Klausur zur Theoretischen Physik 2 (Elektrodynamik) – English version

24. February 2012

Notes or books are **not allowed**; hints are given in the text.
Write on **every** sheet: **Name**, **Matrikel-Nr.**
The exam lasts **90 Minutes**.
The maximum score is **100 Points**.

Exercise 1

A pointlike charge q is located in front of an infinite flat surface, which is conducting and neutral (see Fig. 1).

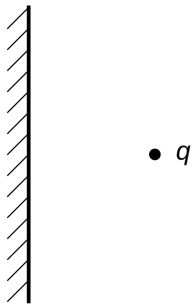

Fig. 1

a) How is the potential $\phi(\vec{r})$ near the charge q? Write the boundary condition for $\phi(\vec{r})$ on the conducting surface. [**5 Points**]

b) Replace the conducting surface by an image charge in such a way to reproduce the conditions on the potential in a). Show that the dipole in Fig. 2 reproduces such an arrangement. Evaluate in this way the electric field $\vec{E}(\vec{r})$ on the conducting surface. [**10 Points**]

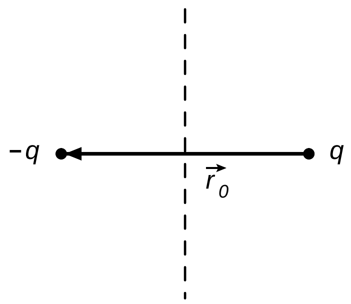

Fig. 2

c) Let F be an arbitrary portion of the conducting surface. Show that the integral $\int_F d\vec{f} \cdot \vec{E}(\vec{r})$ is proportional to the solid angle Ω under which the pointlike charge q sees the portion F. [**5 Points**]

Exercise 2

Consider a flat layer with thickness d in the vacuum (see Fig. 3). The layer (region (II)) has a (real) dielectric constant ε and conductivity σ (the magnetic permeability is $\mu = 1$).* A plane electromagnetic wave, coming from the vacuum (region (I)), falls orthogonally on the surface of the layer. To be determined are the intensities of the reflected wave in the region (I) and of the transmitted wave in the region (III). In particular, go through the following steps:

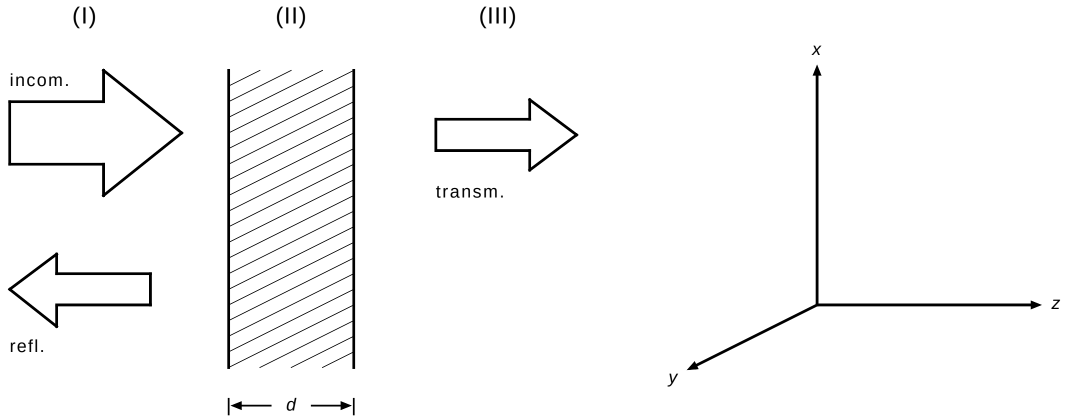

Fig. 3

a) Derive from Maxwell's equations the wave equations for the fields $\vec{E}$ and $\vec{B}$ in the vacuum regions (I) and (III) and show that $E_z = B_z = 0$. [**10 Points**]

b) In the regions (I) and (III), write the solution for $\vec{E}$ that is linearly polarised in the x direction, and the corresponding solution for $\vec{B}$. What is the relation between the frequency ω and the wavenumber k? [**10 Points**]

c) Write Maxwell's equations in the region (II) (i.e. inside the layer).
Hint: use Ohm's law $\vec{J} = \sigma\,\vec{E}$. [**10 Points**]

d) Write the wave equation for $\vec{E}$ in the region (II). For the complex refractive index n, derive the relation $n^2 = \varepsilon + 4\pi\, i\, \sigma/\omega$. [**10 Points**]

e) What are the continuity conditions to be fulfilled by $\vec{E}$ and $\vec{B}$ on the boundary surfaces? Apply these continuity conditions to the situation described in Fig. 3. (One obtains four linear equations for the non-vanishing field amplitudes.) [**15 Points**]

f) Write the reflection coefficient R and the transmission coefficient T of the layer in terms of ratios of squared field amplitudes in the regions (I) and (III). [**5 Points**]

g) For $\sigma = 0$, the refractive index $n = \sqrt{\varepsilon}$ is real. Show that in this case the reflection coefficient is given by

$$R = \frac{\sin^2(nkd)}{\sin^2(nkd) + \gamma^2}, \quad \text{with } \gamma = \frac{2n}{1-n^2}.$$

Discuss this result as a function of the thickness d of the layer. For which values of the wavelength $\lambda = 2\pi/(nk)$, does R reach its maximum? For which values of λ does no reflection occur? [**20 Points**]

*We use the Gauß system of physical units (cgs system of units).