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Department of Electrical, Computer, & Biomedical Engineering
Faculty of Engineering & Architectural Science

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Instructor:	Andy Gean Ye

Assignment/Lab Number:	2
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Toronto Metropolitan University
Department of Electrical, Computer, & Biomedical Engineering
Faculty of Engineering & Architectural Science

ELE 302
Laboratory #2

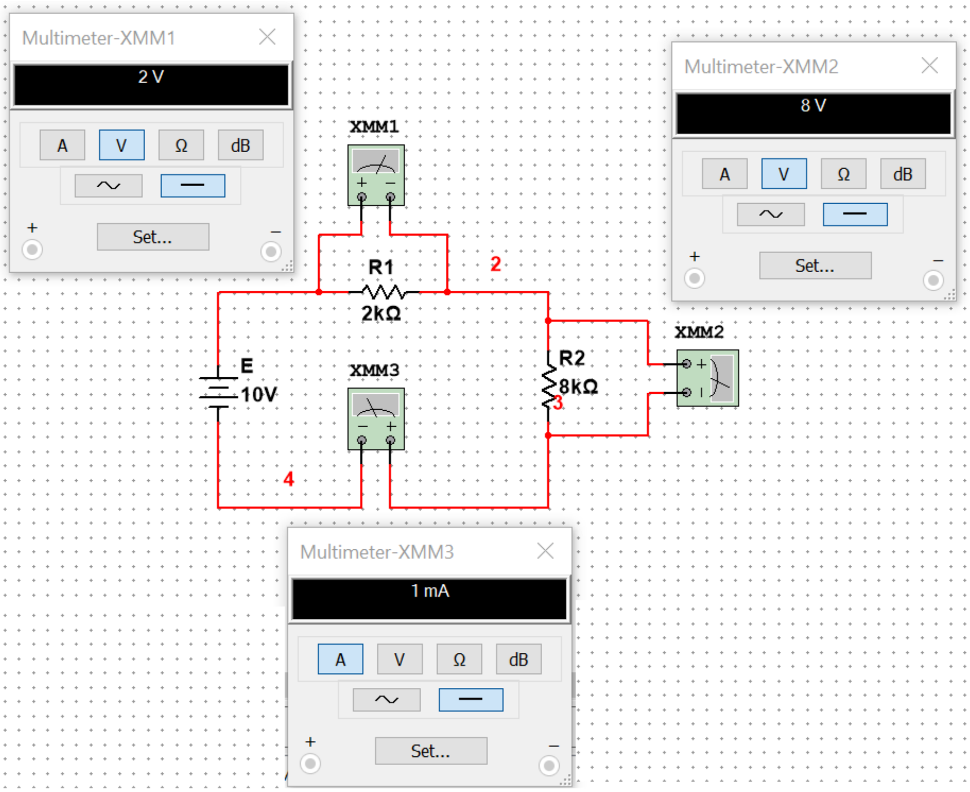
The Step Response of Second-Order Circuits

1.0 INTRODUCTION:

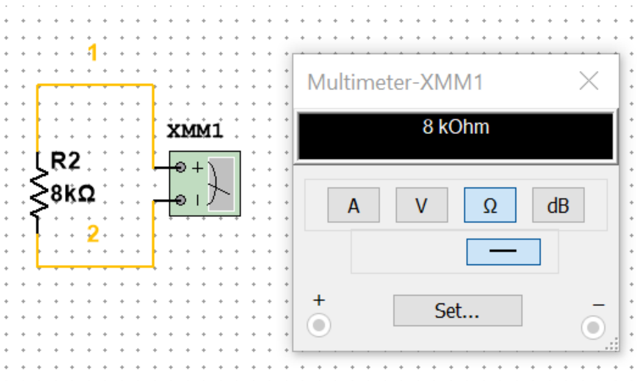
Circuits containing energy-storage elements (capacitors and/or inductors) are knows as dynamic circuits. When switching occurs in a dynamic circuit, the circuit response will go through a transition period prior to settling down to a steady-state value. In applications such as data-acquisition, instrumentation, and computer-control systems, the settling time is an important parameter, as circuits must be allowed to settle to steady state before readings are taken.

Dynamic circuits are often characterized by applying a step-function input. The resulting step-response provides important insights into the response of dynamic circuits in general. By investigating the step response, we discover that it consists of a dc-component called the forced response, and a rapidly vanishing time-varying component, called the natural-response. The form of the time function for the natural component depends on the order and composition of the circuit. The natural response of a second-order circuit is one-out-of-three possible functions known as over damped, critically damped, and under damped, with the under damped case being an exponentially-decaying sinusoid.

This experiment examines the step response of various second-order dynamic circuits. We begin by providing a brief review of the Multisim circuit simulation software and sinusoidal functions.



