Truth Table

	x_1	x_2	x_3	f
0	0	0	0	0
1	0	0	1	0
2	0	1	0	0
3	0	1	1	1
4	1	0	0	0
5	1	0	1	1
6	1	1	0	1
7	1	1	1	1

Design a circuit that provides $f=1$ if more than one inputs are '1'.

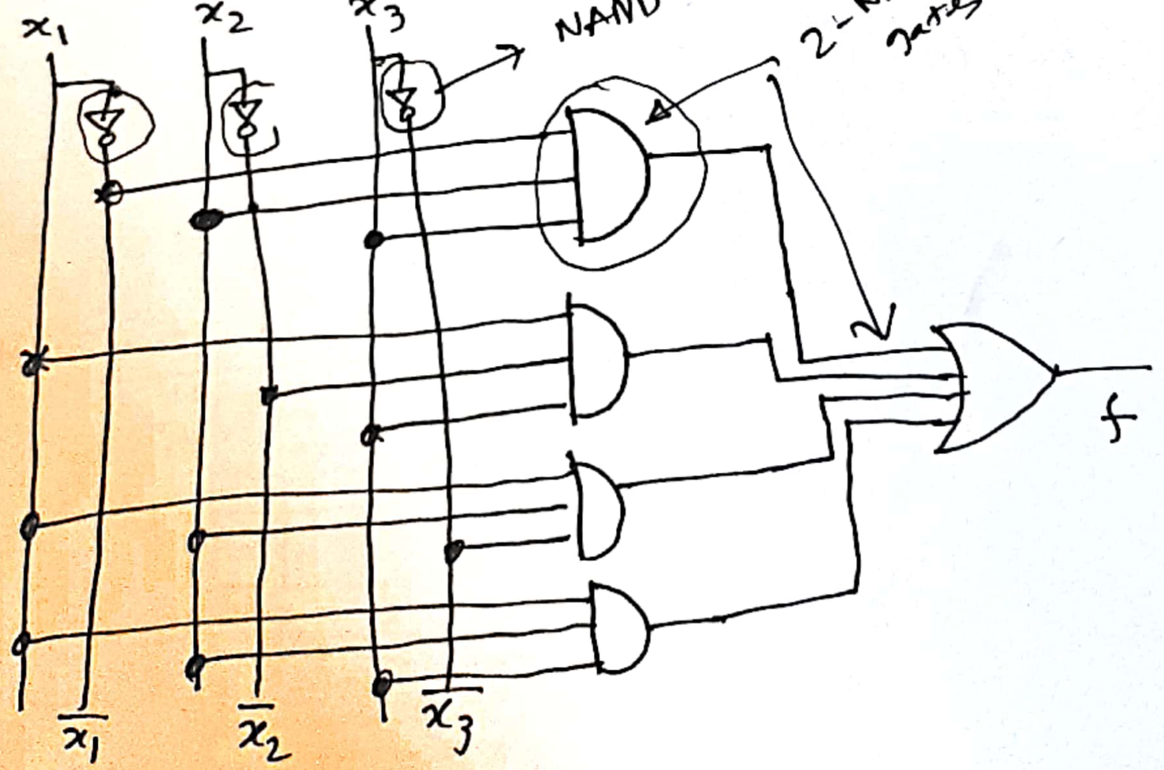
$$f(x_1, x_2, x_3) = \sum m(3,5,6,7)$$

SOP

$$= \sum (m_3, m_5, m_6, m_7)$$

$$f = \bar{x}_1 x_2 x_3 + x_1 \bar{x}_2 x_3 + x_1 x_2 \bar{x}_3 + x_1 x_2 x_3$$

a) Without simplification

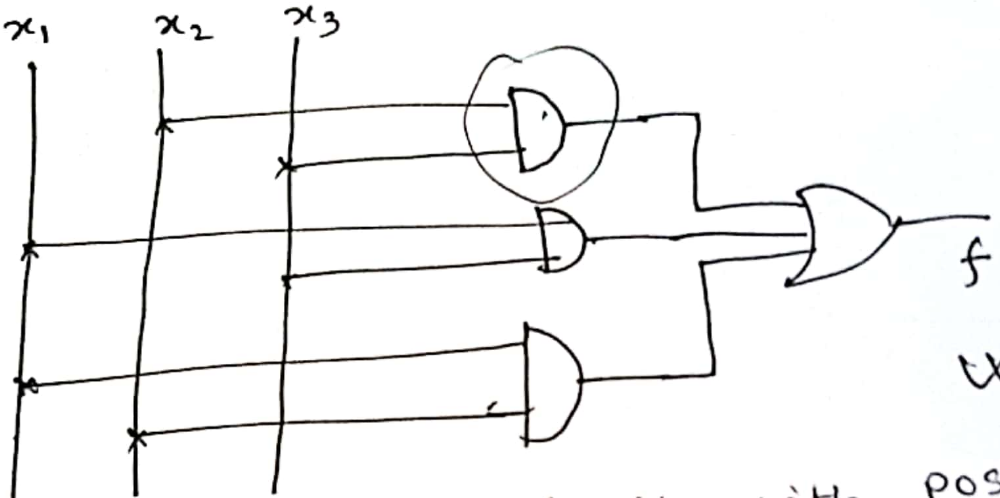


(b) <u>Simplify the expression:</u>

