final_project

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Exerciese 4.25

```r
# Distribution of the standard uniform distribution
f <- function(x, a=0, b=1) dunif(x, a,b)
F <- function(x, a=0, b=1) punif(x, a,b, lower.tail=FALSE)


# then we define the order statistics distribution in exercise 2.4 with
# expectation estimation function and median estimation function
orderStat <- function(x,r,n) {
  return(x * (1 - F(x))^(r-1) * F(x)^(n-r) * f(x))
}


expect <- function(r,n) {
  return((1/beta(r,n-r+1)) * integrate(orderStat,-Inf,Inf, r, n)$value)
}


median <- function(i,n){
  return((i-1/3)/(n+1/3))
}
# then calculate and compare n = 5 and n = 10 in the next block
```

the difference for n = 5 is very small, which means the approximation is great when n = 5.

```r
expect(2.5,5) - median(2.5,5)
```

```
## [1] 0.01041666
```

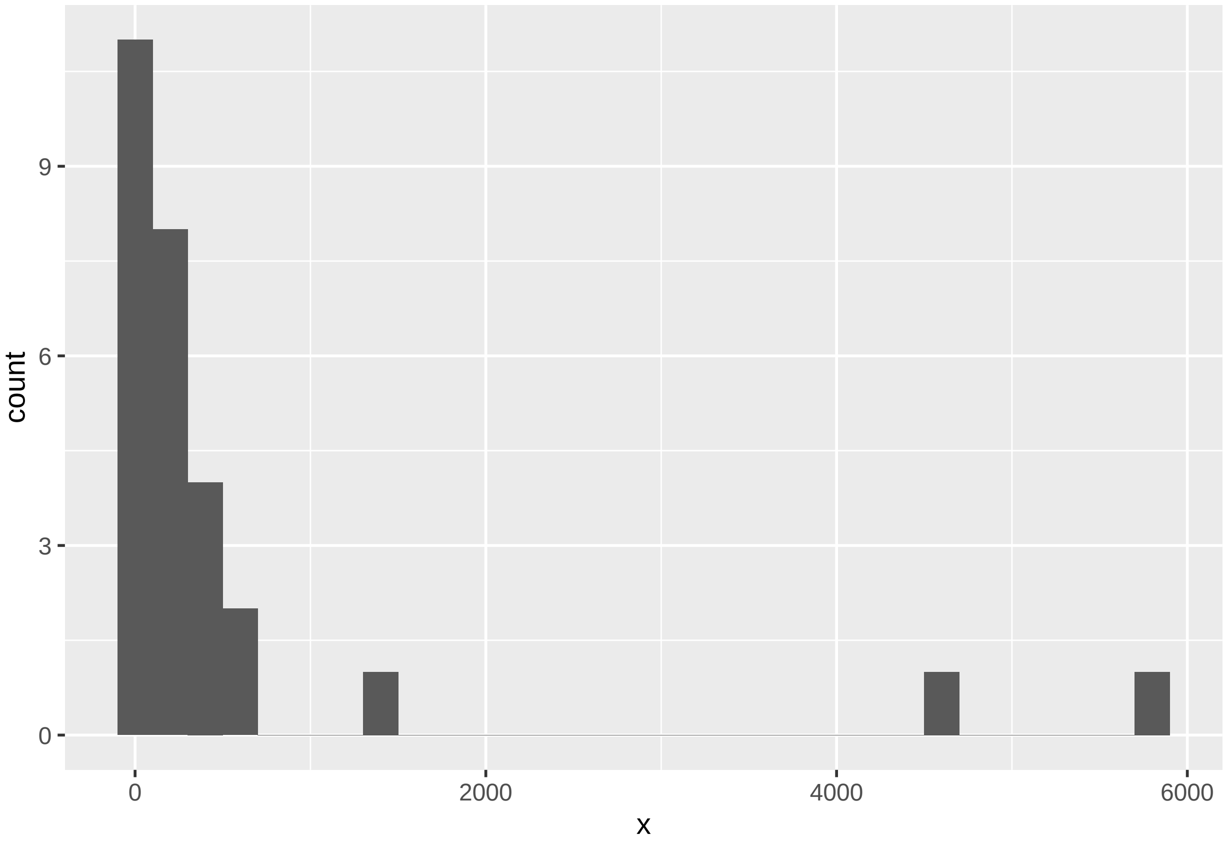
the difference for n = 10 is very small, which means the approximation is great when n = 10.

```r
expect(5,10) - median(5,10)
```

```
## [1] 0.002932548
```