(i)



(ii)

Figure 1.0a: Examples of using MultiSIM

1.1 Multisim

MultiSIM is an electronic schematic capture and simulation program used to analyze circuit behavior. All AC/DC voltages, AC/DC currents, resistance, frequency, phase-shift, time-domain waveform, etc. can be determined using this software. An example circuit simulation measurement is shown below in **Figure 1.0a.** In this simulation, all components are visually laid out in a way that is the same as a circuit diagram. Each DMM configuration is connected the same way that a physical DMM would be connected on the breadboard. Results are obtained by running the simulation and then double clicking on each piece of equipment to read the desired output values. Refer to the MultiSIM software download procedures, related FAQs and video tutorials on the course website (D2L) to get acquainted with proper use of this simulation tool, and become proficient at it.

1.2 Sinusoidal Functions

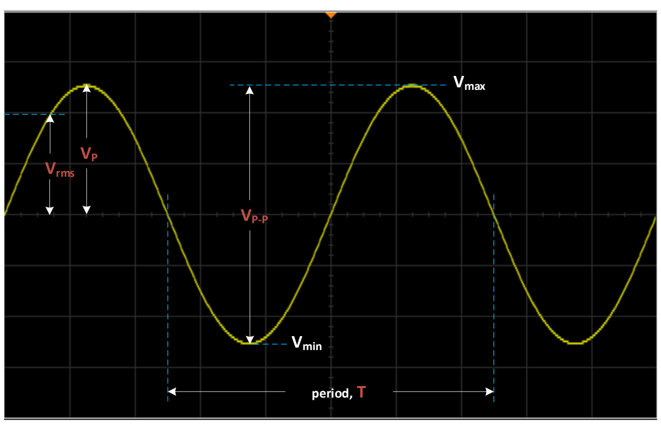


Figure 1.0b: Characterization of a Time-Varying Sinusoidal Signal

A ***sine-wave*** is shown in **Figure 1.0b**. The sinusoidal signal periodically varies with time. It can be characterized by a number of parameters, some of which are shown in **Figure 1.0b**:

- Peak-To-Peak Voltage: $\mathbf{V_{P-P}} = \mathbf{V_{max}} - \mathbf{V_{min}}$
- Amplitude of the sinusoidal signal is defined as its Peak Voltage: $\mathbf{V_P} = \frac{\mathbf{V_{P-P}}}{2}$
- Root-Mean-Squared Voltage: $\mathbf{V_{rms}} = \frac{\mathbf{V_P}}{\sqrt{2}} = 0.707\mathbf{V_P}$
- The period, **T** is defined as the time within which the signal repeats.

- The frequency, **f** is equal to the number of repetitions per unit of time, and can be calculated from the period, **T**: $\mathbf{f}(\mathrm{Hz}) = \frac{1}{\mathbf{T}(\mathrm{sec})}$ or $\mathbf{T}(\mathrm{sec}) = \frac{1}{\mathbf{f}(\mathrm{Hz})}$.
  When frequency is given in radians/sec then the symbol, $\boldsymbol{\omega}$ is used, where $\boldsymbol{\omega} = 2\pi\mathbf{f}$
- Phase Shift, $\boldsymbol{\Theta}$ of one sinusoidal signal with respect to another (of the same frequency) occurs when there is time-offset, $\boldsymbol{\Delta}\mathbf{T}$ between them. Phase-Shift => $\boldsymbol{\theta} = \frac{\Delta\mathrm{T}}{\mathrm{T}} . 2\pi\ (\text{radians}) = \frac{\Delta\mathrm{T}}{\mathrm{T}} . 360°(\text{degrees})$
- Sinusoidal AC voltage as a function of time: $v(\mathrm{t}) = \mathbf{V_P}\cos(\boldsymbol{\omega}\mathrm{t} + \boldsymbol{\theta}) = \mathbf{V_P}\cos(2\pi\mathbf{f}\mathrm{t} + \boldsymbol{\theta})$

2.0 OBJECTIVES:

- To use Multisim circuit simulation software to plot the step response of second-order dynamic circuits.
- To use the oscilloscope to display the step response of second-order dynamic circuits.
- To measure the parameters that characterize the step response of second-order dynamic circuits.

3.0 REQUIRED LAB EQUIPMENT & PARTS:

- Function Generator (FG) and Oscilloscope.
- ELE202 Lab Kit and ELE302 Lab Kit: various components, breadboard, wires and jumpers.