$$f = (\bar{x}_1 x_2 x_3 + x_1 x_2 x_3) + (x_1 \bar{x}_2 x_3 + x_1 x_2 x_3) + (x_1 x_2 \bar{x}_3 + x_1 x_2 x_3)$$

$$= x_2 x_3(\bar{x}_1 + x_1) + x_1 x_3(\bar{x}_2 + x_2) + x_1 x_2(\bar{x}_3 + x_3)$$

$$= x_2 x_3 + x_1 x_3 + x_1 x_2$$



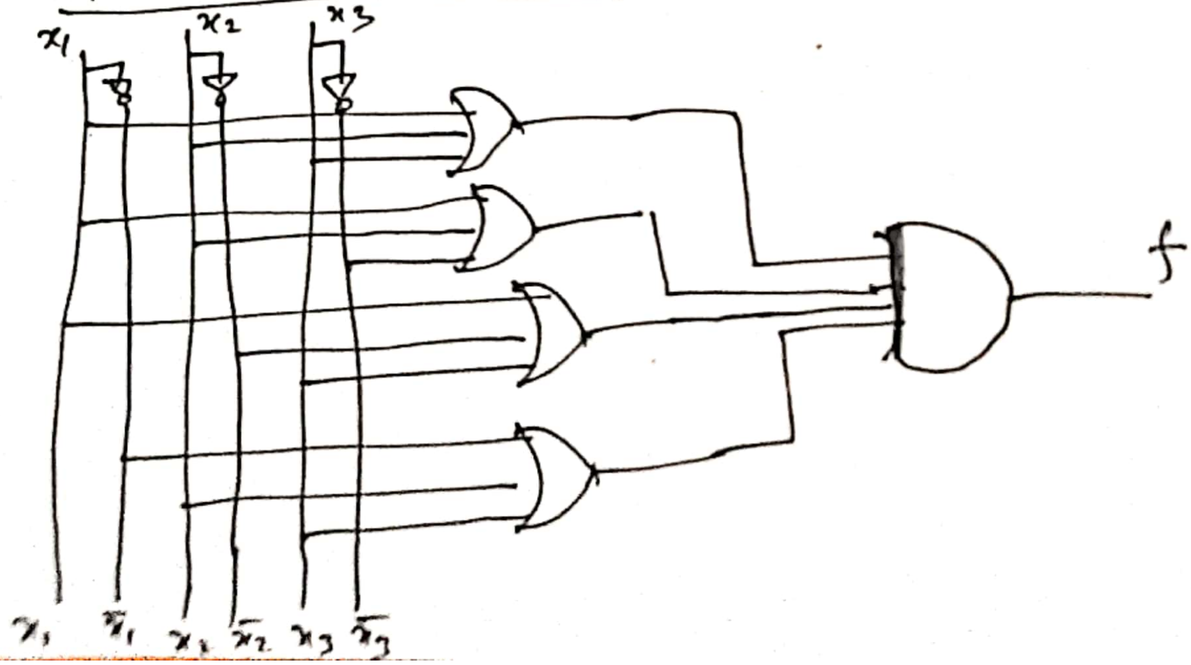
Implement the above function with POS.

POS: $f(x_1, x_2, x_3) = \prod M(0,1,2,4)$

$$= \prod(M_0, M_1, M_2, M_4)$$

$$f = (x_1 + x_2 + x_3)(x_1 + x_2 + \bar{x}_3)(x_1 + \bar{x}_2 + x_3)(\bar{x}_1 + x_2 + x_3)$$

