

Digital Electronics

COE328

Lecture 6

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Logic Function Simplification: Karnaugh Map

In many digital circuits and practical problems, we need to find expressions with minimum variables. We can minimize Boolean expressions of 2,3, and 4 variables very easily using K-map without using any Boolean algebra theorems. K-map can take two forms Sum of Product (SOP) and Product of Sum (POS) according to the need of the problem. K-map is a table-like representation, but it gives more information than TRUTH TABLE. We fill the grids of K-map with 0's and 1's then solve them by making groups.

Solving Problems using K-map

- Steps to solve expression using K-map-

1. Select K-map according to the number of variables. 2, 3, 4
2. Identify minterms or maxterms as given in the problem. $f = m \sum 0, 1, 2, 5, 7$
3. For SOP put 1's in blocks of K-map respective to the minterms (0's elsewhere).
4. For POS put 0's in blocks of K-map respective to the maxterms (1's elsewhere).
5. Make rectangular groups containing total terms in the power of two like 2, 4, 8 ..(except 1) and try to cover as many elements as you can in one group. ✓ ✓ ✓
6. From the groups made in step 5 find the product terms and sum them up for SOP form.

Formation of K-map

$$f = A\bar{B} + AB$$

A \ B	\bar{B} 0	B 1
\bar{A} 0	$\bar{A}\bar{B}$ 0 00	$\bar{A}B$ 1 01
A 1	$A\bar{B}$ 2 10	AB 3 11

(a)

A \ BC	$\bar{B}\bar{C}$ 00	$\bar{B}C$ 01	BC 11	$B\bar{C}$ 10
\bar{A} 0	$\bar{A}\bar{B}\bar{C}$ 0	$\bar{A}\bar{B}C$ 1	$\bar{A}BC$ 3	$\bar{A}B\bar{C}$ 2
A 1	$A\bar{B}\bar{C}$ 4	$A\bar{B}C$ 5	ABC 7	$AB\bar{C}$ 6

(b)

ABC

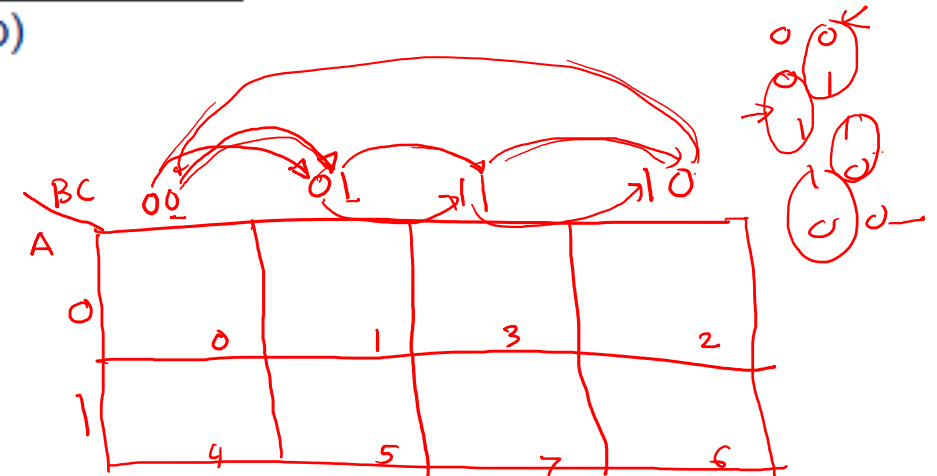
BC	00	01	11	10
0	0	1	1	0
1	0	1	0	1

A	B	C	D
0	0	0	0
0	0	0	1
0	0	1	0
0	0	1	1
0	1	0	0
0	1	0	1
0	1	1	0
0	1	1	1

$\Sigma($

AB \ CD	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	12	13	15	14
10	8	9	11	10

(c)