4.0 PRE-LAB ASSIGNMENT (3 marks with 0.75 marks for each step):

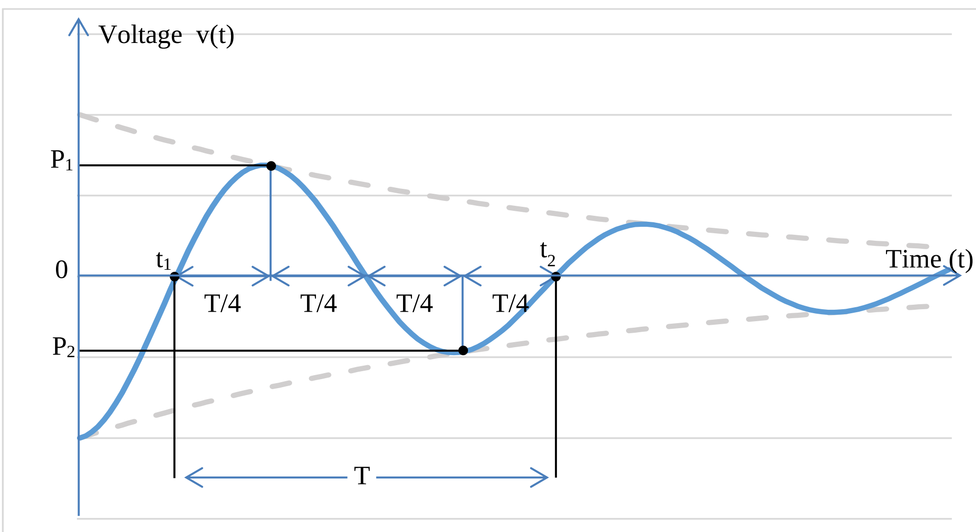


Figure 2.0: An Exponentially Decaying Sinusoidal Function

(a) Step 1: The circuit shown in **Figure 2.0** is an exponentially decaying sinusoidal function that can be characterized by the function: $\mathrm{v(t)} = Ae^{-\sigma t}\cos(\boldsymbol{\omega}\mathrm{t} + \boldsymbol{\theta})$. The figure defines three variables T, P_1, and P_2. t_1 is the time of the first zero-crossing of v(t) and t_2 is time for the third zero-crossing of v(t). T is the time difference between t_1 and t_2. P_1 is the value of v(t) one quarter of the way between the first zero-crossing and the third zero-crossing (i.e. $\mathrm{P_1} = \mathrm{v}(\mathrm{t_1} + \frac{\mathrm{T}}{4})$). Finally, P_2 is the value of v(t) three quarters of the way between the first zero-crossing and the third zero-crossing (i.e. $\mathrm{P_2} = \mathrm{v}(\mathrm{t_1} + 3\frac{\mathrm{T}}{4})$).

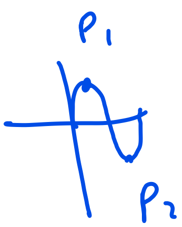
i) Find an expression for ω in terms of T.

Pre-Lab workspace (show your analysis here)

$$\omega = \frac{2\pi}{T}$$

ii) Find an expression for σ in terms of P_1, P_2, and T.

Pre-Lab workspace (show your analysis here)



$P_1 = v\left(\frac{T}{4}\right)$ $\quad v(t) = Ae^{-\sigma t}\cos(\omega t + \theta)$

$P_2 = v\left(\frac{3T}{4}\right)$ $\quad = Ae^{-\sigma t}\sin(t)$

$$\frac{v\left(\frac{3T}{4}\right)}{v\left(\frac{T}{4}\right)} = \frac{\cancel{A}e^{-\sigma\left(\frac{3T}{4}\right)}\cancel{\sin\left(\frac{3\pi}{2}\right)} -1}{\cancel{A}e^{-\sigma\left(\frac{T}{4}\right)}\cancel{\sin\left(\frac{\pi}{2}\right)} 1}$$

$$\frac{v\left(\frac{3T}{4}\right)}{v\left(\frac{T}{4}\right)} = -e^{-\sigma\left(\frac{3T}{4} - \frac{T}{4}\right)}$$

$$\sigma = \frac{-\ln\left(\frac{P_2}{P_1}\right)}{\left(\frac{3T}{4} - \frac{T}{4}\right)}$$

iii) Find an expression of θ in terms of t_1 and T.

Pre-Lab workspace (show your analysis here)

$$\cancel{Ae^{-\sigma t}}\cos(\omega t_1 + \theta)^{-1} = 0^{-1}$$

$$\omega t_1 + \theta = \frac{\pi}{2}$$

$$\theta = \frac{\pi}{2} - \omega t_1$$

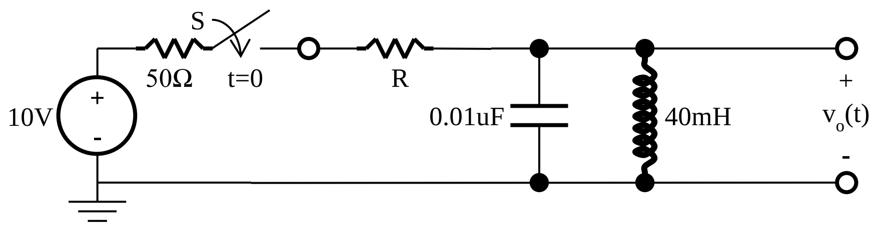
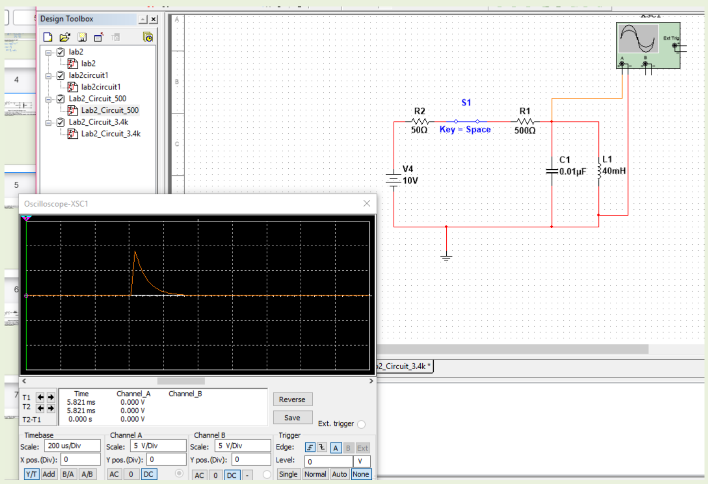


