

ELE 302

Laboratory #2

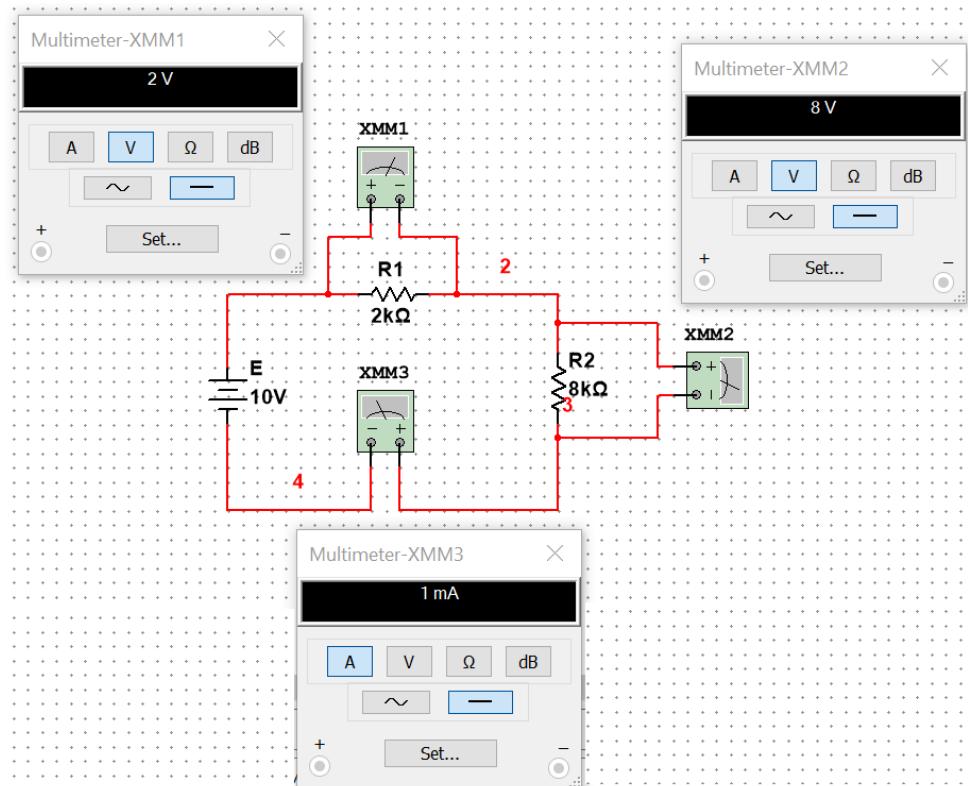
The Step Response of Second-Order Circuits

1.0 INTRODUCTION:

Circuits containing energy-storage elements (capacitors and/or inductors) are known as dynamic circuits. When switching occurs in a dynamic circuit, the circuit response will go through a transition period prior to settling down to a steady-state value. In applications such as data-acquisition, instrumentation, and computer-control systems, the settling time is an important parameter, as circuits must be allowed to settle to steady state before readings are taken.

Dynamic circuits are often characterized by applying a step-function input. The resulting step-response provides important insights into the response of dynamic circuits in general. By investigating the step response, we discover that it consists of a dc-component called the forced response, and a rapidly vanishing time-varying component, called the natural-response. The form of the time function for the natural component depends on the order and composition of the circuit. The natural response of a second-order circuit is one-out-of-three possible functions known as over damped, critically damped, and under damped, with the under damped case being an exponentially-decaying sinusoid.

This experiment examines the step response of various second-order dynamic circuits. We begin by providing a brief review of the Multisim circuit simulation software and sinusoidal functions.



