# Multiplexer

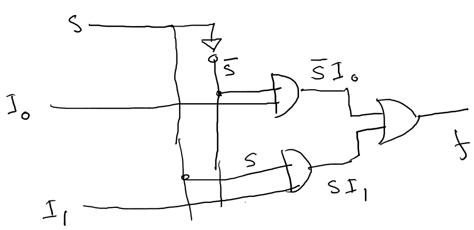
Inputs → Selection lines → 'n' lines   e.g. 2

Inputs → Data inputs → $2^n$   $2^2 = 4$ inputs

Output → one

| Selection | |
|---|---|
| 1 | 2-to-1 |
| 2 | 4 to 1 |
| 3 | 8 to 1 |
| 4 → | 16 to 1 |

## 2-to-1 MUX

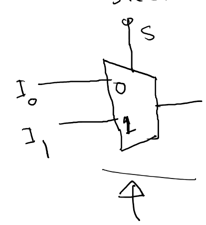



$f = \bar{S} I_0 + S I_1$

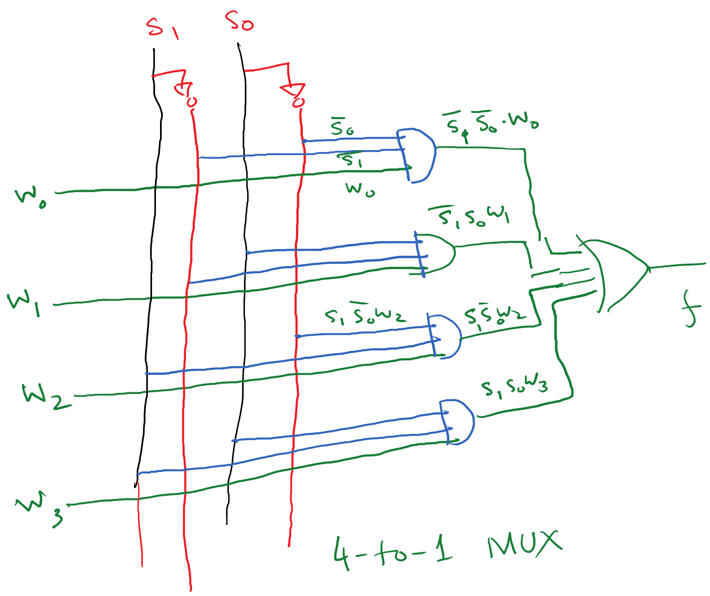
Truth Table

| S | f |
|---|---|
| 0 | $I_0$ |
| 1 | $I_1$ |

Slect line

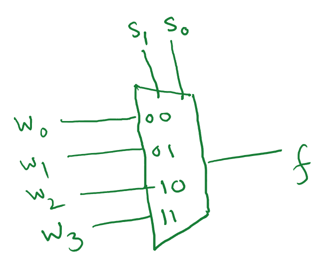



$f = \bar{S} I_0 + S I_1$

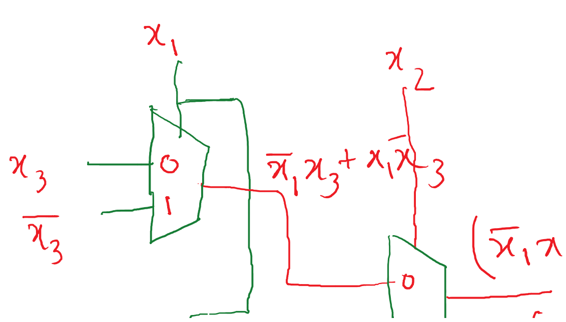



4-to-1 MUX

| $S_1$ | $S_0$ | f |
|---|---|---|
| 0 | 0 | $w_0$ |
| 0 | 1 | $w_1$ |
| 1 | 0 | $w_2$ |
| 1 | 1 | $w_3$ |

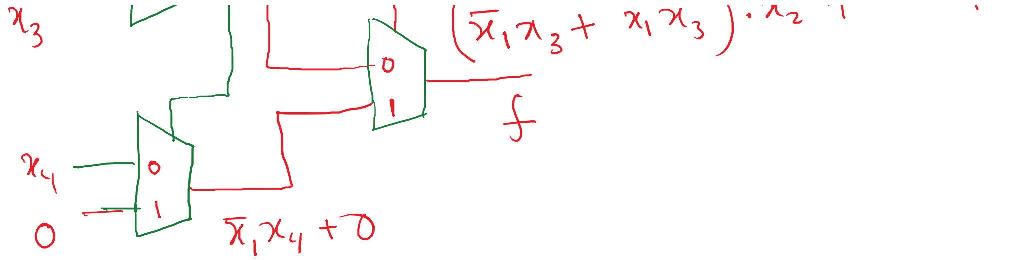



4-to-1 MUX

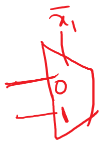



$\bar{x}_1 x_3 + x_1 \bar{x}_3$

$(\bar{x}_1 x_3 + x_1 \bar{x}_3) \cdot \bar{x}_2 + \bar{x}_1 x_4 \cdot x_2$


$x_3$
$(\bar{x}_1 x_3 + x_1 x_3)$
0
1
$f$
$x_4$
0
0
1
$\bar{x}_1 x_4 + 0$

# Shannon's expansion




$$f(x_1, x_2, x_3, \dots x_n) = \bar{x}_1 \underline{f(0, x_2, x_3 \cdots x_n)} + x_1 \underline{f(1, x_2, x_3, \dots x_n)}$$