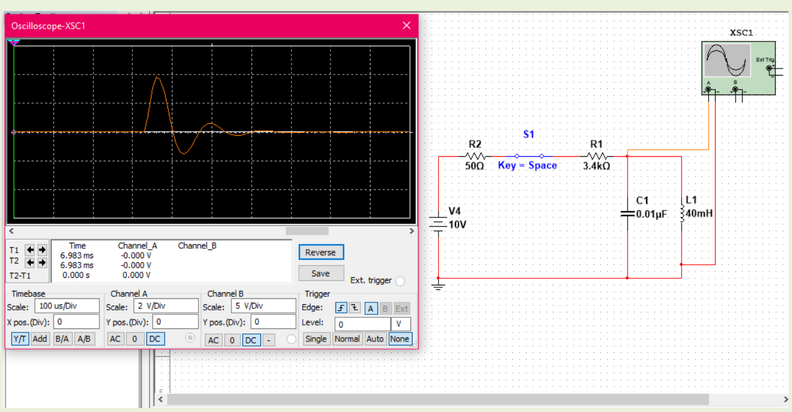
Figure 2.0: Second-Order Bandpass Circuit

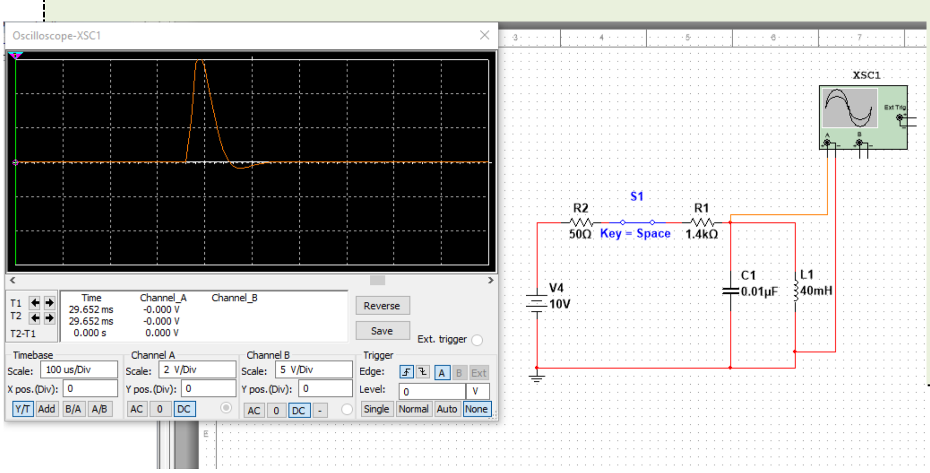
(b) Step 2: Consider the dynamic circuit shown in **Figure 2.0**. The switch S has been open for a long time. At t=0, the switch is closed, where it remains for a long time.

i) Use Multisim to plot the step response [$v_o(t)$ for $t \geq 0^+$] when R=1.4kΩ, 3.4kΩ, and 500Ω.

Pre-Lab workspace (show your analysis here)







ii) Use the plots to calculate the parameters (σ and ω) that characterize the step response as: $\mathbf{v_o}(t) = Ae^{-\sigma t}\sin(\omega t)$ for R=3.4kΩ.

Pre-Lab workspace (show your analysis here)

$$\sigma = \frac{-\ln\left(\frac{V_2}{V_1}\right)}{\left(\frac{T}{2}\right)}$$

$$= \frac{-\ln\left|\frac{-1.543}{3.836}\right|}{\left(\frac{140.127\times10^{-6}}{2}\right)}$$

