Toronto Metropolitan University
Department of Electrical, Computer, & Biomedical Engineering
Faculty of Engineering
& Architectural Science

ELE 302
Laboratory #3

Frequency Response and Bode Plots

1.0 INTRODUCTION:

The sinusoidal-steady-state response of a linear network is a sinusoid of the same frequency as the input excitation, but with different amplitude and phase angle. The ratio of the response-phasor to the excitation-phasor is frequency-dependent, and is called the frequency-response function, H(ω), of the network. Plots of |H(ω)| and ∠H(ω) versus frequency are often used to describe the frequency-selective characteristics of various linear networks such as feedback amplifiers and filters.
The frequency-response function H(ω) is closely related to the transfer function H(s) of the network, as H(ω)= H(s=jω). Both functions are useful in describing different aspects of the behavior of linear networks. While H(s) describes the pole-zero pattern (in the s-plane) of a network's transfer function, H(ω) describes the frequency-selective characteristics associated with such a pattern. Clearly, a change in the pole-zero patterns of H(s) will yield a corresponding change in the frequency-response characteristics for the network.
To quickly visualize how the pole-zero pattern of the transfer function of a network affects the frequency-response characteristics, electrical engineers often use a straight-line approximation technique (known as the Bode method) to simplify the analysis and design of linear networks. Through quick analysis, the designer is then able to evaluate various possibilities before deciding on a suitable network. The Bode method is a conceptual technique that reduces the complete frequency-response characteristics to a sum of elementary straight-line approximations. The straight-line approximations of |H(ω)| in dB and ∠H(ω) in degrees versus frequency [log scale] are said to be the asymptotic Bode plots of the frequency response characteristics.

$$H(\omega)_{\text{dB}} = 20\text{Log}_{10}|\,H(\omega)|$$

This experiment examines the frequency-response characteristics of various linear networks. It also demonstrated the effectiveness of the Bode method in providing a quick visualization of the frequency-response for these networks.

2.0 OBJECTIVES:

- To draw the asymptotic Bode plots that approximates the frequency-response characteristics of various linear networks.
- To measure and plot the magnitude- and phase-frequency responses of the above-mentioned networks.
- To compare the asymptotic Bode plots and the practical measurements.

3.0 REQUIRED LAB EQUIPMENT & PARTS:

- DC Power Supply (PS), Function Generator (FG) and Oscilloscope.
- ELE202 Lab Kit and ELE302 Lab Kit: various components, breadboard, wires and jumpers.

4.0 PRE-LAB ASSIGNMENT (3 marks with 1.5 marks for each step):

$$Z = \frac{1}{\frac{1}{R_1} + \frac{1}{\frac{1}{j\omega C}}} = \frac{1}{\frac{1}{R_1} + j\omega C}$$

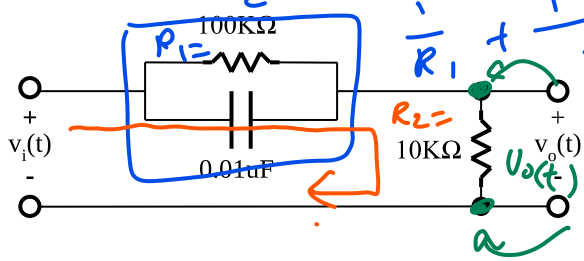


Figure 1.0: An One Pole-One Zero Circuit

(a) Step 1: Consider the network shown in **Figure 1.0**.

i) Derive the transfer function $H(s) = {}^{V_o}/_{V_i}$, where V_o is the phasor representation of $v_o(t)$ and V_i is the phasor representation of $v_i(t)$.

Pre-Lab workspace (show your analysis here)

$$V_i = (Z + R_2)\, i$$

$$= \left(\frac{1}{\frac{1}{R_1} + j\omega C} + R_2\right) i$$

$$V_i = \left(\frac{R_1}{1 + j\omega C R_1} + R_2\right) i$$

$$V_o = R_2\, i$$

$$H(s) = \frac{V_o}{V_i}$$

$$= \frac{R_2 \cancel{i}}{\left(\frac{R_1}{1 + j\omega C R_1} + R_2\right)\cancel{i}}$$

$$= \frac{R_2}{\frac{R_1}{1 + j\omega C R_1} + R_2}$$

$$= \frac{\cancel{R_2}\,(1)\,(1 + j\omega C R_1)}{\cancel{R_2}\left(\frac{R_1}{R_2} \cdot \frac{1}{1 + j\omega C R_1} + 1\right)(1 + j\omega C R_1)}$$

$$= \frac{1 + j\omega C R_1}{\frac{R_1}{R_2} + (1 + j\omega C R_1)}$$

$$Z = \frac{1}{RC} = \frac{1}{(100k)(0.01\mu F)} = 1000$$

$$= \frac{R_2}{R_1}\left(\frac{1 + \frac{j\omega}{\frac{1}{R_1 C}}}{1 + \frac{j\omega}{\frac{1}{R_2 C}}}\right)$$

