Semestrale 03/04 Kemper

Aufgabe 3

$$f : \begin{cases} V \to V \\ f(X) \mapsto X + X^T \end{cases}$$

a) $\forall a \in \mathbb{R} \wedge \forall Y, Z \in V: \quad f(aY+Z) = a \cdot f(Y) + f(Z)$

$$f(aY+Z) = (aY+Z) + (aY+Z)^T$$
$$= aY + Z + (aY)^T + Z^T$$
$$= aY + Z + aY^T + Z^T$$
$$= aY + aY^T + Z + Z^T$$
$$= a(Y+Y^T) + (Z+Z^T)$$
$$= a\, f(Y) + f(Z) \quad \checkmark$$

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Aufgabe 4

$\dim(U_1) = 2$, da $-1 = \lambda \cdot (-1) \Rightarrow \lambda = 1$

aber $2 = 1 \cdot 5$ ↯

also Vektoren aus U_1 lin. unabh.

$\dim(U_2) = 2$ analog zu U_1

$$\begin{array}{c} \begin{pmatrix} -1 & 2 & 3 \\ -1 & 5 & 5 \\ 2 & -2 & 1 \\ -1 & 3 & -2 \end{pmatrix} \end{array} \begin{array}{l} \\ -Z1 \\ +2 \cdot Z1 \\ -Z1 \end{array} \Rightarrow \begin{pmatrix} -1 & 2 & 3 \\ 0 & 3 & 2 \\ 0 & 2 & 5 \\ 0 & 1 & -5 \end{pmatrix} \begin{array}{l} \\ -3 \cdot Z1^* \\ -2 Z2^* \\ \\ \end{array}$$

$$\Rightarrow \begin{pmatrix} -1 & 2 & 3 \\ 0 & 1 & -5 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix} \Rightarrow \begin{pmatrix} +1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\Rightarrow \dim(U_1 + U_2) = 3$$

$$\dim(U_1 + U_2) + \dim(U_1 \cap U_2) = \dim(U_1) + \dim(U_2)$$

$$\Rightarrow \dim(U_1 \cap U_2) = 2 + 2 - 3 = 1$$

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