$$= \frac{}{7.006\times10^{-5}}$$

$$= 12\,998.88$$

$$\sigma = 13\,000$$

$$\omega = \frac{2\pi}{T}$$

$$\omega = \frac{2\pi}{140.127\,\mu s}$$

$$\omega = 44840$$

$$V_o(t) = Ae^{-13kt}\sin(44.8kt)$$

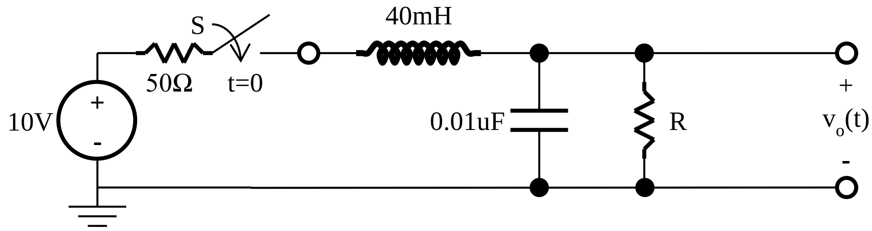
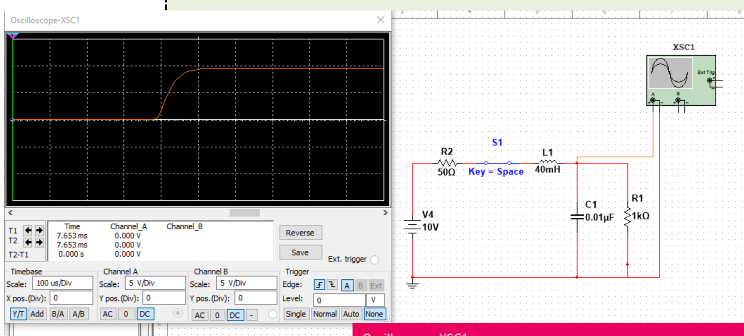


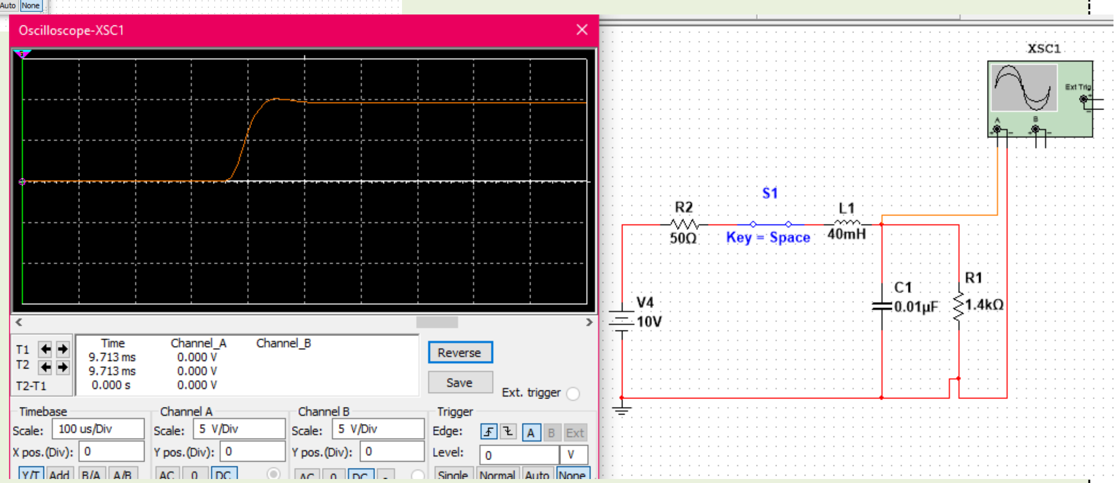
Figure 3.0: Second-Order Lowpass Circuit

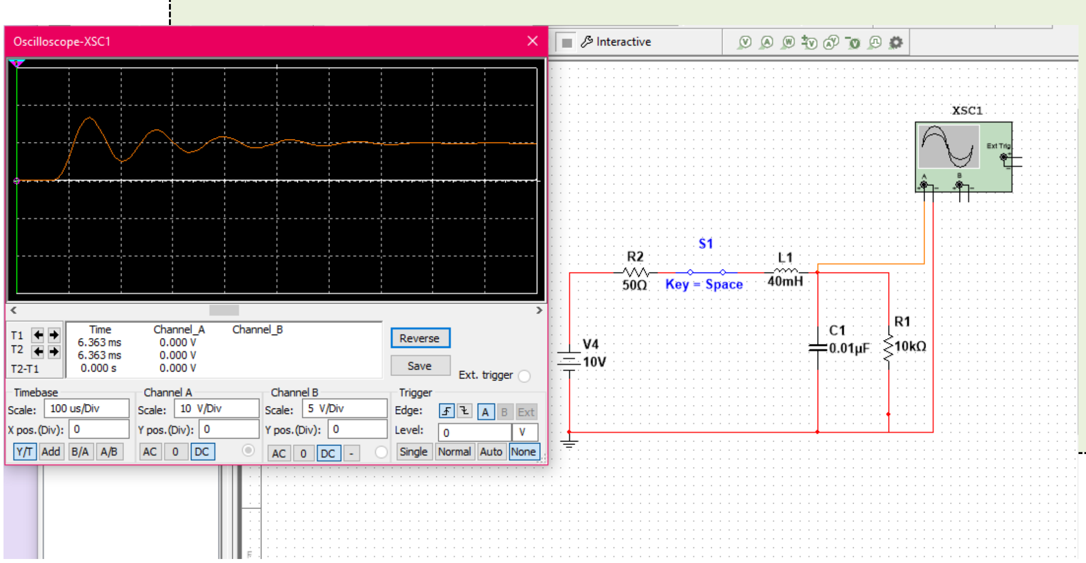
(c) Step 3: Consider the dynamic circuit shown in **Figure 3.0.** The switch S has been open for a long time. At t=0, the switch is closed, where it remains for a long time.

i) Use Multisim to plot the step response [$v_o(t)$ for $t \geq 0^+$] when R=1kΩ, 1.4kΩ, and 10kΩ.

