Digital Electronics COE328

Lecture 6

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Logic Function Simplification: Karnaugh Map

In many digital circuits and practical problems, we need to find expressions with minimum variables. We can minimize Boolean expressions of 2,3, and 4 variables very easily using K-map without using any Boolean algebra theorems. K-map can take two forms Sum of Product (SOP) and Product of Sum (POS) according to the need of the problem. K-map is a table-like representation, but it gives more information than TRUTH TABLE. We fill the grids of K-map with 0’s and 1’s then solve them by making groups.

Solving Problems using K-map

- **Steps to solve expression using K-map-**

1. Select K-map according to the number of variables. 2, 3, 4

2. Identify minterms or maxterms as given in the problem. $f = m\sum(0, 1, 2, 5, 7)$

3. For SOP put 1's in blocks of K-map respective to the minterms (0's elsewhere).

4. For POS put 0's in blocks of K-map respective to the maxterms (1's elsewhere).

5. Make rectangular groups containing total terms in the power of two like 2,4,8 ..(except 1) and try to cover as many elements as you can in one group.

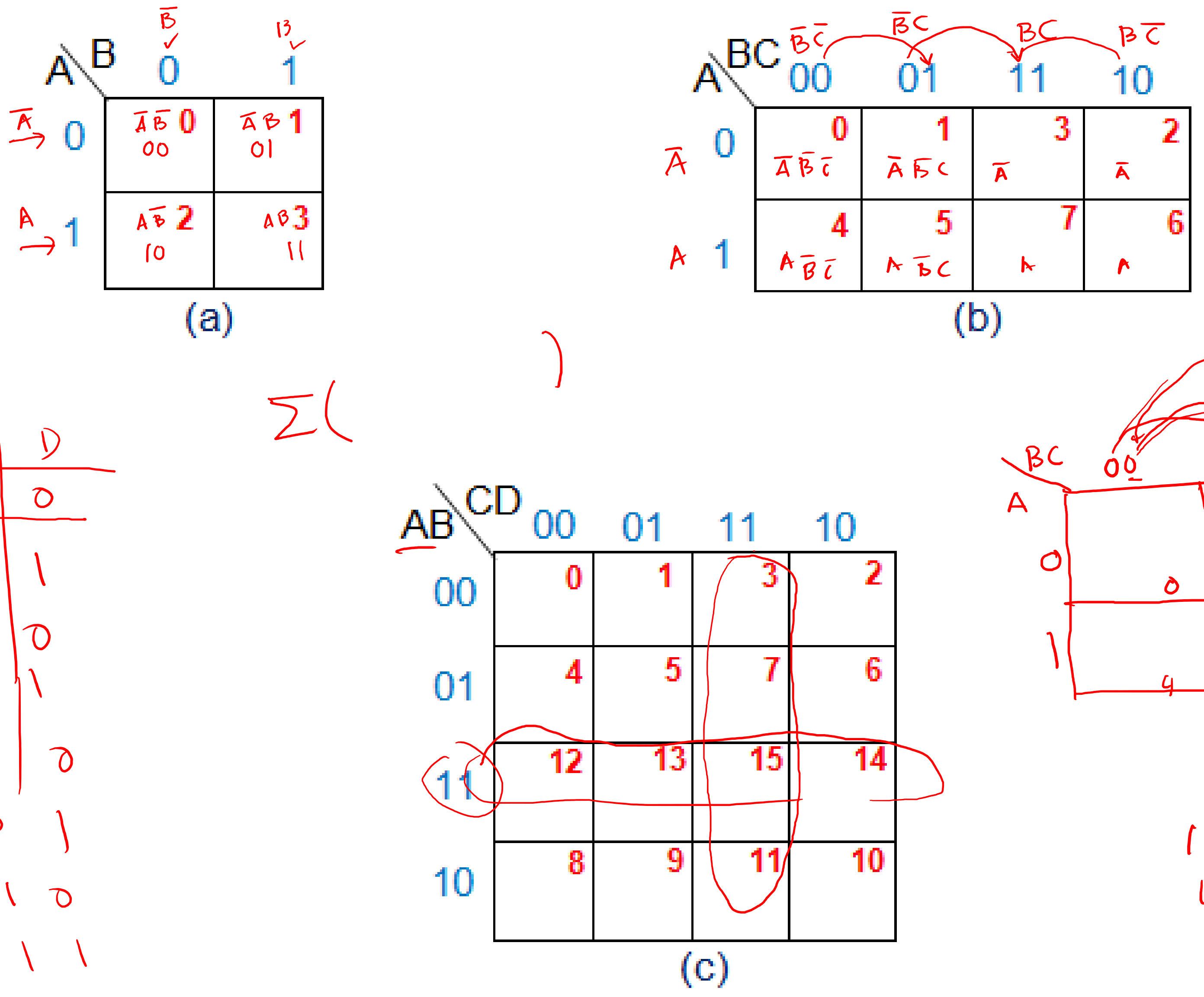
6. From the groups made in step 5 find the product terms and sum them up for SOP form.

