Analysis Vs synthesis

Analysis →
- Stright-forward technique
- Determin the function of an existing logic circuit

synthesis →
- Reverse technique
- Design a circuit to implement a given function.

Goal: →
- Simplify the function
- Implement the function (minimum-cost logic circuit)

Tools:
- Truth Table
- Timing diagram
- Boolean algebra
- Karnaugh map

Procedures:
- Determine sum-of-product function. (SOP)
- or Determine product-of-sum (POS)
- Design a canonical logic circuit
- Simplify the SOP or POS function
- Develop a minimum-cost circuit.

Functionally equivalent circuit.

Boolean Algebra

Axioms

AND
$$0 \cdot 0 = 0$$
$$0 \cdot 1 = 0 = 1 \cdot 0$$
$$1 \cdot 1 = 1$$

OR
$$0 + 0 = 0$$
$$0 + 1 = 1 = 1 + 0$$
$$1 + 1 = 1$$

NOT
$$\bar{0} = 1$$
$$\bar{1} = 0$$

Single variable

$$x \cdot 0 = 0$$
$$x \cdot 1 = x$$
$$x \cdot \bar{x} = 0$$
$$x \cdot x = x$$
$$x + 0 = x$$
$$x + 1 = 1$$
$$x + \bar{x} = 1$$
$$x + x = x$$
$$\bar{\bar{x}} = x$$

Two/Three variables

$$x \cdot y = y \cdot x$$
$$x + y = y + x$$
$$x \cdot (y \cdot z) = (x \cdot y) \cdot z$$
$$x + (y + z) = (x + y) + z$$
$$\boxed{x + y \cdot z = (x+y) \cdot (x+z)}$$
$$x + x \cdot y = x$$

$$x \cdot (1 + y) = x \cdot 1 = x$$

$$x \cdot y + x \cdot \bar{y} = x \cdot (y + \bar{y}) = x \cdot 1 = x$$

$$(x+y) \cdot (x+\bar{y})$$
$$= x \cdot x + x \cdot \bar{y} + x \cdot y + y \cdot \bar{y}$$
$$= x + x(\bar{y} + y) + 0$$
$$= x + x \cdot 1 = x + x = x$$

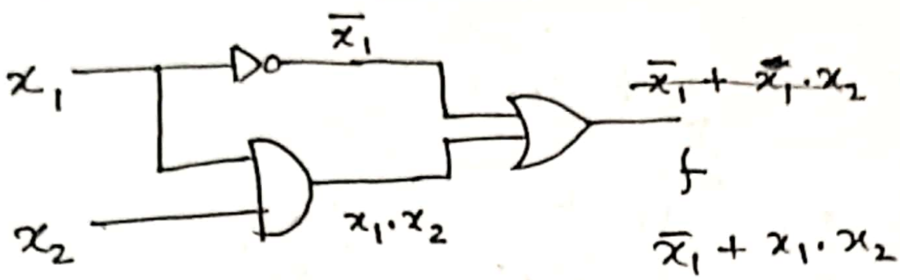
$$\overline{x \cdot y} = \bar{x} + \bar{y}$$ De Morgan's theorem

$$\overline{x + y} = \bar{x} \cdot \bar{y}$$

$$\overline{x + y + z} = \bar{x} \cdot \bar{y} \cdot \bar{z}$$

$$\overline{x + y \cdot z} = \bar{x} \cdot \overline{y \cdot z}$$
$$= \bar{x} \cdot (\bar{y} + \bar{z})$$

Example-1 : Analysis



Given circuit

$\boxed{\text{Function:}}$ $f(x_1, x_2) = \bar{x}_1 + x_1 \cdot x_2$

$1 = x_2 + \bar{x}_2$

$$f = \bar{x}_1 \cdot 1 + x_1 \cdot x_2$$

How do we know to manipulate with this

$$= \bar{x}_1 \cdot (\bar{x}_2 + x_2) + x_1 \cdot x_2$$

$$= \bar{x}_1 \cdot \bar{x}_2 + \bar{x}_1 \cdot x_2 + x_1 \cdot x_2$$

$\boxed{x + x = x}$

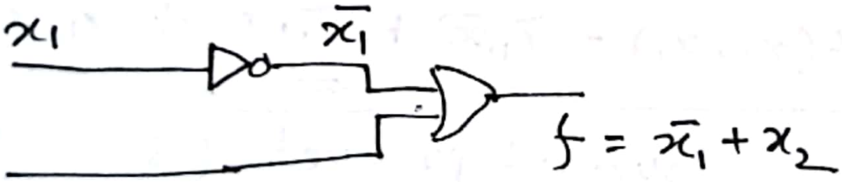
How do we know to repeate this term

Simplification of the function

$$= \bar{x}_1 \cdot \bar{x}_2 + \bar{x}_1 \cdot x_2 + \bar{x}_1 \cdot x_2 + x_1 \cdot x_2$$

$$= \bar{x}_1 (\bar{x}_2 + x_2) + x_2 (\bar{x}_1 + x_1)$$

$$= \bar{x}_1 + x_2$$ ← simplified equation.