$$P = \frac{1}{R_1 C} = \frac{1}{(10k)(0.01\mu F)} = 10\,000$$

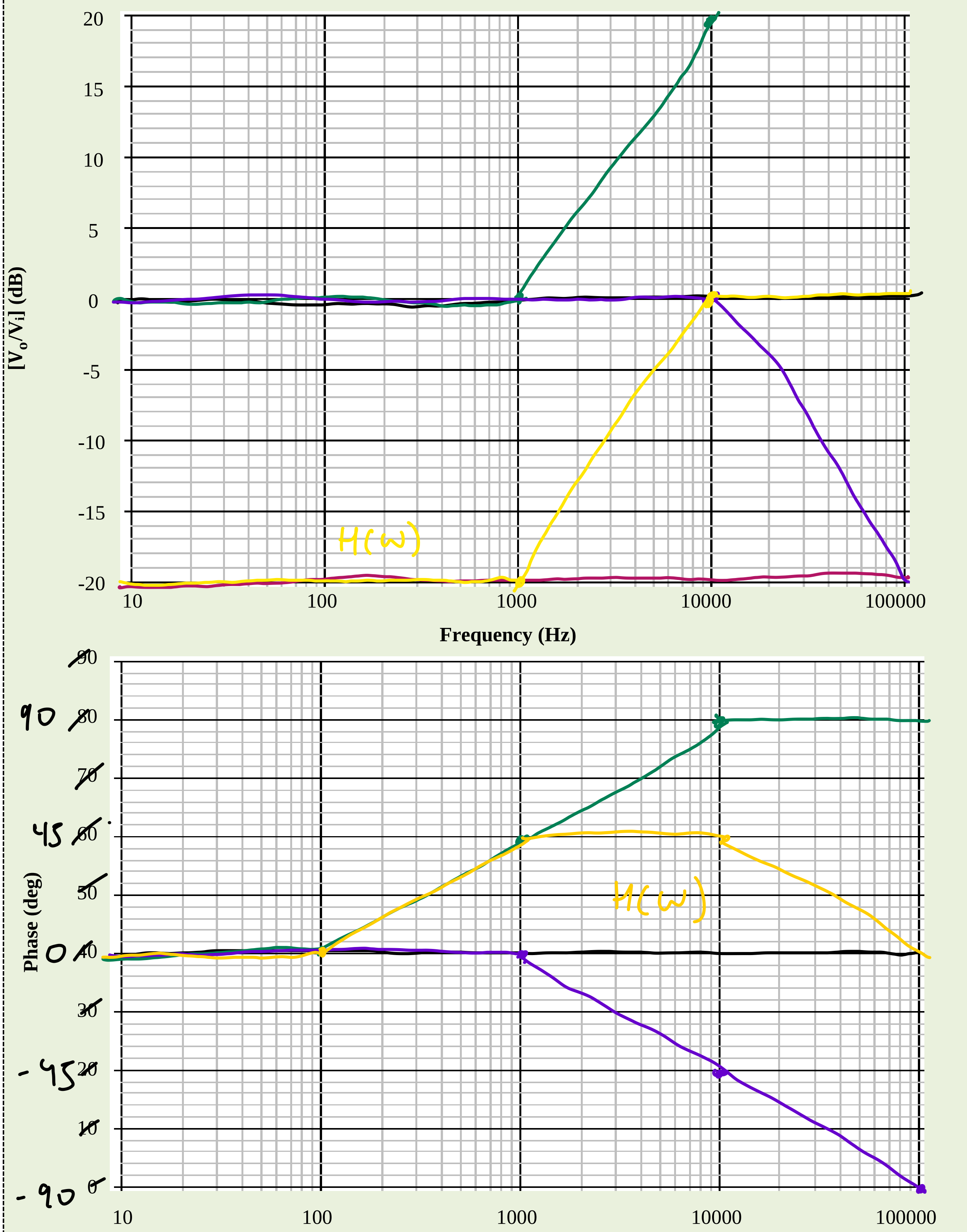
$$\frac{R_2}{R_1} = \frac{10k}{100k} = 0.1$$

$$H(\omega) = 0.1\left(\frac{1 + \frac{j\omega}{1000}}{1 + \frac{j\omega}{10k}}\right)$$

$$H(\omega) = \frac{0.1\left(1+\frac{j\omega}{1000}\right)}{\left(1+\frac{j\omega}{10k}\right)}$$

$20\log(0.1) = -20$

ii) Use **Graph 1.0** to draw the asymptotic Bode plots for the frequency-response of the network.



Graph 1.0

iii) Use Multisim to plot the magnitude in dB and phase in degrees of the frequency-responses of the above circuits, for $10\text{Hz} \leq f \leq 100\text{kHz}$. (Note: Simulate->Instruments->Bode Plotter can be used in creating the plots).

