Toronto Metropolitan University
Department of Electrical, Computer, & Biomedical Engineering
Faculty of Engineering & Architectural Science

ELE 302
Laboratory #4

Filters

1.0 INTRODUCTION:

Filters are frequency-selective networks that permit signals whose frequencies fall within a certain range (called the passband) to pass from the input to the output relatively unchanged, while impede the passage of signals whose frequencies are within other ranges (called the stopbands). Filters are used in a wide range of electrical systems such as radio, telephone, television, power supplies, computer circuits, industrial machinery – just to name but a few.

Filters are classified by the location of their passband: A lowpass filter passes signals whose frequencies are below a certain "cutoff" frequency, while a high-pass filter passes signals whose frequencies are above a desired cutoff frequency. On the other hand, a band-pass filter has a passband within a given range: $\omega_L < \omega < \omega_H$, where ω_L and ω_H are the lower and higher cutoff frequencies. Finally, a notch (bandstop) filter impedes any unwanted signal frequencies (or bands) from passing from the input to the output.

Filter realizations that consist of only passive elements (R, L, and C) are called the passive filters. Such filters work well at high-frequency applications (above 100kHz). For low-frequency applications, such as the audio range, it is desirable to avoid the use of inductors, as the required inductors are usually large, physically bulky, and their characteristics are quite non-ideal. Filter realizations that avoid the use of inductors, and utilize Op-Amps instead, are called active filters.

This experiment examines the frequency-selective characteristics of a second-order low-pass passive filter and a second-order band-pass active filter. In addition, it investigates the effects of the quality factor (Q) variations on their frequency response.

2.0 OBJECTIVES:

- To measure the magnitude- and phase-frequency responses of a second-order lowpass passive filter.
- To measure the magnitude- and phase-frequency responses of a second-order bandpass active filter.
- To investigate the effects of Q variations on the frequency responses of the lowpass and band pass filters.

3.0 REQUIRED LAB EQUIPMENT & PARTS:

- DC Power Supply (PS), Function Generator (FG) and Oscilloscope.
- ELE202 Lab Kit and ELE302 Lab Kit: various components, breadboard, wires and jumpers.
- Digital Multimeter (DMM).

4.0 PRE-LAB ASSIGNMENT (3 marks with 1.5 marks for each step):

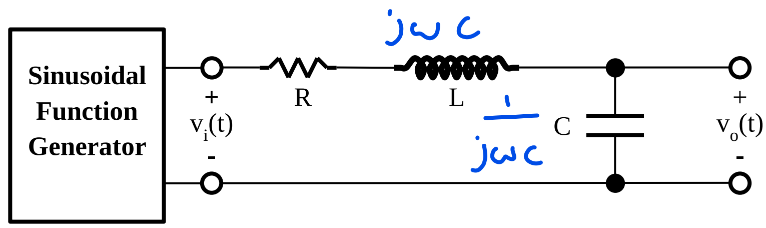


Figure 1.0: Second-Order Lowpass Passive Filter

(a) Step 1: The circuit shown in **Figure 1.0** is a second-order lowpass passive filter.

i) Show that the voltage-transfer function of the filter circuit has the form:

$$H(S) = V_o/V_i = \frac{\omega_o^2}{[S^2+\frac{\omega_o}{Q}S+\omega_o^2]}]$$

Find the expressions for ω_o and Q in terms of R, L, and C.