$$= \bar{x}_1 f_{\bar{x}_1} + x_1 f_{x_1}$$

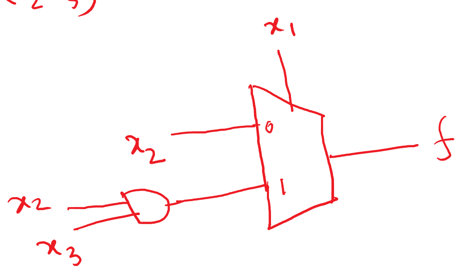
co-factor for $\bar{x}_1$

co-factor for $x_1$

$$f(x_1, x_2, x_3 \dots x_n) = \bar{x}_1 \bar{x}_2 f_{\bar{x}_1 \bar{x}_2} + \bar{x}_1 x_2 f_{\bar{x}_1 x_2} + x_1 \bar{x}_2 f_{x_1 \bar{x}_2} + x_1 x_2 f_{x_1 x_2}$$

$$f = x_1 x_2 x_3 + \bar{x}_1 x_2$$

$$= \bar{x}_1 (x_2) + x_1 (x_2 x_3)$$



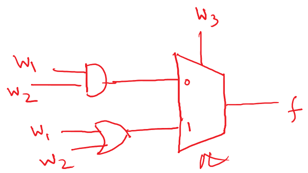
- $f = w_1w_2 + w_1w_3 + w_2w_3$

2-to-1 MUX

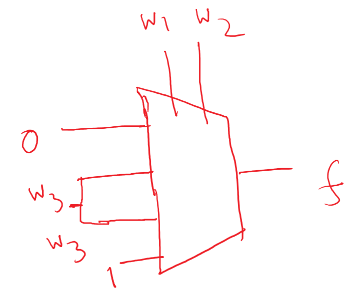
$w_3$ is not specified here

$$f = \bar{w}_3(w_1w_2) + w_3(w_1w_2 + w_1 + w_2)$$

$$= \bar{w}_3(w_1w_2) + w_3[w_1(w_2 + 1) + w_2] = \bar{w}_3(w_1w_2) + w_3(w_1 + w_2)$$




$$f = \bar{w}_1\bar{w}_2(0) + \bar{w}_1w_2(w_3) + w_1\bar{w}_2(w_3) + w_1w_2(1)$$



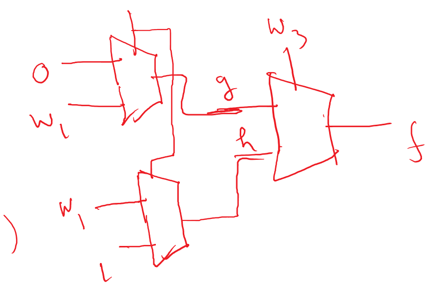

Design using 2-to-1 MUX only

$$f = \bar{w}_3(w_1w_2) + w_3(w_1 + w_2)$$

$$= \bar{w}_3 g + w_3 h$$

$g = w_1w_2$ $\qquad$ $h = w_1 + w_2$

$g = \bar{w}_2(0) + w_2(w_1)$ $\qquad$ $h = \bar{w}_2(w_1) + w_2(1)$



# Design #1: Use 2-to-1 MUX to implement $F(x_1, x_2, x_3) = \sum m(0, 3, 5, 7)$

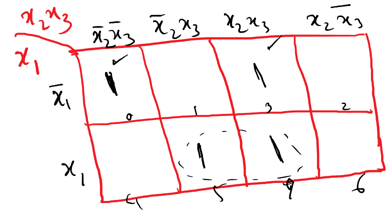
$$f = \bar{x}_1(\bar{x}_2\bar{x}_3 + x_2x_3) + x_1\overbrace{(\bar{x}_2x_3 + x_2x_3)}^{x_3(\bar{x}_2 + x_2)}$$

$$= \bar{x}_1(\underline{\bar{x}_2\bar{x}_3 + x_2x_3}) + x_1x_3$$

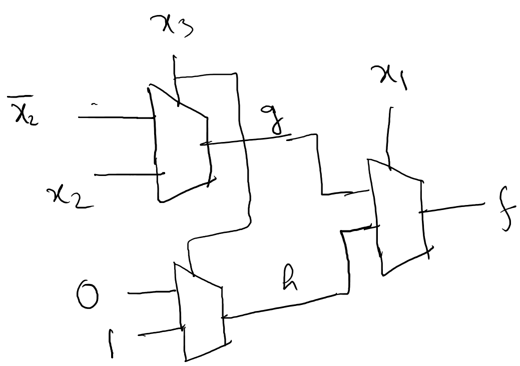
$$= \bar{x}_1 g + x_1 h$$

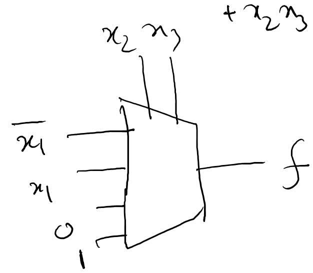
$$g = \bar{x}_2\bar{x}_3 + x_2x_3 \qquad h = x_3$$

$$g = \bar{x}_3(\bar{x}_2) + x_3(x_2) \qquad h = \bar{x}_3(0) + x_3(1)$$




$$f = \bar{x}_2\bar{x}_3(\bar{x}_1) + \bar{x}_2x_3(x_1) + x_2\bar{x}_3(0) + x_2x_3(1)$$



