Toronto Metropolitan University

Department of Electrical, Computer, & Biomedical Engineering
Faculty of Engineering & Architectural Science

ELE 302
Laboratory #5

Mutual Inductance

1.0 INTRODUCTION:

When a time-varying current flows in a coil, it generates a time-varying magnetic flux in the space surrounding it. The time-varying magnetic flux induces a voltage in any conductor linked by this flux. Thus, if a second coil exists in a close physical proximity, an induced voltage will be generated across its terminals; its value is related to the time-varying current in the first coil by a parameter known as mutual inductance. Although the coils are physically separate, their interaction is due to the magnetic coupling that exists between them. The practical application of magnetic-coupling is important, especially in electrical power systems and communication systems.

2.0 OBJECTIVES:

- To determine the dot-convention of a given magnetically-coupled set.
- To measure the resistive and inductive parameters of the set.
- To verify the measured-value of the mutual inductance of the set by using an alternative method: measurement of mutual inductance as a function of self-inductance.
- To examine the concept of impedance reflection between magnetically-coupled networks.

3.0 REQUIRED LAB EQUIPMENT & PARTS:

- Function Generator (FG) and Oscilloscope.
- ELE202 Lab Kit and ELE302 Lab Kit: various components, breadboard, wires and jumpers.
- Magnetically-Coupled Coils (from your TA).

4.0 PRE-LAB ASSIGNMENT (3 marks with 1.5 marks for each step):

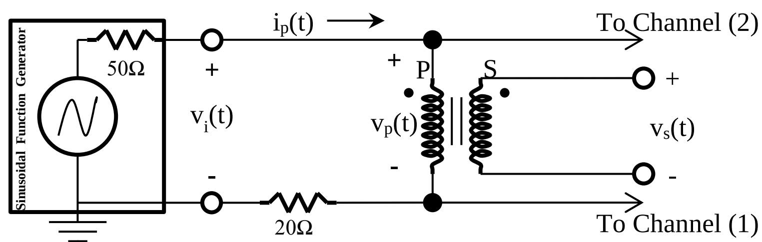


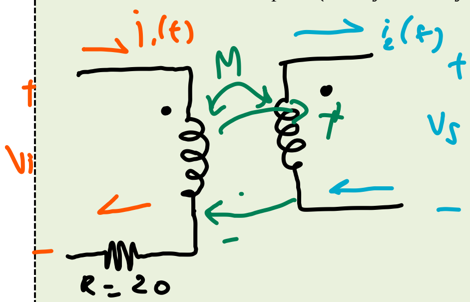
Figure 1.0: Measuring Parameters of Magnetically-Coupled Coils

(a) Step 1: The circuit shown in **Figure 1.0** consists of magnetically-coupled coils, where the primary coil is connected in series with a sinusoidal source and a current-sampling 20Ω-resistor, while the secondary coil is open-circuit; i.e., has no load (NL).

The primary coil has self-inductance L_p and inherent resistance R_p, and the secondary coil has self-inductance L_s and inherent resistance R_s; the mutual inductance between the two coils is M.

i) Determine the expressions for V_i and V_s in terms of I_p and the coils parameters, where V_i, V_s and I_p are the phasors for $v_i(t)$, $v_s(t)$ and $i_p(t)$, respectively.

Pre-Lab workspace (show your analysis here)



$$-V_i + Ri + i \cdot L = 0$$

$$-V_i + Ri_1 + i(-j\omega L) = 0$$

$$V_i = j\omega L i - Ri$$

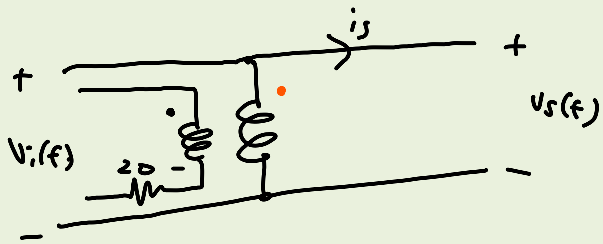
$$V_i = i(j\omega L - R)$$

$$V_s - j\omega M i = 0$$

$$V_s = j\omega M i$$

ii) Suppose that the primary and secondary coils are interchanged; i.e., the primary terminals are now open circuited, and the current flow through L_s is $i_s(t)$. Find the expressions for V_i and V_p in terms of I_s and the coils parameters, where V_i, V_s and I_s are the phasors for $v_i(t)$, $v_s(t)$ and $i_s(t)$, respectively.