Pre-Lab workspace (show your analysis here)

$$Z = R + j\omega L + \frac{1}{j\omega C}$$
$$= R + j\omega L - \frac{j}{\omega C}$$
$$= R + j\left(\omega L - \frac{1}{\omega C}\right)$$
$$\omega L - \frac{1}{\omega C} = 0$$
$$\omega L = \frac{1}{\omega C}$$
$$\omega^2 = \frac{1}{LC}$$
$$\omega = \sqrt{\frac{1}{LC}}$$

$$Q = \omega_o \frac{L}{R}$$
$$= \sqrt{\frac{1}{LC}} \cdot \frac{L}{R}$$
$$Q^2 = \frac{1}{\cancel{L}C} \cdot \frac{L^{\cancel{2}}}{R^2}$$
$$Q^2 = \frac{L}{CR^2}$$
$$Q = \sqrt{\frac{L}{CR^2}}$$
$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$V_o = C = \frac{1}{j\omega C}$$

$$V_i = R + L + C = R + j\omega L + \frac{1}{j\omega C}$$

$$\frac{V_o}{V_i} = \frac{\frac{1}{j\omega C}}{R + j\omega L + \frac{1}{j\omega C}} = \frac{1}{SC} \cdot \frac{1}{R + SL + \frac{1}{SC}}$$
$$= \frac{1}{SCR + S^2LC + 1} \cdot \frac{\frac{1}{LC}}{\frac{1}{LC}}$$
$$= \frac{1}{LC} \cdot \frac{1}{\frac{SCR}{LC} + \frac{S^2LC}{LC} + \frac{1}{LC}}$$

$$Q = \omega_o \frac{L}{R}$$
$$\frac{R}{L} = \frac{\omega_o}{Q}$$

$$= \frac{1}{LC} \cdot \frac{1}{\frac{SR}{L} + S^2 + \frac{1}{LC}}$$

$$\omega_o = \sqrt{\frac{1}{LC}} \qquad = \frac{\omega_o^2}{S^2 + \omega_o^2 + \frac{\omega_o}{Q}S} = H(S)$$
$$\omega_o^2 = \frac{1}{LC}$$

ii) Let L = 0.1H and C = 0.01μF. Select the value of R that will set Q at 0.707. Under this condition, the circuit is said to be a Butterworth (or maximally-flat) lowpass filter.
Determine the value of Q when the value of R is:
* R = 1kΩ
* R = 10kΩ

Pre-Lab workspace (show your analysis here)

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$R = \frac{1}{Q}\sqrt{\frac{L}{C}}$$

$$= \frac{1}{0.707}\sqrt{\frac{0.1\ H}{0.01\mu F}}$$

$$R = 4.473\,K$$

$$Q = \frac{1}{1K}\sqrt{\frac{0.1\ H}{0.01\mu F}}$$

$$Q = 3.16$$

$$Q = \frac{1}{10K}\sqrt{\frac{0.1\ H}{0.01\mu F}}$$

$$Q = 0.316$$

iii) Use Multisim to plot the magnitude (in dB) and phase (in degrees) of [V_o/V_i] for the three Q-values of part ii), over the frequency range 100Hz to 50kHz.

Pre-Lab workspace (show your analysis here)

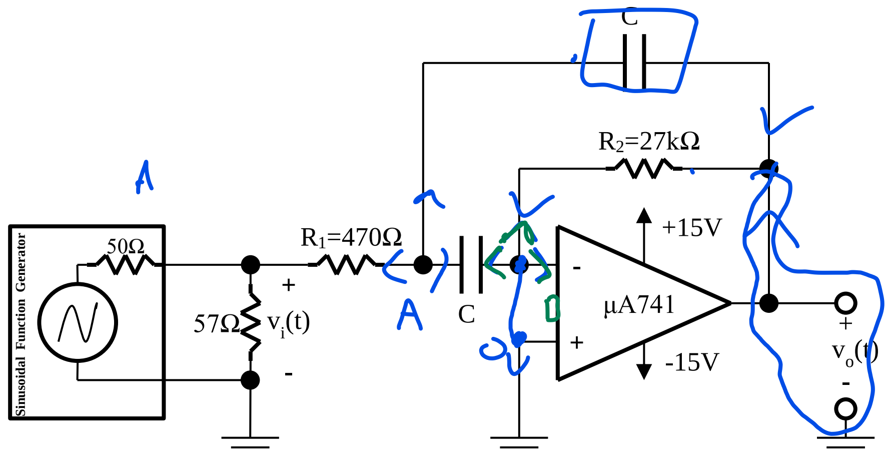


Figure 2.0: Second-Order Bandpass Active Filter

(b) Step 2: The circuit shown in **Figure 2.0** is a second-order bandpass active filter.