1000
1001
1010
1011

1100
1101
1110
1111

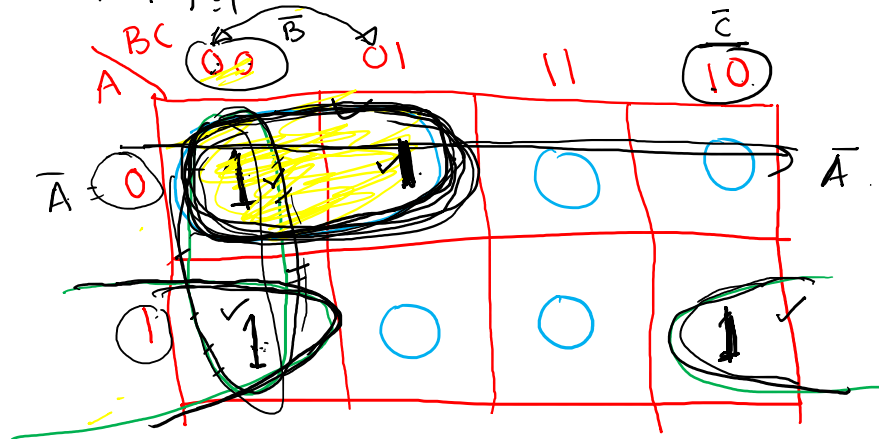
Figure 1 Karnaugh Maps for (a) Two Variables (b) Three Variables (c) Four Variables

Definitions

- Let us consider, $f = \bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C} + \bar{A}\bar{B}C + ABC$
- **Literal:** Each appearance of a variable, either uncomplemented or complemented, is called a literal. For example, $\bar{A}\bar{B}\bar{C}$ has three literals, $x_1\bar{x}_2x_3\bar{x}_4$ has four literals.
- **Implicant:** A product term that implies $f = 1$ is called an implicant of that function. The above function has 4 implicants.
- **Group:** Implicants may be combined (Grouped) to form an implicant with fewer literals.
- **Prime implicant:** A prime implicant is an implicant that cannot be combined further to result in fewer literals.
- **Essential Prime Implicant:** If a prime implicant includes at least one minterm for which $f = 1$ that is not included in any other prime implicants, then it is called an essential prime implicant.
- **Cover:** A collection of implicants that accounts for all valuations for which $f = 1$ is called a cover of that function. Most functions have a number of different covers. The objective of minimization is to obtain a minimal cover.

Example 1: Simplify using K-map

• $f = \bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C} + \bar{A}\bar{B}C + A\bar{B}C$