(i)

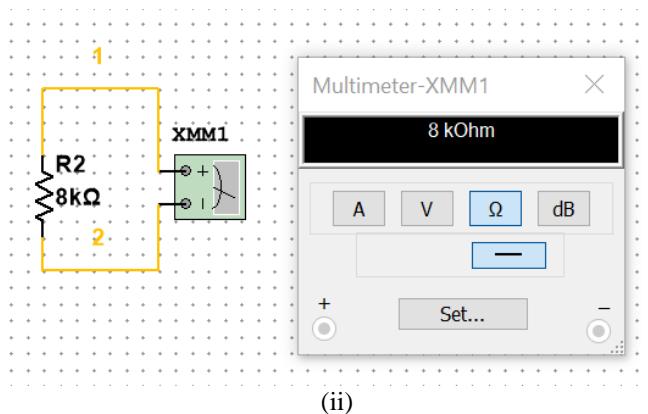


Figure 1.0a: Examples of using MultiSIM

1.1 Multisim

MultiSIM is an electronic schematic capture and simulation program used to analyze circuit behavior. All AC/DC voltages, AC/DC currents, resistance, frequency, phase-shift, time-domain waveform, etc. can be determined using this software. An example circuit simulation measurement is shown below in **Figure 1.0a**. In this simulation, all components are visually laid out in a way that is the same as a circuit diagram. Each DMM configuration is connected the same way that a physical DMM would be connected on the breadboard. Results are obtained by running the simulation and then double clicking on each piece of equipment to read the desired output values. Refer to the MultiSIM software download procedures, related FAQs and video tutorials on the course website (D2L) to get acquainted with proper use of this simulation tool, and become proficient at it.

1.2 Sinusoidal Functions

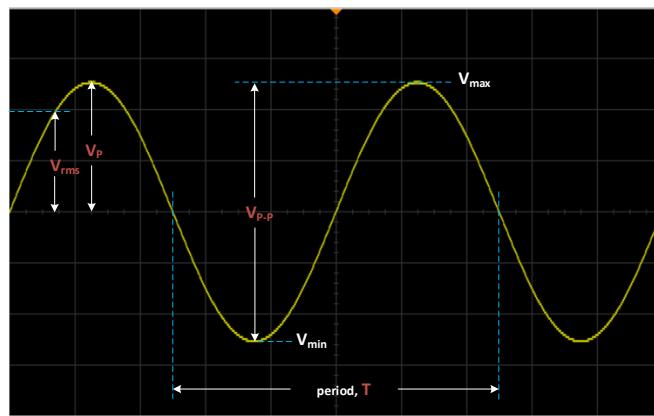


Figure 1.0b: Characterization of a Time-Varying Sinusoidal Signal

A *sine-wave* is shown in **Figure 1.0b**. The sinusoidal signal periodically varies with time. It can be characterized by a number of parameters, some of which are shown in **Figure 1.0b**:

- o Peak-To-Peak Voltage: $V_{P-P} = V_{\max} - V_{\min}$
- o Amplitude of the sinusoidal signal is defined as its Peak Voltage: $V_p = \frac{V_{P-P}}{2}$
- o Root-Mean-Squared Voltage: $V_{rms} = \frac{V_p}{\sqrt{2}} = 0.707V_p$
- o The period, T is defined as the time within which the signal repeats.

- The frequency, f is equal to the number of repetitions per unit of time, and can be calculated from the period, T : $f(\text{Hz}) = \frac{1}{T(\text{sec})}$ or $T(\text{sec}) = \frac{1}{f(\text{Hz})}$.
When frequency is given in radians/sec then the symbol, ω is used, where $\omega = 2\pi f$
- Phase Shift, Θ of one sinusoidal signal with respect to another (of the same frequency) occurs when there is time-offset, ΔT between them. Phase-Shift $\Rightarrow \Theta = \frac{\Delta T}{T} \cdot 2\pi$ (radians) $= \frac{\Delta T}{T} \cdot 360^\circ$ (degrees)
- Sinusoidal AC voltage as a function of time: $v(t) = V_p \cos(\omega t + \Theta) = V_p \cos(2\pi ft + \Theta)$

2.0 OBJECTIVES:

- To use Multisim circuit simulation software to plot the step response of second-order dynamic circuits.
- To use the oscilloscope to display the step response of second-order dynamic circuits.
- To measure the parameters that characterize the step response of second-order dynamic circuits.

3.0 REQUIRED LAB EQUIPMENT & PARTS:

- Function Generator (FG) and Oscilloscope.
- ELE202 Lab Kit and ELE302 Lab Kit: various components, breadboard, wires and jumpers.