Pre-Lab workspace (show your analysis here)

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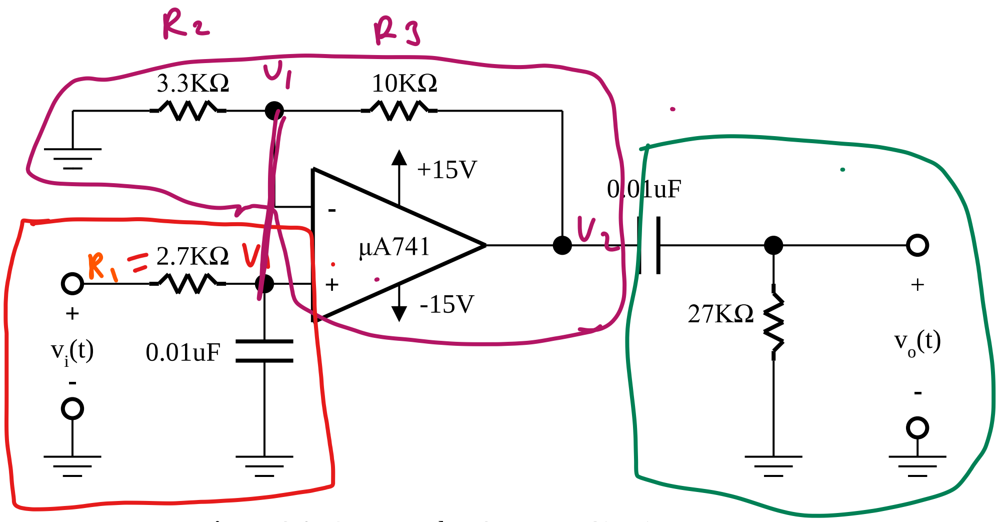


Figure 2.0: A Two Poles-One Zero Circuit

(b) Step 2: Assume that the Op-Amp circuit in **Figure 2.0** is working properly.

i) Derive the transfer function $H(s) = {}^{V_o}/_{V_i}$.

Pre-Lab workspace (show your analysis here)

$$V_i = \left(R_1 + \frac{1}{j\omega C}\right) i$$

$$V_1 = \left(\frac{1}{j\omega C}\right) i$$

$$\frac{V_1}{V_i} = \frac{\frac{1}{j\omega C}\cancel{i}}{R_1 + \frac{1}{j\omega C}\cancel{i}}$$

$$= \frac{1}{R_1 j\omega C + 1}$$

$$\frac{V_o}{V_2} = \frac{R_2}{R_2 + \frac{1}{j\omega C}}$$

$$= \frac{4V_1 R_2}{R_2 + \frac{1}{j\omega C}}$$

$$\frac{V_2 - V_1}{R_3} = \frac{V_1 - 0}{R_2}$$

$$V_2 = V_1\left(\frac{1}{R_2} + \frac{1}{R_3}\right) R_3$$

$$= V_1\left(\frac{10K}{3.3K} + 1\right)$$

$$V_2 = 4V_1$$

$$\frac{V_o}{V_i} = \frac{1}{1 + j\omega R_1 C} \cdot \frac{4 j\omega R_2 C_2}{1 + j\omega R_2 C_2}$$

$$= \frac{4 R_2 C_2 j\omega}{\left(1 + \frac{j\omega}{\frac{1}{R_1 C_1}}\right)\left(1 + \frac{j\omega}{\frac{1}{R_2 C_2}}\right)}$$

$$= \frac{4(27K)(0.01\times10^{-6}) j\omega}{\left(1 + \frac{j\omega}{30K}\right)\left(1 + \frac{j\omega}{4K}\right)}$$

$$= \frac{1.08\times10^{-3} j\omega}{\left(1 + \frac{j\omega}{30K}\right)\left(1 + \frac{j\omega}{4K}\right)}$$