$A = 1$
 $B = 1$
 $C = 0$

$A \cdot B \cdot \bar{C}$
 $1 \cdot 1 \cdot 1 = 1$
 $\bar{C} = 1$

	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
\bar{A}	1	1		
A	1			1

$f = \bar{A}\bar{B} + \bar{B}\bar{C} + A\bar{C}$ ← Prime groups

Essential Prime groups

$f = \bar{A}\bar{B} + A\bar{C}$ ← simplified

Example 2: Simplify using K-map

- $F(x_1, x_2, x_3) = \sum(m_{\underline{0}}, m_{\underline{2}}, m_4, m_6)$

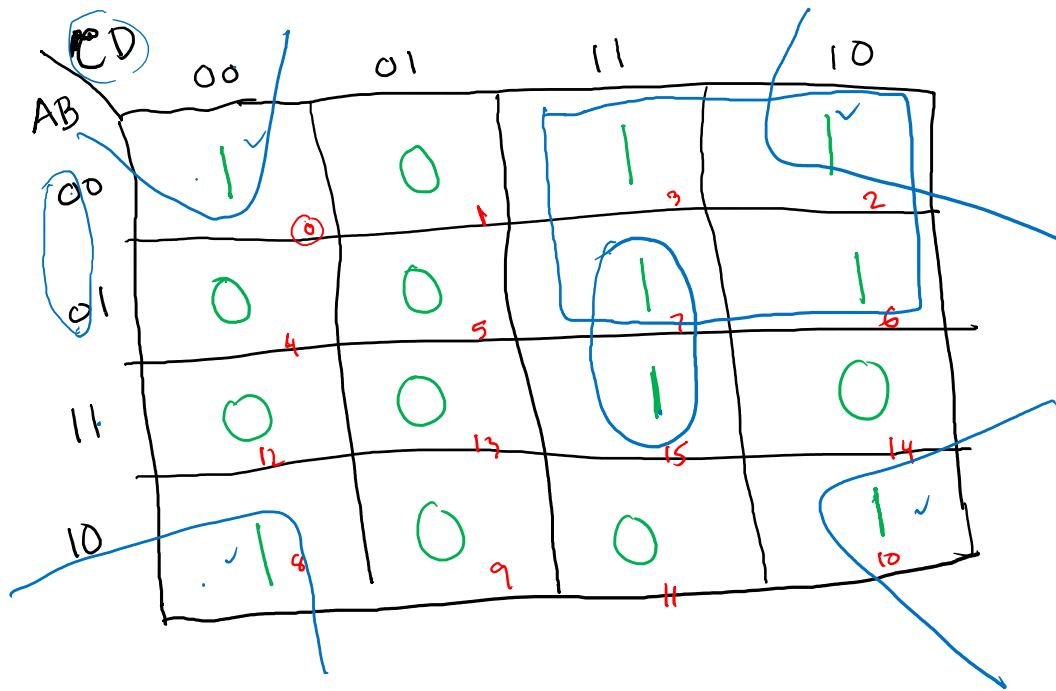
$$2^3 \quad 2^2 \quad 2^1 \\ 8, \underline{4}, 2, \pm$$

$x_2 x_3$					
		00	01	11	10
x_1	0	1	0	0	1
	1	1	0	0	1

$$F = \overline{x_3}$$

Example 3: Simplify using K-map

• $P = \sum(0, 2, 3, 6, 7, 8, 10, 15)$



4-Variable

2 in a group \rightarrow 3 variables

4 in a group \rightarrow 2 variables

8 in a group \rightarrow 1 variable

$$P = \underline{\underline{\bar{A}C}} + \underline{\underline{\bar{B}\bar{D}}} + BCD$$

$$f(\overline{A}BCD)$$

row

$$\boxed{x_1 x_2 x_3 x_4}$$

Example 4: Simplify using K-map

• $f(x_1, x_2, x_3, x_4) = \bar{x}_1 x_2 \bar{x}_4 + \bar{x}_1 x_3 x_4 + \bar{x}_1 x_2 \bar{x}_3 \bar{x}_4 + \bar{x}_1 x_2 \bar{x}_3 x_4 + x_1 x_3 x_4$

Handwritten K-map for $f(x_1, x_2, x_3, x_4)$. The map is a 4x4 grid with columns labeled $x_3 x_4$ (00, 01, 11, 10) and rows labeled $x_1 x_2$ (00, 01, 11, 10). The values in the cells are 0, 0, 1, 0 for the first row; 1, 1, 1, 1 for the second row; 0, 0, 1, 0 for the third row; and 0, 0, 1, 0 for the fourth row. A vertical group of 1s is circled in the middle column (01), and a horizontal group of 1s is circled in the second row (01). Handwritten annotations include $\bar{x}_3 x_3$ and $x_2 \bar{x}_2$ above the columns, and $x_1 \bar{x}_1$ to the left of the rows.

$x_1 x_2 \backslash x_3 x_4$	00	01	11	10
00	0	0	1	0
01	1	1	1	1
11	0	0	1	0
10	0	0	1	0

$f = \bar{x}_1 x_2 + x_3 x_4$ 4 gates

Handwritten derivation showing the simplification of the third and fourth terms of the original function:

$$\bar{x}_1 x_2 \bar{x}_4 (\bar{x}_3 + x_3)$$

$$= \bar{x}_1 x_2 \bar{x}_3 \bar{x}_4 + \bar{x}_1 x_2 x_3 \bar{x}_4$$

Handwritten final simplified function:

