Toronto Metropolitan University

Department of Electrical and Computer Engineering

COE328- Digital Systems

Midterm (Make up) Examination

November 02, 2022 Duration: 120 Minutes

Examiner: Dr. Hossain

Student Last Name: Solution Student First Name: ....

Student Number: .... Section #: .... (Required)

NOTES:

1. Use TMU approved calculators for calculations.
2. A custom made handwritten single sided A4 size formula sheet is permitted.
3. NO QUESTIONS to be asked. If doubt exists as to the interpretation of any question, you are urged to submit with the answer, a clear statement of any logical assumptions made.
4. Use both sides of the sheets. No additional sheets will be supplied

Table 1: Test Score

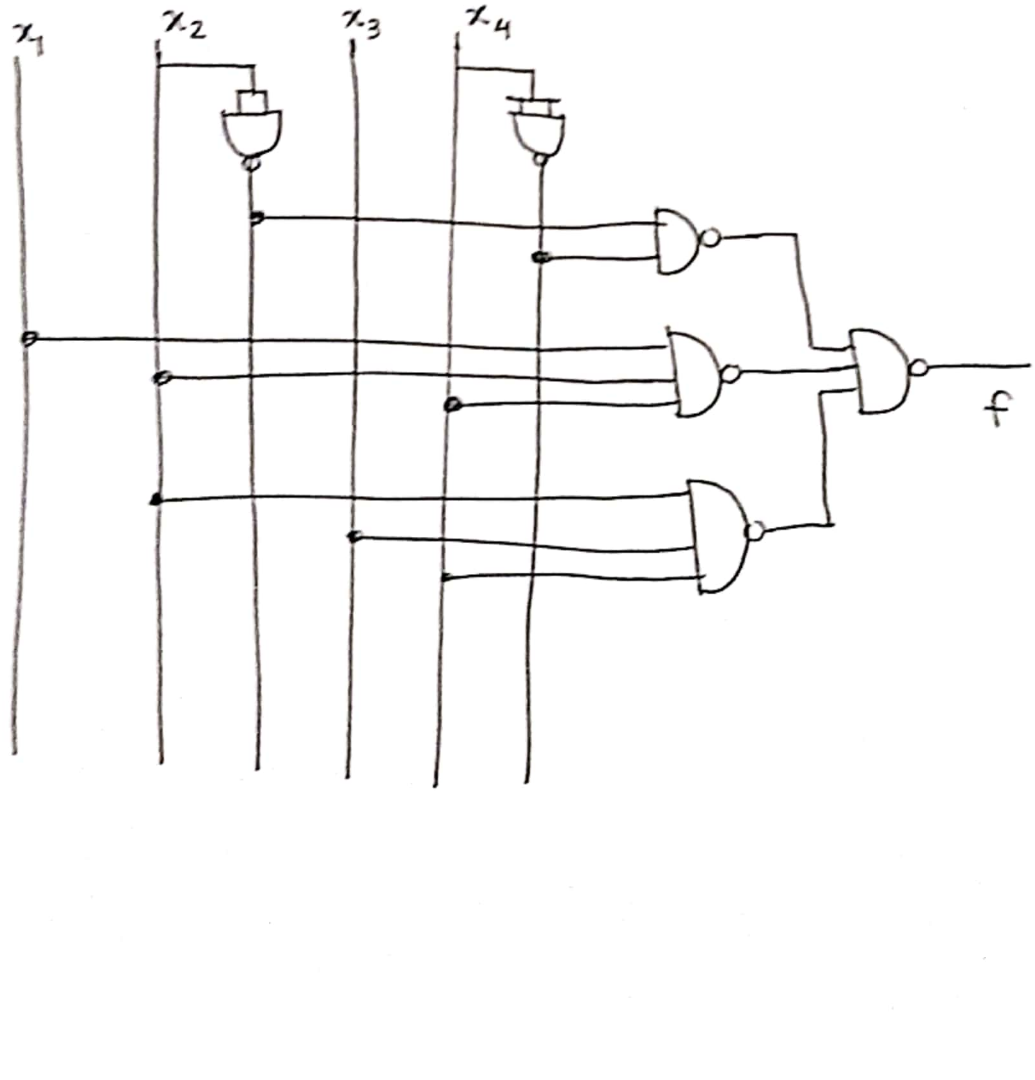
Question	Total Marks	Obtained Marks
Q1	25	
Q2	25	
Q3	15	
Q4	20	
Q5	15	
Total	100	