$$\frac{1}{R_1 C_1} = \frac{1}{(3.3K)(0.01\times10^{-6})}$$

$$= 30.3K$$

$$\frac{1}{R_2 C_2} = \frac{1}{(27K)(0.01\times10^{-6})}$$

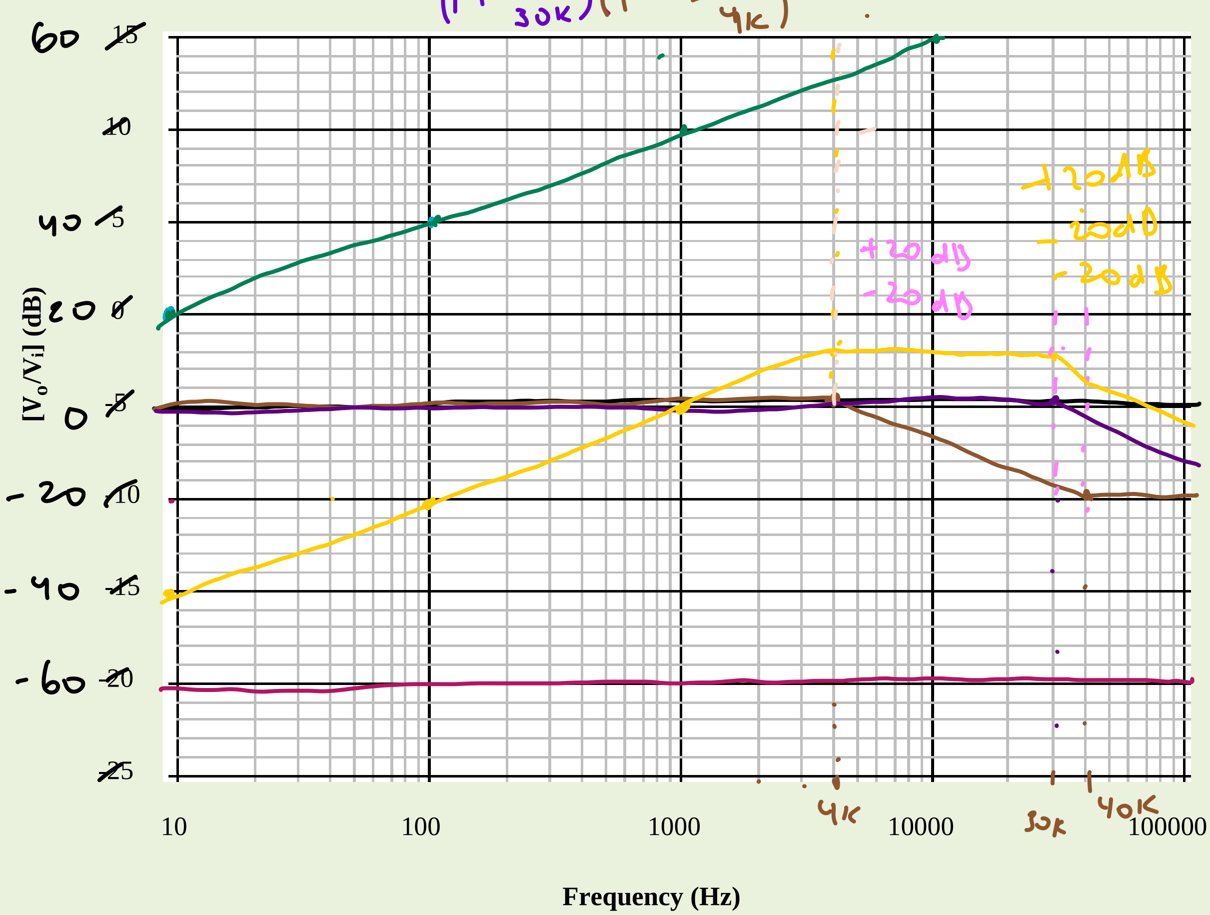
$$= 3.7K$$

$$= 4K$$

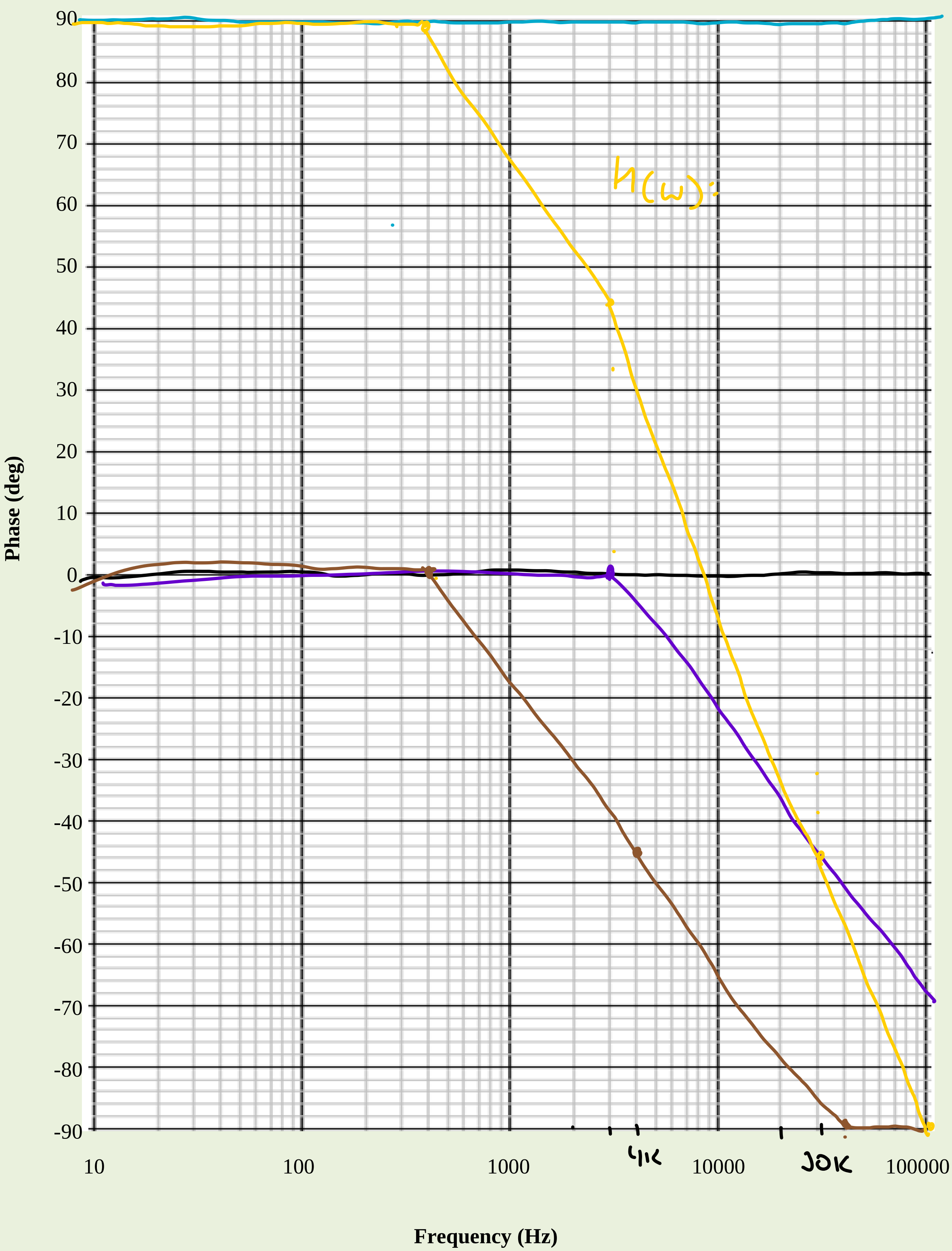
ii) Use **Graph 2.0** to draw the asymptotic Bode plots for the frequency-response of the circuit:

$$H(w) = \frac{1.08 \times 10^{-3}\ jw}{\left(1 + \frac{jw}{30k}\right)\left(1 + \frac{jw}{4k}\right)}$$

$20\log(1.08 \times 10^{-3}) = -60$



Graph 2.0 (*Gain*)



Graph 2.0 (*Phase*)

iii) Use Multisim to plot the magnitude in dB and phase in degrees of the frequency-responses of the above circuits, for $10\text{Hz} \leq f \leq 100\text{kHz}$. (Note: Simulate->Instruments->Bode Plotter can be used in creating the plots).

Pre-Lab workspace (show your analysis here)

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5.0 IN-LAB IMPEMENTATION & MEASUREMENTS (5 marks in total):

Part I: The Frequency Response Associated With [One Pole-One Zero] Transfer Function

(a) Step 1: Connect the circuit shown in **Figure 1.0.**

Connect Channel (1) of the oscilloscope to display $v_i(t)$ and Channel (2) of the oscilloscope to display $v_o(t)$.
Set the trigger source→Channel (1) and Acquire Mode→Averaging.

Adjust the controls of the function generator to provide a **sinusoidal** input voltage $v_i(t)$ of **5V (peak)** at a frequency of 100Hz.

On the Function Generator: Waveform: Sine wave, Amplitude: 5 VPP, Frequency: 100 Hz

(b) Step 2: Use the oscilloscope displays to measure the phase angle $\angle H(\omega)^o$ in degrees, and use either oscilloscope or DMM to measure V_i and V_o. Evaluate the magnitude $|H(\omega)|$ in dB as: $|H(\omega)|$(in dB)=[V_o(in dB) - V_i(in dB)].