Pre-Lab workspace (show your analysis here)







ii) Use the plots to calculate the parameters (σ and ω) that characterize the step response as: $\mathbf{v_o}(t) = B + A\boldsymbol{e}^{-\sigma t}\cos(\omega t + \theta)$ for R=10kΩ.

Pre-Lab workspace (show your analysis here)

$P_1 = 16.756$

$P_2 = 5.166$

$T = 128.148\ \mu s$

$$\sigma = \frac{-\ln\left|\frac{P_2}{P_1}\right|}{\left(\frac{T}{2}\right)}$$

$$= \frac{-\ln\left(\frac{16.756}{5.166}\right)}{\left(\frac{128.148}{2}\right)}$$

$$= \frac{-1.1766}{64.074\times10^{-6}}$$

$$\sigma = -18363.14$$

$$= -18\ K$$

$$\omega = \frac{2\pi}{T}$$

$$= \frac{2\pi}{128.148\times10^{-6}}$$

$$= 49\,030.64$$

$$\omega = 49\ K$$

$$V_o(t) = A + Be^{+18kt}\cos(49k\,t + \theta)$$

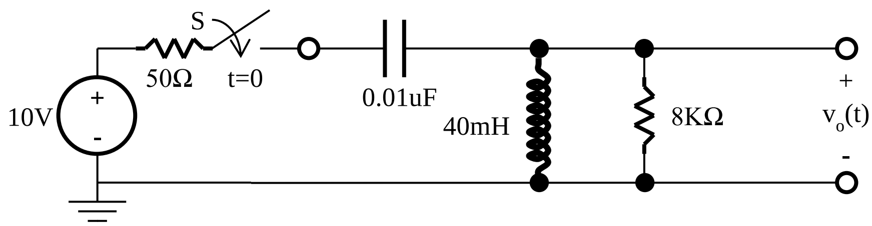
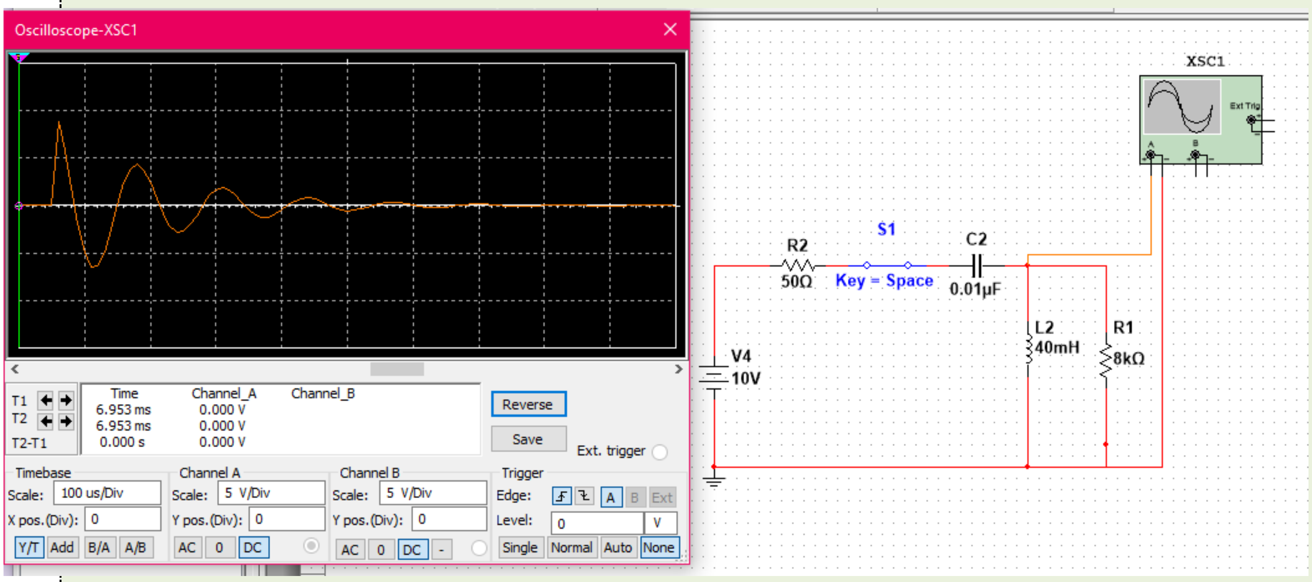


Figure 4.0: Second-Order Highpass Circuit

(d) Step 4: Consider the dynamic circuit shown in **Figure 4.0.** The switch S has been open for a long time. At t=0, the switch is closed, where it remains for a long time.

i) Use Multisim to plot the step response [$v_o(t)$ for $t \geq 0^+$].