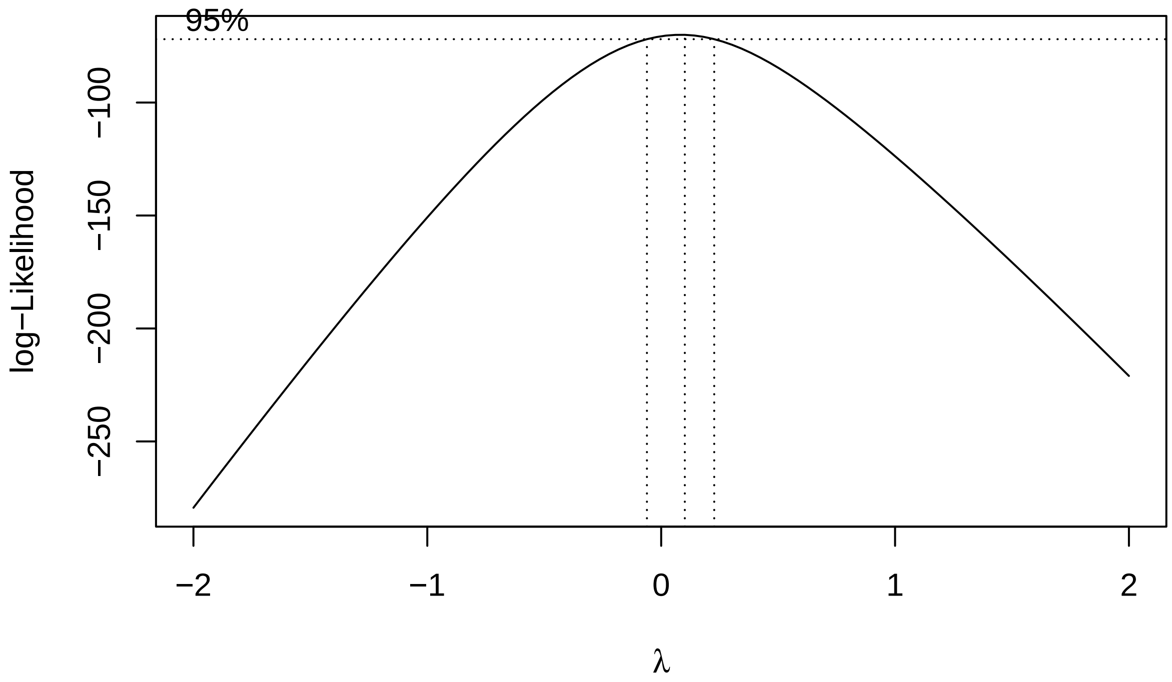
Exercise 4.39

```
data <- data.frame(x = c(0.4,1.0,1.9,3.0,5.5,8.1,12.1,25.6,50.0,56.0,70.0,115.0,
                        115.0,119.5,154.5,157.0,175.0,179.0,180.0,406.0, 419.0,
                        423.0, 440.0, 655.0, 680.0, 1320.0, 4603.0, 5712.0))
ggplot(data) + geom_histogram(aes(x = x), binwidth = 200)
```



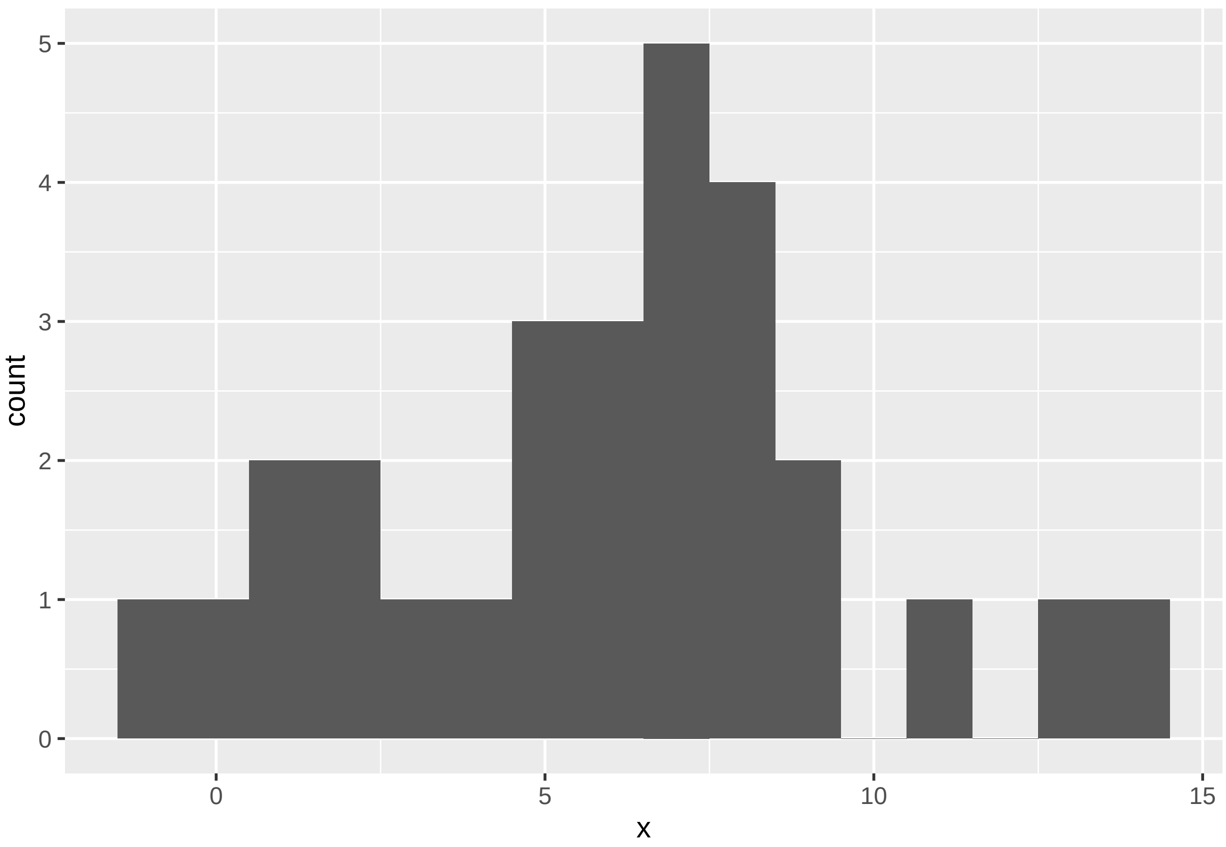
The plot shows that the initial distribution of the data is not normally distributed. Then we conduct the Box–Cox transformation, and plot the lambda graph and extract the optimal lambda, which is 0.1010101.

```
# make Boxcox transformation of the data
box <- boxcox(lm(data$x ~ 1))
```



Then use the optimal lambda to transform the original data, now the new data is nearly normally distributed.

```
# extract lambda
lambda <- box$x[which.max(box$y)]
tran_data <- (data ^ lambda - 1) / lambda
ggplot(tran_data) + geom_histogram(aes(x = x), binwidth = 1)
```