4.0 PRE-LAB ASSIGNMENT (3 marks with 0.75 marks for each step):

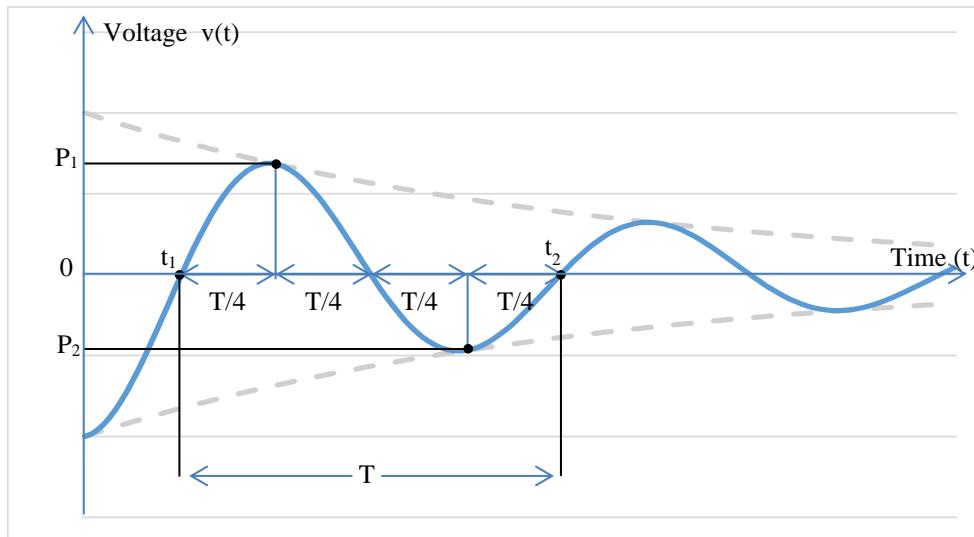


Figure 2.0: An Exponentially Decaying Sinusoidal Function

- (a)** Step 1: The circuit shown in **Figure 2.0** is an exponentially decaying sinusoidal function that can be characterized by the function: $v(t) = Ae^{-\sigma t} \cos(\omega t + \Theta)$. The figure defines three variables T , P_1 , and P_2 . t_1 is the time of the first zero-crossing of $v(t)$ and t_2 is time for the third zero-crossing of $v(t)$. T is the time difference between t_1 and t_2 . P_1 is the value of $v(t)$ one quarter of the way between the first zero-crossing and the third zero-crossing (i.e. $P_1 = v(t_1 + \frac{T}{4})$). Finally, P_2 is the value of $v(t)$ three quarters of the way between the first zero-crossing and the third zero-crossing (i.e. $P_2 = v(t_1 + 3\frac{T}{4})$).

- i) Find an expression for ω in terms of T.

Pre-Lab workspace (show your analysis here)

$$\omega = 2\pi f$$

$$\omega = \frac{2\pi}{T}$$

- ii) Find an expression for σ in terms of P_1 , P_2 , and T.

Pre-Lab workspace (show your analysis here)

$$P_1 = v(t_1 + \frac{T}{4})$$

$$P_2 = v(t_1 + 3\frac{T}{4})$$

$$\begin{aligned} P_1 &= A e^{-\sigma(t_1 + \frac{T}{4})} \cos(\omega(t_1 + \frac{T}{4}) + \phi) \\ &= A e^{-\sigma t_1 - \frac{6\pi}{4}} \cdot \cos(\omega t_1 + \omega \frac{T}{4} + \phi) \end{aligned}$$

$$P_1 = -e^{-\sigma t_1} \cdot e^{-\frac{6\pi}{4}} \sin(\omega t_1 + \phi)$$

$$P_2 = A e^{-\sigma(t_1 + 3\frac{\pi}{4})} \cos(\omega(t_1 + 3\frac{\pi}{4}) + \phi)$$

$$P_2 = A e^{-\sigma t_1 - \frac{6\pi}{4}} \sin(\omega t_1 + \phi)$$

$$\frac{P_1}{P_2} = \frac{-e^{-\sigma t_1} \cdot e^{-\frac{6\pi}{4}}}{e^{-\sigma t_1 - \frac{6\pi}{4}}} = -e^{\frac{\sigma T}{2}} \left| \frac{P_1}{P_2} \right| = e^{\frac{\sigma T}{2}}$$

- iii) Find an expression of θ in terms of t_1 and T.