Q1. (a) Simplify the SOP function $f(x_1, x_2, x_3, x_4) = \sum m(0, 2, 5, 7, 8, 10, 15)$ [Marks 2]

$x_1x_2 \backslash x_3x_4$	00	01	11	10
00	1	0	0	1
01	0	0	1	0
11	0	1	1	0
10	1	0	0	1

$$f = \bar{x}_2\bar{x}_4 + x_1x_2x_4 + x_2x_3x_4$$

Q1. (b) Implement the simplified function using NAND gates only. **[4 Marks]**



Q1. (c) Implement the simplified function using multiplexers only. Use 4-to-1 and 2-to-1 multiplexers.

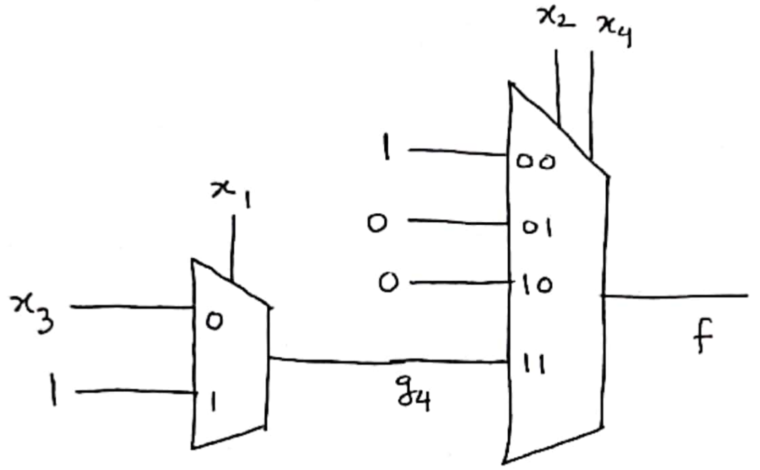
[10 Marks]

This part can be solved in several ways.

$$f = \bar{x}_2\bar{x}_4 + x_1x_2x_4 + x_2x_3x_4$$
$$= \bar{x}_2\bar{x}_4(1) + \bar{x}_2x_4(0) + x_2\bar{x}_4(0) + x_2x_4(x_1+x_3)$$

$g_1 = 1$ $g_2 = 0$ $g_3 = 0$ $g_4 = x_1 + x_3$

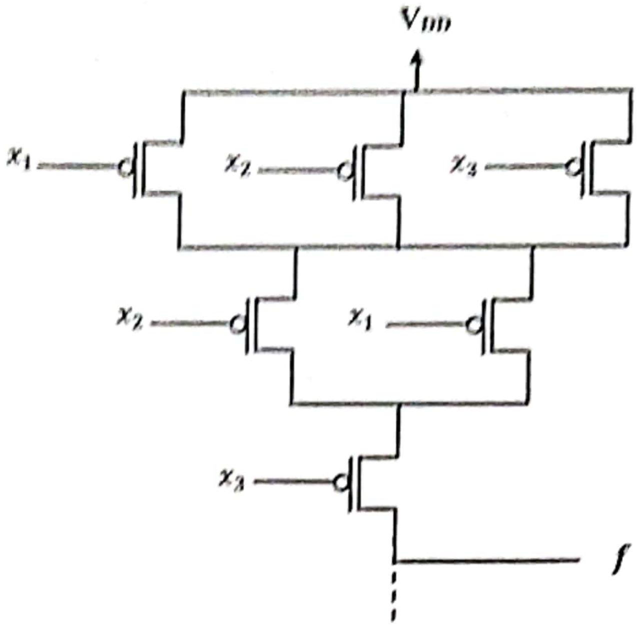
$g_4 = \bar{x}_1(x_3) + x_1(1)$



Q1. (d) Find the complement of the simplified function using DeMorgan's theorem. **[3 Marks]**

$$\bar{f} = \overline{\bar{x}_2\bar{x}_4 + x_1x_2x_4 + x_2x_3x_4}$$
$$= (x_2 + x_4)(\bar{x}_1 + \bar{x}_2 + \bar{x}_4)(\bar{x}_2 + \bar{x}_3 + \bar{x}_4)$$

Q2. A logic function is implemented in CMOS, Its Pull Up Network (PMOS circuit) is given in the following figure.



