Numbers

- Unsigned
  - only positive
- signed
  - Positive or negative

Number Systems

- Decimal
- Binary
- Octal
- Hexadecimal

Decimal : Base 10 . e.g. 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

D: $d_{n-1} \cdots\cdots d_2 d_1 d_0$

Value: $d_{n-1} \times 10^{n-1} + \cdots + d_2 \times 10^2 + d_1 \times 10^1 + d_0 \times 10^0$

Example : $(5873)_{10}$

$D = 5 \times 10^3 + 8 \times 10^2 + 7 \times 10^1 + 3 \times 10^0$

$= 5000 + 800 + 70 + 3$

Binary: Base 2 e.g

Digits (or bit) : 0, 1

B: $b_{n-1} \cdots b_2 b_1 b_0$

Value $B = b_{n-1} \times 2^{n-1} + \cdots + b_2 \times 2^2 + b_1 \times 2^1 + b_0 \times 2^0$

Example: $(101010)_2$

$B = 1 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0$

$= 32 + 0 + 8 + 0 + 2 + 0$

$= (42)_{10}$

Octal : Base 8 (Compressed form of a binary)

Digits : 0, 1, 2, 3, 4, 5, 6, 7

Example : $(572)_8$

Value : $= 5 \times 8^2 + 7 \times 8^1 + 2 \times 8^0 = 5 \times 64 + 56 + 2 = 320 + 56 + 2 = 378$

$(572)_8 = (378)_{10}$

Hexadecimal : Base 16

Digits : 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F

A → 10
B → 11
C → 12

Digits : 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F

B → 11
C → 12
D → 13
E → 14
F → 15

Example : $(10A5)_{16}$

Value : $1\times16^3 + 0\times16^2 + 10\times16^1 + 5\times16^0$

$= (4261)_{10}$

Decimal to Binary

$(45)_{10} = (101101)_2$

2	45	
2	22 r	1 → LSB (Least significant bit)
2	11 –	0
2	5 –	1
2	2 –	1
2	1 –	0
	0	1 → MSB (Most significant bit)

101101

Decimal to octal

8	45	
8	5 –	5
	0 –	5

$(45)_{10} = (55)_8$

10 — A
11 — B
12 — C
13 — D

45
32
13

Decimal to Hexadecimal

16	45	
16	2 –	D
	0 –	2

$(45)_{10} = (2D)_{16}$

12

Binary to octal

$(101\ 110\ 111\ 010)_2$

$(5\ 6\ 7\ 2)_8$

4	2	1	Decimal
0	0	0	0
0	0	1	1
0	1	0	2
0	1	1	3
1	0	0	4
1	0	1	5
1	1	0	6
1	1	1	7

octal

2^3	2^2	2^1	2^0	
8	4	2	1	
1	1	0	0	
1	0	1	1	= 11
1	1	1	1	

05 = 5

Binary to hexadecimal

$(1011\ 1011\ 1010)_2$

$(BBA)_{16}$

8	4	2	1	
1	0	1	1	11 → B
1	0	1	0	10 → A

$(325)_{10} \rightarrow (\quad)_2 \longrightarrow (\quad)_8 \longrightarrow (\quad)_{16}$

Octal $(475)_8$ Each octal digit must be

Octal

$(425)_8$

100 010 101

Each octal digit must be converted into a three digit binary.

$(100010101)_2$

Binary

↓

Hexa → $(115)_{16}$

Addition

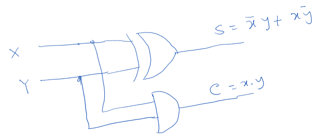
$$\begin{array}{r} 2 \\ +3 \\ \hline 05 \end{array} \qquad \begin{array}{r} 5 \\ +7 \\ \hline 12 \end{array} \qquad \begin{array}{r} 0 \\ +0 \\ \hline 00 \end{array} \qquad \begin{array}{r} 0 \\ +1 \\ \hline 01 \end{array} \qquad \begin{array}{r} 1 \\ +0 \\ \hline 01 \end{array} \qquad \begin{array}{r} 1 \\ +1 \\ \hline 10 \end{array}$$