Exercise 4.27

Part a

```
Jan<-c(0.15,0.25,0.10,0.20,1.85,1.97,0.80,0.20,0.10,0.50,0.82,0.40,
       1.80,0.20,1.12,1.83,0.45,3.17,0.89,0.31,0.59,0.10,0.10,0.90,
       0.10,0.25,0.10,0.90)


Jul<-c(0.30,0.22,0.10,0.12,0.20,0.10,0.10,0.10,0.10,0.10,0.10,0.17,
       0.20,2.80,0.85,0.10,0.10,1.23,0.45,0.30,0.20,1.20,0.10,0.15,
       0.10,0.20,0.10,0.20,0.35,0.62,0.20,1.22,0.30,0.80,0.15,1.53,
       0.10,0.20,0.30,0.40,0.23,0.20,0.10,0.10,0.60,0.20,0.50,0.15,
       0.60,0.30,0.80,1.10,0.20,0.10,0.10,0.10,0.42,0.85,1.60,0.10,
       0.25,0.10,0.20,0.10)

summary(Jan)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##  0.1000  0.1875  0.4250  0.7196  0.9000  3.1700
```

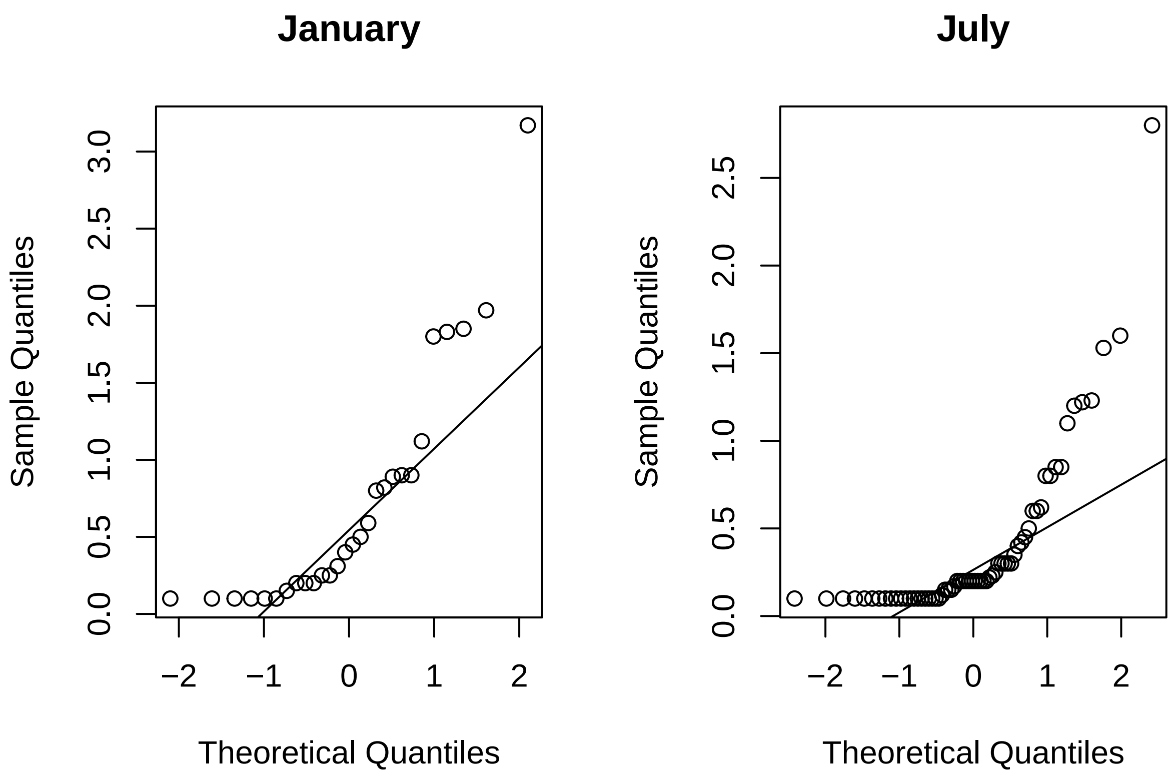
```
summary(Jul)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max. 
##  0.1000  0.1000  0.2000  0.3931  0.4275  2.8000
```

The two collections of data share the same minimum value, but generally value of the January data are all bigger than the data of July.

Part b

```
par(mfrow = c(1, 2))
qqnorm(Jan, pch = 1, main = "January")
qqline(Jan)
qqnorm(Jul, pch = 1, main = "July")
qqline(Jul)
```



According to the QQ plot, both two data are not follow the normal distribution.

part c

```r
Jan.fit <-  fitdist(Jan,'gamma','mle')
July.fit <-  fitdist(Jul,'gamma','mle')
summary(Jan.fit)
```

```
## Fitting of the distribution ' gamma ' by maximum likelihood
## Parameters :
##       estimate Std. Error
## shape 1.056222  0.2497495
## rate  1.467650  0.4396202
## Loglikelihood:  -18.7616   AIC:  41.5232   BIC:  44.18761
## Correlation matrix:
##           shape      rate
## shape 1.0000000 0.7893943
## rate  0.7893943 1.0000000
```