Pre-Lab workspace (show your analysis here)



ii) Use the plot to calculate the parameters (σ and ω) that characterize the step response as: $\mathbf{v_o}(t) = A\boldsymbol{e^{-\sigma t}}\cos(\omega t + \theta)$.

Pre-Lab workspace (show your analysis here)

$P_1 = 8.376$

$P_2 = -6.407$

$T = 101.98045$

$$\sigma = \frac{-\ln\left(\frac{P_2}{P_1}\right)}{T/2}$$

$= 0.267979$

$\sigma = 5255.52$

$\sigma = 53k$

$$\omega = \frac{2\pi}{T}$$

$= 61\ 611.66$

$\omega = 62k$

$$v_o(t) = Ae^{-53kt}\cos(62kt + \theta)$$

5.0 IN-LAB IMPEMENTATION & MEASUREMENTS (5 marks in total):

Part I: The Step Response of a Second-Order Bandpass Circuit

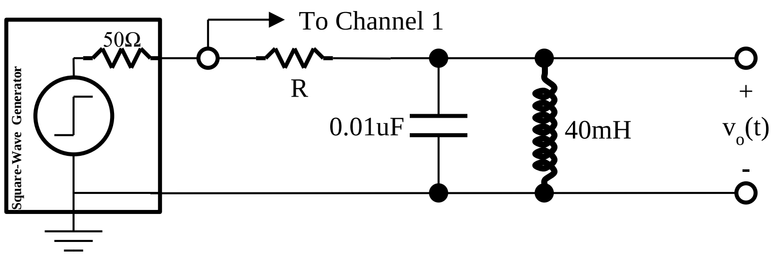


Figure 5.0: Second-Order Bandpass Filter

(a) Step 1: Connect Channel (1) of the oscilloscope to display the open-circuit voltage $v_s(t)$ of the function generator. Set the following:

- Trigger: source→Channel (1), and slope→rising.

Adjust the controls of the function generator to provide a square-wave signal $v_s(t)$ with a peak-to-peak value of 10V and DC offset of 5V at a frequency of 20Hz.
On FG: Waveform → Square, Frequency → 20 Hz, Amplitude → 5 Vpp*, Offset → 2.5 V*

*Due to FG matching impedance

Next, set the following:

- Channel (2): Vertical-position→one division above the bottom of the screen, coupling→dc, and V/div→1V.
- Time: Time/div→50μs when $v_s(t)$ is rising.
- Blank off Channel (1).

(b) Step 2: Construct the circuit shown in **Figure 5.0**. Set R=3.4kΩ. Connect Channel (2) to display the step response $v_o(t)$. Plot $v_o(t)$ on **Graph 1.0.**

(c) Step 3: Use the cursors on the oscilloscope to measure points on the $v_o(t)$ display to calculate the parameters (σ and ω) that characterize the step response as:

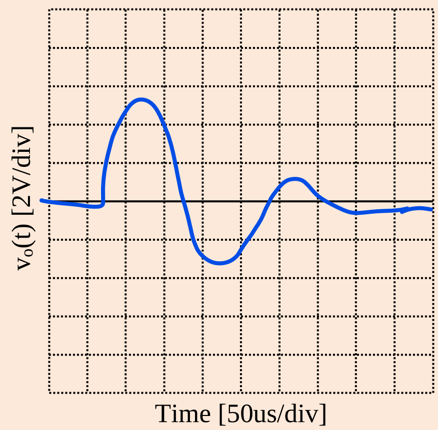
$$\mathbf{v_o}(t) = A\boldsymbol{e}^{-\boldsymbol{\sigma t}}\sin(\omega t)$$

(d) Step 4: Record your results in **Table 1.0.**

(e) Step 5: Demonstrate the correct operation of your setup to your TA. (1 mark)

(f) Step 6: Set R=1.4kΩ, and repeat as in Step 2 and Step 3. Record your results in **Table 1.0.**

(g) Step 7: Set R=500Ω, and repeat as in Step 2.



34k

Graph 1.0 (0.5 marks)

P1 P2

Table 1.0 (0.5 marks)

R (KΩ)	**T**	**v_o(T/4)**	**v_o(3T/4)**	**σ**	**ω**
3.4	125us	9.0	-1.273	18.3k	50.3k
1.4	130us	5.4	0.02	86.1k	48.3k