i) Show that the voltage-transfer function of the filter circuit has the form:

$$H(S) = V_o/V_i = \frac{AS}{[S^2 + \frac{\omega_o}{Q}S + \omega_o^2]}]$$

Find the expressions for A, ω_o and Q in terms of C, R_1, and R_2.

Pre-Lab workspace (show your analysis here)

$$\frac{V_A - V_i}{R_1} + \frac{V_A - V_o}{C} + \frac{V_A - 0}{C} = 0$$

$$\frac{V_A - V_i}{R_1} + \frac{V_A - V_o}{\frac{1}{SC}} + \frac{V_A}{\frac{1}{SC}} = 0$$

$$\frac{V_A - V_i}{R_1} + SCV_A - SCV_o + SCV_A = 0$$

$$\left(\frac{V_A}{R_1} + 2SCV_A = SCV_o + \frac{V_i}{R_1}\right)R_1$$

$$V_A(1 + 2SCR_1) = V_i + SCR_1V_o$$

$$\left(\frac{-V_o}{SCR_2}\right)(1 + 2SCR_1) = V_i + SCR_1V_o$$

$$\frac{-V_o}{SCR_2} - \frac{V_o 2SCR_1}{SCR_2} - SCR_1V_o = V_i$$

$$-V_o\left(\frac{1}{SCR_2} + \frac{2SCR_1}{SCR_2} + SCR_1\right) = V_i$$

$$-V_o\left(\frac{1 + 2SCR_1 + (SC)^2R_1R_2}{SCR_2}\right) = V_i$$

$$\frac{V_o}{V_i} = \frac{-SCR_2}{1 + 2SCR_1 + S^2C^2R_1R_2} \cdot \frac{\frac{1}{C^2R_1R_2}}{\frac{1}{C^2R_1R_2}}$$

$$= \frac{\frac{-S}{CR_1}}{\frac{1}{C^2R_1R_2} + \frac{2S}{CR_2} + S^2}$$

$$\frac{\cancel{V_B} - V_A}{C} + \frac{\cancel{V_B} - V_o}{R_2} = 0$$

$$\frac{-V_A}{\frac{1}{SC}} - \frac{V_o}{R_2} = 0$$

$$-V_ASC = \frac{V_o}{R_2}$$

$$V_A = \frac{-V_o}{SCR_2}$$

$$\frac{2}{CR_2} = \frac{\omega_0}{Q} \qquad Q = \frac{\omega_0 CR_2}{2}$$

$$\omega_0 = \frac{1}{C\sqrt{R_1R_2}} \qquad A = \frac{-1}{CR_1}$$

ii) Let C = 0.01μF. Find the values of $|H(S=j\omega_o)|$ in dB, ω_o, and Q when:

* $R_1 = 470\Omega$ and $R_2 = 27k\Omega$
* $R_1 = 1k\Omega$ and $R_2 = 12k\Omega$

Pre-Lab workspace (show your analysis here)

$$Q = \frac{1}{2}\sqrt{\frac{R_2}{R_1}}$$

$$= 3.79$$

$$\omega_0 = \frac{1}{C} \cdot \frac{1}{\sqrt{R_1 R_2}}$$

$$= 28\,071.73 \text{ rad/s}$$

$$A = \frac{-1}{CR_1}$$

$$= -2.127 \times 10^5$$

$$H(s) = \frac{As}{\omega_0^2 + \frac{\omega_0}{Q}s + s^2}$$

$$= \frac{-2.127\times10^5\,(j\omega_0)}{(28071)^2 + \frac{(28071)}{3.79}(j\omega_0) + (j\omega_0)^2}$$

