Chapter 4

October 9, 2022 10:05 AM

***4.1** Find the minimum-cost SOP and POS forms for the function $f(x_1, x_2, x_3) = \sum m(1, 2, 3, 5)$.

$x_1 \backslash x_2x_3$	00	01	11	10
0	0	1	1	1
1	0	1	0	0

SOP: $f(x_1, x_2, x_3) = \bar{x}_2 x_3 + \bar{x}_1 x_2$

$x_1 \backslash x_2x_3$	00	01	11	10
0	0	1	1	1
1	0	1	0	0

POS $\bar{f}(x_1, x_2, x_3) = \bar{x}_2\bar{x}_3 + x_1 x_2$

$f = \overline{\bar{x}_2\bar{x}_3 + x_1x_2} = (x_2 + x_3)(\bar{x}_1 + \bar{x}_2)$

***4.2** Repeat problem 4.1 for the function $f(x_1, x_2, x_3) = \sum m(1, 4, 7) + D(2, 5)$.

$x_1 \backslash x_2x_3$	00	01	11	10
0	0	1	0	D
1	1	D	1	0

$f = \bar{x}_2 x_3 + x_1 \bar{x}_2 + x_1 x_3$

$x_1 \backslash x_2x_3$	00	01	11	10
0	0	1	0	D
1	1	D	1	0

$\bar{f} = \bar{x}_1 x_2 + x_2 \bar{x}_3 + \bar{x}_1 \bar{x}_3$

$f = \overline{\bar{x}_1 x_2} \cdot \overline{x_2 \bar{x}_3} \cdot \overline{\bar{x}_1 \bar{x}_3}$

$= (x_1 + \bar{x}_2)(\bar{x}_2 + x_3)(x_1 + x_3)$

4.3 Repeat problem 4.1 for the function $f(x_1, \ldots, x_4) = \Pi M(0, 1, 2, 4, 5, 7, 8, 9, 10, 12, 14, 15)$.

$x_1x_2 \backslash x_3x_4$	00	01	11	10
00	0	0	1	0
01	0	0	0	1
11	0	1	0	0
10	0	0	1	0

$\bar{f} = \bar{x}_2\bar{x}_4 + \bar{x}_1\bar{x}_3 + x_1\bar{x}_4 + \bar{x}_2\bar{x}_3 + x_2x_3x_4$

$f = \overline{\bar{x}_2\bar{x}_4 + \bar{x}_1\bar{x}_3 + x_1\bar{x}_4 + \bar{x}_2\bar{x}_3 + x_2x_3x_4}$

$= (x_2 + x_4)(x_1 + x_3)(\bar{x}_1 + x_4)(x_2 + x_3)(\bar{x}_2 + \bar{x}_3 + \bar{x}_4)$

4.4 Repeat problem 4.1 for the function $f(x_1, \ldots, x_4) = \sum m(0, 2, 8, 9, 10, 15) + D(1, 3, 6, 7)$.

$x_1x_2 \backslash x_3x_4$	00	01	11	10
00	1	D	D	1
01	0	0	D	D
11	0	0	1	0
10	1	1	0	1

$f = \bar{x}_2\bar{x}_4 + \bar{x}_2\bar{x}_3 + x_2x_3x_4 \leftarrow$ SOP

$x_1x_2 \backslash x_3x_4$	00	01	11	10
00	1	D	D	1
01	0	0	D	D
11	0	0	1	0
10	1	1	0	1

$\bar{f} = x_2\bar{x}_3 + x_2\bar{x}_4 + \bar{x}_2x_3x_4$

$f = \overline{x_2\bar{x}_3 + x_2\bar{x}_4 + \bar{x}_2x_3x_4}$

$= (\bar{x}_2 + x_3)(\bar{x}_2 + x_4)(x_2 + \bar{x}_3 + \bar{x}_4) \leftarrow$ POS

***4.9** A four-variable logic function that is equal to 1 if any three or all four of its variables are equal to 1 is called a *majority* function. Design a minimum-cost SOP circuit that implements this majority function.

A	B	C	D	F
0	0	0	0	0
0	0	0	1	0

$F = \sum m(7, 11, 13, 14, 15)$

AB \ CD: 00 01 11 10

A	B	C	D	F
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

$F = \Sigma m(7, 11, 13, 14, 15)$

AB \ CD	00	01	11	10
00	0	0	0	0
01	0	0	1	0
11	0	1	1	1
10	0	0	1	0

$F = ABD + ABC + BCD + ACD$

4.10 Derive a minimum-cost realization of the four-variable function that is equal to 1 if exactly two or exactly three of its variables are equal to 1; otherwise it is equal to 0.

x_1	x_2	x_3	x_4	f
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	0
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	0

x_1x_2 \ x_3x_4	00	01	11	10
00	0	0	1	0
01	0	1	1	1
11	1	1	0	1
10	0	1	1	1

$f = x_1x_2\bar{x}_3 + x_1\bar{x}_2x_4 + x_1x_3\bar{x}_4 + \bar{x}_1x_2x_3 + \bar{x}_1x_3x_4 + x_2\bar{x}_3x_4$

