

Exercise set 1, Statistical Methods in Physics and Engineering, 2025

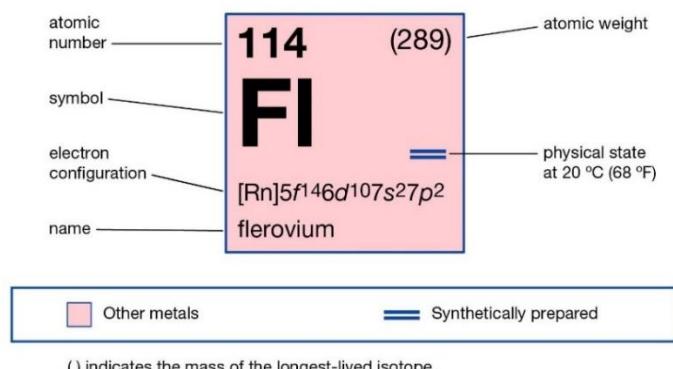
Assessment criteria: for grade “pass” (3), solutions to all exercises marked “mandatory” must be handed in before **10 am on November 24th**. In addition, the student must choose and solve the “advanced” exercises from either this set or a later one. For “pass with distinction” (4 or 5), solutions to all exercises marked “mandatory” and addition, the student must choose two out of three “advanced” sets of exercises from this and the coming sets. In this set, exercises 3 b-d count as one set of advanced exercises.

All exercises must be correct (after iterating with the teacher) on **January 30th**.

Literature: G. Cowan, *Statistical Data Analysis*, chapter 1, 5, 6 and 9.

1. Maximum likelihood (Mandatory) Many processes in nature (*e.g.* radioactive decays, concentration of a chemical substance in a human body), and society (*e.g.* page requests on Wikipedia, time between phone calls) occur continuously and independent of each other, at an overall constant rate. These processes follow an exponential probability density function.

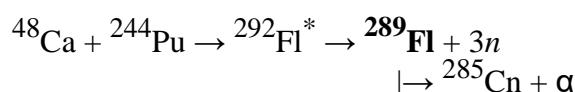
Flerovium



In a fictive experiment, we study the superheavy element Flerovium (²⁸⁹Fl). Having 114 protons and 185 neutrons (cf. ²³⁸U has Z = 92, N = 146), this isotope can be produced in Calcium-Plutonium reactions and since it is unstable, it decays *via* α -decay into Copernicium.

The full production and decay process is written

© Encyclopædia Britannica, Inc.



Six such decays were recorded:

Decay	Time (s)
1	3.88927
2	0.06820
3	0.55236
4	2.17826
5	9.02553
6	0.90834

- Derive the Maximum-Likelihood (ML) estimator for the lifetime τ of the ²⁸⁹Fl isotope, calculate it and give the asymmetric uncertainties $\Delta\tau_+$ and $\Delta\tau_-$.
- With two lines of python code, you can generate a sample of N events with a lifetime τ :

```
import numpy as np
print(np.random.exponential(τ, N))
```

Generate a sample with the lifetime you obtained in a). Check the *consistency* of the ML estimator by estimating the lifetime for a sample of 10, 100, 1000 and 10000 events. Please also calculate the uncertainties for $N \geq 100$.

More exercises on the next page

- c) Calculate the bias of the Maximum-Likelihood estimator for a sample size of 50 events.

2. Combining uncertainties (Mandatory): The Particle Data Group (PDG) has compiled available data on the neutrino mass squared (see table below and here (<https://pdg.lbl.gov/>).) and calculated a world average. When two uncertainties are given, the first uncertainty is statistical and the second is systematic. However, the PDG only included a few measurements, mainly because many of the estimators from older measurements are negative.

- a) Combine all available measurements to calculate an “all-inclusive” world average. Assume that all measurements are independent and for the measurement labelled “Aker 19”, symmetrise the uncertainty by taking the average of the upper and the lower uncertainty. Statistical and systematic uncertainties from the same measurement are added in quadrature.

The most precise measurement so far is provided by the KATRIN experiment ([Nature Physics 18, 160-166 \(2022\)](#)) and found it to be $m_\nu^2 = 0.26 \pm 0.34 \text{ eV}^2 \text{c}^{-4}$. What upper mass limits of this quantity do you obtain for a confidence level of 90% and 95%, respectively

- b) when using the classical frequentist method, *i.e.* by integrating the P.D.F.
 c) when using the Bayesian method.

