## Exercise set 2, Statistical Methods in Physics, 2025

Assessment criteria: for "pass" (Grade 3), solutions to all exercises marked "mandatory" must be handed in before **10.00 am** on **December 11th**. In addition, the student must choose and solve the "advanced" exercises from either this set or a from one of the other sets. For "passed with distinction" (Grade 4 or 5) , solutions to all exercises marked "mandatory" must be solved before **10.00 am** on **December 10th** and in addition, the student must choose two out of three "advanced" sets of exercises from this and the other exercise sets.

All exercises must be correct (after iterating with the teacher) on **January 30th.**

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1. Method of moments, consistency and bias: The Λ_c^+ baryon is a particle consisting of three quarks just like the proton, but where one of the light *up* quarks is replaced by the more than 500 times heavier *charm* quark. The Λ_c^+ can be produced together with its antiparticle in electron-positron annihilation: $e^+e^- \to \Lambda_c^+ \bar{\Lambda}_c^-$. Such an experiment has been carried out at the BESIII experiment in China at a center-of-mass energy of 4.5745 GeV. The Λ_c^+ is produced at an angle θ with respect to the e^+e^- beam according to the distribution $W(\cos\theta)= 1+\alpha\cos^2\theta$. In Phys. Rev. Lett. 120, 132001 (2018), the BESIII collaboration found the parameter α to be = - 0.13 $\pm$ 0.12 $\pm$0.08.
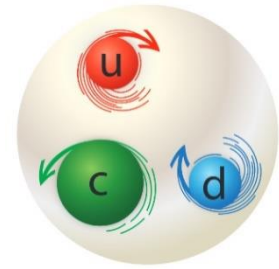
a) **Mandatory:** Express α in terms of the moment $<\cos^2\theta>$. What is the estimator of the moment $<\cos^2\theta>$ in this case?

b) **Mandatory:** Express the variance of the estimator in terms of the estimators of the moments $<\cos^2\theta>$ and $<\cos^4\theta>$.

2. Poisson upper limits (Mandatory): A large pharmaceutical company wants to test a new medicine in a Phase II study where 500 people were recruited as a test sample.

a) None of these 500 people show any symptoms of a certain rare but possible side effect. Assume (somewhat unrealistically) that these symptoms cannot occur for any other reason (*i.e.* the background is zero). Based on this, estimate the 95% C.L. upper limit of the risk (quantified in %) of obtaining this side effect as a consequence of the medicine.

b) After the successful Phase II study, it is time for Phase III. Now, 50 000 people are tested. What is the 95% C.L. upper limit of the risk of getting the side-effect, if the results are the same as in a), i.e. no one show any symptoms of the side-effect ?

c) What if 5 people out of 50 000 indeed show symptoms of the side effect, but that a placebo study predicts that 8 out of 50 000 people should get symptoms for other reasons than as a side-effect from the medicine? Estimate (an approximation is sufficient) the 95% C.L. upper limit using the **Bayesian** approach.

More exercises on the next page

3. Hypothesis test: When the 1987 Supernova (SN1987A) occurred, interactions from neutrinos originating from the supernova were seen in two large underground experiments: the IMB experiment in the US and the Kamiokande (KAM) experiment in Japan. The events in the detectors are expected to produce recoil particles with an angular distribution of the form

$$\frac{d\sigma}{d(\cos\theta)} \propto 1 + \alpha\cos\theta$$

where $\alpha \approx 0.1$, with respect to the direction of the supernova. The measured events are tabulated below and both experiments were sensitive to the full angular range. Perform a Kolmogorov-Smirnov test (NOT using pre-written software!) to find out whether or not we can reject the hypothesis that

a) **(Mandatory)** the experimental results from the two experiments are compatible with each other at 5% and 1% significance. Please include all steps in your solution, in particular, how you define your test statistic and the critical value.

b) **(Mandatory)** the experimental results are compatible with the expected angular distribution at 5% and 1% significance, treating all the data as coming from the same source (i.e. forming one common sample out of the two).

IMB event	Recoil Angle (degrees)
1	80
2	44
3	56
4	65
5	33
6	52
7	42
8	104
KAM event	
1	18
2	32
3	30
4	38

(The other KAM events are below the IMB energy threshold.)
Eventual uncertainties in the angular determination can be neglected.

4. **Least Square Fitting**: The electronics circuit to the right induces a signal measured at V_c. Given that the capacity C is known, one can determine the internal resistance R and inductance L of the black box "Little Henry" by measuring q(w) :

$$\cot \theta = (L/R)\omega - (1/RC)\omega^{-1}$$
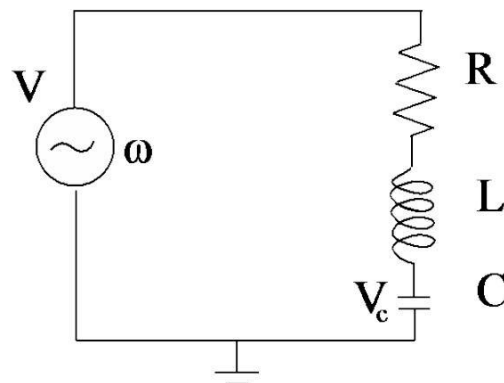
Using $\alpha_1 = \frac{\omega_0 L}{R}, \alpha_2 = \frac{1}{\omega_0 RC}, \; x \equiv \frac{\omega}{\omega_0}$ where $\omega_0 = 1$ rad/s, this equation may be written as

$$y = \cot\theta = \alpha_1 x - \alpha_2 x^{-1}$$

A measurement at five frequencies by connecting a known capacitor with C = 0.02 µF to the circuit, gave the following results:

Y	σy	x	σx
-4.017	0.5	22000	440
-2.742	0.25	22930	470
-1.1478	0.08	23880	500
1.491	0.09	25130	530
6.873	1.90	26390	540

a) **Mandatory:** Determine the values of L and R, and their uncertainties, of "Little Henry" neglecting the uncertainties in x. What is the χ^2 of the fit?

b) **Mandatory:** Plot the covariance ellipse and extract the correlation coefficient using the intersect method.

   Advanced: Determine the values of L and R (with uncertainties) of "Little Henry" neglecting the errors in y. What is the χ^2 of the fit? Plot the covariance ellipse to extract the uncertainties and covariance.
   Hint. This problem is non-linear, which requires a numerical solution. You can use standard software packages for this.

c) **Advanced:** Determine the values of L and R (with uncertainties) of "Little Henry", taking into account **both** the uncertainties in x and y, using the method of effective variance. What is the χ^2 of the fit?

d) **Advanced:** Plot the results of the fits together with the data. Do you observe any trend in the uncertainties and the χ^2 for the cases a-c? Is this expected?

Please read the guidelines on the next page

Guidelines for solutions

1. The solutions should be provided in PDF format (not as a notebook) in a private message either on Studium or Slack.
2. The solutions should include the step-by-step explanations and motivations.
3. If you generate your pdf file from a python notebook, please make sure that you add blocks of text for explanations and analytical mathematical expressions,
4. Please include code snippets in your pdf file if relevant for your solution. You do not need to use python but can use ROOT, MATLAB or another language of choice.
5. If you want to write the explanations by hand, please provide a clear and clean text.
6. Hand-drawn figures are only suitable for explaining concepts and cannot be used for extracting numbers. If you use a graphical method for estimating a quantity, please display the figure from the software in which you extract the numerical solution.
7. You are allowed to collaborate, but should hand in your solutions individually and it should be clarified with whom you have collaborated.
8. In general, generative AI is **not** allowed for solving exercises, **BUT** so-called code completion is allowed.