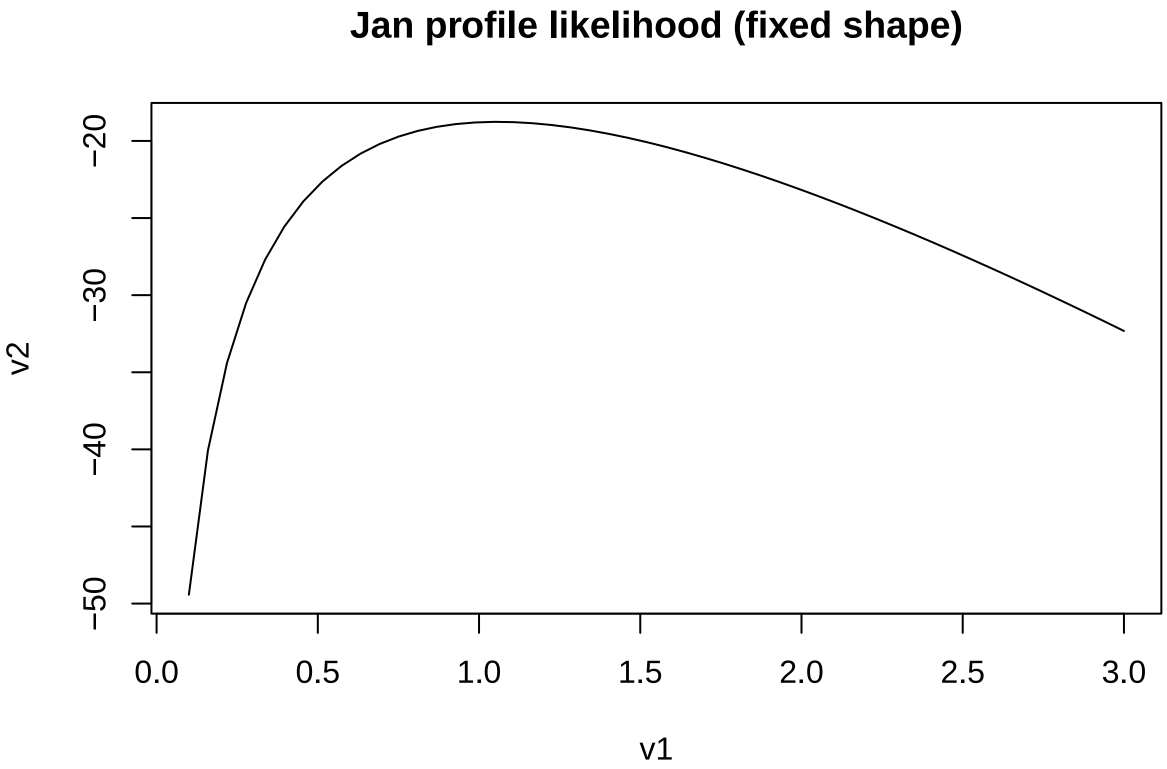
```r
summary(July.fit)
```

```
## Fitting of the distribution ' gamma ' by maximum likelihood
## Parameters :
##       estimate Std. Error
## shape 1.196419  0.1891196
## rate  3.043403  0.5936302
## Loglikelihood:  -3.634886   AIC:  11.26977   BIC:  15.58754
## Correlation matrix:
##           shape      rate
## shape 1.0000000 0.8103948
## rate  0.8103948 1.0000000
```

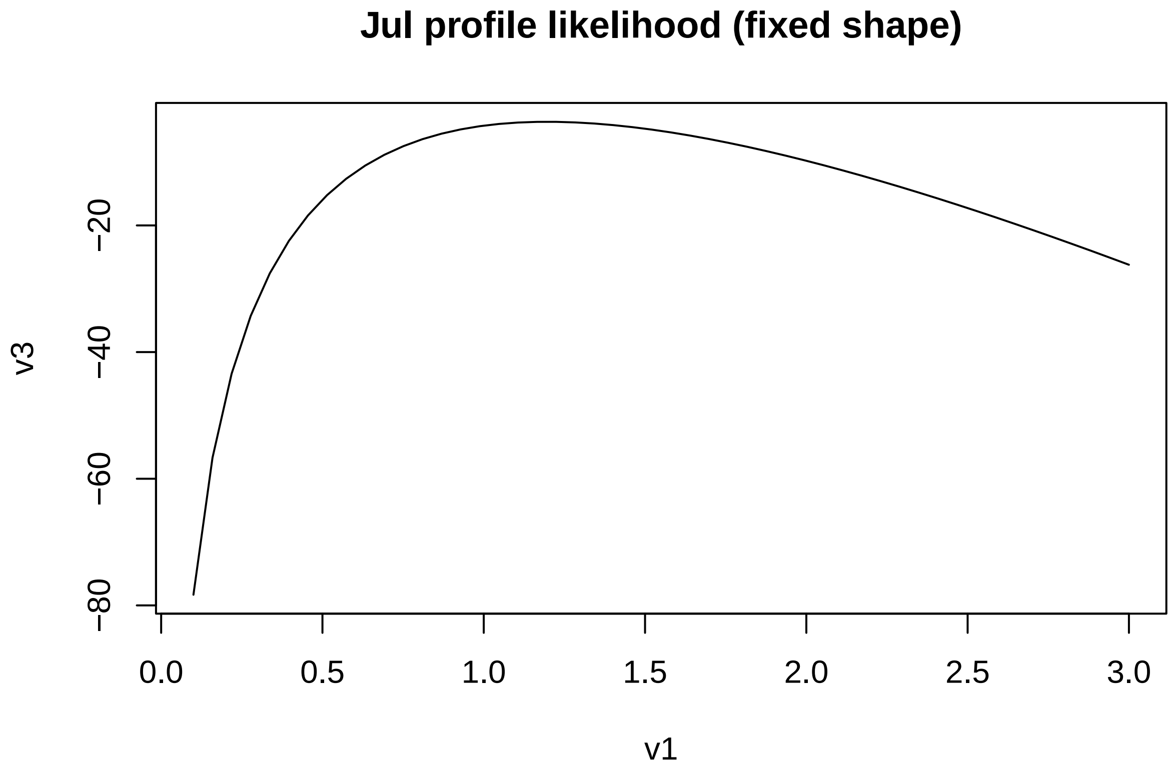
We can see the model for July with a smaller AIC than model for January. Then we plot the profile likelihood of both months with the fixed shape.

```r
x <- Jan
prof_log_lik=function(a){
   b=(optim(1,function(z) -sum(log(dgamma(x,a,z)))))$par
   return(-sum(log(dgamma(x,a,b))))
 }

v1 <- seq(.1, 3, length=50)
v2 <- -Vectorize(prof_log_lik)(v1)
plot(v1, v2, type="l", main = 'Jan profile likelihood (fixed shape)')
```

