Esercizio su Gauss naif e fattorizz. LU

$$\begin{cases} x_1+x_2-2x_3=1 \\ x_1+x_3=0 \\ -2x_1+x_2-x_3=-3 \end{cases} \quad m_{21}=-1 \quad m_{31}=2 \quad \begin{cases} x_1+x_2-2x_3=1 \\ -x_2+3x_3=-1 \\ 3x_2-5x_3=-1 \end{cases} \quad m_{32}=3$$

$$\begin{cases} x_1+x_2-2x_3=1 \\ -x_2+3x_3=-1 \\ 4x_3=-4 \end{cases} \rightarrow \begin{array}{l} x_3=-1 \\ -x_2+3=-1 \rightarrow x_2=-2 \\ x_1-2-2=1 \rightarrow x_1=1 \end{array} \quad \underline{x}=\begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix}$$

$$L_1=\begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix} \quad L_2=\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 3 & 1 \end{pmatrix}$$

$$L_1^{-1}=\begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix} \quad L_2^{-1}=\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -3 & 1 \end{pmatrix} \rightarrow L=L_1^{-1}L_2^{-1}=\begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ -2 & -3 & 1 \end{pmatrix}$$

$$L_2L_1A=U=\begin{pmatrix} 1 & 1 & -2 \\ 0 & -1 & 3 \\ 0 & 0 & 4 \end{pmatrix} \qquad \text{Risolviamo: } \begin{cases} Ly=b \\ Ux=y \end{cases}$$

$Ly=b$: $\begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ -2 & -3 & 1 \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}=\begin{pmatrix} 1 \\ 0 \\ -3 \end{pmatrix}$

$y_1=1$

$y_1+y_2=0 \rightarrow y_2=-1$

$-2y_1-3y_2+y_3=-3 \rightarrow y_3=-4$

$$\underline{y}=\begin{pmatrix} 1 \\ -1 \\ -4 \end{pmatrix}$$

$Ux=y$ $\begin{pmatrix} 1 & 1 & -2 \\ 0 & -1 & 3 \\ 0 & 0 & 4 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}=\begin{pmatrix} 1 \\ -1 \\ -4 \end{pmatrix}$

$x_3=-1$

$-x_2+3x_3=-1 \rightarrow x_2=-2$

$x_1+x_2-2x_3=-4 \rightarrow x_1=1$

$$\underline{x}=\begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix}$$

Metodo di Gauss naif

$$\begin{cases} x+y+z=6 \\ 2x+3y-z=5 \\ x+2y+5z=20 \end{cases}$$

$m_{21}^{(1)} = -2$

$m_{31}^{(1)} = -1$

$$\begin{cases} x+y+z=6 \\ y-3z=-7 \\ y+4z=14 \end{cases}$$

$m_{32}^{(2)} = -1$

$$\begin{cases} x+y+z=6 \\ y-3z=-7 \\ 7z=21 \end{cases}$$

$\rightarrow z=3$

$\rightarrow y = 9-7=2$

$\rightarrow x=6-3-2=1$

$$sol = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

Metodo di Gauss con pivot parziale

$$\begin{cases} 2x+3y-z=5 \\ x+y+z=6 \\ x+2y+5z=20 \end{cases}$$

$m_{21}^{(1)} = -1/2$

$m_{31}^{(1)} = -1/2$

$$\begin{cases} 2x+3y-z=5 \\ -\frac{1}{2}y+\frac{3}{2}z=\frac{7}{2} \\ \frac{1}{2}y+\frac{11}{2}z=\frac{35}{2} \end{cases}$$

$m_{32}^{(2)} = 1$

$$\begin{cases} 2x+3y-z=5 \\ -\frac{1}{2}y+\frac{3}{2}z=\frac{7}{2} \\ 7z=21 \end{cases}$$

$\rightarrow z=3$

$-\frac{1}{2}y = \frac{7}{2}-\frac{3}{2}=-1$

$\downarrow$

$y=2$

$2x=-6+3+5=2 \rightarrow x=1$

$$sol = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

Metodo di Gauss con pivot totale

$$\begin{cases} 5\tilde{x} + 2\tilde{y} + \tilde{z} = 20 \\ -\tilde{x} + 3\tilde{y} + 2\tilde{z} = 5 \\ \tilde{x} + \tilde{y} + \tilde{z} = 6 \end{cases} \qquad \begin{matrix} \tilde{x} = 3 \\ \tilde{y} = 4 \\ \tilde{z} = 1 \end{matrix}$$

$m_{21}^{(1)} = 1/5$
$m_{31}^{(1)} = -\frac{1}{5}$

$$\begin{cases} 5\tilde{x} + 2\tilde{y} + \tilde{z} = 20 \\ \frac{17}{5}\tilde{y} + \frac{11}{5}\tilde{z} = 9 \\ \frac{3}{5}\tilde{y} + \frac{4}{5}\tilde{z} = 2 \end{cases} \qquad m_{32}^{(2)} = -3/17$$

$$\begin{cases} 5\tilde{x} + 2\tilde{y} + \tilde{z} = 20 \\ \frac{17}{5}\tilde{y} + \frac{11}{5}\tilde{z} = 9 \\ \frac{7}{17}\tilde{z} = \frac{7}{17} \rightarrow \tilde{z} = 1 \end{cases}$$

$\frac{17}{5}\tilde{y} = 9 - \frac{11}{5} = \frac{34}{5} \rightarrow \tilde{y} = 2$

$5\tilde{x} + 4 + 1 = 20 \rightarrow \tilde{x} = 3$

$$\Rightarrow \tilde{sol} = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} \rightarrow sol = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

Matrice di permutazione

$$P = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad PAP = \begin{pmatrix} 5 & 2 & 1 \\ -1 & 3 & 2 \\ 1 & 1 & 1 \end{pmatrix}, \quad PbP = \begin{pmatrix} 20 \\ 5 \\ 6 \end{pmatrix}$$