<u>Without simplification.</u>



2

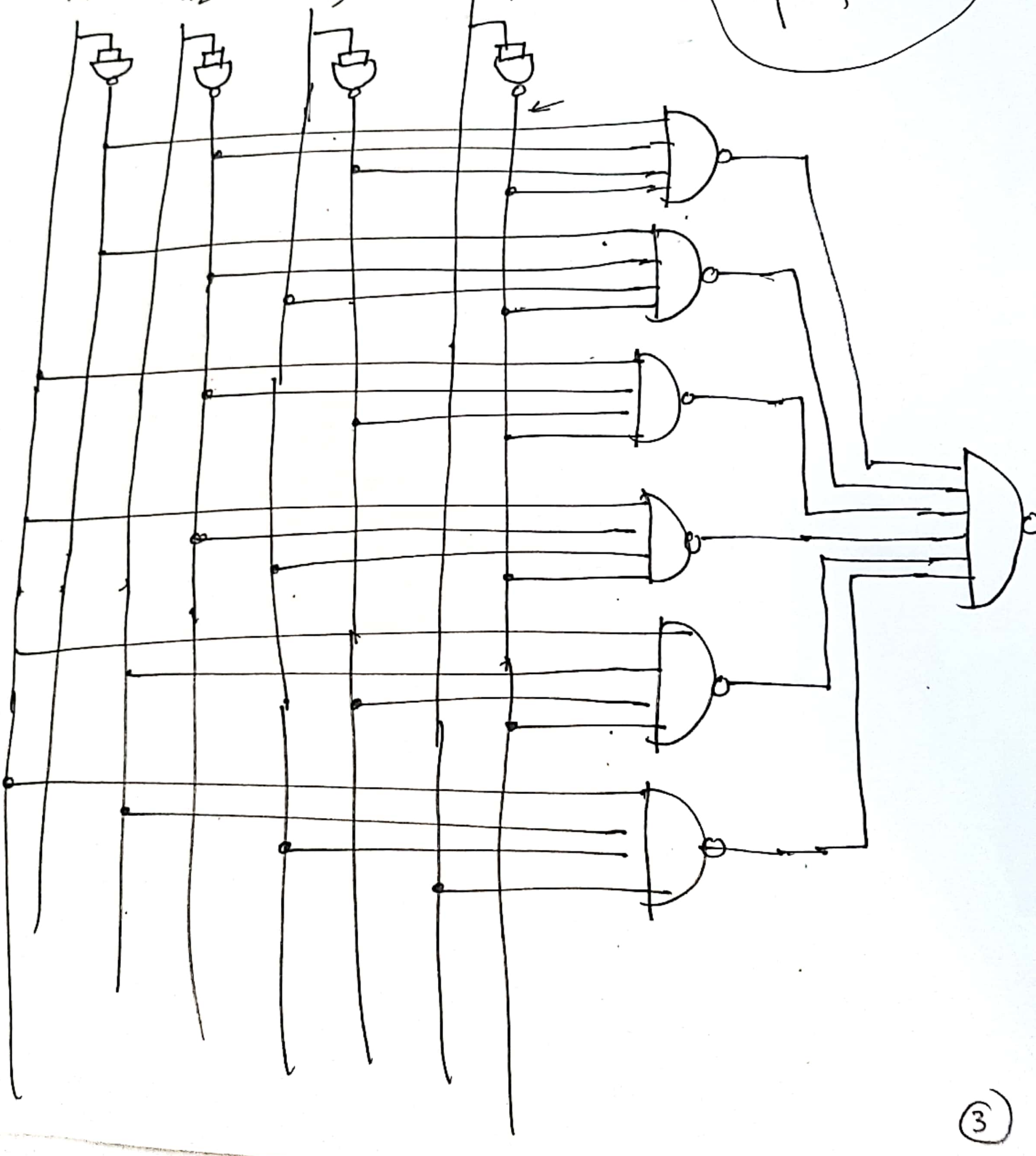
Implement the function :-

$$F = \sum m(0, 2, 8, 10, 13, 15)$$

$$F = \bar{x}_1\bar{x}_2\bar{x}_3\bar{x}_4 + \bar{x}_1\bar{x}_2 x_3\bar{x}_4 + x_1\bar{x}_2\bar{x}_3\bar{x}_4 + x_1\bar{x}_2 x_3\bar{x}_4 + x_1 x_2\bar{x}_3 x_4 + x_1 x_2 x_3 x_4$$

with NAND gates only

x_1	x_2	x_3	x_4
8	4	2	1
0	0	0	0
0	0	1	0
1	0	0	0
1	0	1	0
1	1	0	1



Simplified form

$$F = \bar{x}_2 \bar{x}_3 \bar{x}_4 (\bar{x}_1 + x_1) + \cancel{\bar{x}_2 \bar{x}_3 \bar{x}_4 (\bar{x}_1 + x_1)}$$