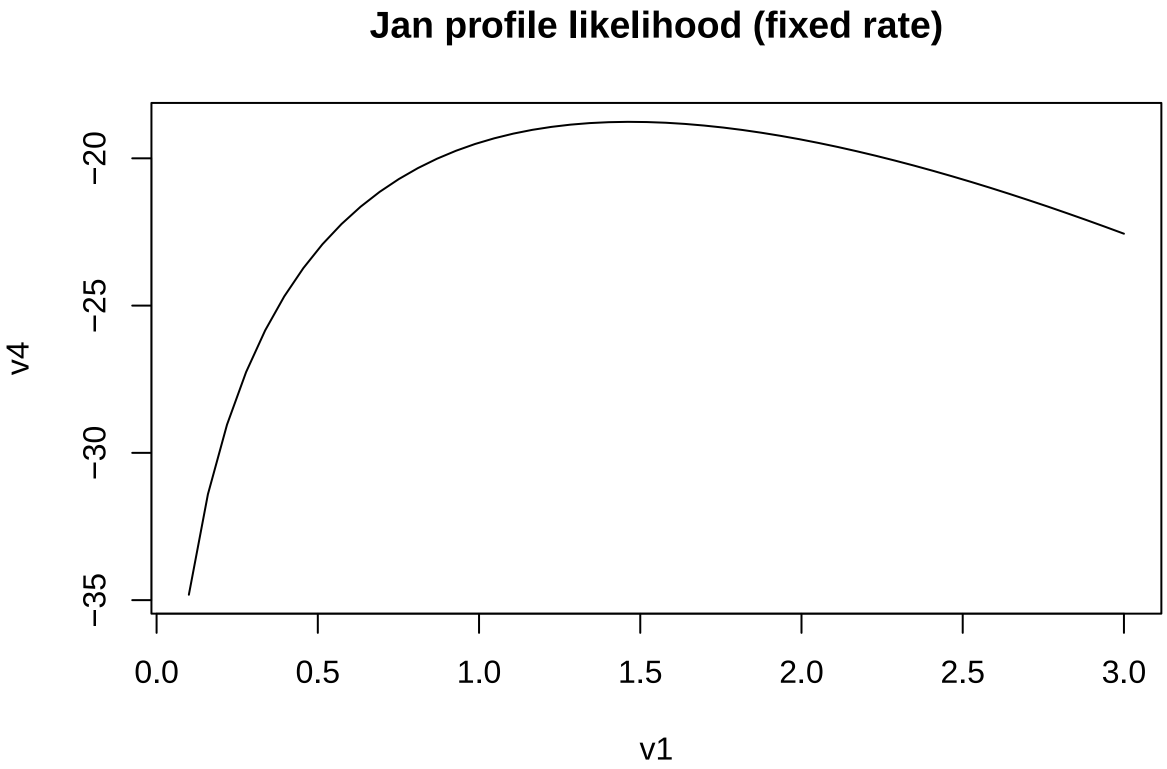


```
x <- Jul
v3 <-  -Vectorize(prof_log_lik)(v1)
plot(v1, v3, type="l", main='Jul profile likelihood (fixed shape)')
```

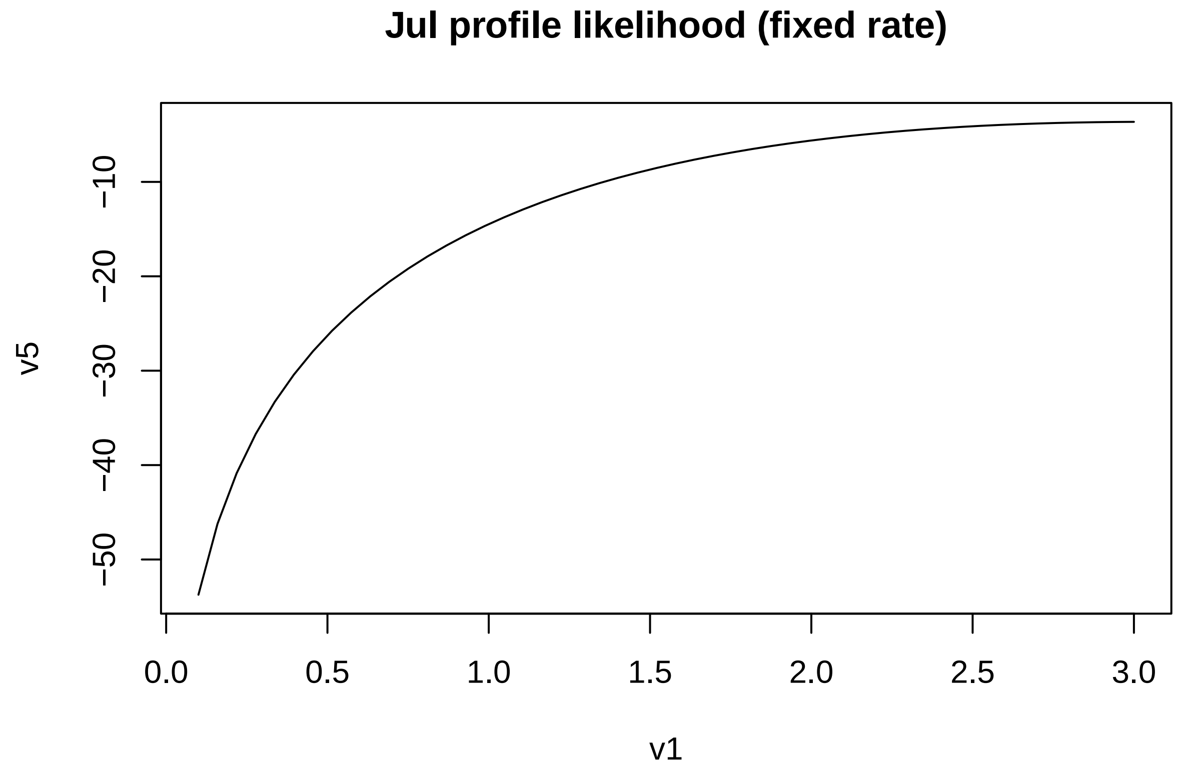


Then plot the profile likelihood of both mouth with the fixed rate.

```
x <- Jan
prof_log_lik=function(z){
   a=(optim(1,function(a) -sum(log(dgamma(x,a,z)))))$par
   return(-sum(log(dgamma(x,a,z))))
 }
v4 <- -Vectorize(prof_log_lik)(v1)
plot(v1, v4, type="l",main='Jan profile likelihood (fixed rate)')
```