Q2. (a) Find the logic function, f. **[5 Marks]**

$$f = (\bar{x}_1 + \bar{x}_2 + \bar{x}_3) \cdot (\bar{x}_2 + \bar{x}_1) \cdot \bar{x}_3$$

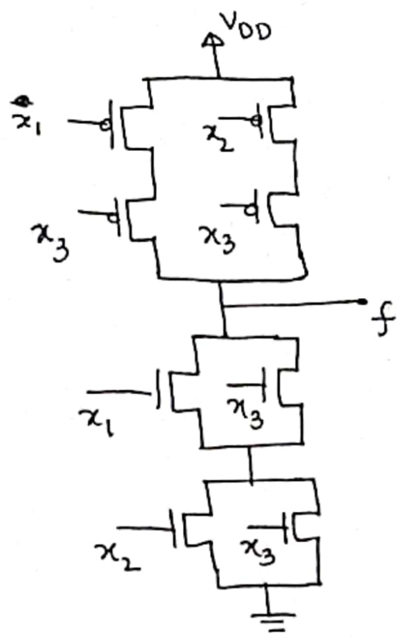
Q2. (b) Simplify the function using Boolean algebra. **[5 Marks]**

$$f = ((\bar{x}_1 + \bar{x}_2) + \bar{x}_3)(\bar{x}_1 + \bar{x}_2) \cdot \bar{x}_3$$
$$= (\bar{x}_1 + \bar{x}_2) \cdot \bar{x}_3$$
$$= \bar{x}_1 \bar{x}_3 + \bar{x}_2 \cdot \bar{x}_3$$

Q2. (c) Implement the simplified circuit in part (b) in CMOS. Show both PUN and PDN. [8 Marks]

$f = \bar{x}_1\bar{x}_3 + \bar{x}_2\bar{x}_3$ (for PUN)

$\bar{f} = (x_1+x_3)(x_2+x_3)$ (for PDN)



You also can implement

$f = \bar{x}_3(\bar{x}_1 + \bar{x}_2)$ PUN

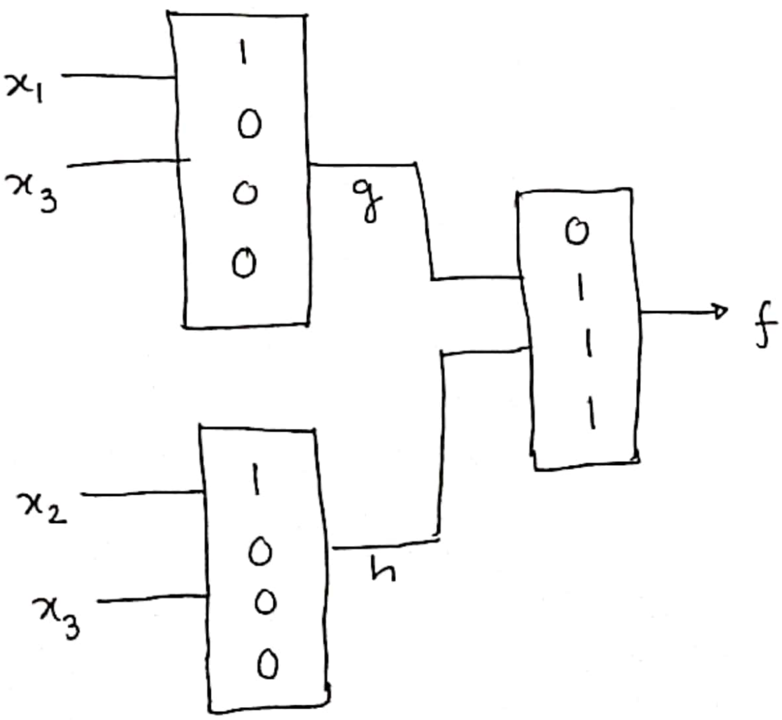
$\bar{f} = x_3 + x_1 \cdot x_2$ PDN

Q2. (d) Write the simplified function in part (b) in SOP form. Implement the SOP function using 2-input LUTs. [7 Marks]

$f = \bar{x}_1\bar{x}_3 + \bar{x}_2\bar{x}_3$

$= g + h$

$g = \bar{x}_1\bar{x}_3$ $h = \bar{x}_2\bar{x}_3$



ollowing signed numbers are given.

(a) A decimal number $(37)_{10}$. Convert it into signed-magnitude binary form, 2's complement form and hexadecimal form. Consider an 8-bit representation where the MSB will indicate the sign of the number.