Pre-Lab workspace (show your analysis here)

$$A e^{-\sigma t_1} \cos(\omega t_1 + \phi) = 0$$

$$\cos(\omega t_1 + \phi) = 0$$

$$\omega t_1 + \phi = \pm \frac{\pi}{2} (2n+1) \quad n=0,$$

$$\phi = -\frac{\pi}{2} - \omega t_1$$

$$\phi = -\frac{\pi}{2} - \frac{2\pi}{T} t_1$$

$$\sigma = \frac{2}{T} \ln \left| \frac{P_2}{P_1} \right|$$

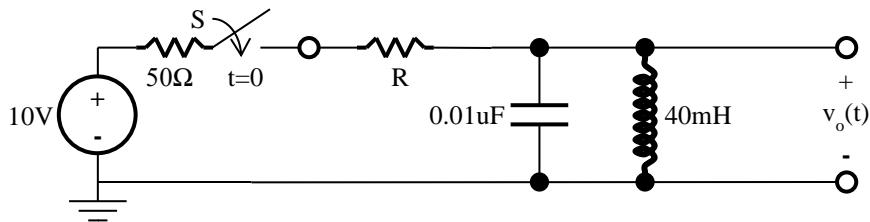


Figure 2.0: Second-Order Bandpass Circuit

(b) Step 2: Consider the dynamic circuit shown in **Figure 2.0**. The switch S has been open for a long time.

At $t=0$, the switch is closed, where it remains for a long time.

- Use Multisim to plot the step response [$v_o(t)$ for $t \geq 0^+$] when $R=1.4\text{k}\Omega$, $3.4\text{k}\Omega$, and 500Ω .

Pre-Lab workspace (show your analysis here)

- ii) Use the plots to calculate the parameters (σ and ω) that characterize the step response as: $v_o(t) = Ae^{-\sigma t} \sin(\omega t)$ for $R=3.4k\Omega$.

Pre-Lab workspace (show your analysis here)

$$T = 1.894 \text{ ms}$$

$$P_1 = 3.831 \text{ V}$$

$$P_2 = -1.53 \text{ V}$$

$$\omega = \frac{2\pi}{T}$$

$$= \frac{2\pi}{138.94 \text{ ms}}$$

$$\omega = 45222.3 \text{ rad/s}$$

$$\sigma = \frac{-2}{138.94 \text{ ms}} \ln \left| \frac{3.831}{-1.53} \right|$$

$$= 13212.3$$

$$v_o(t) = A e^{-13212.3t} \sin(45222.3t)$$

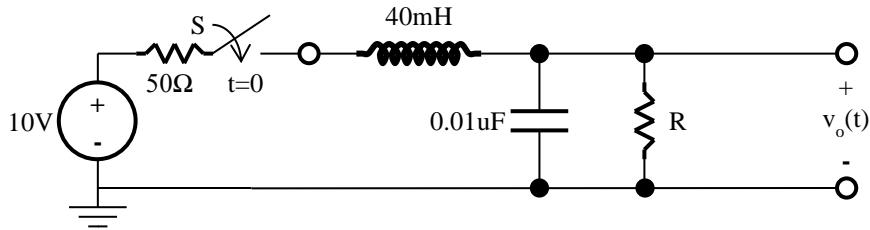


Figure 3.0: Second-Order Lowpass Circuit

(c) Step 3: Consider the dynamic circuit shown in **Figure 3.0**. The switch S has been open for a long time.

At $t=0$, the switch is closed, where it remains for a long time.

- Use Multisim to plot the step response [$v_o(t)$ for $t \geq 0^+$] when $R=1\text{k}\Omega$, $1.4\text{k}\Omega$, and $10\text{k}\Omega$.

Pre-Lab workspace (show your analysis here)

- ii) Use the plots to calculate the parameters (σ and ω) that characterize the step response as: $v_o(t) = B + Ae^{-\sigma t} \cos(\omega t + \theta)$ for $R=10k\Omega$.

