COE328: Chapter 2 Practice problems

September 17, 2022 8:10 AM

Answers to problems marked by an asterisk are given at the back of the book.

2.1 Use algebraic manipulation to prove that $x + yz = (x + y) \cdot (x + z)$. Note that this is the distributive rule, as stated in identity 12b in section 2.5.

2.2 Use algebraic manipulation to prove that $(x + y) \cdot (x + \overline{y}) = x$.

***2.7** Determine whether or not the following expressions are valid, i.e., whether the left- and right-hand sides represent the same function.

(a) $\overline{x}_1x_3 + x_1x_2\overline{x}_3 + \overline{x}_1x_2 + x_1\overline{x}_2 = \overline{x}_2x_3 + x_1\overline{x}_3 + x_2\overline{x}_3 + \overline{x}_1x_2x_3$

(b) $x_1\overline{x}_3 + x_2x_3 + \overline{x}_2\overline{x}_3 = (x_1 + \overline{x}_2 + x_3)(x_1 + x_2 + \overline{x}_3)(\overline{x}_1 + x_2 + \overline{x}_3)$

(c) $(x_1 + x_3)(\overline{x}_1 + \overline{x}_2 + \overline{x}_3)(\overline{x}_1 + x_2) = (x_1 + x_2)(x_2 + x_3)(\overline{x}_1 + \overline{x}_3)$

2.8 Draw a timing diagram for the circuit in Figure 2.19a. Show the waveforms that can be observed on all wires in the circuit.

2.9 Repeat problem 2.8 for the circuit in Figure 2.19b.

2.10 Use algebraic manipulation to show that for three input variables x_1, x_2, and x_3

$$\sum m(1, 2, 3, 4, 5, 6, 7) = x_1 + x_2 + x_3$$

2.11 Use algebraic manipulation to show that for three input variables x_1, x_2, and x_3

$$\Pi M(0, 1, 2, 3, 4, 5, 6) = x_1x_2x_3$$

***2.12** Use algebraic manipulation to find the minimum sum-of-products expression for the function $f = x_1x_3 + x_1\overline{x}_2 + \overline{x}_1x_2x_3 + \overline{x}_1\overline{x}_2\overline{x}_3$.

2.13 Use algebraic manipulation to find the minimum sum-of-products expression for the function $f = x_1\overline{x}_2\overline{x}_3 + x_1x_2x_4 + x_1\overline{x}_2x_3\overline{x}_4$.

2.14 Use algebraic manipulation to find the minimum product-of-sums expression for the function $f = (x_1 + x_3 + x_4) \cdot (x_1 + \overline{x}_2 + x_3) \cdot (x_1 + \overline{x}_2 + \overline{x}_3 + x_4)$.

***2.15** Use algebraic manipulation to find the minimum product-of-sums expression for the function $f = (x_1 + x_2 + x_3) \cdot (x_1 + \overline{x}_2 + x_3) \cdot (\overline{x}_1 + \overline{x}_2 + x_3) \cdot (x_1 + x_2 + \overline{x}_3)$.

***2.20** Design the simplest sum-of-products circuit that implements the function $f(x_1, x_2, x_3) = \sum m(3, 4, 6, 7)$.

2.21 Design the simplest sum-of-products circuit that implements the function $f(x_1, x_2, x_3) = \sum m(1, 3, 4, 6, 7)$.

2.22 Design the simplest product-of-sums circuit that implements the function $f(x_1, x_2, x_3) = \Pi M(0, 2, 5)$.

***2.23** Design the simplest product-of-sums expression for the function $f(x_1, x_2, x_3) = \Pi M(0, 1, 5, 7)$.

2.24 Derive the simplest sum-of-products expression for the function $f(x_1, x_2, x_3, x_4) = x_1\overline{x}_3\overline{x}_4 + x_2\overline{x}_3x_4 + x_1\overline{x}_2\overline{x}_3$.

