

Calcolo di autovalori

$$A = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 3 \\ 2 & 1 & 1 \end{pmatrix}$$

$$p(\lambda) = \det(A - \lambda I) = \det \begin{pmatrix} 1-\lambda & 2 & 1 \\ 0 & 1-\lambda & 3 \\ 2 & 1 & 1-\lambda \end{pmatrix} =$$

$$= (1-\lambda)^3 + 12 - 2(1-\lambda) - 3(1-\lambda) = -(\lambda^3 - 3\lambda^2 - 2\lambda - 8)$$

$$\lambda_1 = 4 \quad \begin{array}{c|ccc|c} & 1 & -3 & -2 & -8 \\ 4 & & 4 & 4 & 8 \\ \hline & 1 & 1 & 2 & // \end{array}$$

$$p(\lambda) = -(\lambda - 4)(\lambda^2 + \lambda + 2)$$

$$\lambda_2 = \frac{-1 - i\sqrt{7}}{2}, \quad \lambda_3 = \frac{-1 + i\sqrt{7}}{2} \quad |\lambda_2| = |\lambda_3| = \sqrt{2}$$

$$\text{Genscheitkreis: } \gamma = \gamma_i = \{z \in \mathbb{C} : |z-1| \leq 3\}$$

$$|\lambda_2 - 1| = |\lambda_3 - 1| = \left| \frac{-1 \pm i\sqrt{7}}{2} - 1 \right| = \left| -\frac{3}{2} \pm \frac{i\sqrt{7}}{2} \right| = 2 < 3$$