Pre-Lab workspace (show your analysis here)

$$T = 169.61 \text{ us} \quad P_1 = 16.708 \text{ V} \quad P_2 = 5.207 \text{ V}$$

$$t_{-1} = 0$$

$$\omega = \frac{2\pi}{T}$$

$$\omega = 37044.9$$

$$\sigma = \frac{2}{T} \ln \left| \frac{P_1}{P_2} \right| \\ = 13747.8$$

$$\alpha = -\frac{\pi}{2} - \frac{2\pi}{T} t_1$$

$$\beta = -\frac{\pi}{2}$$

$$V = B + A e^{-13747.8 t} \cos(37044.9 - \frac{\pi}{2})$$

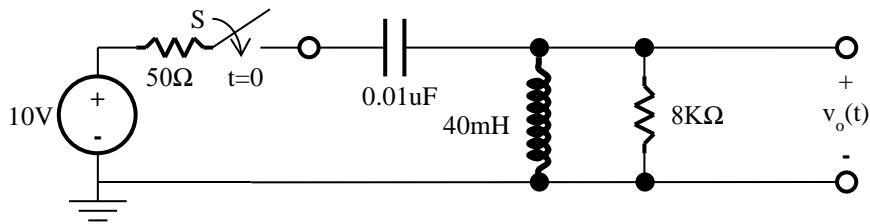


Figure 4.0: Second-Order Highpass Circuit

(d) Step 4: Consider the dynamic circuit shown in **Figure 4.0**. The switch S has been open for a long time.

At $t=0$, the switch is closed, where it remains for a long time.

- Use Multisim to plot the step response [$v_o(t)$ for $t \geq 0^+$].

Pre-Lab workspace (show your analysis here)

- ii) Use the plot to calculate the parameters (σ and ω) that characterize the step response as: $v_0(t) = Ae^{-\sigma t} \cos(\omega t + \theta)$.

Pre-Lab workspace (show your analysis here)

$$T = 100.372 \text{ us}$$

$$p_1 = 8.714 \text{ V}$$

$$p_2 = -6.411 \text{ V}$$

$$t_1 = 0$$

$$\omega = \frac{2\pi}{T}$$

$$= 62598.98$$

$$\sigma = \frac{-p_1}{2} - \frac{\omega^2}{4} t_1$$

$$\sigma = \frac{-8.714}{2 \cdot 100.372 \text{ us}} \cdot \ln \left| \frac{8.714}{-6.411} \right|$$

$$\sigma = 6115.56$$

$$v = A e^{-6115.56t} \cos(62598.98t - \frac{\pi}{2})$$

5.0 IN-LAB IMPLEMENTATION & MEASUREMENTS (5 marks in total):

Part I: The Step Response of a Second-Order Bandpass Circuit

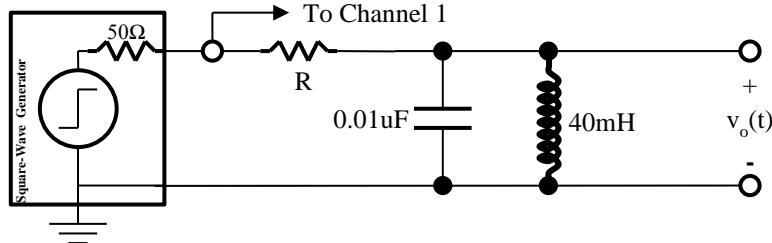


Figure 5.0: Second-Order Bandpass Filter

- (a)** Step 1: Connect Channel (1) of the oscilloscope to display the open-circuit voltage $v_s(t)$ of the function generator. Set the following:

- Trigger: source → Channel (1), and slope → rising.

Adjust the controls of the function generator to provide a square-wave signal $v_s(t)$ with a peak-to-peak value of 10V and DC offset of 5V at a frequency of 20Hz.

Next, set the following:

- Channel (2): Vertical-position → one division above the bottom of the screen, coupling → dc, and V/div → 1V.
- Time: Time/div → 50μs when $v_s(t)$ is rising.
- Blank off Channel (1).

