Chapter 5: 5.1-5.4, 5.10, 5.21

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***5.1** Determine the decimal values of the following unsigned numbers:
(a) $(0111011110)_2$
(b) $(1011100111)_2$
(c) $(3751)_8$
(d) $(A25F)_{16}$
(e) $(F0F0)_{16}$

(a) $V = 0\times2^9 + 1\times2^8 + 1\times2^7 + 1\times2^6 + 0\times2^5 + 1\times2^4 + 1\times2^3 + 1\times2^2 + 1\times2^1 + 0\times2^0 = (478)_{10}$

(b) $V = 2^0 + 2^1 + 2^2 + 2^5 + 2^6 + 2^7 + 2^9 = (743)_{10}$

(c) $V = 1\times8^0 + 5\times8^1 + 7\times8^2 + 3\times8^3 = (2025)_{10}$

(d) $V = 15\times16^0 + 5\times16^1 + 2\times16^2 + 10\times16^3 = (41567)_{10}$

(e) $V = 0 + 15\times16^1 + 0 + 15\times16^3 = (61680)_{10}$

***5.2** Determine the decimal values of the following 1's complement numbers:
(a) 0111011110
(b) 1011100111
(c) 1111111110

(a) $\underset{+}{0}\ \overbrace{\underset{8}{1}\ \underset{7}{1}\ \underset{6}{1}\ \underset{5}{0}\ \underset{4}{1}\ \underset{3}{1}\ \underset{2}{1}\ \underset{1}{1}\ \underset{0}{0}}^{\cancel{\&}\text{ magnitude}} = +\ 2^8 + 2^7 + 2^6 + 2^4 + 2^3 + 2^2 + 2^1 = +478$

(b) $1\ \overbrace{\underset{8}{1}\ \underset{7}{0}\ \underset{6}{0}\ \underset{5}{0}\ \underset{4}{1}\ \underset{3}{1}\ \underset{2}{0}\ \underset{1}{0}\ \underset{0}{0}} = -(2^8 + 2^4 + 2^3) = -280$

(c) $1\ \overbrace{0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1} = -1$

***5.3** Determine the decimal values of the following 2's complement numbers:
(a) 0111011110
(b) 1011100111
(c) 1111111110

(a) $\underset{+}{0}\ \overbrace{1\ 1\ 1\ 0\ 1\ 1\ 1\ 1\ 0}^{\text{Magnitude}} = +(2^8 + 2^7 + 2^6 + 2^4 + 2^3 + 2^2 + 2^1) = +478$

(b) $\underline{1}\ \overbrace{1\ 0\ 0\ 0\ 1\ 1\ 0\ 0\ 1}^{\text{2's complement}} = -(2^8 + 2^4 + 2^3 + 2^0) = -281$

(c) $1\ \overbrace{0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 0} = -2$

***5.4** Convert the decimal numbers 73, 1906, −95, and −1630 into signed 12-bit numbers in the following representations:
(a) Sign and magnitude
(b) 1's complement
(c) 2's complement

2 | 73
2 | 36 — 1
2 | 18 — 0
2 | 9 — 0
2 | 4 — 1
2 | 2 — 0
2 | 1 — 0
0 — 1

73 = 1001001

1906 = 11101110010
95 = 1011111
1630 = 11001011110

Decimal		Sign & Magnitude					1's Complement					2's complement				
73	000	001	001	001		000	001	001	001		000	001	001	001		
1906	011	101	110	010		011	101	110	010		011	101	110	010		
-95	100	001	011	111		111	110	100	000		111	110	100	001		
-1630	111	001	011	110		100	110	100	001		100	110	100	010		

5.10 In section 5.5.4 we stated that a carry-out signal, c_k, from bit position $k-1$ of an adder circuit can be generated as $c_k = x_k \oplus y_k \oplus s_k$, where x_k and y_k are inputs and s_k is the sum bit. Verify the correctness of this statement.

See the class note

5.21 Suppose that we want to determine how many of the bits in a three-bit unsigned number are equal to 1. Design the simplest circuit that can accomplish this task.

x_1	x_2	x_3	No. of '1'	y_1	y_0
0	0	0	0	0	0
0	0	1	1	0	1
0	1	0	1	0	1
0	1	1	2	1	0
1	0	0	1	0	1
1	0	1	2	1	0
1	1	0	2	1	0
1	1	1	3	1	1

binary equivalent of number of '1'

simplify for y_0

$x_1 \backslash x_2x_3$	00	01	11	10
0	(0)	1 (1)	(3)	1 (2)
1	1 (4)	(5)	1 (7)	(6)

$$y_0 = x_1\bar{x}_2\bar{x}_3 + \bar{x}_1\bar{x}_2x_3 + x_1x_2x_3 + \bar{x}_1x_2\bar{x}_3$$

$$= x_1(\bar{x}_2\bar{x}_3 + x_2x_3) + \bar{x}_1(\bar{x}_2x_3 + x_2\bar{x}_3)$$

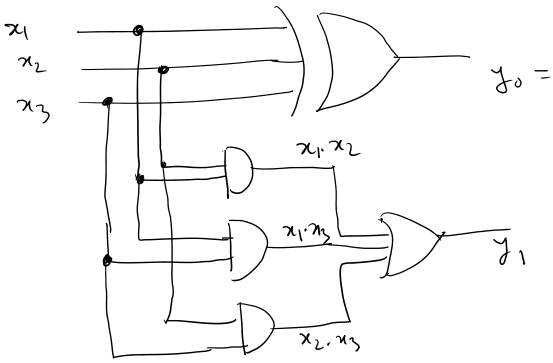
$$= x_1(\overline{x_2 \oplus x_3}) + \bar{x}_1(x_2 \oplus x_3)$$

$$y_0 = x_1 \oplus x_2 \oplus x_3$$

Simplify for y_1

$x_1 \backslash x_2x_3$	00	01	11	10
0			1	
1		1	1	1

$$y_1 = x_2x_3 + x_1x_2 + x_1x_3$$



$y_0 = x_1 \oplus x_2 \oplus$