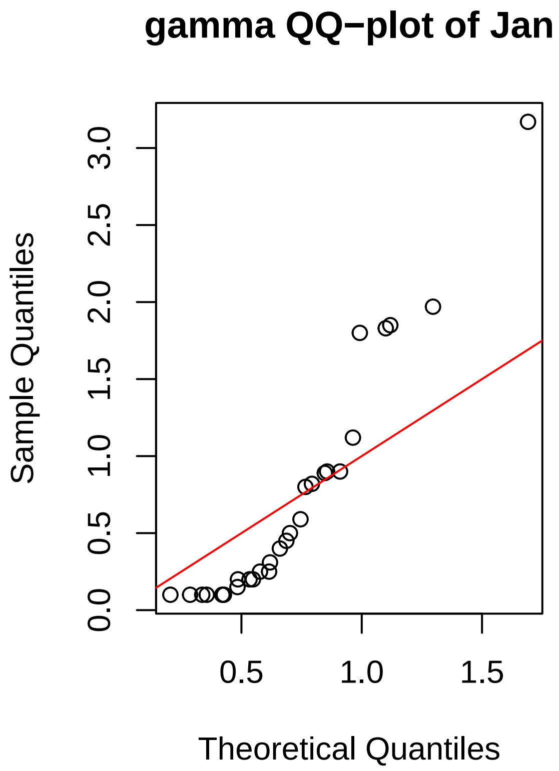
```
x <- Jul
v5 <- -Vectorize(prof_log_lik)(v1)
plot(v1, v5, type="l",main='Jul profile likelihood (fixed rate)')
```

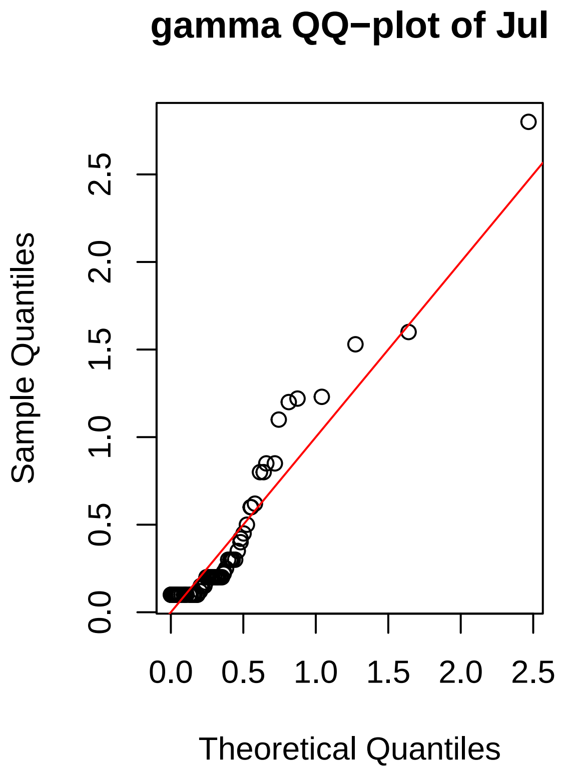


part d

Here we plot the gamma QQ-plot for both months and we can see that the Gamma distribution is more adequate than the normal distribution for these two collections of data.

```r
fit_Jan <- Jan.fit
fit_Jul <- July.fit
par(mfrow = c(1, 2))
mean_Jan <- fit_Jan$estimate[1]/fit_Jan$estimate[2]
var_Jan <- (fit_Jan$sd)^2
probabilities = (1:length(Jan))/(length(Jan)+1)
gamma.quantiles = qgamma(probabilities, shape = mean_Jan^2/var_Jan, scale = var_Jan/mean_Jan)
plot(sort(gamma.quantiles), sort(Jan), xlab = 'Theoretical Quantiles', ylab = 'Sample Quantiles', main =
abline(0,1, col = "red")
mean_Jul <- fit_Jul$estimate[1]/fit_Jul$estimate[2]
var_Jul <- (fit_Jul$sd)^2
probabilities = (1:length(Jul))/(length(Jul)+1)
gamma.quantiles = qgamma(probabilities, shape = mean_Jul^2/var_Jul, scale = var_Jul/mean_Jul)
plot(sort(gamma.quantiles), sort(Jul), xlab = 'Theoretical Quantiles', ylab = 'Sample Quantiles', main =
abline(0,1, col = "red")
```