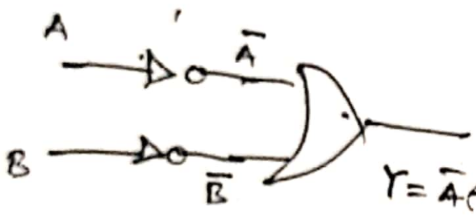
$$\cancel{x_1 x_3 (\bar{x}_2 \bar{x}_4 + x_2 x_4)}$$

$$\bar{x}_2 x_3 \bar{x}_4 (\bar{x}_1 + x_1) + x_1 x_2 x_4 (\bar{x}_3 + x_3)$$

$$= \bar{x}_2 \bar{x}_3 \bar{x}_4 + \cancel{\bar{x}_2 \bar{x}_3 \bar{x}_4} + \bar{x}_2 x_3 \bar{x}_4 + x_1 x_2 x_4$$

$$= \bar{x}_2 \bar{x}_4 (\bar{x}_3 + x_3) + x_1 x_2 x_4 = \bar{x}_2 \bar{x}_4 + x_1 x_2 x_4$$

8	4	2	1
0	0	0	0
0	1	0	0



$$Y = \bar{A} + \bar{B}$$

$$= \overline{A \cdot B}$$

NAND

$$\overline{\bar{x}_2 \bar{x}_4} + x_2 x_4$$

$$\bar{A}\,\bar{B} + A\,B$$

$$Y + \bar{Y} = 1$$

$$Y = A + B$$

$$Y = \bar{A} \cdot \bar{B}$$

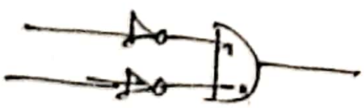
$$\bar{Y} = \overline{\bar{A} \cdot \bar{B}}$$

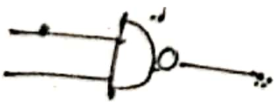
$$\overline{AB} + AB$$

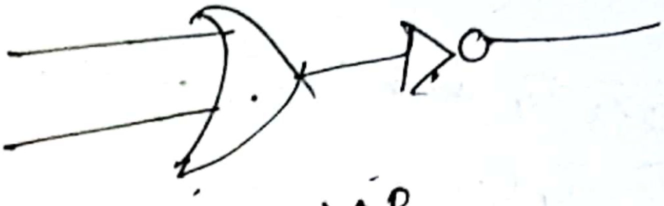
$$\overline{A \cdot B} = \bar{A} + \bar{B}$$