$$H(j\omega_0) = 28.73$$

$$= 29.16 \text{ dB}$$

$$Q = \frac{1}{2}\sqrt{\frac{R_2}{R_1}}$$

$$= 1.73$$

$$\omega_0 = \frac{1}{C} \cdot \frac{1}{\sqrt{R_1 R_2}}$$

$$= 28867 \text{ rad/s}$$

$$A = \frac{-1}{CR_1}$$

$$= \frac{-1}{0.01\mu F \cdot 1k}$$

$$A = -10^5$$

$$H(\omega) = \frac{As}{\omega_0^2 + \frac{\omega_0}{Q}s + s^2}$$

$$= \frac{-10^5 \cdot \omega}{(28867)^2 + \frac{(28867)}{1.73}s + s^2}$$

$$H(\omega_0) = 5.9908$$

$$= 15.561 \text{ dB}$$

iii) Use Multisim to plot the magnitude (in dB) and phase (in degrees) of [V_o/V_i] for the two cases of part ii) over the frequency range of 500Hz to 50kHz.

Pre-Lab workspace (show your analysis here)

5.0 IN-LAB IMPEMENTATION & MEASUREMENTS (5 marks in total):

Part I: The Frequency Responses of the Lowpass Passive Filters

(a) Step 1: Construct the circuit shown in **Figure 1.0**: Set L = 0.1H, C = 0.01μF, and R = 4kΩ [note that the internal resistance of the inductor in addition to the value of R is the total resistance value that should be used to design a Butterworth (Q = 0.707) lowpass filter]. You can use the Potentiometer for the resistance.

Connect Channels (1) and (2) of the oscilloscope to display the input voltage and output voltage respectively, with the trigger source at Channel (1). Set both channels to AC coupling. Set the controls of the function generator to provide a **sinusoidal** input voltage of 6V (peak) at a frequency of 5kHz.

(b) Step 2: Use the oscilloscope displays to measure the phase angle $\angle H(\omega)$ in degrees, and use DMM to measure the dB-values of the input and output voltage, and evaluate the magnitude $|H(\omega)|$ in dB as:

$$|H(\omega)|_{dB} = |V_o|_{dB} - |V_i|_{dB}$$

Record your results in **Table 1.0**.

(c) Step 3: Repeat as in Step 2 for each frequency setting in **Table 1.0**. Use **Graph 1.0** to plot the magnitude $|H(\omega)|$ in dB and phase $\angle H(\omega)$ in degrees versus frequency in Hz.