2.25 Derive the simplest sum-of-products expression for the function $f(x_1, x_2, x_3, x_4, x_5) = \overline{x}_1\overline{x}_3\overline{x}_5 + \overline{x}_1\overline{x}_3\overline{x}_4 + \overline{x}_1x_4x_5 + x_1\overline{x}_2\overline{x}_3x_5$. (Hint: Use the consensus property 17a.)

2.26 Derive the simplest product-of-sums expression for the function $f(x_1, x_2, x_3, x_4) = (\overline{x}_1 + \overline{x}_3 + \overline{x}_4)(\overline{x}_2 + \overline{x}_3 + x_4)(x_1 + \overline{x}_2 + \overline{x}_3)$. (Hint: Use the consensus property 17b.)

2.27 Derive the simplest product-of-sums expression for the function $f(x_1, x_2, x_3, x_4, x_5) = (\overline{x}_2 + x_3 + x_5)(x_1 + \overline{x}_3 + x_5)(x_1 + x_2 + x_5)(x_1 + \overline{x}_4 + \overline{x}_5)$. (Hint: Use the consensus property 17*b*.)

***2.28** Design the simplest circuit that has three inputs, x_1, x_2, and x_3, which produces an output value of 1 whenever two or more of the input variables have the value 1; otherwise, the output has to be 0.

2.29 Design the simplest circuit that has three inputs, x_1, x_2, and x_3, which produces an output value of 1 whenever exactly one or two of the input variables have the value 1; otherwise, the output has to be 0.

2.30 Design the simplest circuit that has four inputs, x_1, x_2, x_3, and x_4, which produces an output value of 1 whenever three or more of the input variables have the value 1; otherwise, the output has to be 0.

2.31 For the timing diagram in Figure P2.3, synthesize the function $f(x_1, x_2, x_3)$ in the simplest sum-of-products form.

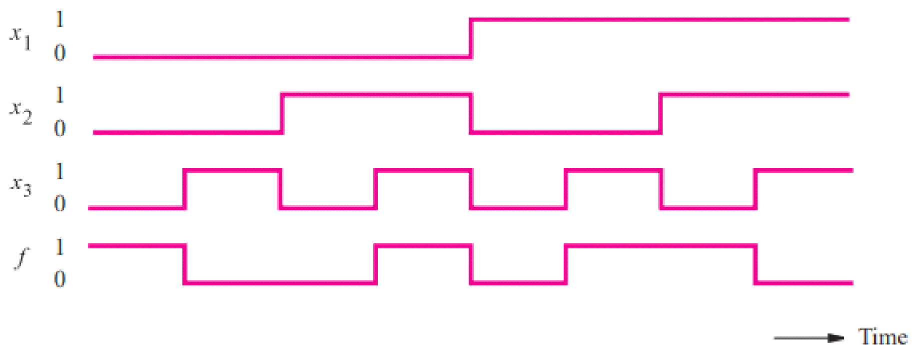

Figure P2.3 A timing diagram representing a logic function.

***2.32** For the timing diagram in Figure P2.3, synthesize the function $f(x_1, x_2, x_3)$ in the simplest product-of-sums form.

***2.33** For the timing diagram in Figure P2.4, synthesize the function $f(x_1, x_2, x_3)$ in the simplest sum-of-products form.

2.34 For the timing diagram in Figure P2.4, synthesize the function $f(x_1, x_2, x_3)$ in the simplest product-of-sums form.

2.35 Design a circuit with output f and inputs x_1, x_0, y_1, and y_0. Let $X = x_1x_0$ be a number, where the four possible values of X, namely, 00, 01, 10, and 11, represent the four numbers 0, 1, 2, and 3, respectively. (We discuss representation of numbers in Chapter 5.) Similarly, let $Y = y_1y_0$ represent another number with the same four possible values. The output f should be 1 if the numbers represented by X and Y are equal. Otherwise, f should be 0.

(a) Show the truth table for f.

(b) Synthesize the simplest possible product-of-sums expression for f.