Simplified circuit

$f = \bar{x}_1 + x_1 \cdot x_2$

$\bar{x}_1 \rightarrow A$, $x_1 \cdot x_2 \rightarrow B$

Midterm

Oct 24

x_1	x_2	f	A	B
0	0	1	1	0
0	1	1	1	0
1	0	0	0	0
1	1	1	0	1

Forming Truth Table from a given function

Product terms $\rightarrow$ minterms

	x_1	x_2	Minterm
$0 \leftarrow$	0	0	1 $\bar{x}_1 \cdot \bar{x}_2$
$1 \leftarrow$	0	1	1 $\bar{x}_1 \cdot x_2$
$2 \leftarrow$	1	0	0 $x_1 \cdot \bar{x}_2$
$3 \leftarrow$	1	1	1 $x_1 \cdot x_2$

0

$\bar{x}_1 \cdot \bar{x}_2$

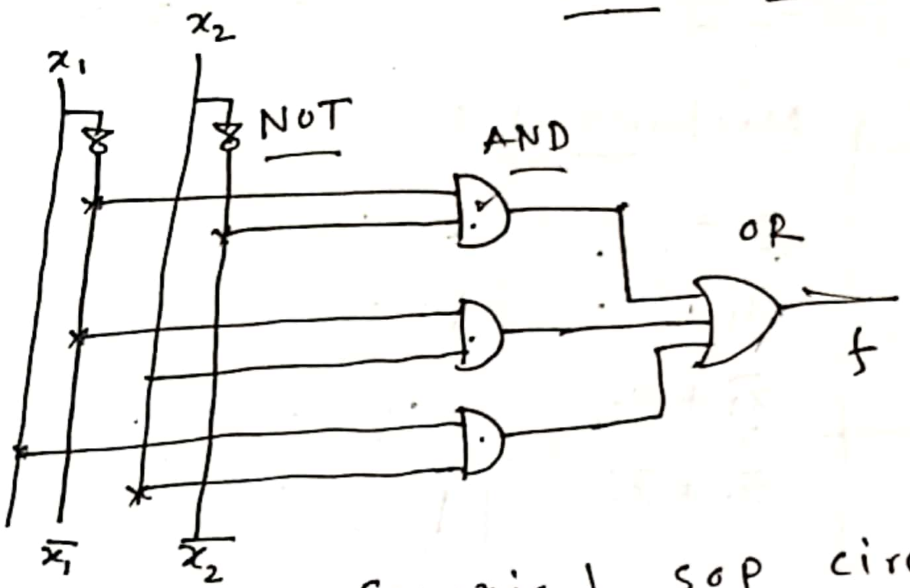
$$\boxed{f(x_1, x_2) = \bar{x}_1 \cdot \bar{x}_2 + \bar{x}_1 \cdot x_2 + x_1 \cdot x_2}$$

Sum of product (SOP) function

$$f(x_1, x_2) = \sum m(0, 1, 3)$$

SOP Function

$$f(x_1, x_2) = \bar{x}_1 \cdot \bar{x}_2 + \bar{x}_1 \cdot x_2 + x_1 \cdot x_2$$



Canonical SOP circuit

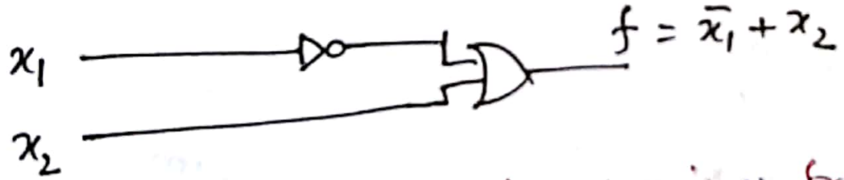
$$f = \bar{x}_1 \bar{x}_2 + \bar{x}_1 x_2 + x_1 x_2$$

Simplification

$$= \bar{x}_1 \bar{x}_2 + \bar{x}_1 x_2 + \bar{x}_1 x_2 + x_1 x_2$$

$$= \bar{x}_1 (\bar{x}_2 + x_2) + (\bar{x}_1 + x_1) x_2$$

$$= \bar{x}_1 + x_2$$



Simplified circuit from the SOP

POS : Product - of - sum

P (AND) OS (OR)