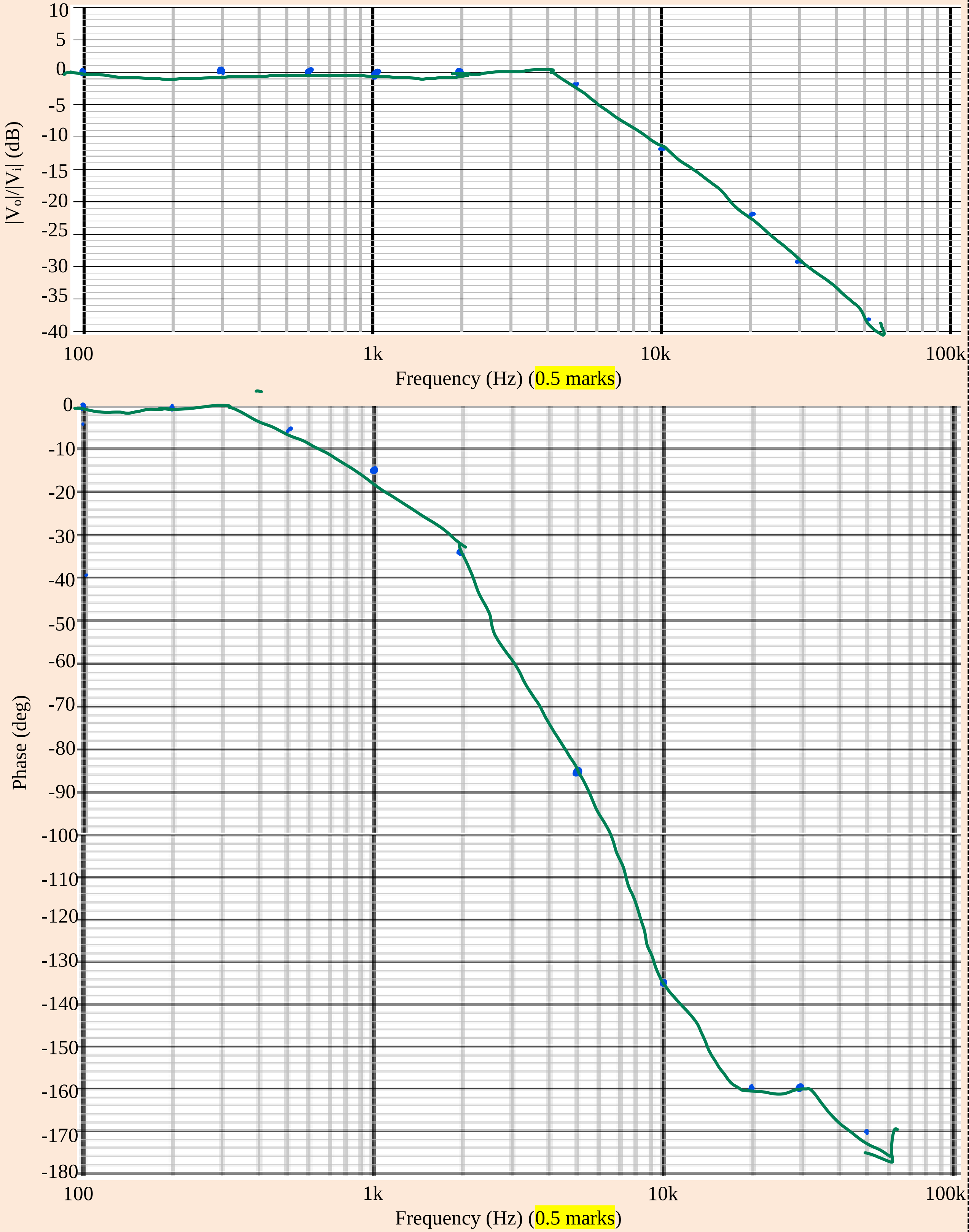
Note: Make sure you re-adjust the input voltage to 6V (peak) when you change the frequency.

Use your plots to find the location of the low-cutoff frequency f_o. Fill in the corresponding entry for f_o in **Table 1.0**.

(d) Step 4: Demonstrate the correct operation of your experiment setup to your TA. (1 mark)\

Table 1.0 (0.5 marks)

Frequency (Hz)	$\lvert V_o\rvert$ (dB)	$\lvert V_i\rvert$ (dB)	$\lvert H(\omega)\rvert$ (dB)	$\angle H(\omega)$ (degrees)
f_o = 4k	20.9	22.34	−1.44	−68.8
100	22.2		−0.07	−2.62
200				−3.91
500				−5.52
1k				−15.29
2k	22.21		−0.13	−34.62
5k	19.64		−2.7	−85.83
10k	10.47		−11.87	−135.9
20k	−0.53		−22.87	−160.57
30k	−7.53		−29.87	−160.9
50k	−16.3		−38.64	−170.24



|Vo|/|Vi| (dB)
Frequency (Hz) (0.5 marks)
Phase (deg)
Frequency (Hz) (0.5 marks)

Graph 1.0

Part II: The Frequency Responses of the Bandpass Active Filters

(e) Step 5: Connect the circuit shown in **Figure 2.0**, with $R_1 = 470\Omega$, $R_2 = 27k\Omega$ and $C = 0.01\mu F$. Please use the resistors and capacitors from your lab kit.