- (b)** Step 2: Construct the circuit shown in **Figure 5.0**. Set $R=3.4k\Omega$. Connect Channel (2) to display the step response $v_o(t)$. Plot $v_o(t)$ on **Graph 1.0**.

- (c)** Step 3: Use the cursors on the oscilloscope to measure points on the $v_o(t)$ display to calculate the parameters (σ and ω) that characterize the step response as:

$$v_o(t) = A e^{-\sigma t} \sin(\omega t)$$

- (d)** Step 4: Record your results in **Table 1.0**.

- (e)** Step 5: Demonstrate the correct operation of your setup to your TA. (1 mark)

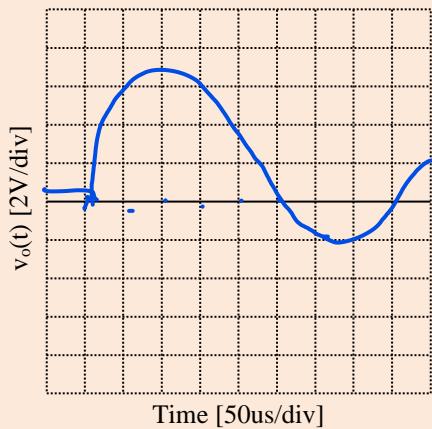
- (f)** Step 6: Set $R=1.4k\Omega$, and repeat as in Step 2 and Step 3. Record your results in **Table 1.0**.

- (g)** Step 7: Set $R=500\Omega$, and repeat as in Step 2.

$$V_{O_1} = V_o \left(\frac{\pi}{4} \right)$$

$$V_{O_2} = V_o \left(\frac{3\pi}{4} \right)$$

T=0 & rts # 3 - across # 1



Graph 1.0 (0.5 marks)

Table 1.0 (0.5 marks)

R ($k\Omega$)	T	$v_o(T/4)$	$v_o(3T/4)$	σ	ω
3.4	$120 \mu s$	4 V	-1.25 V	19.4×10^3	52.3×10^3
1.4	$82 \mu s$	6.42 V	187.5 mV	86.2×10^3	76.6×10^3

Part II: The Step Response of a Second-Order Lowpass Circuit

$$V_{o_1} = 4V$$

$$V_{o_2} = -1.25V$$

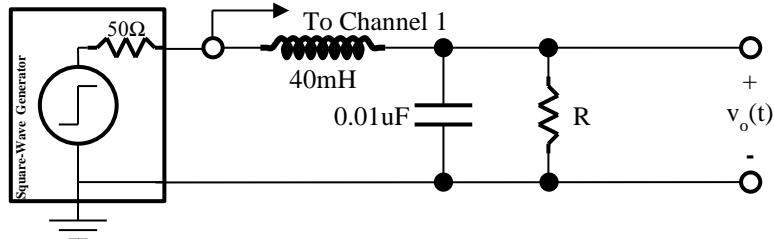


Figure 6.0: Second-Order Lowpass Filter

(h) Step 8: Connect the circuit shown in **Figure 6.0**. Set $R=10k\Omega$. Use Channel (2) to display the step response $v_o(t)$. Plot $v_o(t)$ on **Graph 2.0**.

(i) Step 9: Use the cursors on the oscilloscope to measure points on the $v_o(t)$ display to calculate (and record in **Table 2.0**) the parameters (σ and ω) that characterize the step response as:

$$v_o(t) = B + A e^{-\sigma t} \cos(\omega t + \theta)$$

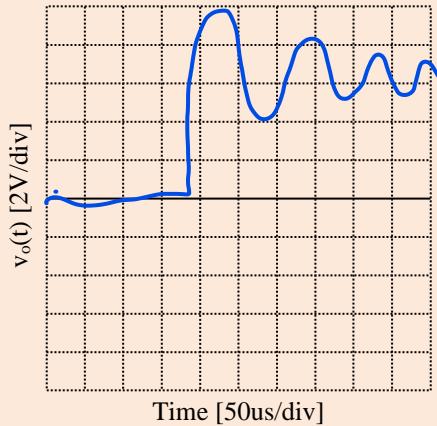
$$6 = \frac{2}{120 \times 10^6} \ln \left| \frac{4}{1.25} \right|$$

(j) Step 10: Set $R=1.4k\Omega$, and repeat as in Step 8.