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- $F = \bar{x}_1\bar{x}_2\bar{x}_4 + \bar{x}_1 x_2 x_3 + \underline{x_2\ x_4} + x_1\ x_2 + x_3\ x_4$

$$F = \bar{x}_1\left(\bar{x}_2\bar{x}_4 + x_2x_3 + x_2x_4 + x_3x_4\right) + x_1\left(x_2x_4 + x_2 + x_3x_4\right)$$

$$= \bar{x}_1 g + x_1 h$$

$$g = \bar{x}_2\bar{x}_4 + x_2x_3 + \underline{\underline{x_2x_4}} + x_3x_4$$

$$= \bar{x}_2\bar{x}_4(1) + \bar{x}_2x_4(x_3) + x_2\bar{x}_4(x_3) + x_2x_4(1)$$

$$h = x_2x_4 + x_2 + x_3x_4$$

$$h = \bar{x}_2\bar{x}_4(0) + \bar{x}_2x_4(x_3) + x_2\bar{x}_4(1) + x_2x_4(1)$$

16 to - 1

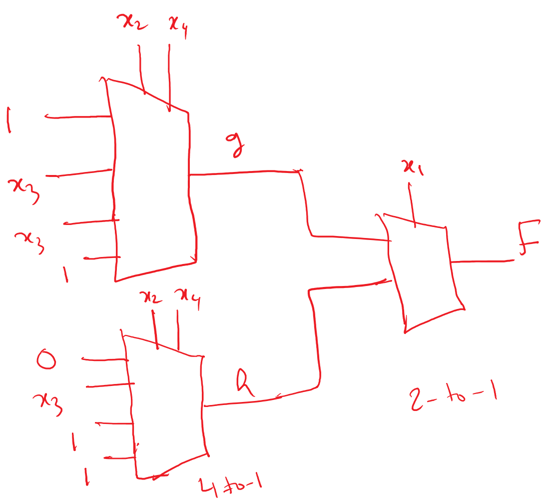
Using 2 to - 1

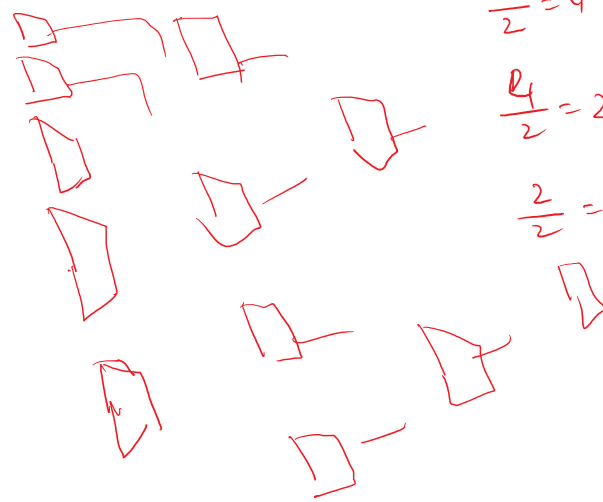
$\frac{16}{2} = 8$ ✓

$\frac{8}{2} = 4$ ✓

$\frac{4}{2} = 2$ ✓

$\frac{2}{2} = 1$ ✓



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- $F = \bar{x}_1\bar{x}_2\bar{x}_4 + \bar{x}_1 x_2 x_3 + x_2\, x_4 + x_1\, x_2 + x_3\, x_4$

$= \bar{x}_1\bar{x}_2(\bar{x}_4 + x_3x_4) + \bar{x}_1 x_2 (x_3 + x_4 + x_3 x_4) + x_1\bar{x}_2 (x_3 x_4) + x_1 x_2 (1)$

$= \bar{x}_1\bar{x}_2\, g_1 + \bar{x}_1 x_2\, g_2 + x_1\bar{x}_2\, g_3 + x_1 x_2\, g_4$

$g_1 = \bar{x}_4 + x_3 x_4$

$g_2 = x_3 + x_4 + x_3 x_4 = x_3(1 + x_4) + x_4 = x_3 + x_4$

$g_3 = x_3 x_4$

$g_4 = 1$

$g_1 = \bar{x}_4 (1) + x_4 (x_3)$

$g_2 = \bar{x}_4 (x_3) + x_4 (1)$

$g_3 = \bar{x}_4 (0) + x_4 (x_3)$

$g_4 = 1 = \bar{x}_4 + x_4$