DVP - M2 - Physik, 4.9.06, -1-

1. Aufgabe.

1. $P_2 : P_3$ :

$$(X^3 + 2X^2 + X + 2) : (X^2 + 1) = X + 2$$

$$\begin{array}{l} X^3 \qquad\quad + X \\ \hline - \quad 2X^2 \qquad + 2 \\ \quad\;\; 2X^2 \qquad + 2 \\ \hline \qquad\qquad 0 \end{array}$$

$\Rightarrow (X+2) \cdot P_3 = P_2$ (2)

$$\left.\begin{array}{l} P_1 = P_2\, Q + 20\, P_3 \\ P_2 = P_3 \cdot (X+2) \end{array}\right\} \Rightarrow$$ $P_3$ ist größter gem. Teiler von $P_1$ und $P_2$ (2)

2. $P_2(X) \underset{1.}{=} (X+2)(X^2+1) = (X+2)(X+i)(X-i)$ (2)

3. $P_1(-2) = \underset{0}{\underbrace{P_2(-2)}}\, Q(-2) + 20 \cdot P_3(-2) = 20 \cdot ((-2)^2+1)$
$= 100$ (1)

4. $(X-2), (X-3), (X^2+1)$ sind Teiler von $P_1$

$\underset{\text{grad } P_1 = 4}{\Longrightarrow}$ $P_1(X) = \alpha(X-2)(X-3)(X^2+1)$. (2)

Aus 3. folgt: $100 = P_1(-2) = \alpha(-4)(-5)(4+1)$
$= \alpha \cdot 100$

$\Rightarrow \alpha = 1$. (3)

$\Rightarrow P_1(X) = (X-2)(X-3)(X^2+1)$

$( = (x^2 - 5X + 6)(X^2+1) = X^4 - 5x^3 + 7x^2 - 5X + 6)$

Bem.: $Q(X) = X - 7$

## 2. Aufgabe $z = cb$

(1) $z^2 = -a = (cb)^2 = c^2 b^2 = c^2 a$

$\Leftrightarrow c^2 = -1 \Leftrightarrow c = \pm i$. Lösungen $z = \pm i b$ 1

(2) $z^2 = ia = c^2 b^2 = c^2 a \Leftrightarrow c^2 = i$

$\Leftrightarrow c = \pm \frac{1}{\sqrt{2}}(1+i)$. Lösungen: $z = \pm \frac{1}{\sqrt{2}}(1+i)b$ 2

(3) $z^2 = -ia = c^2 b^2 = c^2 a \Leftrightarrow c^2 = -i$

$\Leftrightarrow c = \pm \frac{1}{\sqrt{2}}(-1+i)$. Lösungen: $z = \pm \frac{1}{\sqrt{2}}(-1+i)a$ 2

3. Aufgabe

1. $x = 0$ : $0 \in ]-\frac{1}{n}, \frac{1}{n}[ \quad \forall n \in \mathbb{N}$

$\Rightarrow \varphi_n(0) = \frac{1}{n} \Rightarrow f(0) = \lim_{n\to\infty} \frac{1}{n} = 0$ (1)

$x \neq 0 \Rightarrow \exists \; N \in \mathbb{N}$ mit $|x| > \frac{1}{N}$

Für $n \geq N$ gilt: $|x| > \frac{1}{N} \geq \frac{1}{n}$

$\Rightarrow \varphi_n(x) = 1$ für $n \geq N$

$\Rightarrow f(x) = \lim_{n\to\infty} \varphi_n(x) = 1.$ (2)

$$f(x) = \begin{cases} 1 & \text{für } x \in [-1, 1] \setminus \{0\} \\ 0 & \text{" } x = 0. \end{cases}$$ (1)

2.

$$\varphi_n(x) - f(x) = \begin{cases} 0 & \text{für } |x| \geq \frac{1}{n} \\ \frac{1}{n} - 1 & \text{" } 0 < |x| < \frac{1}{n} \\ \frac{1}{n} & \text{" } x = 0 \end{cases}$$ (1)

$$\Rightarrow \|\varphi_n - f\|_s = \max(0, 1 - \frac{1}{n}, \frac{1}{n}) = \begin{cases} 1 & \text{für } n = 1 \\ 1 - \frac{1}{n} & \text{" } n \geq 2 \end{cases}$$ (1)