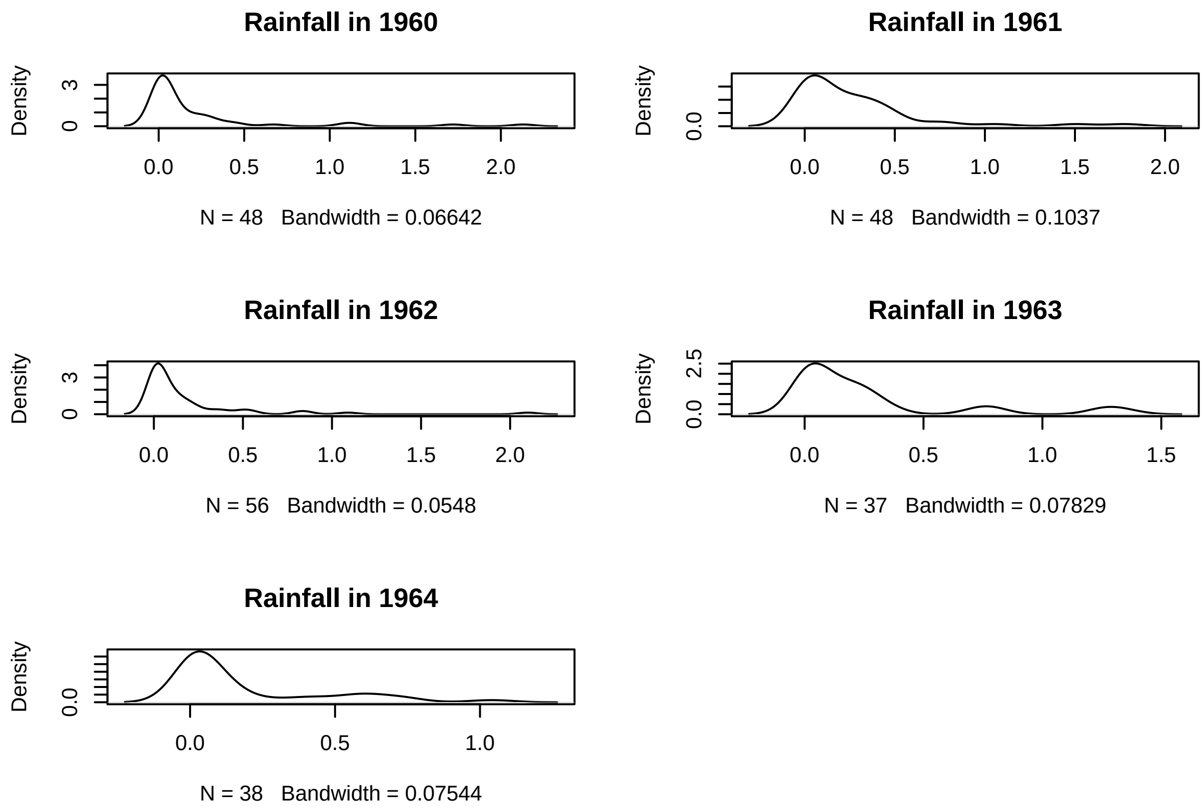
Illinois rainfall

part a

Questions: Use the data to identify the distribution of rainfall produced by the storms in southern Illinois. Estimate the parameters of the distribution using MLE. Prepare a discussion of your estimation, including how confident you are about your identification of the distribution and the accuracy of your parameter estimates.

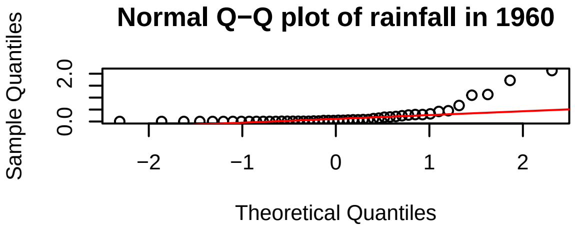
Distribution of the data using density plots:

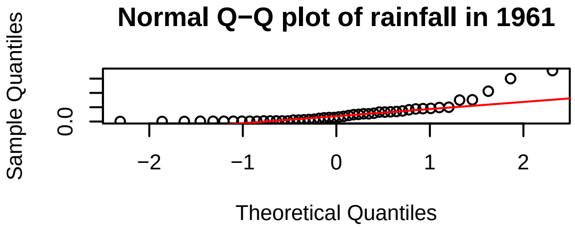
```
rain <- read.xlsx('Illinois_rain_1960-1964.xlsx')
years <- c(1960, 1961, 1962, 1963, 1964)
par(mfrow = c(3, 2))
for (i in 1:5){
   plot(density(na.omit(rain[,i])), main = paste("Rainfall in", years[i]))
}
```

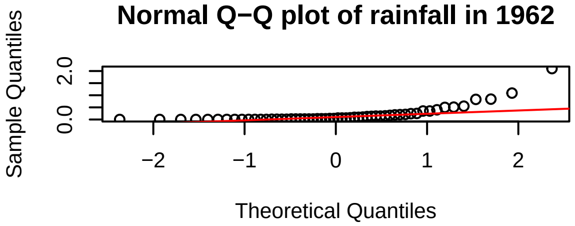


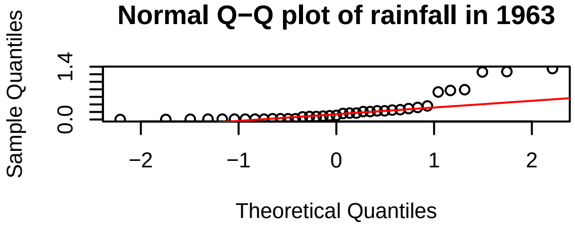
### QQplot check for nomality

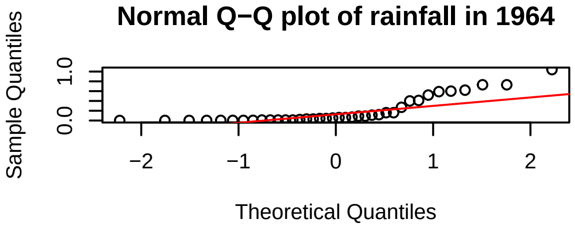
```r
par(mfrow = c(3, 2))
for (i in 1:5) {
  temp <- na.omit(rain[,i])
  qqnorm(temp, pch = 1, main = paste("Normal Q-Q plot of rainfall in", years[i]))
  qqline(temp, col = "red", lwd = 1)
}
```











The data are similar to normal distribution but with a wider deviance so we use gamma distribution.

```
fit <- fitdist(c(na.omit(unlist(rain))), 'gamma', method='mle')
summary(fit)
```