Carry Sum

X	Y	C	S
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

$S = \bar{X}Y + X\bar{Y} = X \oplus Y$

$C = X \cdot Y$



Half-adder

Block diagram

1 ← Carry

$$\begin{array}{r} 25 \\ +17 \\ \hline 42 \end{array}$$

C_{i+1} S_i

C_i	x_i	y_i	C_{i+1}	S_i
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

$\overline{\bar{x}y + x\bar{y}}$ Ex-OR

$= \overline{\bar{x}y} \cdot \overline{x\bar{y}}$

$= (\bar{\bar{x}} + \bar{y}) \cdot (\bar{x} + \bar{\bar{y}})$

$= (x + \bar{y})(\bar{x} + y)$

Ex-NOR $= xy + \bar{x}\bar{y}$

$S_i = \bar{C_i}\bar{x_i}y_i + \bar{C_i}x_i\bar{y_i} + C_i\bar{x_i}\bar{y_i} + C_i x_i y_i$

$= \bar{C_i}(\bar{x_i}y_i + x_i\bar{y_i}) + C_i(\bar{x_i}\bar{y_i} + x_i y_i)$

$= \bar{C_i}(x_i \oplus y_i) + C_i \cdot \overline{x_i \oplus y_i}$

$= x_i \oplus y_i \oplus C_i$

↑ 3 input X-OR gate

output is high when odd numbers of input(s) are high

C_i \ $x_i y_i$	00	01	11	10
0	0	1	0	1
1	1	0	1	0

$\bar{x}y + x\bar{y}$ ↓ Ex-OR

$xy + \bar{x}\bar{y}$ Ex-NOR

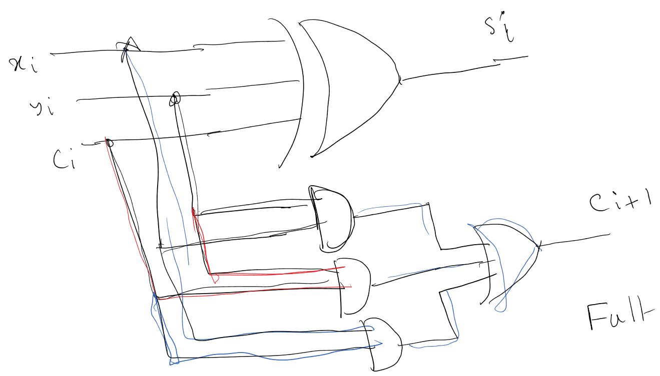
$C_{i+1} = \bar{C_i}x_i y_i + C_i\bar{x_i}y_i + C_i x_i \bar{y_i} + C_i x_i y_i$

$= x_i y_i(\bar{C_i} + C_i)$

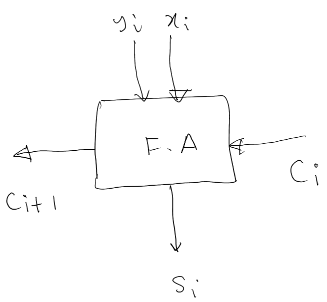
C_i \ $x_i y_i$	00	01	11	10
0	0	0	1	0
1	0	1	1	1

$$S_i = x_i \oplus y_i \oplus C_i$$

$$C_{i+1} = x_i y_i + C_i y_i + C_i x_i$$



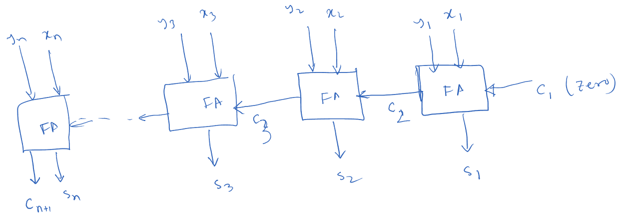
Full adder circuit



$$\begin{array}{r} 11 \leftarrow \text{carry} \\ 357 \\ +485 \\ \hline 842 \end{array}$$

sum

Ripple carry adder



* Design a Full-adder with Half adder(s)

$$\begin{array}{r} 1 \\ 1 \\ \hline 1 \end{array}$$

$1 + 1 + 1$

$2 + 1 = 3$

$x \oplus y \oplus C$

2 + 1 = 3