MATH1853 Part IV Assignment 5

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Question 1.

Since $N(1,4)$, then $\mu = 1$ and $\sigma = 2$.

(a) $Z < \dfrac{3-1}{2} = 1$, and find from the table that

$$P(X < 3) = P(Z < 1) = \phi(1) \approx 0.8413$$

(b) $Z \leq \dfrac{1.5-1}{2} = 0.25$, and find from the table that

$$P(X \leq 1.5) = P(Z \leq 0.25) = \phi(0.25) \approx 0.5987$$

(c)

$$P(1.5 < X < 3) = P(1.5 \leq X < 3) = P(X < 3) - P(X < 1.5) \approx 0.8413 - 0.5987 = 0.2426$$

Question 2.

$$P(|Z| \leq k) = P(-k \leq Z \leq k) = 0.85$$

and

$$P(Z \leq -k) = \frac{1 - 0.85}{2} = 0.075$$

and

$$P(Z \leq k) = 0.85 + 0.075 = 0.925$$

from the table

$$k \approx 1.4$$

Question 3.

(a)

$$P(X = x) = \binom{n}{x} p^n (1-p)^{n-x}$$

where $n = 100$ and $p = 0.1$.

$$\begin{aligned} P(7 \leq X < 11) &= P(X = 7) + P(X = 8) + P(X = 9) + P(X = 10) \\ &= 0.08889524636812617 + 0.11482302655882966 + 0.13041627707916453 + 0.13186534682448858 \\ &= 0.46599989683 \end{aligned}$$

(b) $\mu = np = 100 \cdot 0.1 = 10$
$\sigma^2 = np(1-p) = 10(1-0.1) = 9$, and hence $\sigma = 3$
So: $N(10, 9)$

(c) Apply Continuity Correction:

$$P_{discrete}(7 \leq X < 11) = P_{continuous}(6.5 < X < 10.5)$$

For $P(X < 6.5)$, $Z < \dfrac{6.5 - 10}{3} = -1.1666$, and find from the table that

$$P(X < 6.5) = P(Z < -1.1666) = \phi(-1.1666) \approx 0.1210$$

For $P(X < 10.5)$, $Z < \dfrac{10.5 - 10}{3} = 0.1666$, and find from the table that

$$P(X < 10.5) = P(Z < 0.1666) = \phi(0.1666) \approx 0.5675$$

Finally

$$P(6.5 < X < 10.5) = P(X < 10.5) - P(X < 6.5) = 0.5675 - 0.1210 = 0.4465$$

Question 4.

Estimate $p' = \dfrac{310}{10000} = 0.031$
By C.L.T.,

$$\bar{X}_n = p'$$

and since sample size is greater than 30,

$$\bar{X}_n \approx N(p, p(1-p)/n)$$

then

$$\frac{\bar{X}_n - p}{\sqrt{p(1-p)/n}} \approx Z$$

$$\frac{p' - p}{\sqrt{p(1-p)/n}} \approx Z$$

$$P(|Z| < k) = 0.9$$

$$P(Z < k) = \frac{1 + 0.9}{2} = 0.95$$

$$k = 1.645$$

finally the Confidence Interval:

$$\left[p' - Z_{\alpha/2}\sqrt{\frac{p'(1-p')}{n}}, p + Z_{\alpha/2}\sqrt{\frac{p'(1-p')}{n}}\right]$$

$$\left[0.031 - 1.645 \times \sqrt{\frac{0.031 \times 0.969}{10000}}, 0.031 + 1.645 \times \sqrt{\frac{0.031 \times 0.969}{10000}}\right]$$

$$[0.02814, 0.03385]$$