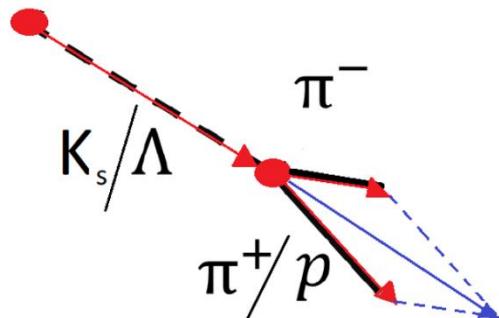
ν MASS SQUARED (electron based)

Given troubling systematics which result in improbably negative estimators of $m_{\nu_e}^{2(\text{eff})} \equiv \sum_i |\mathbf{U}_{ei}|^2 m_{\nu_i}^2$, in many experiments, we use only KRAUS 05, LOBASHEV 99, and AKER 22 for our average.

VALUE (eV ²)		DOCUMENT ID	TECN	COMMENT
0.08 ± 0.30 OUR AVERAGE				
0.1 ± 0.3	¹ AKER	22	SPEC	${}^3\text{H}$ β decay
- 0.67 ± 2.53	² ASEEV	11	SPEC	${}^3\text{H}$ β decay
- 0.6 ± 2.2 ± 2.1	³ KRAUS	05	SPEC	${}^3\text{H}$ β decay
• • • We do not use the following data for averages, fits, limits, etc. • • •				
- 1.0 ± 0.9	⁴ AKER	19	SPEC	${}^3\text{H}$ β decay
- 1.9 ± 3.4 ± 2.2	⁵ LOBASHEV	99	SPEC	${}^3\text{H}$ β decay
- 3.7 ± 5.3 ± 2.1	⁶ WEINHEIMER	99	SPEC	${}^3\text{H}$ β decay
- 22 ± 4.8	⁷ BELESEV	95	SPEC	${}^3\text{H}$ β decay
129 ± 6010	⁸ HIDDEMANN	95	SPEC	${}^3\text{H}$ β decay
313 ± 5994	⁸ HIDDEMANN	95	SPEC	${}^3\text{H}$ β decay
- 130 ± 20 ± 15	⁹ STOEFL	95	SPEC	${}^3\text{H}$ β decay
- 31 ± 75 ± 48	¹⁰ SUN	93	SPEC	${}^3\text{H}$ β decay
- 39 ± 34 ± 15	¹¹ WEINHEIMER	93	SPEC	${}^3\text{H}$ β decay
- 24 ± 48 ± 61	¹² HOLZSCHUH	92B	SPEC	${}^3\text{H}$ β decay
- 65 ± 85 ± 65	¹³ KAWAKAMI	91	SPEC	${}^3\text{H}$ β decay
- 147 ± 68 ± 41	¹⁴ ROBERTSON	91	SPEC	${}^3\text{H}$ β decay

[More exercises on the next page](#)

3. Error propagation (Advanced): An experiment measuring the production of neutral strange particles has observed a “ V^0 ” event, *i.e.* a neutral and hence invisible particle decaying into two particles with opposite charge (in old bubble-chamber experiments, these events look like a V, which is where the name comes from). Two particles can give rise to this kind of detector signature; either a Λ hyperon decaying into a proton and a pion and K_s meson decaying into two pions:



Information about Λ , K_s and π^\pm can be found under “Particle Listings” on the PDG webpage (<https://pdg.lbl.gov/>).

The flight direction of the V^0 is determined from the measured production and decay points. The momenta and angles of the decay particles are also measured.

The experimentally measured quantities, given in Cartesian coordinates, are:

Production point: $(x, y, z) = (-44.40 \pm .14, -1.80 \pm .17, -16.20 \pm .26)$ mm
 Decay point: " $(-28.80 \pm .15, -5.30 \pm .16, -16.00 \pm .26)$ mm

- a) Calculate the direction vector of the V^0 particle in spherical coordinates (r, θ, ϕ) along with its uncertainties.
- b) Give the analytical and numerical expression for the covariance matrix.
- c) What do off-diagonal elements of the covariance matrix mean?

You may use software packages for the algebra, but you need to present each step and clarify which assumptions you make etc.

Guidelines for solutions

1. The solutions should be provided in pdf format (not as a notebook) in a private message either on Studium or Slack.
2. The solutions should include the step-by-step explanations of your solution, in particular relevant assumptions, which theorem or simplification you use etc. Your procedure should be clearly motivated.
3. If you generate your pdf file from a python notebook, please make sure that you add blocks of text for explanations and analytical mathematical expressions,
4. Please include code snippets in your pdf file if relevant for your solution. You do not need to use python – you can use e.g. ROOT, MATLAB or whatever you are comfortable with as long as it is suitable for solving your problems.
5. If you want to write the explanations by hand, please provide a clear and clean text.
6. Hand-drawn figures are only suitable for explaining concepts and cannot be used for extracting numbers. If you use a graphical method for estimating a quantity, please display the figure from the software in which you extract the numerical solution.
7. You are allowed to collaborate, but it should be clarified with whom you have collaborated.
8. In general, generative AI is **not** allowed for solving exercises, **BUT** so-called code completion is allowed.