Pre-Lab workspace (show your analysis here)



$V_i = j\omega M i$

$V_s = R_s i + 20 i + j\omega M i$

$= (R_s + 20 + j\omega M)\, i$

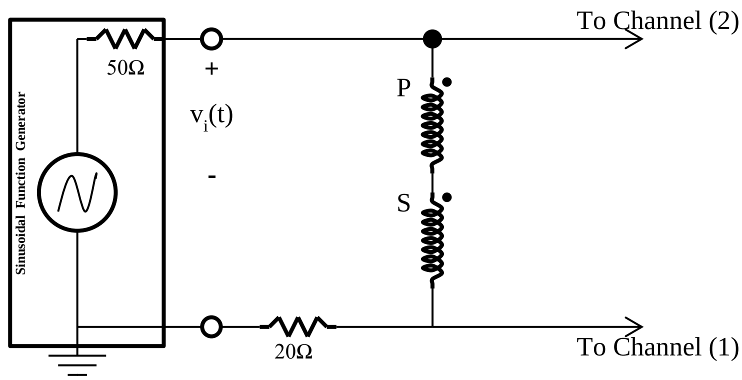


Figure 2.0: Measuring Mutual Inductance as Self-Inductance

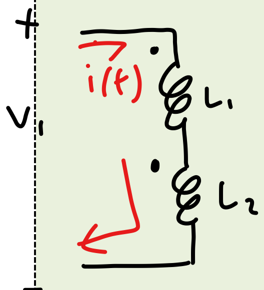
(b) Step 2: The primary and secondary coils shown in **Figure 2.0** are connected in series-aiding with the current i(t) entering each of the coils from its dotted terminal. Let the total inductance of this arrangement be L_A. Upon interchanging the secondary terminals connections, however, total inductance of the series-opposing arrangement becomes L_B.

Show that the mutual inductance M is:

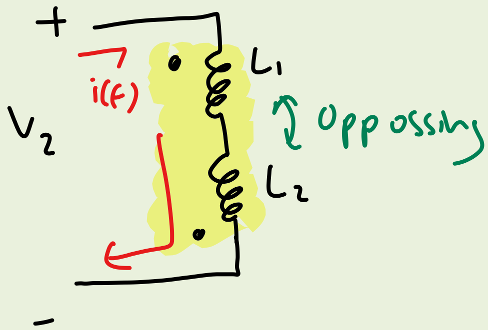
$$M = \frac{[L_A - L_B]}{4}$$

Pre-Lab workspace (show your analysis here)

LA:



LB:



$$V_1 = L_T \frac{di}{dt}$$

$$= (L_1 + L_2 + 2M)\frac{di}{dt}$$

$$V_1 = L_A \frac{di}{dt}$$

$$V_2 = L_T \frac{di}{dt}$$

$$= (L_1 + L_2 - 2M)\frac{di}{dt}$$

$$V_2 = L_B \frac{di}{dt}$$

$$L_A - L_B = (L_1 + L_2 + 2M) - (L_1 + L_2 - 2M)$$

$$= \cancel{L_1} - \cancel{L_1} + \cancel{L_2} - \cancel{L_2} + 2M + 2M$$

$$L_A - L_B = 4M$$

$$M = \frac{L_A - L_B}{4}$$

5.0 IN-LAB IMPEMENTATION & MEASUREMENTS (5 marks in total):

Part I: Measurements of the Parameters of the Magnetically-Coupled Coils

(a) Step 1: Connect the circuit shown in **Figure 1.0**. Connect Channel (1) of the oscilloscope to display the voltage across the 20Ω-current-sampling resistor (please use the 20Ω-resistor from the green-resistance box), and Channel (2) to display the voltage $v_i(t)$. By setting the trigger source at Channel (1) on the *falling edge*, the input current is now selected as the reference sinusoid.