[3 Marks]

(b) A signed-magnitude binary number $(1\ 10\ 1\ 1\ 0\ 0\ 1\ 1\ 0\ 0\ 1\ 0)_2$. Find its equivalent decimal value, 2's complement value, and hexadecimal value.

[6 Marks]

(c) A 2's complement binary number $(1\ 0\ 1\ 1\ 0\ 0\ 1\ 1\ 0\ 1\ 10\ 0)_2$. Find its equivalent decimal value, signed-magnitude value, and hexadecimal value.

[6 Marks]

Show your work and write your answers in the Table below.

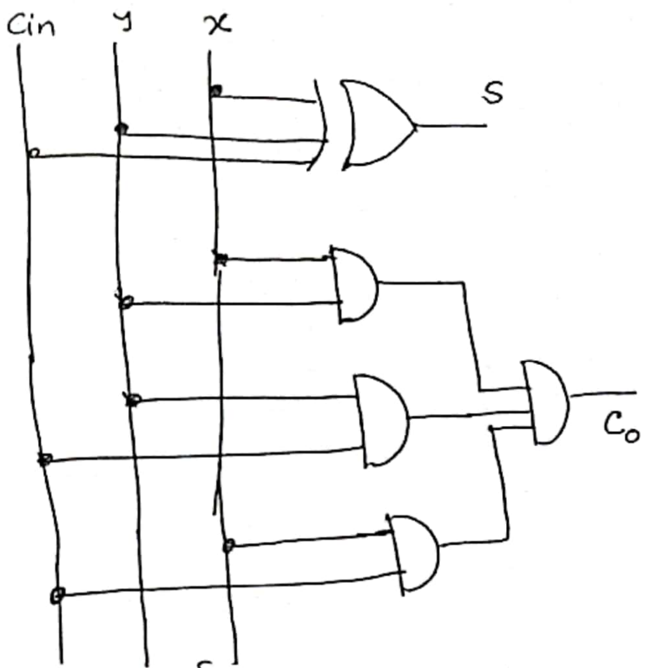
Decimal	$(37)_{10}$	-2866	-2452
Signed-magnitude	00100101	1 10 1 1 0 0 1 1 0 0 1 0	1100110010100
2's Complement	00100101	10100110011110	1 0 1 1 0 0 1 1 0 1 1 0 0
Hexadecimal	$(25)_{16}$	- B32	-994

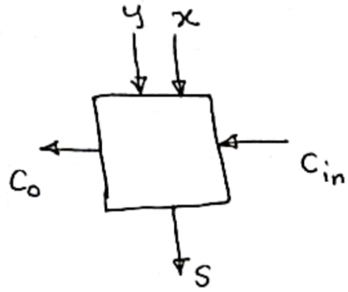
2 | 37
2 | 18 − 1
2 | 9 − 0
2 | 4 − 1
2 | 2 − 0
2 | 1 − 0
0 − 1

100101

Block

Q4. (a) Draw the circuit diagram of a full adder. Label all input and output lines. Consider the inputs are x, y, c_{in} (c_{in} = carry input) and the outputs are s (sum) and c_o (carry output). **[3 Marks]**





(Midtem)

For (Make-up test)

Q4. (b) Complete the Truth Table of the full adder. Also, find the logic expressions for the s (sum) and c_o(carry output) outputs. Simplify your expressions. **[5 Marks]**