(k) Step 11: Set $R=1k\Omega$, and repeat as in Step 8.

$$B_{3.4} = 19385.846$$

$$B_{1.4} = 86180.3549$$



Graph 2.0 (0.5 marks)

Table 2.0 (0.5 marks)

R (KΩ)	T	v _o (T/4)	v _o (3T/4)	σ	ω
10	122ms	16.2	5.80	16.8 × 10 ³	51.5 × 10 ³

Part III: The Step Response of a Second-Order Highpass Circuit

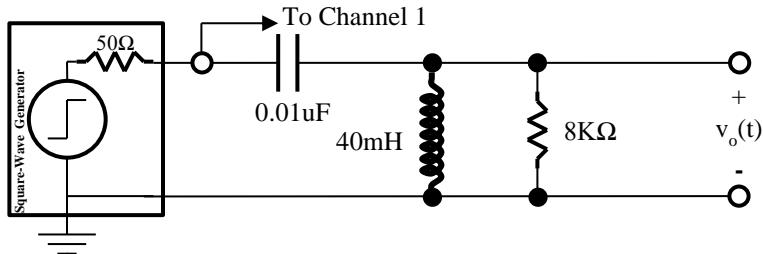
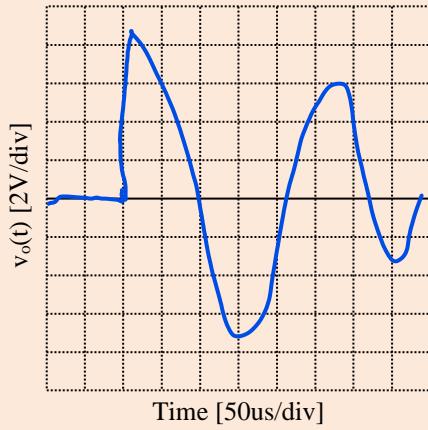


Figure 7.0: Second-Order Highpass Filter

- (l) Step 12: Connect the circuit shown in **Figure 7.0**. Connect Channel (2) to display the step response $v_o(t)$. Plot $v_o(t)$ on **Graph 3.0**.
- (m) Step 13: Use the cursors on the oscilloscope to measure points on $v_o(t)$ display to calculate (and record in **Table 3.0**) the parameters (σ and ω) that characterize the step response as:

$$v_o(t) = Ae^{-\sigma t} \cos(\omega t + \theta)$$

- (n) Step 14: Demonstrate the correct operation of your setup to your TA. (1 mark)



Graph 3.0 (0.5 marks)

Table 3.0 (0.5 marks)

R (KΩ)	T	v _o (T/4)	v _o (3T/4)	σ	ω
8	115.6 μs	9.63	-6.38 V	7.12 × 10 ³	54.3 × 10 ³

4.7 + 3.3

1K + 0.5K

6.0 POST-LAB QUESTIONS (2 marks in total, 2/3 marks for each question):

- (1) By examining your plots on **Graph 1.0**, answer the following:
- What are the effects of varying the value of R on the step response of a second-order bandpass circuit?

- It appears that varying R in a second order circuit affects the $V_o(\frac{T}{4})$ and $V_o(\frac{3T}{4})$ voltages, since a change in R also caused a change in the graph's amplitude.

It also seems that a $|K| > R$ value generates a sharper initial curve

- (2) By examining your plots on **Graph 2.0**, answer the following:
- What are the effects of varying the value of R on the step response of a second-order lowpass circuit?

- It seems that a smaller R generates a small 'hook' in the beginning of the function, but fails to affect the majority of the function.

However, a larger R value is able to affect the entire wave and produce a familiar sin graph that alternates above the y-axis.

- (3) Suppose that the $8k\Omega$ -resistor is removed from the circuit in Fig (2.6), what effects will this have on the step response?

- The shape of the graph will be largely unaffected, however the amplitude will increase because all current will flow through the inductor which increases final voltage.