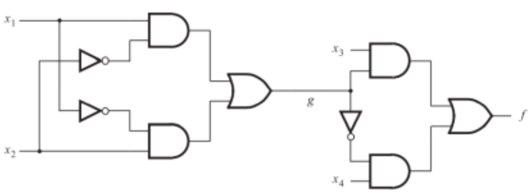
x_1x_2 \ x_3x_4	00	01	11	10
00	0	0	1	0
01	0	1	1	1
11	1	1	0	1
10	0	1	1	1

$\bar{f} = \bar{x}_1\bar{x}_2\bar{x}_3 + \bar{x}_1\bar{x}_3\bar{x}_4 + \bar{x}_2\bar{x}_3\bar{x}_4 + \bar{x}_1\bar{x}_2\bar{x}_4 + x_1x_2x_3x_4$

$f = (x_1+x_2+x_3)(x_1+x_3+x_4)(x_2+x_3+x_4)(x_1+x_2+x_4)(\bar{x}_1+\bar{x}_2+\bar{x}_3+\bar{x}_4)$

POS is the cost-minimum function.

***4.14** Implement the logic circuit in Figure 4.23 using NAND gates only.



$g = x_1\bar{x}_2 + \bar{x}_1x_2$

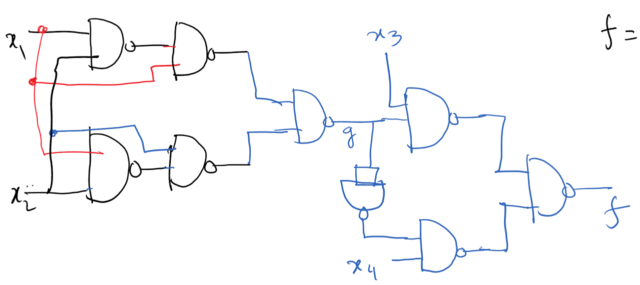
$= \overline{\overline{x_1\bar{x}_2 + \bar{x}_1x_2}} = \overline{\overline{x_1\bar{x}_2}\cdot\overline{\bar{x}_1x_2}}$

$= \overline{\overline{x_1(\bar{x}_1+\bar{x}_2)}\cdot\overline{(\bar{x}_1+\bar{x}_2)x_2}} = (x_1 \uparrow (\bar{x}_1+\bar{x}_2)) \uparrow (x_2 \uparrow (\bar{x}_1+\bar{x}_2))$

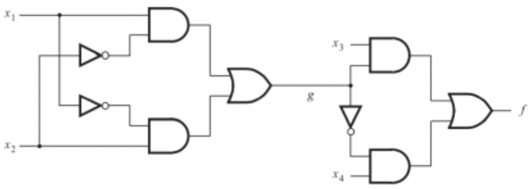
$= (x_1 \uparrow (x_1 \uparrow x_2)) \uparrow ((x_1 \uparrow x_2) \uparrow x_2)$

$\uparrow$ is a NAND operator.

$f = x_3g + \bar{g}x_4 = \overline{\overline{x_3g + \bar{g}x_4}} = \overline{\overline{x_3g}\cdot\overline{\bar{g}x_4}} = (x_3 \uparrow g) \uparrow ((g \uparrow g) \uparrow x_4)$



***4.15** Implement the logic circuit in Figure 4.23 using NOR gates only.



$$g = x_1\bar{x}_2 + \bar{x}_1 x_2$$
$$= \bar{\bar{x}}_1 \bar{x}_2 + \bar{x}_1 \bar{\bar{x}}_2$$
$$= \overline{\bar{x}_1 + x_2} + \overline{x_1 + \bar{x}_2}$$
$$= (\bar{x}_1 \downarrow x_2) + (x_1 \downarrow \bar{x}_2)$$
$$= (x_1 \downarrow x_1) \downarrow x_2 + x_1 \downarrow (x_2 \downarrow x_2)$$
$$\bar{g} = ((x_1 \downarrow x_1) \downarrow x_2) \downarrow (x_1 \downarrow (x_2 \downarrow x_2))$$

$$f = x_3 g + \bar{g} x_4$$
$$\bar{f} = \overline{x_3 g + \bar{g} x_4} = (\bar{x}_3 \downarrow \bar{g}) \downarrow (g \downarrow \bar{x}_4)$$
$$= ((x_3 \downarrow x_3) \downarrow (g \downarrow g)) \downarrow (g \downarrow (x_4 \downarrow x_4))$$

Finally, $\bar{f} = f \downarrow f$

***4.21** Find the minimum-cost circuit for the function $f(x_1, \ldots, x_4) = \sum m(0, 4, 8, 13, 14, 15)$. Assume that the input variables are available in uncomplemented form only. (Hint: use functional decomposition.)

$x_1x_2 \backslash x_3x_4$	00	01	11	10
00	1			
01	1			
11		1	1	1
10	1			

$$f = \bar{x}_1\bar{x}_3\bar{x}_4 + \bar{x}_2\bar{x}_3\bar{x}_4 + x_1x_2x_3 + x_1x_2x_4$$
$$= (\bar{x}_1 + \bar{x}_2)\bar{x}_3\bar{x}_4 + x_1x_2(x_3 + x_4)$$
$$= \overline{x_1x_2} \cdot (\overline{x_3 + x_4}) + x_1x_2(x_3 + x_4)$$
$$= \bar{g}\bar{h} + gh \qquad g = x_1x_2 \qquad h = x_3 + x_4$$
$$= \overline{g \oplus h}$$