the initial states and the types of boundary conditions?

Open-ended part

Please note:

1. **The aim is for your group to do its own, original calculations.** A literature review and, perhaps, reproduction of other people's results may be important steps along the way, but these are not the final aim.
2. The cellular automata topic has run in PHYS379 for many years with great success. There are plenty of different projects you can do within this topic. If you are uncertain about which topic to choose, I advise you to choose this one.

Ideas include:

- One-dimensional Boolean cellular automata with larger neighbourhoods. See Problem 14.3 of Ref.[3] and the text immediately above it.
- Coarse graining of a one-dimensional cellular automaton. See Problem 14.4 of Ref.[3] and the text immediately above it.
- Traffic models. See Problem 14.5 of Ref.[3] and the text above it.
- Two dimensional automata: `Conway's game of life' [7], see also Problem 14.6 of Ref.[3] and the text above it.
- Probabilistic cellular automata: forest fire models. See Problems 14.10, 14.11 of Ref.[3].
- Sandpile and earthquake models, e.g. the Bak-Tang-Wiesenfeld model [8,9], the Olami-Feder-Christensen model [10], the Burridge-Knopoff model [3]. See Problems 14.7, 14.8 and 14.9, and Project 14.26 of Ref.[3]. What is self-organised criticality?
- Cellular automata models of fluid flow: lattice gas models [11]. See Section 14.6 and Project 14.25 of Ref.[3]. How do the results of simulation agree with your expectations from analytic results? Lattice Boltzmann methods [12,13].
- Cellular automata models of galaxy formation [14-16]. See Section 19.2 of Ref.[3].
- Cellular automata and music.
- Cellular automata and cryptography.
- If you study a cellular automata model of fluid flow or heat flow, say, it may be possible to compare your model with the `exact' solution of partial differential

equations using the finite difference method (this will typically allow you to write the problem as a linear algebra problem, i.e. inverting a matrix). Alternatively, you could use the finite element method with a pre-written package such as MATLAB PDE Modeler.

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