$$\overline{A + B}$$

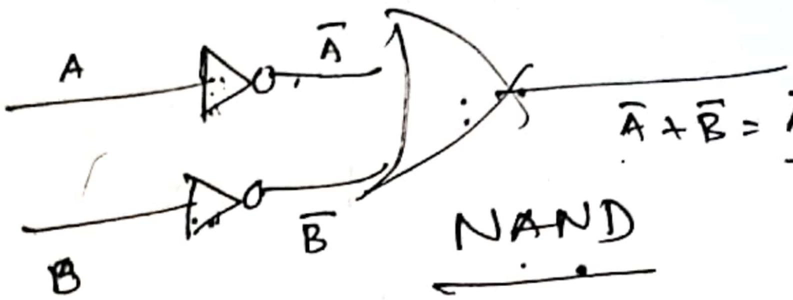
$$\overline{x_2 + x_4} + x_2 x_4$$







NOR

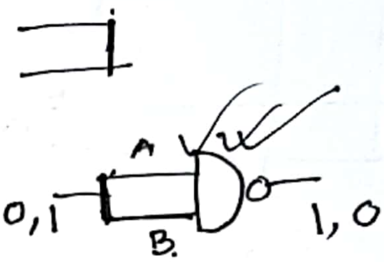


$\bar{A} + \bar{B} = \overline{A \cdot B}$

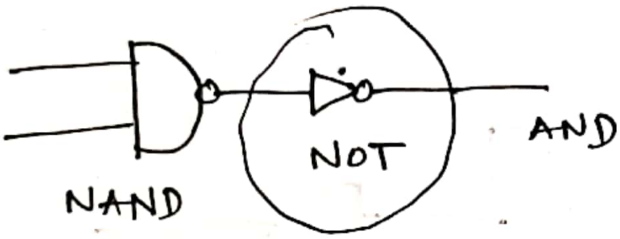
NAND

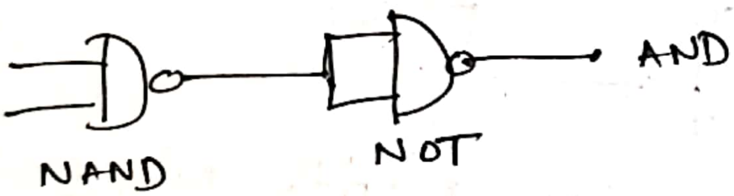
NAND

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0



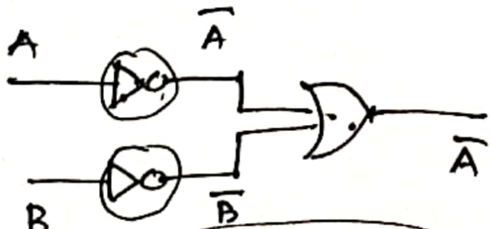
NAND → NOT



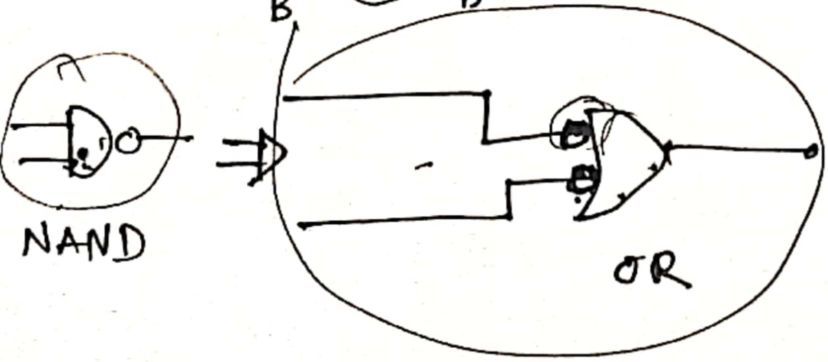


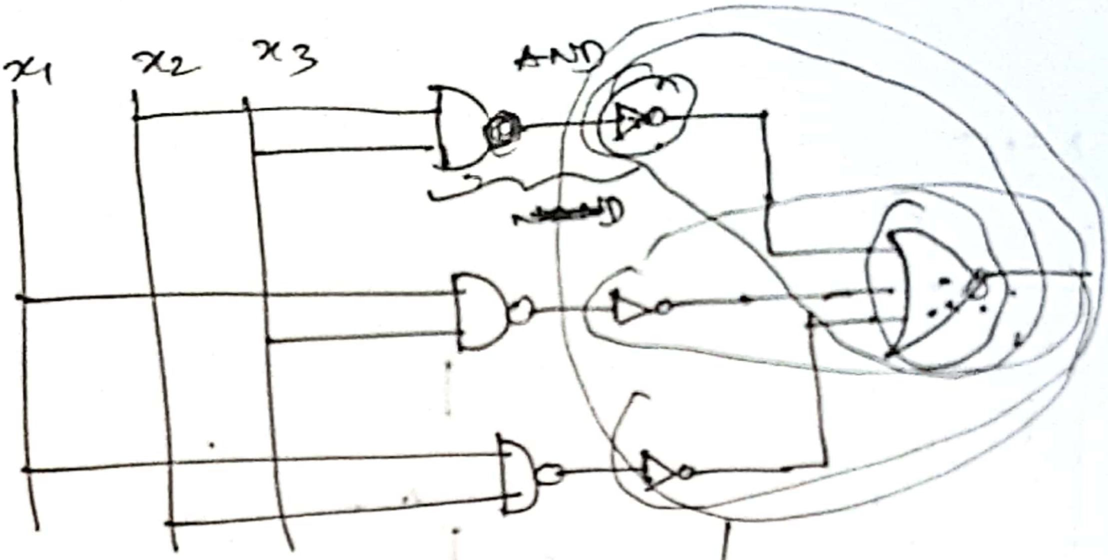
$$Y = \overline{A \cdot B} = \bar{A} + \bar{B}$$