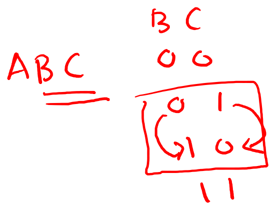
Formation of K-map

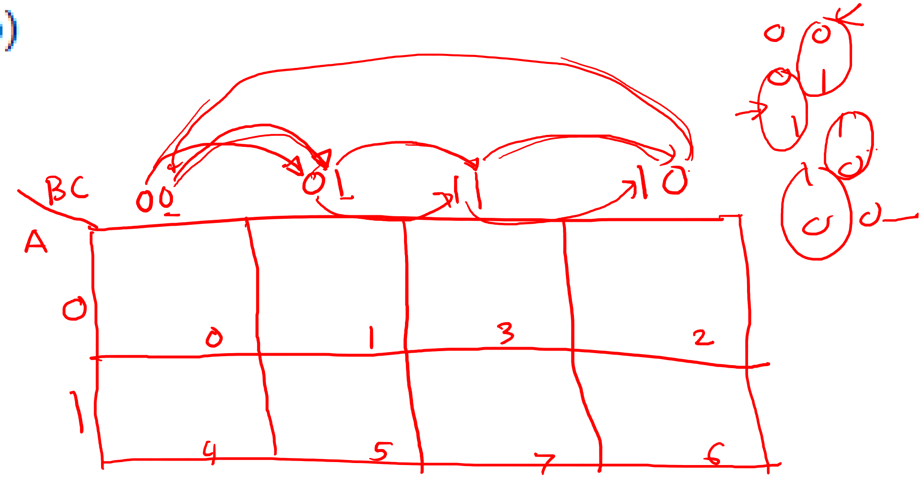
$f = A\bar{B} + AB$

A	B	C	D
0	0	0	0
0	0	0	1
0	0	1	0
0	0	1	1
0	1	0	0
0	1	0	1
0	1	1	0
0	1	1	1

$\Sigma($ $)$







1 0 0 0
1 0 0 1
1 0 1 0
1 0 1 1

1 1 0 0
1 1 0 1
1 1 1 0
1 1 1 1

Figure 1 Karnaugh Maps for (a) Two Variables (b) Three Variables (c) Four Variables

Definitions

- Let us consider, $f = \bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C}$+ $\bar{A}\bar{B}C$+ ABC
- **Literal:** Each appearance of a variable, either uncomplemented or complemented, is called a literal. For example, $\bar{A}\bar{B}\bar{C}$ has three literals, $x_1\bar{x}_2x_3\bar{x}_4$ has four literals.
- **Implicant:** A product term that implies $f = 1$ is called an implicant of that function. The above function has 4 implicants.
- **Group**: Implicants may be combined (Grouped) to form an implicant with fewer literals.
- **Prime implicant:** A prime implicant is an implicant that cannot be combined further to result in fewer literals.
- **Essential Prime Implicant:** If a prime implicant includes at least one minterm for which f =1 that is not included in any other prime implicants, then it is called an essential prime implicant.
- **Cover**: A collection of implicants that accounts for all valuations for which f =1 is called a cover of that function. Most functions have a number of different covers. The objective of minimization is to obtain a minimal cover.