NOT - OR - AND

	x_1	x_2	Maxterms	f
0	0	0	$x_1 + x_2$	1
1	0	1	$\bar{x}_1 + \bar{x}_2$	1
2	1	0	$\bar{x}_1 + x_2$	0
3	1	1	$\bar{x}_1 + \bar{x}_2$	1

POS Function: $f(x_1, x_2) = \pi M(2)$

$$\boxed{f = \bar{x}_1 + x_2}$$

Another Example

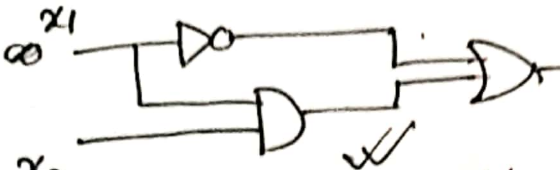
$$\boxed{f = x_1 x_2 + \bar{x}_1 x_2}$$ → Given function

Truth Table

x_1	x_2	f
0	0	0
0	1	1
1	0	0
1	1	1

SOP → $f = \Sigma m(1, 3)$ (sum)

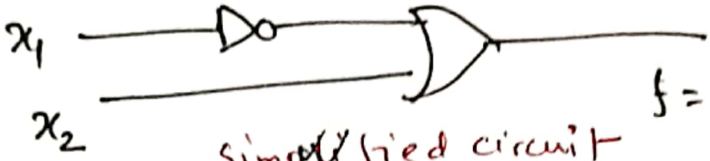
POS → $f = \pi M(0, 2)$



Original circuit

$\boxed{f = \bar{x}_1 + x_1 \cdot \bar{x}_2}$

$\bar{x}_1 + x_1 \cdot x_2$



simplified circuit

$f = \bar{x}_1 + \bar{x}_2$

A + B

Truth Table of The original circuit (see page#4)