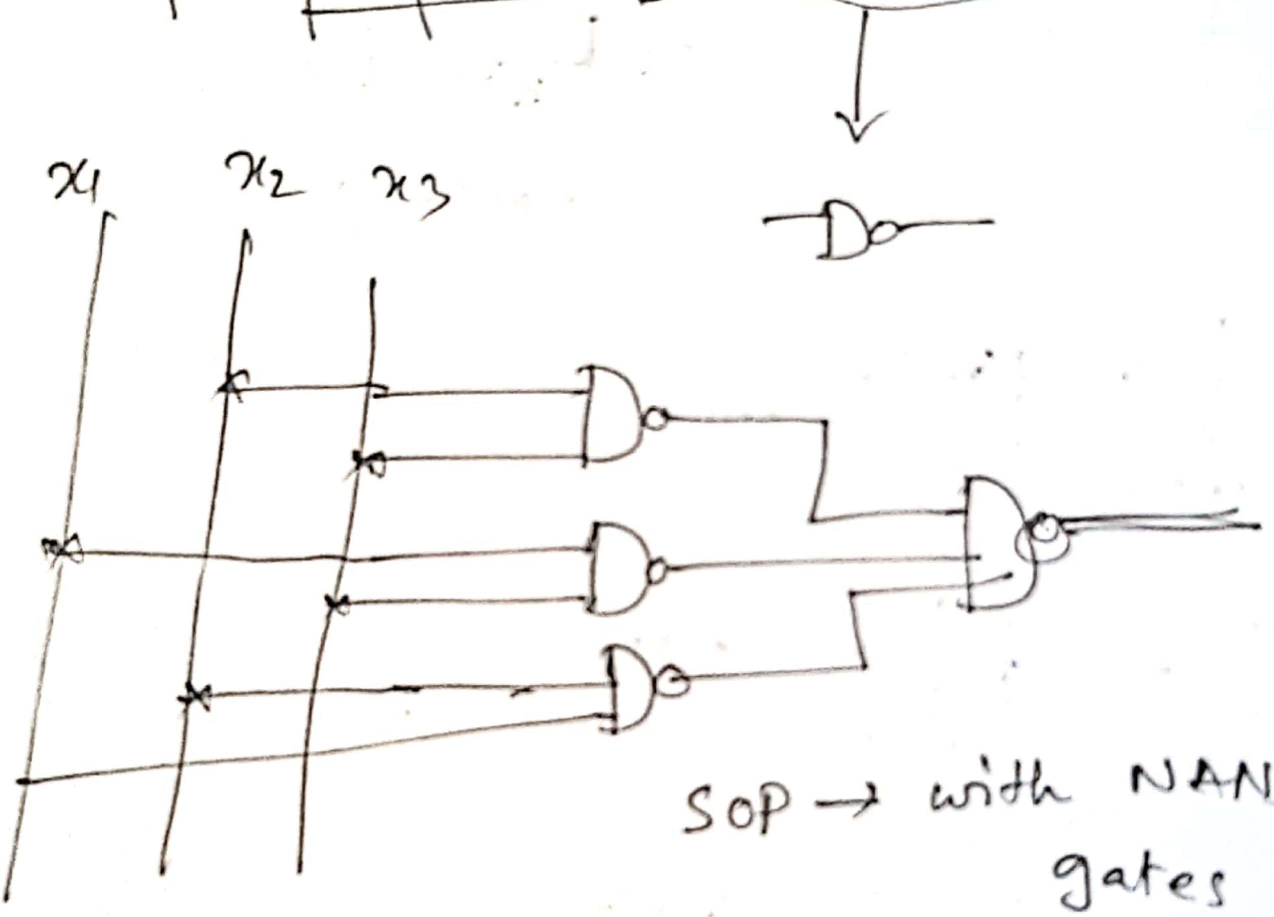
OR



$\bar{A} + \bar{B} = \overline{A \cdot B}$

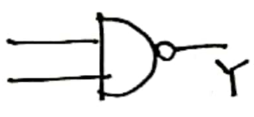


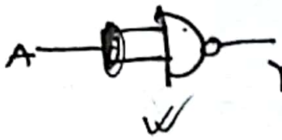




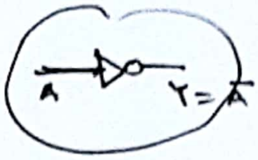
SOP → with NAND gates only.

Using NAND gate to make NOT gate



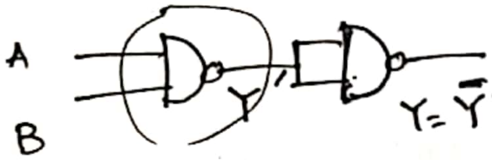


$Y = \overline{A \cdot A} = \overline{A}$



NAND → AND + NOT

~~NOT + AND~~



$Y = \overline{Y'} = \overline{\overline{A \cdot B}} = A \cdot B$

$Y' = \overline{A \cdot B}$

$\overline{\overline{x}}$ $\overline{\overline{0}}$

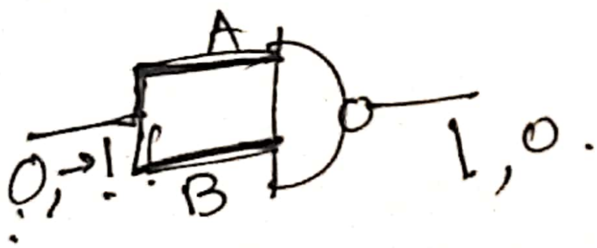
$Y = \overline{\overline{A \cdot B}} = \overline{A \cdot B}$ (NAND)