Record your results in **Table 1.0.**

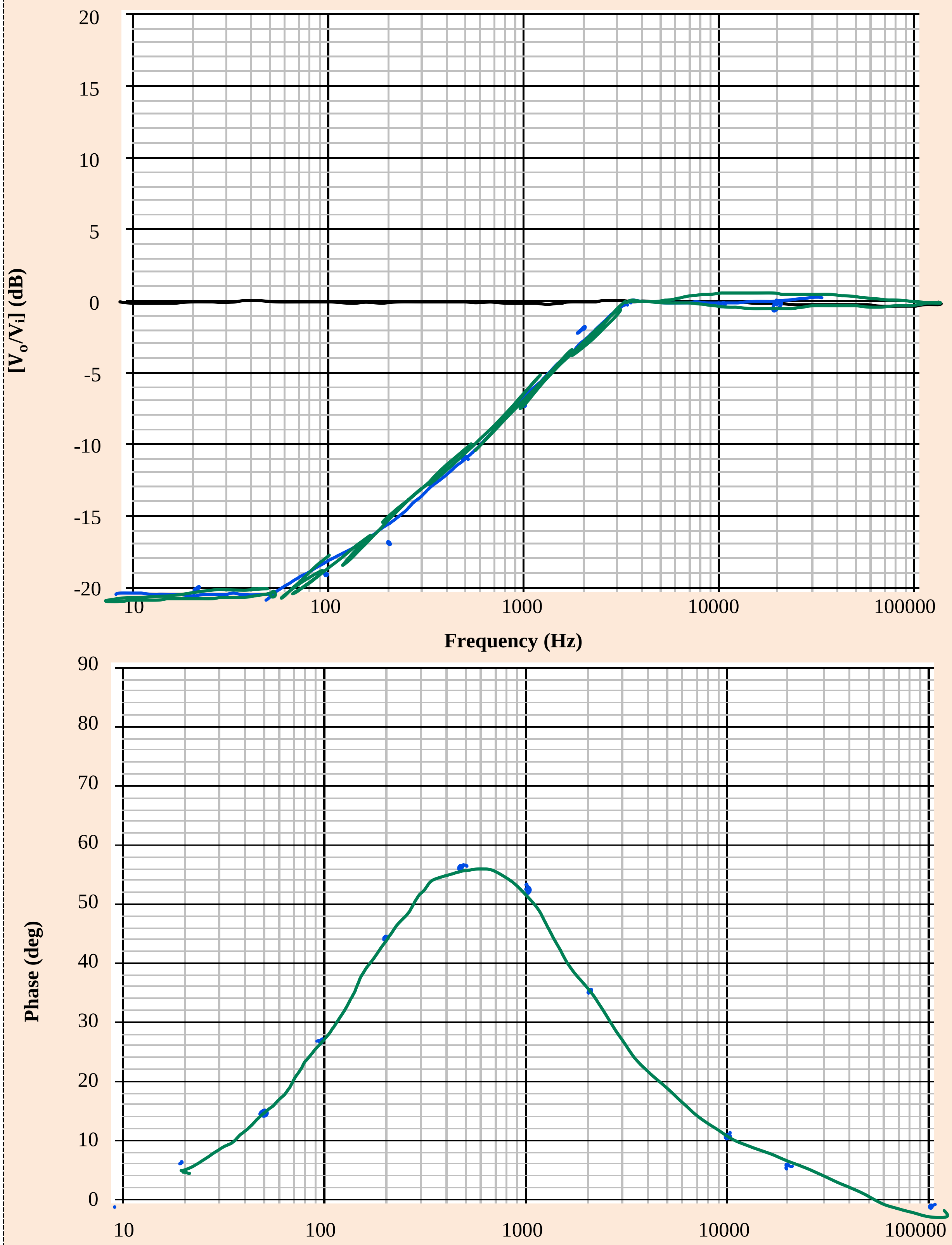
(c) Step 3: Repeat as in Step 2 for each frequency setting in **Table 1.0.**

(d) Step 4: Use **Graph 3.0** to plot the magnitude $|H(\omega)|$ in dB and phase $\angle H(\omega)^o$ in degrees versus frequency in Hz. Use your plot to determine the locations of the corner frequencies: f_Z and f_P. Mark these frequencies on **Graph 3.0.**

(e) Step 5: Demonstrate Step 1 to Step 4 to your TA. (1 mark)

Table 1.0 (0.5 marks)

	Ch2	Ch1	Ch2-Ch1	Phase -(1->2)
Frequency (Hz)	**$|V_o|$ (dB)**	**$|V_i|$ (dB)**	**$|H(\omega)|$ (dB)**	**$\angle H(\omega)$ (degrees)**
20	0.2567	20.906	-20.65	6.02
50	0.5876	20.906	-20.32	15.2
100	1.51		-19.396	27.8
200	3.806		-17.1	43.7
500	9.34		-11.57	56
1K	13.9		-7.0	52
2K	18.3		-2.61	35
10K	20.59		-0.316	10.31
20K	20.75		0.1	6.31
50K	20.58		-0.326	1.5
100K			-0.326	0



Graph 3.0 (0.5 marks for each plot)

Part II: The Frequency Response Associated with [Two Poles-One Zero] Transfer Function

(f) Step 6: Connect the circuit shown in **Figure 2.0.** Connect Channel (1) of the oscilloscope to display $v_i(t)$ and Channel (2) of the oscilloscope to display $v_o(t)$; set the trigger source→Channel (1). Adjust the controls of the function generator to provide a **sinusoidal** input voltage $v_i(t)$ of **0.5V (peak)** at a frequency of 100Hz.