Example 1: Simplify using K-map

- $f = \bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C} + \bar{A}\bar{B}C + AB\bar{C}$

0 0 0   1 . 0 . 0   0 0 1   1 1 0

$1 \cdot 1 \cdot 1 = 1$

$A = 1$ $A \cdot B\bar{C}$

$B = 1$ $1 \cdot 1 \cdot 1 = 1$

$C = 0$ $\bar{C} = 1$

A \ BC	00 ($\bar{B}$)	01	11	10 ($\bar{C}$)
0 ($\bar{A}$)	1	1	0	0
1	1	0	0	1

A \ BC	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
$\bar{A}$	1	1		
A	1			1

$\boxed{f = \bar{A}\bar{B} + \bar{B}\bar{C} + A\bar{C}}$ ← Prime groups

Essential Prime groups

$f = \bar{A}\bar{B} + A\bar{C}$ ← simplified

Example 2: Simplify using K-map

- $F(x_1, x_2, x_3) = \sum(m_o, m_2, m_4, m_6)$

x_1 \ x_2x_3	00	01	11	10
0	1	0	0	1
1	1	0	0	1

$2^3 \quad 2^2 \quad 2^1$

$8, \; 4, \; 2, \; 1$

$F = \overline{x_3}$

Example 3: Simplify using K-map

- $P = \sum(0, 2, 3, 6, 7, 8, 10, 15)$

AB \ CD	00	01	11	10
00	1 (0)	0 (1)	1 (3)	1 (2)
01	0 (4)	0 (5)	1 (7)	1 (6)
11	0 (12)	0 (13)	1 (15)	0 (14)
10	1 (8)	0 (9)	0 (11)	1 (10)

$P = \bar{A}C + \bar{B}\bar{D} + BCD$

4-Variable

2 in group → 3 Variables

4 in a group → 2 Variable

8 in a group → 1 variable

$f(\bar{A}BCD)$

row

$\boxed{x_1 x_2} x_3 x_4$

Example 4: Simplify using K-map

- $f(x_1, x_2, x_3, x_4) = \bar{x}_1 x_2 \bar{x}_4 + \bar{x}_1 x_3 x_4 + \bar{x}_1 x_2 \bar{x}_3 \bar{x}_4 + \bar{x}_1 x_2 \bar{x}_3 x_4 + x_1 x_3 x_4$

