Metoda lui Cholesky

$$\begin{pmatrix} u_{11} & & \\ u_{21} & u_{22} & \\ u_{31} & u_{32} & u_{33} \end{pmatrix} \cdot \begin{pmatrix} u_{11} & u_{21} & u_{31} \\ & u_{22} & u_{32} \\ & & u_{33} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{31} \\ a_{21} & a_{22} & a_{32} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

$$u_{11} = \sqrt{a_{11}}$$

$$u_{ij} = \left(a_{ij} - \sum_{k=1}^{j-1} u_{ik} u_{kj} \right) / u_{jj} \qquad \begin{matrix} i = 2, n \\ j = 1, i-1 \end{matrix}$$

$$u_{ii} = \left(a_{ii} - \sum_{k=1}^{i-1} u_{ik}^2 \right)^{1/2} \qquad i = 2, n$$

$$a_{11} = u_{11}^2 \rightarrow u_{11} = \sqrt{a_{11}}$$

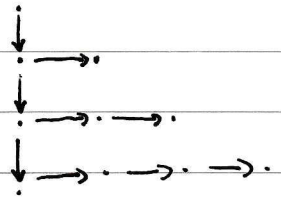
$$a_{21} = u_{21} u_{11} \rightarrow u_{21} = a_{21} / u_{11}$$

$$a_{22} = u_{21}^2 + u_{22}^2 \rightarrow u_{22} = \sqrt{a_{22} - u_{21}^2}$$

$$a_{31} = u_{31} u_{11} \rightarrow u_{31} = a_{31} / u_{11}$$

$$a_{32} = u_{31} u_{21} + u_{32} u_{22} \rightarrow u_{32} = \frac{a_{32} - u_{31} u_{21}}{u_{22}}$$

$$a_{33} = u_{31}^2 + u_{32}^2 + u_{33}^2 \rightarrow u_{33} = \sqrt{a_{33} - u_{31}^2 - u_{32}^2}$$



Exercício sul metodo di Cholesky

$$A = \begin{pmatrix} 4 & 2 & 4 \\ 2 & 2 & 4 \\ 4 & 4 & 9 \end{pmatrix} \qquad |A| \neq 0$$

$u_{11} = \sqrt{a_{11}} = 2$ $\qquad i=1$

$u_{21} = a_{21} / u_{11} = 2/2 = 1$ $\qquad i=2, j=1$

$u_{22} = \sqrt{a_{22} - u_{21}^2} = \sqrt{2-1} = 1$ $\qquad i=2$

$u_{31} = a_{31} / u_{11} = 4/2 = 2$ $\qquad i=3, j=1$

$u_{32} = \dfrac{a_{32} - u_{31} u_{21}}{u_{22}} = \dfrac{4-2}{1} = 2$ $\qquad i=3, j=2$

$u_{33} = \sqrt{a_{33} - u_{31}^2 - u_{32}^2} = \sqrt{9-4-4} = 1$ $\qquad i=3$

Verifico: $\begin{pmatrix} 2 & 0 & 0 \\ 1 & 1 & 0 \\ 2 & 2 & 1 \end{pmatrix} \begin{pmatrix} 2 & 1 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 4 & 2 & 4 \\ 2 & 2 & 4 \\ 4 & 4 & 9 \end{pmatrix}$

M. Thomas

$$\begin{pmatrix} a_1 & c_1 & 0 \\ b_2 & a_2 & c_2 \\ 0 & b_3 & a_3 \end{pmatrix} = \begin{pmatrix} \alpha_1 & 0 & 0 \\ b_2 & \alpha_2 & 0 \\ 0 & b_3 & \alpha_3 \end{pmatrix} \begin{pmatrix} 1 & \gamma_1 & 0 \\ 0 & 1 & \gamma_2 \\ 0 & 0 & 1 \end{pmatrix}$$

$a_1 = \alpha_1 \rightarrow \alpha_1 = a_1$

$c_1 = \alpha_1 \gamma_1 \rightarrow \gamma_1 = c_1/\alpha_1$

$a_2 = b_2 \gamma_1 + \alpha_2 \rightarrow \alpha_2 = a_2 - b_2 \gamma_1$

$c_2 = \alpha_2 \gamma_2 \rightarrow \gamma_2 = c_2/\alpha_2$

$a_3 = b_3 \gamma_2 + \alpha_3 \rightarrow \alpha_3 = a_3 - b_3 \gamma_2$