$$f = \bar{x}_1 x_2 + x_1 x_2 x_3 x_4$$

Example 5: Simplify using K-map

- $Q = \prod (0, 1, 4, 8, 9, 12, 15)$

↑ pos
put 0

CD \ AB	00	01	11	10
00	0	1	1	1
01	0	1	1	1
11	0	1	0	1
10	0	0	1	1

$$\bar{Q} = \bar{C}\bar{D} + \bar{B}\bar{C} + ABCD$$

$$\begin{aligned}
 Q &= \overline{\bar{C}\bar{D} + \bar{B}\bar{C} + ABCD} \\
 &= (\overline{\bar{C}\bar{D}})(\overline{\bar{B}\bar{C}})(\overline{ABCD}) \\
 &= (\bar{C} + \bar{D})(\bar{B} + \bar{C})(\bar{A} + \bar{B} + \bar{C} + \bar{D}) \\
 &= (C + D)(B + C)(\bar{A} + \bar{B} + \bar{C} + \bar{D})
 \end{aligned}$$

Example 6: Simplify using K-map

- Optimization of Incompletely Specified Functions
- Gives more flexibility and therefore better optimization
- DO NOT CARE conditions could be 0 or 1
- Example: $P = \sum 2, 4, 5, 6, 10 + D(12, 13, 14, 15)$

$$P = B\bar{C} + \bar{C}\bar{D}$$

CD \ AB	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	12	13	15	14
10	8	9	11	10

Handwritten K-map for the function $P = \sum 2, 4, 5, 6, 10 + D(12, 13, 14, 15)$. The map shows the following values:

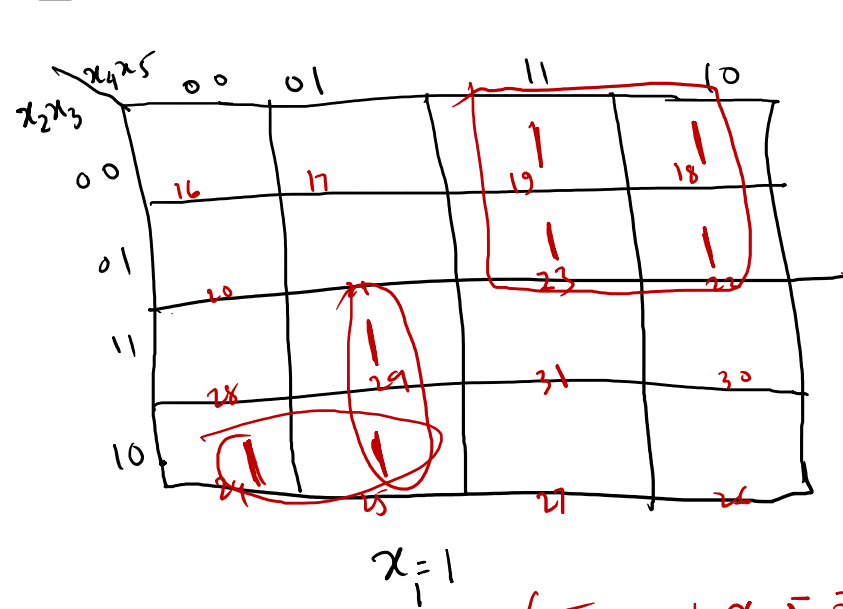
- Row 00: 0, 1, 3, 2
- Row 01: 4, 5, 7, 6
- Row 11: 12, 13, 15, 14
- Row 10: 8, 9, 11, 10

Groupings (circles) are shown for the following prime implicants:

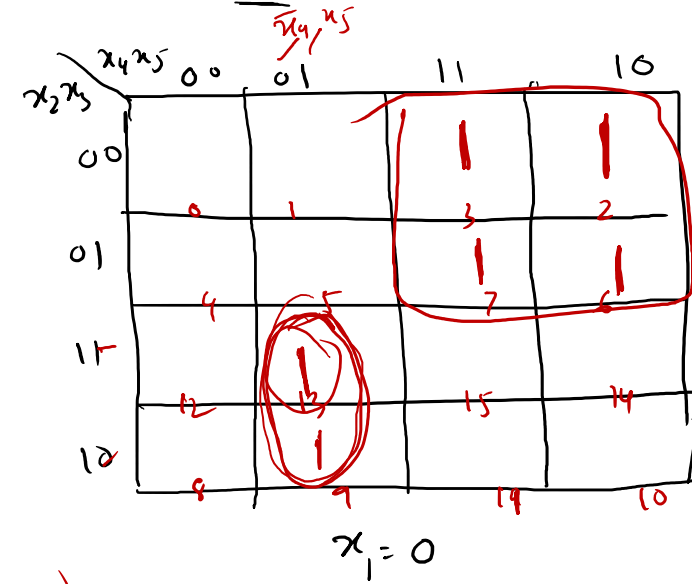
- Group 1: Cells (00, 01), (01, 01), (11, 01), (10, 01) - This group is circled and represents the term $\bar{C}\bar{D}$.
- Group 2: Cells (01, 00), (01, 01), (11, 00), (11, 01) - This group is circled and represents the term $B\bar{C}$.