Set the function generator to provide a sinusoidal input voltage $v_i(t)$ of maximum value at a frequency of 50kHz. [$v_i(t)$ maximum should be approximately 0.6V]

(b) Step 2: Use the oscilloscope to measure the phase angle φ of $v_i(t)$ relative to $i_p(t)$, and the peak values of $v_i(t)$ and $v_{20\Omega}(t)$. ***Note that, after pushing the "Auto Scale" button, you might have to adjust the horizontal time scale knob and the trigger knob to get a stable read out on the oscilloscope.***

Determine the no-load input impedance $Z_{i(NL)}$ seen by the source, as:

$$Z_{i(NL)}=\frac{V_i}{I_p}=20+R_p+jX_p$$

Find the numerical values of X_p ($\omega L_p=2\pi f L_p$) and R_p. Record your results in **Table 1.0**.

Table 1.0 (0.5 marks)

φ (degree)	$\lvert V_i \rvert$ (mV)	$\lvert I_p \rvert$ (mA)	$Z_{i(NL)}$ (Ω)	X_p (Ω)	R_p (Ω)
72.57	311.13	12.73	24.4∠73	23.28	7.31

(c) Step 3: Use Channel (2) to display the open-circuit voltage $v_s(t)$, and measure the phase-angle of $v_s(t)$ relative to $i_p(t)$.

Suppose that the top terminal of the primary coil in **Figure 1.0** is selected as the dotted terminal, then based on your phase measurement, which terminal of the secondary coil should be dotted? ……………………………

Measure the peak value of $v_s(t)$. Find the numerical value of the mutual inductance M, and record your results in **Table 2.0**.

Table 2.0 (0.5 marks)

$\lvert V_s \rvert$ (mV)	$\angle V_{s\ to\ i}$ (degrees)	X_M (Ω)
141.42	90.0	18.38

(d) Step 4: Interchange the primary and secondary coils; the primary terminals are now open circuited, and the reference phasor is I_s, instead of I_p. Repeat your measurements as in Step 2 and Step 3. Evaluate the numerical values of X_s, R_s, and X_M ($\omega M=2\pi f M$) , and record your results in **Table 1.0** and **Table 2.0.**

Table 3.0 (0.5 marks)

φ (degree)	$\lvert V_i \rvert$ (mV)	$\lvert I_s \rvert$ (mA)	$Z_{i(NL)}$ (Ω)	X_s (Ω)	R_s (Ω)
41.1	393.98	16.97	23.33	15.34	17.6

Table 4.0 (0.5 marks)

$\lvert V_p \rvert$ (mV)	$\angle V_{p \text{ to } i}$ (degrees)	X_M (Ω)
509.12	90.0	25.45

(e) Step 5: Draw the equivalent circuit of the magnetically-couple coils in the time domain indicating the values of its parameters. Use a numerical value for M that is equal to the average value for M found from **Table 2.0** and **Table 4.0.**

Find the value of the coefficient of coupling:

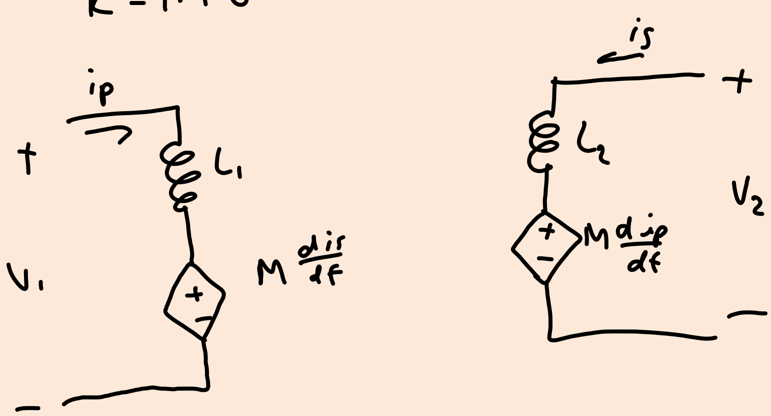
$$k = \frac{M}{\sqrt{L_p L_s}} = \frac{X_M}{\sqrt{X_p X_s}}$$