x_1	x_2	f
0	0	1
0	1	1
1	0	0
1	1	1

$\bar{x}_1$

Truth Table of the simplified circuit

x_1	x_2	f
0	0	1
0	1	1
1	0	0
1	1	1

Functionally equivalent circuit

A	B
1	0
1	1
0	0
0	1

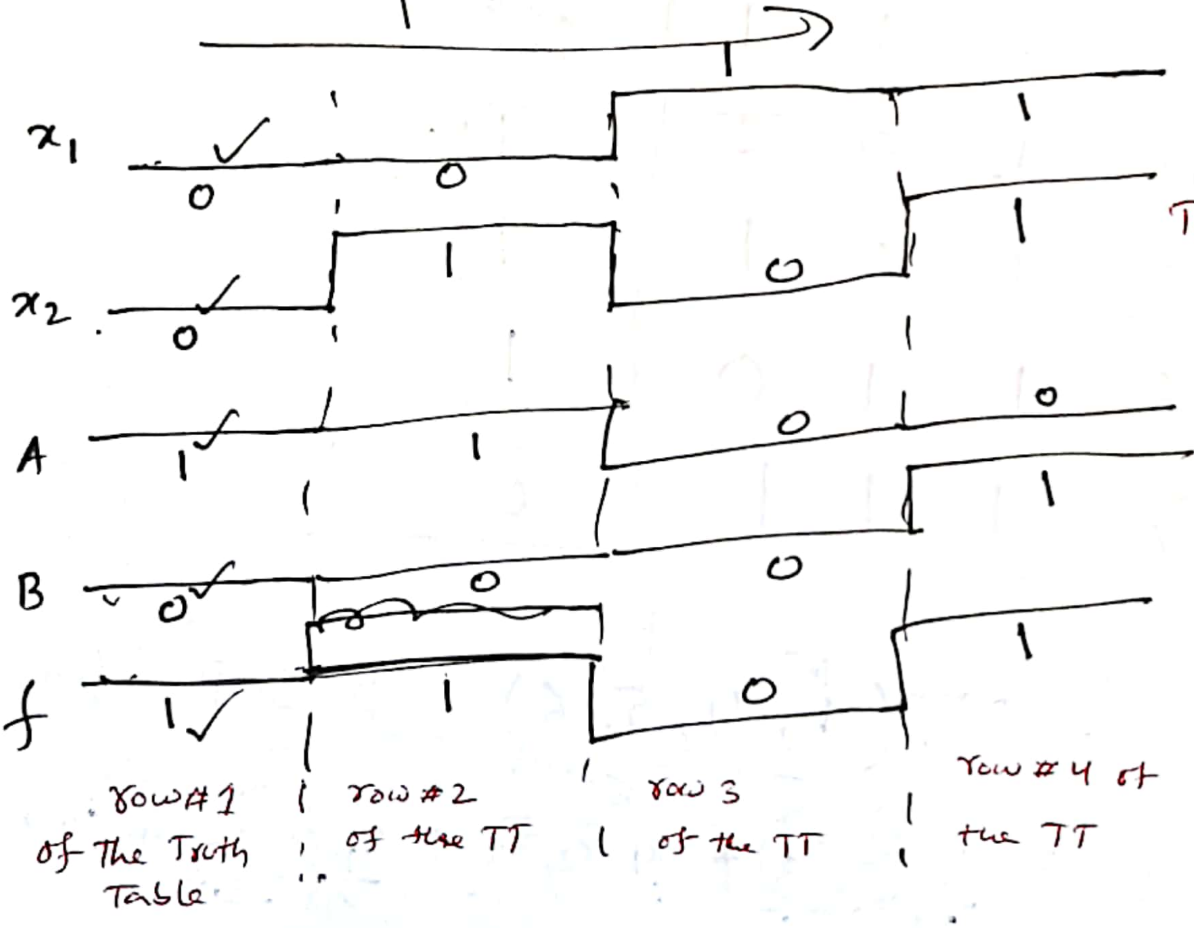
Timing diagram

Top → bottom

Truth Table

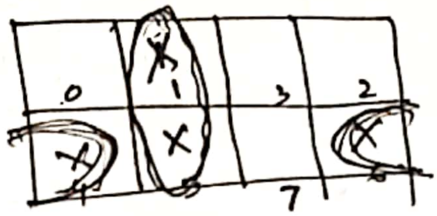
x_1	x_2	f	A	B
0	0	1	1	0
0	1	1	1	0
1	0	0	0	0
1	1	1	0	1

Left to right



Timing diagram

Another Example



A Given Truth Table

	x_1	x_2	x_3	f
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	0
4	1	0	0	1
5	1	0	1	1
6	1	1	0	1
7	1	1	1	0

$f = \Sigma(1, 4, 5, 6)$ SOP function

Simplified equation

$$= \bar{x}_1 \bar{x}_2 x_3 + x_1 \bar{x}_2 \bar{x}_3 + x_1 \bar{x}_2 x_3 + x_1 x_2 \bar{x}_3$$

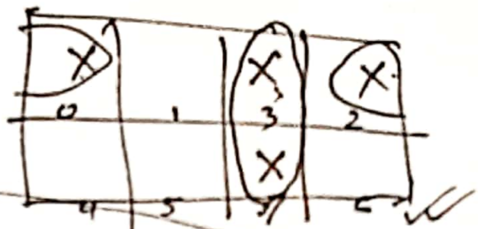
$$= \bar{x}_2 x_3 (\bar{x}_1 + x_1) + x_1 \bar{x}_3 (\bar{x}_2 + x_2)$$

$$= \bar{x}_2 \bar{x}_3 + x_1 \bar{x}_3$$

$\boxed{\text{Draw the simplified circuit}}$

POS

$$f = \pi(0, 2, 3, 7)$$

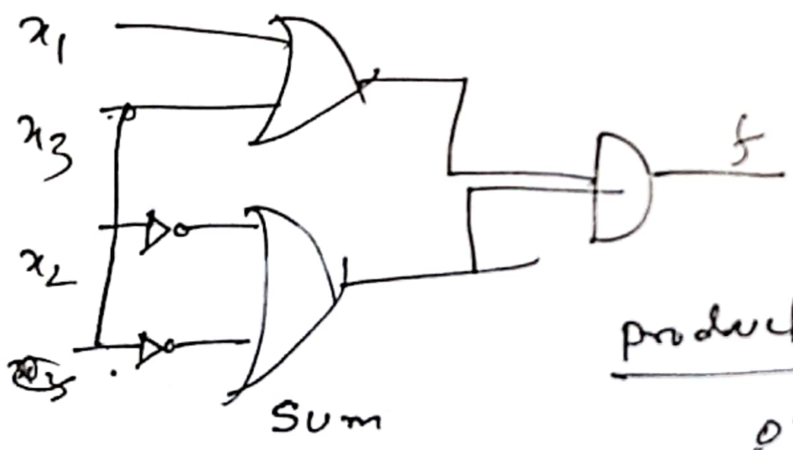


$$f = (x_1 + x_2 + x_3)(x_1 + \bar{x}_2 + x_3)(x_1 + \bar{x}_2 + \bar{x}_3)(\bar{x}_1 + \bar{x}_2 + \bar{x}_3)$$

$$= [(x_1 + x_3) + x_2] \cdot [(x_1 + x_3) + \bar{x}_2] \cdot [x_1 + (\bar{x}_2 + \bar{x}_3)][\bar{x}_1 + (\bar{x}_2 + \bar{x}_3)]$$

$$(X \quad Y)\ (X + \bar{Y})$$

$$= (x_1 + x_3)(\bar{x}_2 + \bar{x}_3)$$



product