$Y = A \cdot B$ (AND)

$\overline{\overline{A \cdot B}} = A \cdot B$

NAND + NOT = AND

$\boxed{\text{OR} \longrightarrow \text{NAND}}$

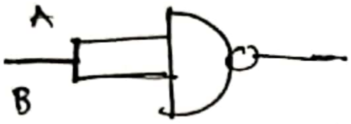


NAND

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

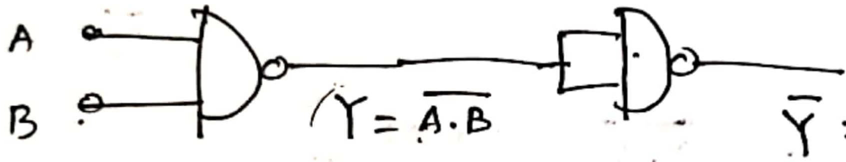
$Y = \overline{A \cdot B}$ $\qquad$ $Y = \overline{A}$

$= \overline{A \cdot A} = \overline{A}$



NOT gate.

NAND + NOT = AND



$Y = \overline{A \cdot B}$ $\qquad$ $\overline{Y} = \overline{\overline{A \cdot B}} = A \cdot B$

AND

~~POS~~

$$\overset{0\,0\,0\,0}{\bar{x}_1\bar{x}_2\bar{x}_3\bar{x}_4} + \overset{0\,0\,1\,0}{\bar{x}_1\bar{x}_2 x_3\bar{x}_4} + \overset{1\,0\,0\,0}{x_1\bar{x}_2\bar{x}_3\bar{x}_4} + \overset{1\,0\,1\,0}{x_1\bar{x}_2 x_3\bar{x}_4}$$

$$+ \underset{1\,1\,0\,1}{x_1x_2\bar{x}_3x_4} + \underset{1\,1\,1\,1}{x_1x_2x_3x_4}$$

$$= \bar{x}_1\bar{x}_2\bar{x}_4(\bar{x}_3+x_3) + x_1\bar{x}_2\bar{x}_4(\bar{x}_3+x_3) \cancel{+\bar{x}_2x_3\bar{x}_4(\bar{x}_1+x_1)}$$

$$+ x_1x_2x_4(\bar{x}_3+x_3)$$

$$= \bar{x}_1\bar{x}_2\bar{x}_4 + x_1\bar{x}_2\bar{x}_4 + x_1x_2x_4\cancel{(\bar{x}_3+x_3)}$$