$x_2 / \bar{x}_2$

$\bar{x}_3 + x_3$ $\quad x_2 / \bar{x}_2$

x_1x_2 \ x_3x_4	00	01	11	10
00	0	0	1	0
01	1	1	1	1
11	0	0	1	0
10	0	0	1	0

$\boxed{f = \bar{x}_1 x_2 + x_3 x_4}$ 4 gates

1

$\bar{x}_1 x_2 \bar{x}_4 (\bar{x}_3 + x_3)$

$= \bar{x}_1 x_2 \bar{x}_3 x_4 + \bar{x}_1 x_2 x_3 \bar{x}_4$

$f = \bar{x}_1 x_2 + \boxed{x_1 x_2 x_3 x_4}$

Example 5: Simplify using K-map

- $Q = \prod(0, 1, 4, 8, 9, 12, 15)$

POS

Put = 0

AB \ CD	00	01	11	10
00	0	0	1	1
01	0	1	1	1
11	0	1	0	1
10	0	0	1	1

$$\bar{Q} = \bar{C}\bar{D} + \bar{B}\bar{C} + ABCD$$

$$Q = \overline{\bar{C}\bar{D} + \bar{B}\bar{C} + ABCD}$$

$$= (\overline{\bar{C}\bar{D}})(\overline{\bar{B}\bar{C}})(\overline{ABCD})$$

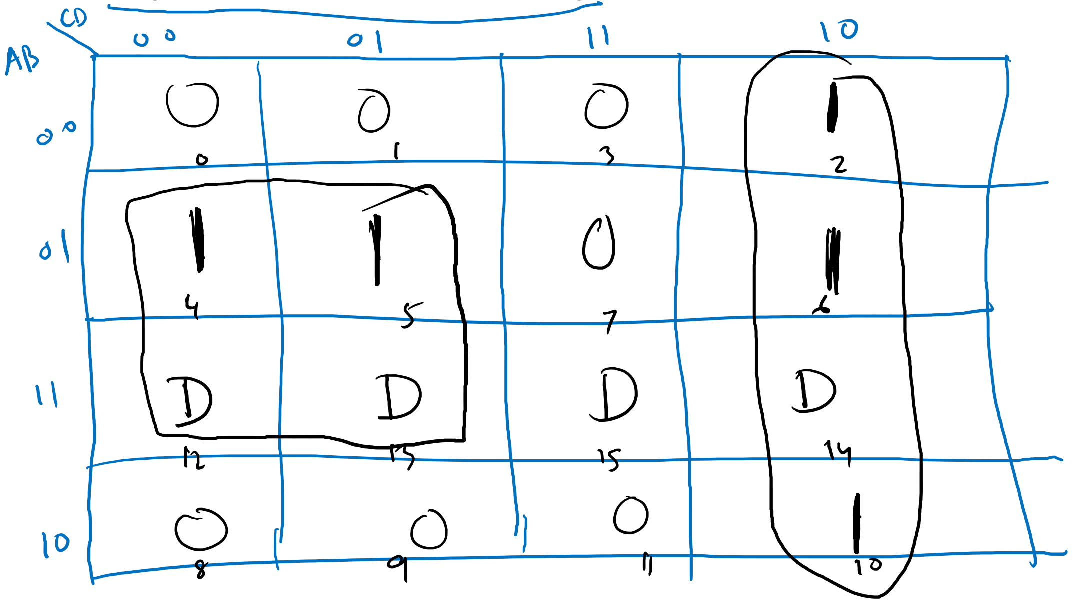
$$= (\bar{\bar{C}} + \bar{\bar{D}})(\bar{\bar{B}} + \bar{\bar{C}})(\bar{A} + \bar{B} + \bar{C} + \bar{D})$$

$$= (C + D)(B + C)(\bar{A} + \bar{B} + \bar{C} + \bar{D})$$

Example 6: Simplify using K-map

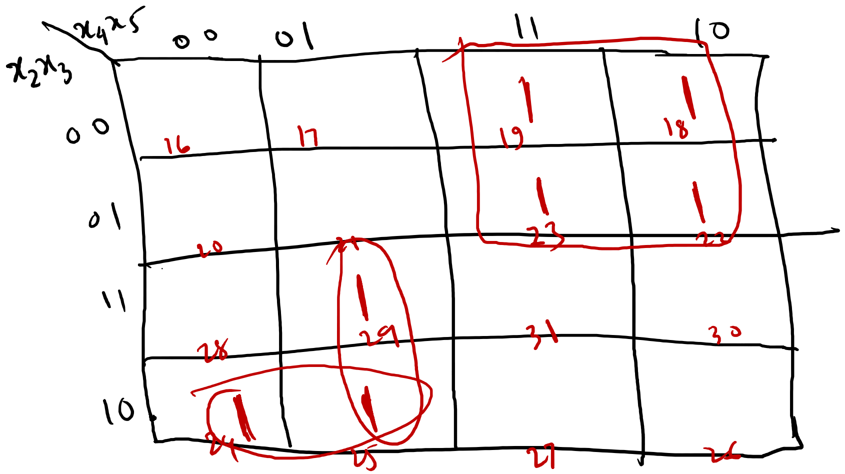
- Optimization of Incompletely Specified Functions
- Gives more flexibility and therefore better optimization
- DO NOT CARE conditions could be 0 or 1
- Example: P=∑ 2, 4, 5, 6, 10 + D(12, 13, 14, 15)