On the Function Generator: Waveform: Sine wave, Amplitude: 0.5 VPP, Frequency: 100 Hz

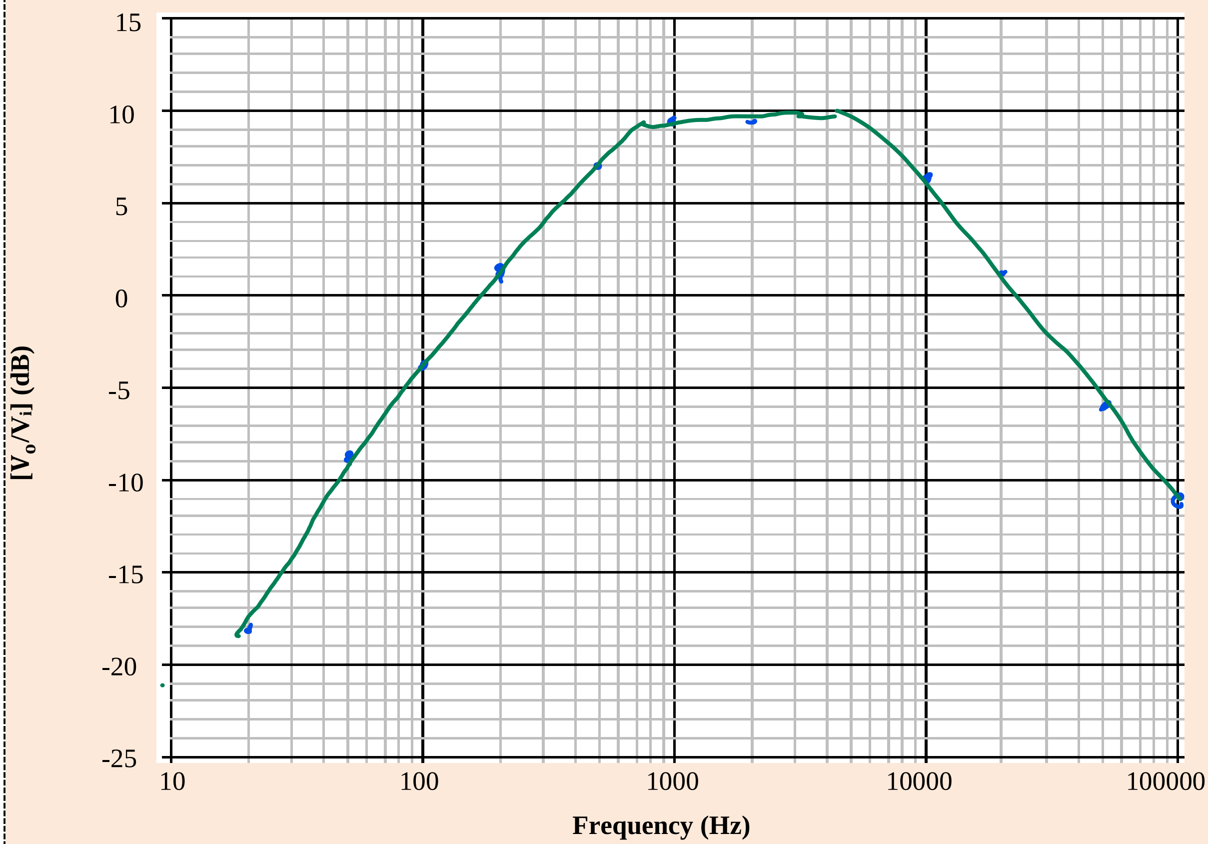
(g) Step 7: Repeat the measurements as in Step 2 and Step 3, and record your results in **Table 2.0.**

(h) Step 8: Use **Graph 4.0** to plot the magnitude $|H(\omega)|$ in dB and phase $\angle H(\omega)$ in degrees versus frequency in Hz. Use your plots to determine the locations of the corner frequencies: f_{P1} and f_{P2}. Mark these frequencies on **Graph 4.0.**