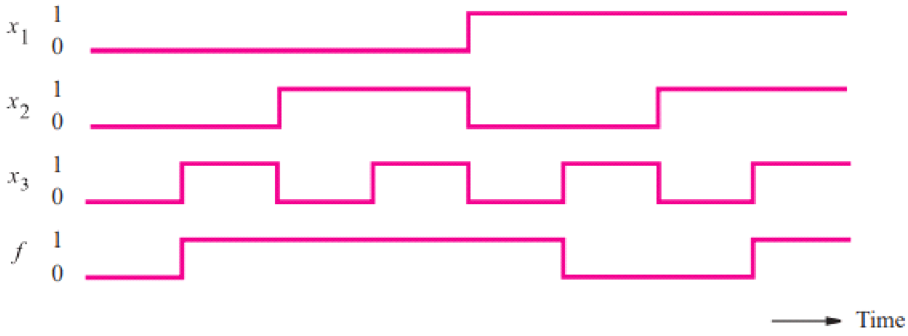

Figure P2.4 A timing diagram representing a logic function.

2.36 Repeat problem 2.35 for the case where f should be 1 only if $X \geq Y$.
(a) Show the truth table for f.
(b) Show the canonical sum-of-products expression for f.
(c) Show the simplest possible sum-of-products expression for f.

2.37 Implement the function in Figure 2.26 using only NAND gates.

2.38 Implement the function in Figure 2.26 using only NOR gates.

2.39 Implement the circuit in Figure 2.35 using NAND and NOR gates.

***2.40** Design the simplest circuit that implements the function $f(x_1, x_2, x_3) = \sum m(3, 4, 6, 7)$ using NAND gates.

2.41 Design the simplest circuit that implements the function $f(x_1, x_2, x_3) = \sum m(1, 3, 4, 6, 7)$ using NAND gates.

***2.42** Repeat problem 2.40 using NOR gates.

2.43 Repeat problem 2.41 using NOR gates.

2.44 Use algebraic manipulation to derive the minimum sum-of-products expression for the function $f = x_1\bar{x}_3 + x_1x_2 + \bar{x}_1\bar{x}_2 + \bar{x}_2x_3$.

2.45 Use algebraic manipulation to derive the minimum sum-of-products expression for the function $f = \bar{x}_1\bar{x}_2x_3 + x_1x_3 + x_2x_3 + x_1x_2\bar{x}_3$.

2.46 Use algebraic manipulation to derive the minimum product-of-sums expression for the function $f = x_2 + x_1x_3 + \bar{x}_1\bar{x}_3$.

2.47 Use algebraic manipulation to derive the minimum product-of-sums expression for the function $f = (x_1 + x_2 + x_3)(x_1 + \bar{x}_2 + x_3)(\bar{x}_1 + x_2 + x_3)(\bar{x}_1 + \bar{x}_2 + x_3)(x_1 + x_2 + \bar{x}_3 + x_4)$.

2.48 (a) Use a schematic capture tool to draw schematics for the following functions

$$f_1 = x_2\bar{x}_3\bar{x}_4 + \bar{x}_1x_2x_4 + \bar{x}_1x_2x_3 + x_1x_2x_3$$
$$f_2 = x_2\bar{x}_4 + \bar{x}_1x_2 + x_2x_3$$

(b) Use functional simulation to prove that $f_1 = f_2$.

2.49 (a) Use a schematic capture tool to draw schematics for the following functions

$$f_1 = (x_1 + x_2 + \bar{x}_4) \cdot (\bar{x}_2 + x_3 + \bar{x}_4) \cdot (\bar{x}_1 + x_3 + \bar{x}_4) \cdot (\bar{x}_1 + \bar{x}_3 + \bar{x}_4)$$
$$f_2 = (x_2 + \bar{x}_4) \cdot (x_3 + \bar{x}_4) \cdot (\bar{x}_1 + \bar{x}_4)$$

(b) Use functional simulation to prove that $f_1 = f_2$.