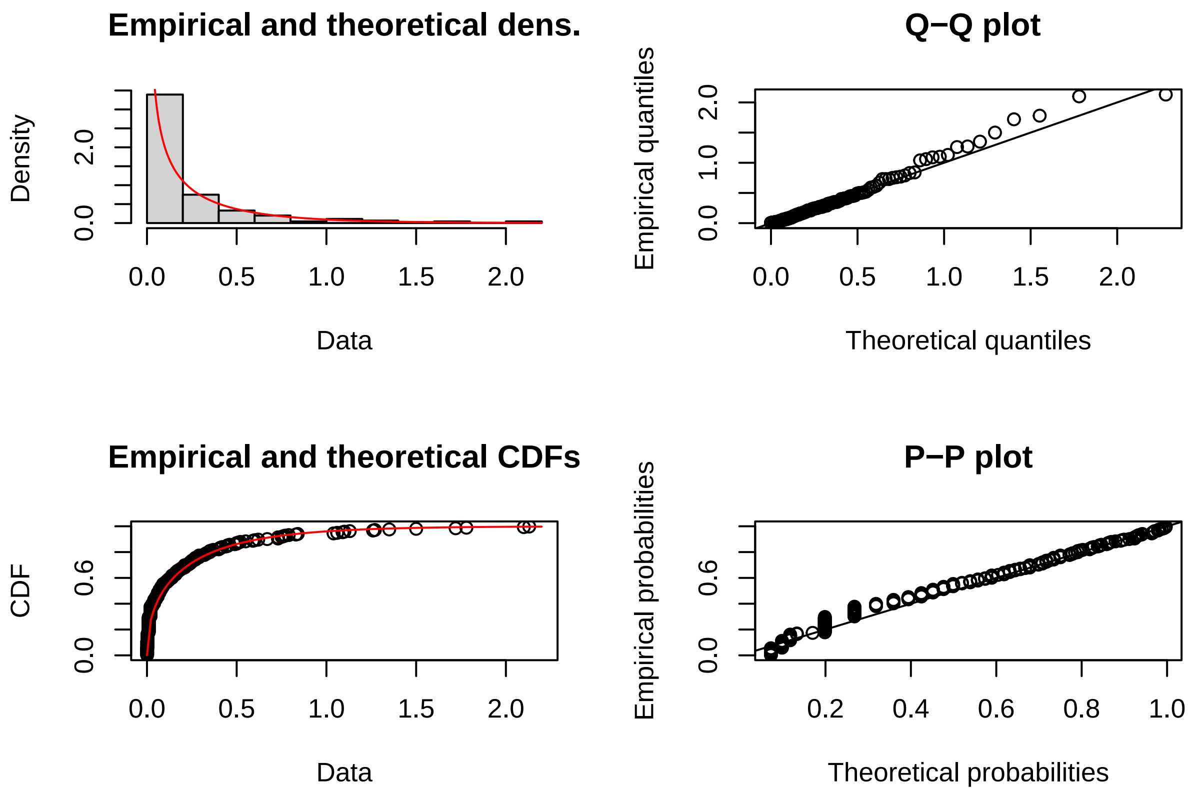
```
## Fitting of the distribution ' gamma ' by maximum likelihood
## Parameters :
##        estimate Std. Error
## shape 0.4408386  0.0337663
## rate  1.9648409  0.2474440
## Loglikelihood:  185.3477   AIC:  -366.6954   BIC:  -359.8455
## Correlation matrix:
##           shape      rate
## shape 1.0000000 0.6082109
## rate  0.6082109 1.0000000
```

```
summary(bootdist(fit))
```

```
## Parametric bootstrap medians and 95% percentile CI
##          Median      2.5%     97.5%
## shape 0.4439784 0.3842745 0.5146969
## rate  1.9937043 1.5870845 2.5610346
```

The model fit Gamma distribution with shape = 0.36 and rate = 1.66 and 95% CI of shape as wel as rate shows the result is statistically significant.

```
plot(fit)
```



According to the baseline in the plot, we can see the model fit the data distribution well.

Part b

Questions: Using this distribution, identify wet years and dry years. Are the wet years wet because there were more storms, because individual storms produced more rain, or for both of these reasons?

We compare the mean rainfall of storm in each year to overall data distribution and consider the number of stroms in each year to define the year is a dry year or wet year.

We calcalate the 95% CI of overall mean, which calculate from the paameter of the distibution. So we can say that when the aveage rainfall of one year lower than the lower bound of 95%CI of overall mean value, we will identify this year as a dry year, otherwise this year will be wet year.

```
overall_mean <- fit$estimate[1] / fit$estimate[2]
overall_CI_L <- (fit$estimate[1] + 2*fit$sd[1])/(fit$estimate[2] + 2*fit$sd[2])
overall_CI_U <- (fit$estimate[1] - 2*fit$sd[1])/(fit$estimate[2] - 2*fit$sd[2])

each_mean <- apply(rain, 2, mean, na.rm = TRUE)
num_storm <- apply(!is.na(rain), 2, sum) %>% round(1)
result <- as.data.frame(t(data.frame(mean = c(each_mean, overall_mean),
                     storm = c(num_storm, mean(num_storm)))))
result$`95%CI` <- c(overall_CI_L, overall_CI_U)
names(result)[6]='overall mean'
```

```
result <- cbind(rownames(result), result)
flextable(result)
```

rownames(result)	1960	1961	1962	1963	1964	overall mean	95%CI
mean	0.2202917	0.2749375	0.18475	0.2624324	0.1871053	0.2243635	0.2066777
storm	48.0000000	48.0000000	56.00000	37.0000000	38.0000000	45.4000000	0.2539578

We can found from the table, 1962 and 1964 are dry year and 1960,1963,1961 are wet year. While in 1962 there are 56 strom, we can consider that the participant of each storm not get high value.

Part c

Question: To what extent do you believe the results of your analysis are for generalization? What do you think the next steps would be after the analysis? An article by Floyd Huff, one of the authors of the 1967 report is included.

Answer: Although it shows that our model is fitting well, it is only based on five years' data. So, if we really want to generalize our findings, we need more rainfall data from multiple years and multiple locations, and then rebuild our model. So, the next step would be to collect more data and update the model. In the article by Floyd Huff, it provides strong evidence that even a 100- year record of point rainfall may be misleading in estimating the frequency of extreme rainfall events. This shows that it is really hard to generalize the result and build a weather forecast system based only on partial data.

Reference:

1. https://github.com/qPharmetra/qpToolkit/blob/master/R/qqGamma.r
2. https://www.r-bloggers.com/2015/11/profile-likelihood/
3. https://stackoverflow.com/questions/24211595/order-statistics-in-r?msclki d=fd6683dac56711ecbfcea9bd8a172395
4. Special thanks to my folk Xihao Cao who has provided many ideas and advice to this project. He helps me to get a deeper insight into the Gamma distribution and how it differs from the normal distribution. Meanwhile, I also have learned how to apply the gamma distribution when normal distribution does not work.