Connect Channels (1) and (2) of the oscilloscope to display the input and output voltage respectively with the trigger source at Channel (1). Set both channels to AC coupling. Set Channel (2) on inv, as the polarity of the output voltage in **Figure 2.0** is inverted w.r.t. that of the input voltage. Set the controls of the function generator to provide a sinusoidal input voltage of 0.5V (peak-to-peak) at a frequency of 5kHz.

(f) Step 6: Use the oscilloscope displays to measure the phase angle $\angle H(\omega)$ in degrees, and use DMM to measure the dB-values of the input and output voltages, and evaluate the magnitude $|H(\omega)|$ in dB. Record your results in **Table 2.0**.

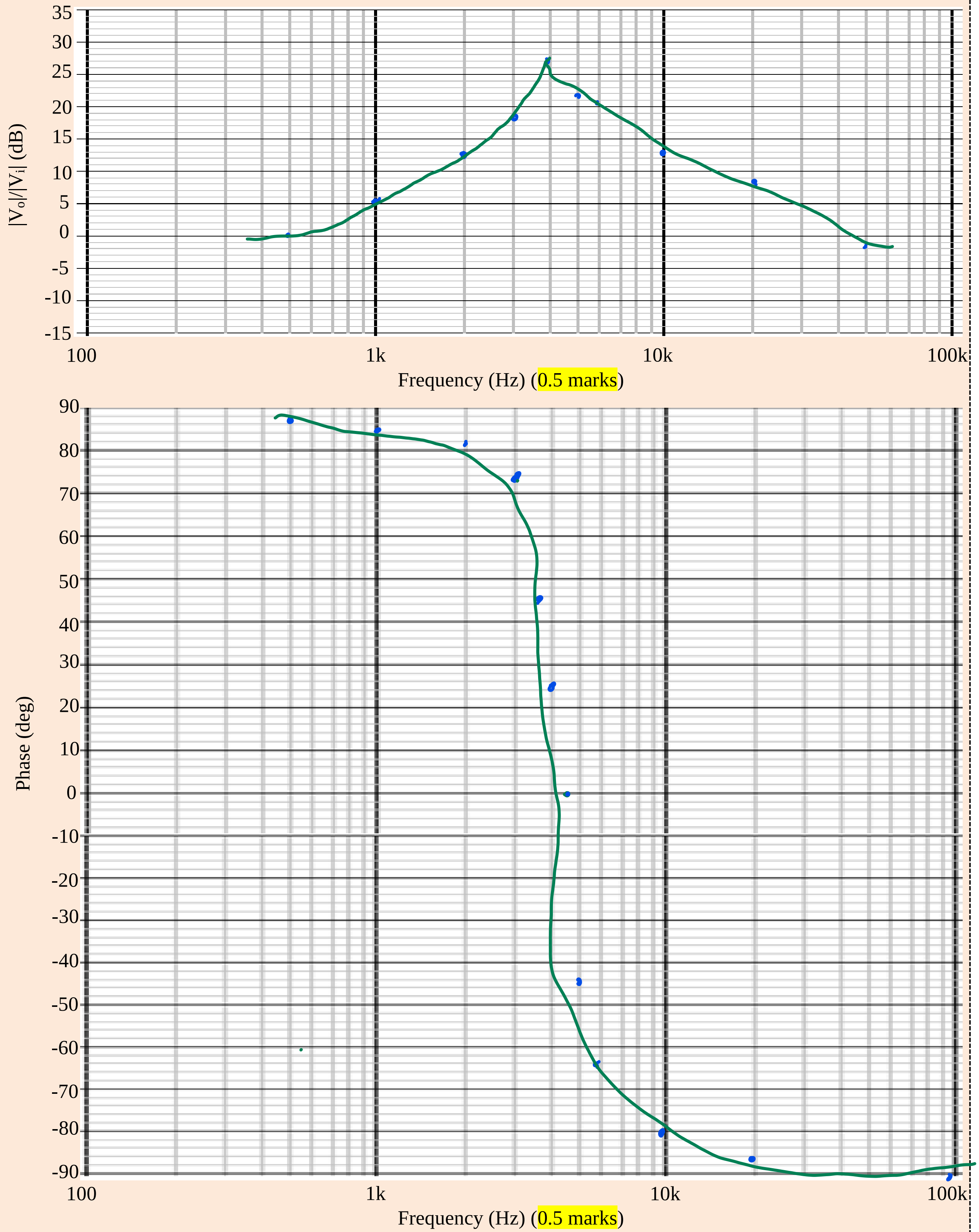
(g) Step 7: Scan the frequency spectrum to locate the central frequency f_o, the low-cutoff frequency f_L, and the high-cutoff frequency f_H, **as defined by their phase values in Table 2.0**. Record the corresponding voltages and $|H(\omega)|$.