$$K = \frac{X_M}{\sqrt{X_p X_s}}$$

$$= \frac{\frac{(18.38+25.45)}{2}}{\sqrt{(23.28)(15.39)}}$$

$$= \frac{21.915}{18.8975}$$

$$K = 1.16$$



(f) Step 6: Demonstrate Step 1 to Step 5 to your TA. (1 mark)

Part II: Measurement of the Mutual Inductance as a Self-Inductance

(g) Step 7: Modify your circuit connections as shown in **Figure 2.0**, where the coils are connected in series-aiding arrangement. Repeat your measurements as in Step 2, and determine the value of X_A. Record your results in **Table 5.0**. Note that angle φ in this case is the phase angle of $v_i(t)$ relative to i(t).

Table 5.0 (0.5 marks)

φ (degree)	$\|V_i\|$ (mV)	$\|I\|$ (mA)	$Z_i = V_i / I$ (Ω)	X_A (Ω)
83	388.9	2.3	169.1	167.8

(h) Step 8: Interchange the connections of the secondary terminals; the coils are now in series-opposing form. Repeat your measurements as in Step 2, and determine the value of X_B. Record your results in **Table 6.0:**

Table 6.0 (0.5 marks)

φ (degree)	$\|V_i\|$ (mV)	$\|I\|$ (mA)	$Z_i = V_i / I$ (Ω)	X_B (Ω)
47	434.9	12.8	33.8	24.7

Determine the value of X_M as:

$$= \frac{167.8 - 24.7}{4} = 35.78$$

$$X_M = \frac{X_A - X_B}{4}$$

(i) Step 9: Demonstrate Step 7 and Step 8 to your TA. (1 mark)

6.0 POST-LAB QUESTIONS (2 marks in total, 1 mark for each question):

(1) In comparing the different methods of measuring the value of the mutual inductance between the two coils, which one would you consider to be more reliable? Explain your answer.

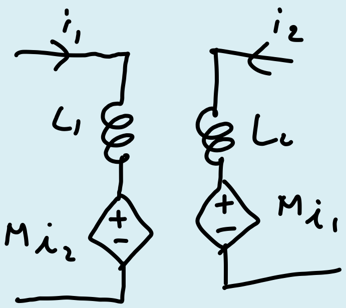
- Inductors connected in series produces a more accurate M, since M is not solely reliant on mag field, which can be influenced by ext forces.

(2) Suppose the coefficient of coupling between the two coils is close to unity, what would be the value of the turn ratio of the coils set?

M

$$K = \frac{M}{\sqrt{L_1 L_2}} \qquad \frac{N_2}{N_1} = ?$$

$$M = k\sqrt{L_1 L_2}$$



$$L = N\phi$$

$$N_1\phi = L_1 i_1 + M i_2 \qquad N_2\phi = L_2 i_2 + M i_1$$

$$N_1\cancel{\phi} = L\,\cancel{i_1} + \cancel{k}\,\cancel{i_2}\sqrt{L_1 \cancel{L_2}} \qquad N_2\cancel{\phi} = L_2\cancel{i_2} + \cancel{k}\sqrt{\cancel{L_1} L_2}\;\cancel{i_1}$$

$$\frac{N_2}{N_1} = \frac{\sqrt{L_2}}{\sqrt{L_1}}$$

$$\frac{N_2}{N_1} = \sqrt{\frac{L_2}{L_1}}$$