$\Rightarrow \lim_{n\to\infty} \|\varphi_n - f\|_s = 1$

$\Rightarrow (\varphi_n)$ konvergiert nicht glm. gegen $f$. (1)

3. $f$ Treppenfkt., da $f|[-1, 0[ = 1$, $f|]0, 1] = 1$ (1)

4. $f$ Regelfkt., da Treppenfkt. (1)

4. Aufgabe

1. Quotientenkrit.

$$\frac{|x^{2k+3}|(2k+1)}{(2k+3)|x^{2k+1}|} = \frac{2k+1}{2k+3}|x|^2 \xrightarrow[k\to\infty]{} |x|^2 \overset{!}{<} 1$$ ①

$\Leftrightarrow |x|^2 < 1 \Leftrightarrow |x| < 1$. Folglich $\rho = 1$ ①

2. $$f'(x) = \sum_{k=0}^{\infty}(-1)^k x^{2k} = \sum_{k=0}^{\infty}(-x^2)^k = \frac{1}{1-(-x^2)}$$ ①

geom. Reihe

$$\Rightarrow f'(x) = \frac{1}{1+x^2}$$ ②

$$\left.\begin{aligned} \Rightarrow f(x) &= \arctan x + c, \\ f(0) = 0 &\Rightarrow c = 0 \end{aligned}\right\} \Rightarrow f(x) = \arctan x$$ ①

3. $$\lim_{x\to 0} \frac{3\arctan x - 3x + x^3}{x^5} \overset{\text{"}\frac{0}{0}\text{" Hôpital}}{=}$$

$$= \lim_{x\to 0} \frac{\frac{3}{1+x^2} - 3 + 3x^2}{5x^4} = \lim_{x\to 0} \frac{3-3-3x^2+3x^2+3x^4}{5x^4(1+x^2)} =$$

$$= \lim_{x\to 0} \frac{3}{5}\,\frac{1}{1+x^2} = \frac{3}{5}$$ ②

4. $$g(x) = \begin{cases} \dfrac{3\arctan x - 3x + x^3}{x^5} & \text{für } x \neq 0 \\ \dfrac{3}{5} & \text{für } x = 0 \end{cases}$$

(a). $3\arctan x - 3x + x^3 =$

$$= 3x - 3\cdot\frac{x^3}{3} + 3\sum_{k=2}^{\infty}(-1)^k \frac{x^{2k+1}}{2k+1} - 3x + x^3$$

$$= \sum_{k=0}^{\infty}(-1)^k\, 3\cdot\frac{x^{2k+5}}{2k+5}, \quad |x| < 1$$

$$\Rightarrow g(x) = \sum_{k=0}^{\infty} 3\cdot(-1)^k \frac{x^{2k}}{2k+5} = T_{g,0}(x), \quad |x|<1 \qquad (3)$$

(Anm.: $g(0) = \frac{3}{5}$)

(b) $g^{(2k+1)}(0) = 0$ für $k = 0, 1, 2, \ldots$

$$\frac{g^{(2k)}(0)}{(2k)!} = 3\cdot(-1)^k \cdot \frac{1}{2k+5}$$

$$\Rightarrow g^{(2k)}(0) = (-1)^k \cdot 3 \cdot \frac{(2k)!}{2k+5} \qquad (3)$$

(Folgt aus $T_{g,0}$)

5.

(a) $F(x) = \int_0^x \arctan t \, dt \underset{\text{P}}{=} t \arctan t \Big|_{t=0}^{x} - \int_0^x \frac{t\,dt}{1+t^2} =$

$u' = 1, \; v = \arctan t \quad \Big| \quad v = 1+t^2$

$u = t, \; v' = \frac{1}{1+t^2} \quad \Big| \quad \frac{dv}{dt} = 2t$

$= x \arctan x - \frac{1}{2} \ln |v| \Big|_{v=1}^{1+x^2} =$

$= x \arctan x - \frac{1}{2} \ln(1+x^2) \qquad (2)$

$(= x \arctan x - \ln\sqrt{1+x^2})$

(b) $F(x) = \int_0^x f(t)\,dt \underset{\text{Nr. 1}}{=} \sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1} \int_0^x t^{2k+1}\,dt$

$$\Rightarrow F(x) = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)(2k+2)} x^{2k+2} = T_{F,0}(x), \; |x|<1 \qquad (2)$$

$$\left( T_{F,0}(x) = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{(2k-1)\,2k} x^{2k}, \; |x|<1 \right)$$