$P = B\bar{C} + C\bar{D}$

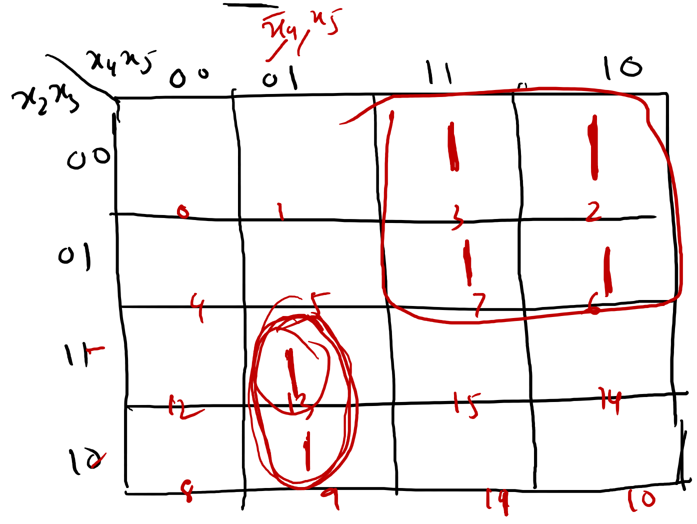


Example 7: Simplify using K-map

- K-Map for 5 input variables
- P=∑ 2, 3, 6, 7, 9, 13, 18, 19, 22, 23, 24, 25, 29



$x_1 = 1$



$x_1 = 0$

$\bar{x}_1(\bar{x}_2 x_4 + x_2 \bar{x}_4 x_5)$

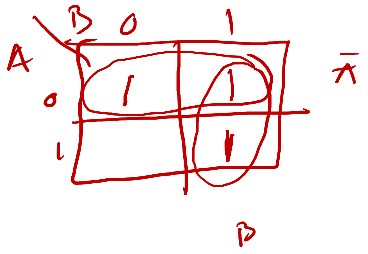
$x_1(\bar{x}_2 x_4 + x_2 \bar{x}_3 \bar{x}_4 + x_2 \bar{x}_4 x_5)$

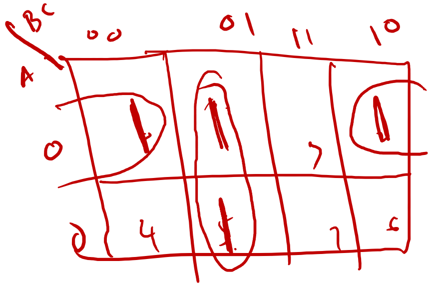
$P = \bar{x}_1 \bar{x}_2 x_4 + \bar{x}_1 x_2 \bar{x}_4 x_5 + x_1 \bar{x}_2 x_4 + x_1 x_2 \bar{x}_3 \bar{x}_4 + x_1 x_2 \bar{x}_4 x_5$

Example 8: Simplify using K-map

- F1= ∑(1, 3, 5, 6, 7, 13, 15)
- F2=∑(2, 3, 5, 6, 7, 10, 11, 13, 14)
- F3= ∏(1, 3, 6, 9, 11, 13, 15)
- Implement F1 and F2 using NAND gates only.
- Implement F3 using NOR gates only.

$\boxed{\bar{A} + AB = \bar{A} + B}$





$f = \bar{B}C + \bar{A}\bar{C}$

$f = \sum(0, 1, 2, 5)$

$f = \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}C$

$= (\bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C}) + (\bar{A}\bar{B}C + A\bar{B}C)$

$= \bar{A}\bar{C}(\bar{B} + B) + \bar{B}C(\bar{A} + A)$

$= \bar{A}\bar{C} + \bar{B}C$