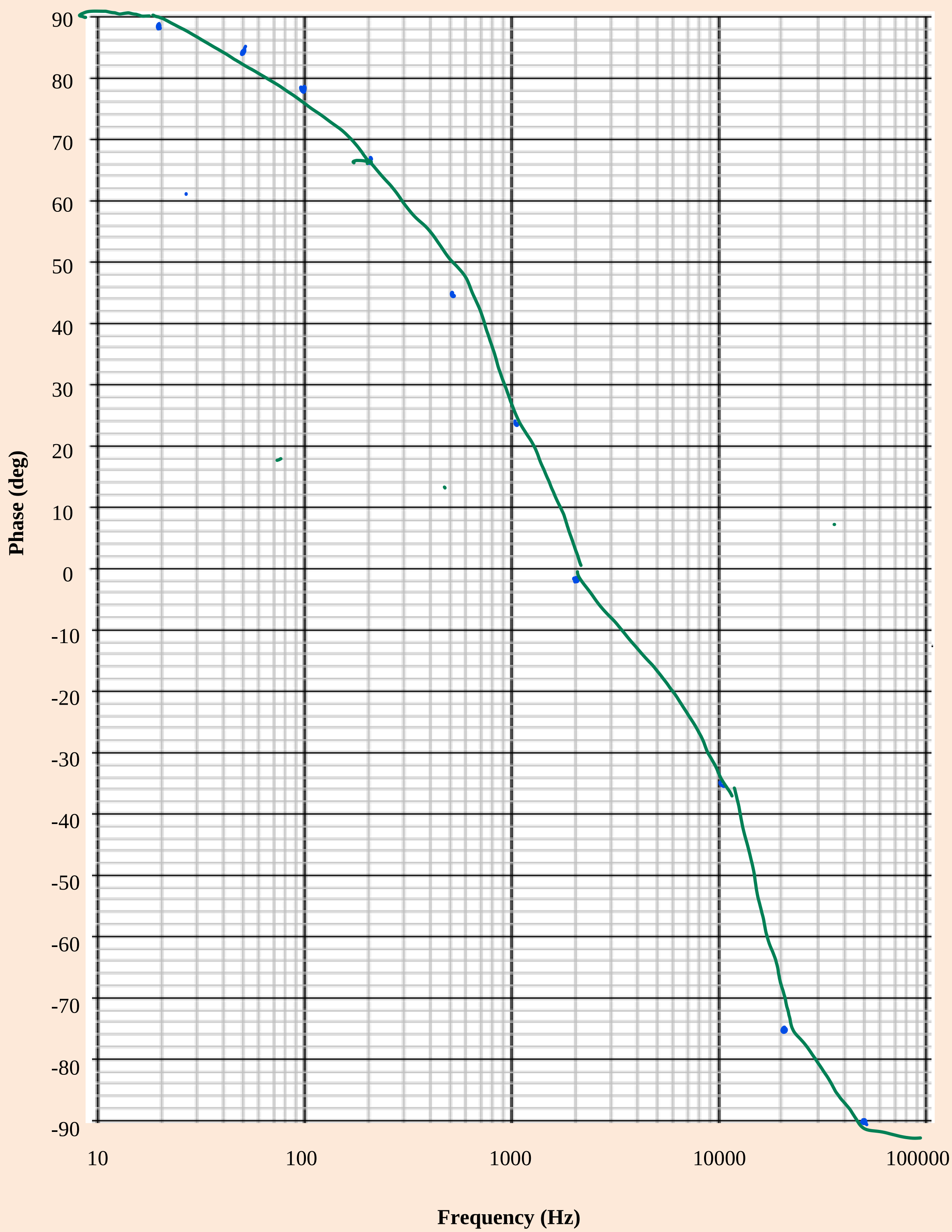
(i) Step 9: Demonstrate Step 6 to Step 8 to your TA. (1 mark)

Table 2.0 (0.5 marks)

Frequency (Hz)	$\lvert V_o \rvert$ (dB)	$\lvert V_i \rvert$ (dB)	$\lvert H(\omega) \rvert$ (dB)	$\angle H(\omega)$ (degrees)
20	-15.97	1.06	-17.03	86.85
50	-8.825		-9.885	83.25
100	-3.35		-4.41	79.6
200	2.6		1.54	68.2
500	8.46		7.4	45.18
1K	10.98		9.92	24.81
2K	11.82		10.76	-1.32
10K	7.35		6.29	-35.01
20K	2.345		1.285	-75.32
50K	-5.19		-6.25	-94.2
100K	-11.05		-12.11	-114.5



Graph 4.0 *Gain* (0.5 marks)



Graph 4.0 ***Phase*** (0.5 marks)

6.0POST-LAB QUESTIONS (2 marks in total, 2/3 marks for each question):

(1) By considering all the asymptotic & measured plots, answer the following:

a) At which frequencies does the asymptotic plot yield the maximum error? What would you do to minimize this error?

b) At what frequency range does the error become negligible?

a) The max error occur at f where the graph is changing slope, since it's difficult to predict where to draw the graph. To minimize errors, more data points should be taken around the f where the slope changes.

b) At f ranges where slope is linear, since the change is constant.

(2) What is the maximum rate of attenuation of | H(ω)| in dB/decade for your plots on in **Graph 3.0**?

Max attenuation of H(ω) in dB/dec is at
100k f.

$$\frac{|V_o|}{|V_i|} = 0.984 \text{ dB/dec}$$

(3) By considering your plots on **Graph 4.0,** answer the following:

a) What is the maximum rate of attenuation of | H(ω)| in dB/decade?

b) Suppose that the 2.7kΩ resistor and the 0.01μF capacitor at the input part of the circuit in **Figure 2.0** were interchanged, what effect would this have on the frequency-response characteristics?

a) Max H(ω) is @ 20 Hz

$$\frac{|V_o|}{|V_i|} = \frac{|15.97|}{|1.06|}$$

$$= 15.06$$

b) If the 2.7K resistor and 0.01 uF are interchanged, dB and phase gain would increase, which increases slope of bode graphs, but graph would remain in high pass