Example 7: Simplify using K-map

- K-Map for 5 input variables
- $P = \sum 2, 3, 6, 7, 9, 13, 18, 19, 22, 23, 24, 25, 29$



$$x_1 (\bar{x}_2 x_4 + x_2 \bar{x}_3 \bar{x}_4 + x_2 \bar{x}_4 x_5)$$

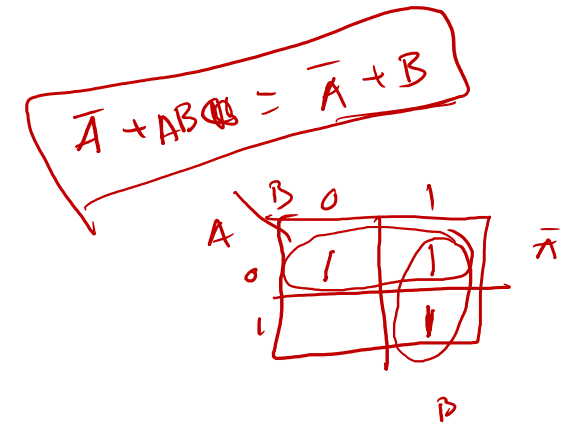
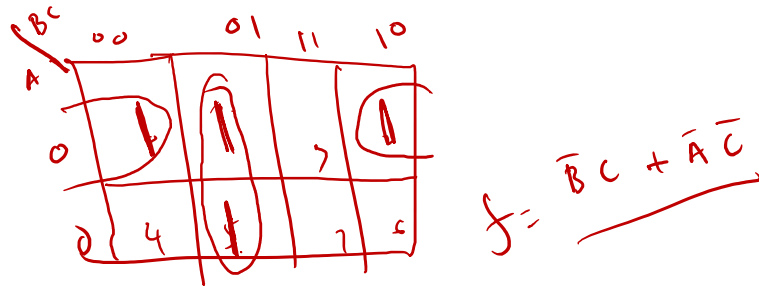


$$\bar{x}_1 (\bar{x}_2 x_4 + x_2 \bar{x}_4 x_5)$$

$$P = \bar{x}_1 \bar{x}_2 x_4 + \bar{x}_1 x_2 \bar{x}_4 x_5 + x_1 \bar{x}_2 x_4 + x_1 x_2 \bar{x}_3 \bar{x}_4 + x_1 x_2 \bar{x}_4 x_5$$

Example 8: Simplify using K-map

- $F1 = \sum(1, 3, 5, 6, 7, 13, 15)$
- $F2 = \sum(2, 3, 5, 6, 7, 10, 11, 13, 14)$
- $F3 = \prod(1, 3, 6, 9, 11, 13, 15)$
- Implement F1 and F2 using NAND gates only.
- Implement F3 using NOR gates only.



$$\begin{aligned}
 f &= \sum(0, 1, 2, 5) \\
 f &= \overline{A}\overline{B}\overline{C} + \overline{A}\overline{B}C + \overline{A}B\overline{C} + A\overline{B}C \\
 &= (\overline{A}\overline{B}\overline{C} + \overline{A}\overline{B}C) + (\overline{A}B\overline{C} + A\overline{B}C) \\
 &= \overline{A}\overline{C}(\overline{B} + B) + \overline{B}C(\overline{A} + A) \\
 &= \underline{\overline{A}\overline{C} + \overline{B}C}
 \end{aligned}$$