Part II: The Step Response of a Second-Order Lowpass Circuit

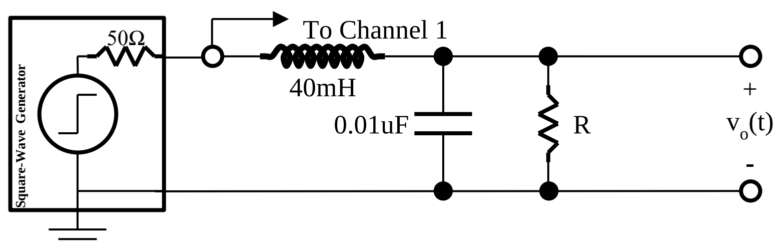


Figure 6.0: Second-Order Lowpass Filter

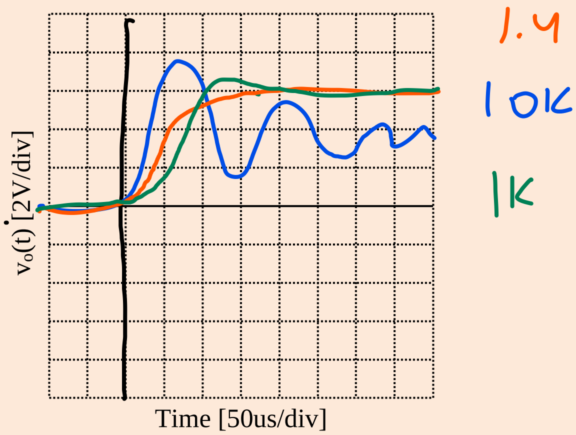
(h) Step 8: Connect the circuit shown in **Figure 6.0.** Set R=10kΩ. Use Channel (2) to display the step response v_o(t). Plot v_o(t) on **Graph 2.0.**

(i) Step 9: Use the cursors on the oscilloscope to measure points on the v_o(t) display to calculate (and record in **Table 2.0**) the parameters (σ and ω) that characterize the step response as:

$$\mathbf{v_o}(t) = B + A\boldsymbol{e}^{-\boldsymbol{\sigma t}}\cos(\omega t + \theta)$$

(j) Step 10: Set R=1.4kΩ, and repeat as in Step 8.

(k) Step 11: Set R=1kΩ, and repeat as in Step 8.



Graph 2.0 (0.5 marks)

Table 2.0 (0.5 marks)

R (KΩ)	T	$v_o(T/4)$	$v_o(3T/4)$	σ	ω
10	120 ms	9.8	6.5	7.4 K	52.4K

Part III: The Step Response of a Second-Order Highpass Circuit

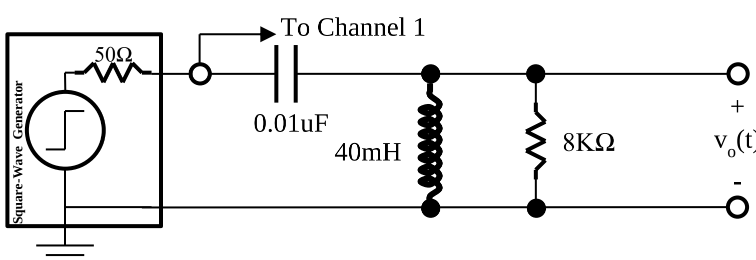


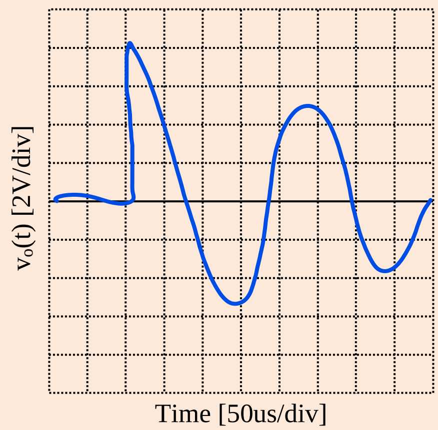
Figure 7.0: Second-Order Highpass Filter

(l) Step 12: Connect the circuit shown in **Figure 7.0.** Connect Channel (2) to display the step response $v_o(t)$. Plot $v_o(t)$ on **Graph 3.0.**

(m) Step 13: Use the cursors on the oscilloscope to measure points on $v_o(t)$ display to calculate (and record in **Table 3.0**) the parameters (σ and ω) that characterize the step response as:

$$\mathbf{v_o}(t) = Ae^{-\sigma t}\cos(\omega t + \theta)$$

(n) Step 14: Demonstrate the correct operation of your setup to your TA. (1 mark)



Graph 3.0 (0.5 marks)

Table 3.0 (0.5 marks)

R (KΩ)	T	v_o(T/4)	v_o(3T/4)	σ	ω
8	120us	-6.4	3.0	12.6K	52.4K

6.0POST-LAB QUESTIONS (2 marks in total, 2/3 marks for each question):

(1) By examining your plots on **Graph 1.0,** answer the following:

a) What are the effects of varying the value of R on the step response of a second-order bandpass circuit?

- The varying. R seems to change the Amplitude of the graph, thus affecting the P_1 & P_2 values. It also seems to effect the sharpness of the graph

(2) By examining your plots on **Graph 2.0,** answer the following:

a) What are the effects of varying the value of R on the step response of a second-order lowpass circuit?

A smaller R creates a small "hook" that appears in the beginning of the function. This doesn't effect the rest of the function

A larger R affects the entire graph and forces the sin graph portion of the graph above the y-axis

(3) Suppose that the 8kΩ-resistor is removed from the circuit in Fig (2.6), what effects will this have on the step response?

When the 8k is removed, all current will flow through the inductor, which will increase the graph amplitude and final voltage,