Truth Table

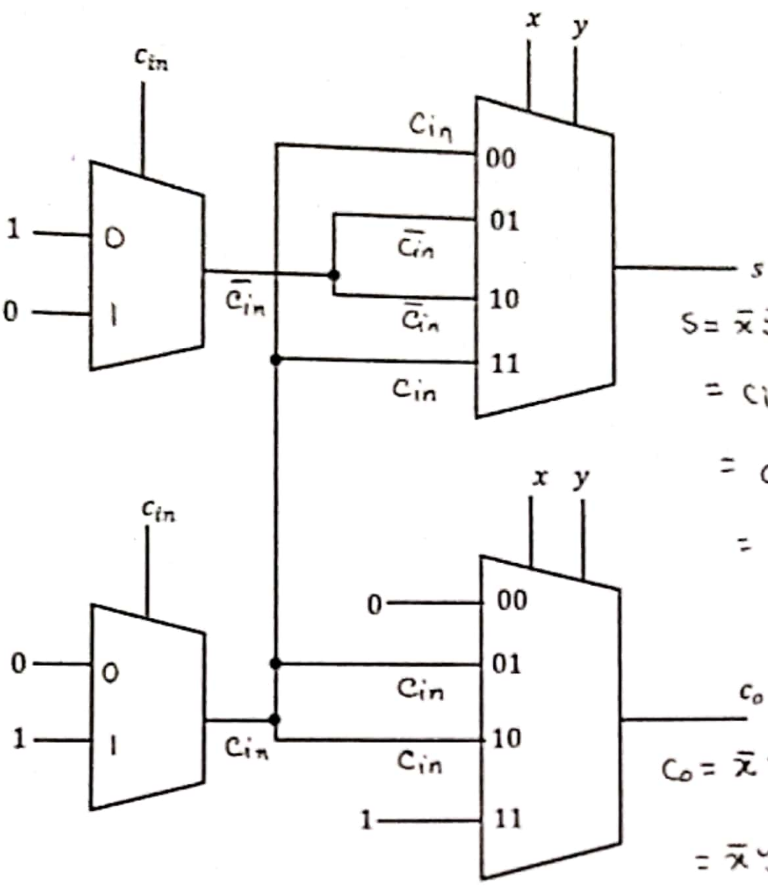
x	y	c_{in}	s	c_o
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

$$S = \bar{x}\bar{y}C_{in} + \bar{x}\cdot y\,\overline{C_{in}} + x\bar{y}\,\overline{C_{in}} + xyC_{in}$$
$$= C_{in}(\bar{x}\bar{y} + xy) + \overline{C_{in}}(\bar{x}y + x\bar{y})$$
$$= C_{in}\cdot\overline{x\oplus y} + \overline{C_{in}}\cdot x\oplus y$$
$$= x\oplus y\oplus C_{in}$$

$$C_o = \bar{x}yC_{in} + x\bar{y}C_{in} + xy\overline{C_{in}} + xyC_{in}$$
$$= xy + xC_{in} + yC_{in}$$

Use K-map

Q3 (c) Show that the following multiplexer circuit can function as a full adder. **[12 Marks]**



$$S = \bar{x}\bar{y}\,C_{in} + \bar{x}y\,\bar{C}_{in} + x\bar{y}\,\bar{C}_{in} + xy\,C_{in}$$

$$= C_{in}(\bar{x}\bar{y} + xy) + \bar{C}_{in}(\bar{x}y + x\bar{y})$$

$$= C_{in}\,\overline{x \oplus y} + \bar{C}_{in}\; x \oplus y$$

$$= x \oplus y \oplus C_{in}$$

(Same before)

$$C_o = \bar{x}y\,C_{in} + x\bar{y}\,C_{in} + xy$$

$$= \bar{x}y\,C_{in} + x\bar{y}\,C_{in} + xy(C_{in} + \bar{C}_{in})$$

$$= y\,C_{in}(\bar{x} + x) + x\,C_{in}(y + \bar{y}) + xy$$

$$C_o = y\,C_{in} + x\,C_{in} + xy$$ (same before)

Q5. A 4-bit ASU is used to add two signed numbers X and Y. The values of X and Y are given in the Table below. Covert X and Y into 2's complement form $x_3x_2x_1x_0$ and $y_3y_2y_1y_0$, respectively. Find the output $c_4s_3s_2s_1s_0$. Comment on ASU's output; is its output correct or wrong? Why? What logic expression can indicate an arithmetic overflow?

[15 Marks]

Show your work and present your answers on the Table below.

	X	Y	X				Y				S=X+Y					Arithmetic Overflow?
			x_3	x_2	x_1	x_0	y_3	y_2	y_1	y_0	c_4	s_3	s_2	s_1	s_0	
Correct	+3	+3	0	0	1	1	0	0	1	1	0	0	1	1	0	No overflow
Correct	+6	-2	0	1	1	0	1	1	1	0	1	0	1	0	0	No overflow
wrong	+7	+3	0	1	1	1	0	0	1	1	0	1	0	1	0	Overflow
wrong	-7	-3	1	0	0	1	1	1	0	1	1	0	1	1	0	Overflow

sign bit affected

unused

$$C_{ov} = C_3 \oplus C_4 \quad \text{or}$$

$$C_{ov} = S_3 \oplus x_3 \oplus y_3 \oplus C_4$$