(h) Step 8: Repeat Step 6 for each frequency setting in **Table 2.0**. Use **Graph 2.0** to plot the magnitude $|H(\omega)|$ in dB and phase $\angle H(\omega)$ in degrees versus frequency in Hz. Double check your input voltage when you change the frequency.

(i) Step 9: Demonstrate the correct operation of your experimental setup to your TA. (1 mark)

Table 2.0 (0.5 marks)

Frequency (Hz)	$\|V_o\|$ (dB)	$\|V_i\|$ (dB)	$\|H(\omega)\|$ (dB)	$\angle H(\omega)$ (degrees)
f_o = 4.5k	12.36	-12.19	24.5	0^o
f_L= 3.8k	10.86		23	$+45^o$
f_H= 5k	↓		↓	-45^o
500	-12.29		-0.15	88
1k	-6.57		5.57	86.7
2k	0.57		12.71	82
3k	6.77		18.91	73
4k	13.33		27.47	25
5k	9.827		21.97	-44
6k	8.627		20.8	-64
10k	0.995		13.09	-81
20k	-6.43		5.71	-87
50k	-14.75		-2.61	-91



35
30
25
20
15
10
5
0
-5
-10
-15
|Vo|/|Vi| (dB)
100
1k
10k
100k
Frequency (Hz) (0.5 marks)
90
80
70
60
50
40
30
20
10
0
-10
-20
-30
-40
-50
-60
-70
-80
-90
Phase (deg)
100
1k
10k
100k
Frequency (Hz) (0.5 marks)

Graph 2.0

6.0POST-LAB QUESTIONS (2 marks in total, 2/3 marks for each question):

(1) By considering your plots on **Graph 1.0** and the Multisim plots from prelab, answer the following:

a) How is the maximum rate of attenuation of | H(ω)| affected by Q?

b) What is so special about the magnitude response when Q = 1/√2?

c) How do the phase responses compare as a function of Q?

a) $Q = \frac{\omega_0}{B}$. ∴ if Q increase, B decreases, and r.o.A will increase.

b) At Q = 0.707, P absorbed is extactly ½ power

c) Phases for Q = 0.707 are ± 45°.

(2) What effect does the value of Q have on the frequency responses of the band-pass filter circuit?

Q effects rate of attenuation. High Q = high A, and steeper slopes. Low Q = flatter graphs

(3) Is it practically feasible to realize a band-pass characteristic with f_o = 5kHz and Q = 100, using the active-filter circuit shown in **Figure 2.0** with C = 0.01 μF? Explain your answer.

$$Q = 100$$

$$= \frac{1}{2} \cdot \omega_0 C R$$

$$\frac{2Q}{\omega_0 C} = R \qquad \omega_0 = 2\pi f$$

$$R = \frac{\cancel{2}(100)}{\cancel{2}\pi(5k)(0.01\times10^{-6})}$$

$$R = 636.62\times10^{3}\ \Omega$$

$$R = 636.6k\Omega$$

extremely high R value but technically possible