$$= \bar{x}_2\bar{x}_4 + x_1x_2x_4$$

8	4	2	1
0	0	0	0
0	0	1	0

NOT

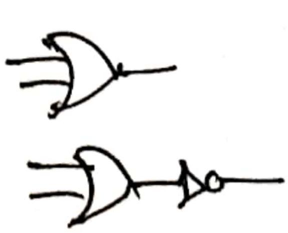
T.T

x_1	x_2	x_3	x_4
0	0	0	0
0	0	0	1
0	0	1	0
0	0	1	1

$x + x + x = x$

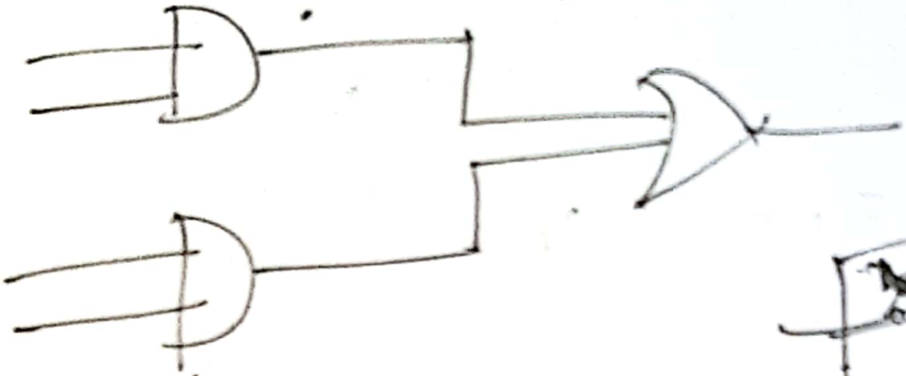
$(x_1 + x_2 + x_3)$

NOR → OR

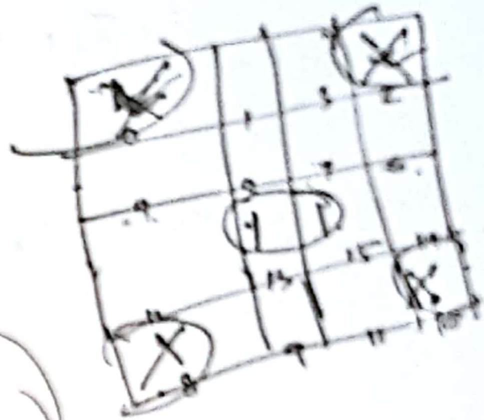


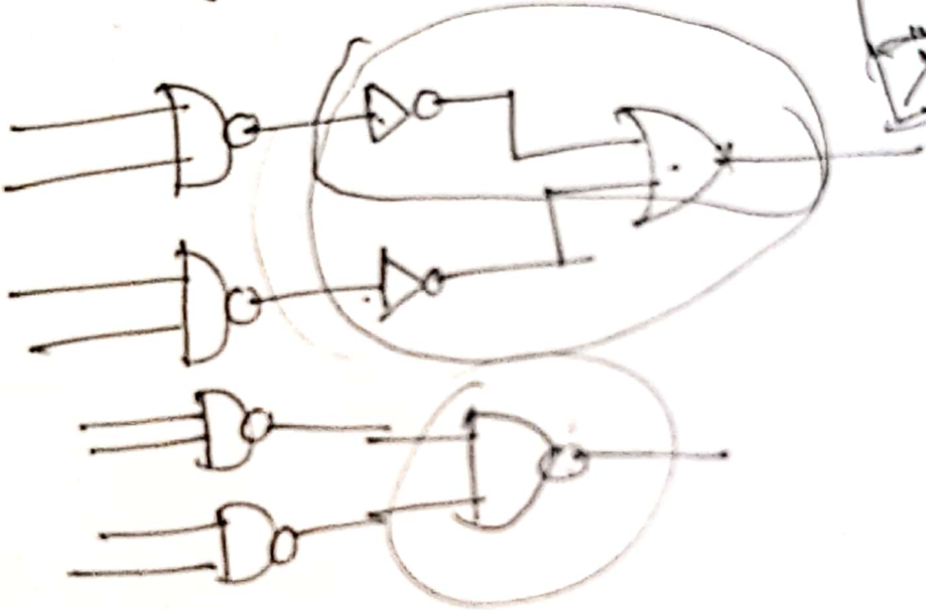
NOR

POS



$\Downarrow$

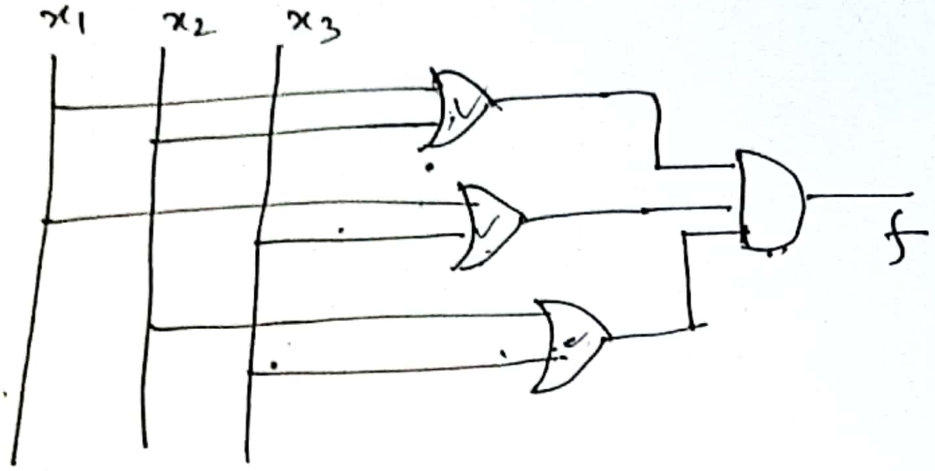




$$\overline{A+B} = \overline{A} \oplus \overline{B}$$

NAND = (NOT A) OR (NOT B)

$f = (x_1 + x_2)(x_1 + x_3)(x_2 + x_3)$



$$\left(\underbrace{x_1 + x_2}_{X} + \underbrace{x_3}_{Y}\right)\left(\underbrace{x_1 + x_2}_{X} + \underbrace{\bar{x}_3}_{\bar{Y}}\right)$$

(SOP)

$$\boxed{(x+y)(x+\bar{y})}$$

$= x.x + x.\bar{y} + x.y + \cancel{y.\bar{y}}$ 0

$= x + x(\bar{y} + y)$

$= x + x = x$

☐ Draw the circuit with NAND gate only

☐ Draw the (POS) circuith with NOR gate only.