

University of Illinois at UrbanaChampaign
Department of Industrial and Enterprise Systems Engineering
IE 533 Big Graphs and Social Networks (Spring 2023)

Homework #1
Assigned 1/19 Thursday
Due 1/26 Thursday

1. Company ZZ wants to assign its staff members efficiently among the jobs and wants to utilize their time. Based on people skills and job requirements, we have a utility of u_{ij} for assigning person i to job j . The manager of ZZ wants to assign people to jobs to maximize overall utility. How would you formulate this problem as a network flow problem?
 - a. (Bonus) Now assume that you have job assignments to be made for multiple time periods. How would you change your formulation to meet this new situation while still attempting to maximize the overall utility.
2. A global supply chain company has n factories (indexed by i) and m warehouses (indexed by j). Each factory has a certain supply available and each warehouse as a certain demand requirement. Assume that the total supply is greater than the total demand for sake of simplicity. The cost to ship one unit of item from factory i to warehouse j is c_{ij} . The supply chain manager wants to minimize total shipment cost. Formulate the problem as a network flow model.
 - a. (Bonus) If there are multiple parts indexed by k and the unit shipping costs are c_{ijk} , and you have a unit size of d_{ijk} , you want to limit the shipments from each warehouse to a (say trucksize) D . Revise your above model.
3. Paper and wood products companies need to define cutting schedules that will maximize the total wood yield of their forests over some planning period. Suppose that a company with control of p forest units wants to identify the best cutting schedule over a planning horizon of k years. Forest unit i has a total acreage of a_i units, and studies that the company has undertaken predict that this unit will have w_{ij} tons of woods available for harvesting in the j^{th} year. Based on its prediction of economic conditions, the company believes that it should harvest at least l_j tons of wood in year j . Due to the availability of equipment and personnel, the company can harvest at most U_j tons of wood in year j . Formulate the problem of determining a schedule with maximum wood yield as a network flow problem.
4. For a network $G = (V, E)$ with source node s and terminal node t , let $c_{ij} \geq 0$ be the arc capacity for arc $(i, j) \in E$.
 - a. Write down the LP formulation for the max-flow problem.
 - b. Write the LP dual of the formulation.
 - c. In your own words, provide an intuition behind the dual problem.

5. The figure below shows an instance of the multi-commodity flow problem. The network has 2 commodities, and a source and sink node for each commodity. There are 6 transshipment nodes. The arc costs are given alongside. In all but one arc (from node 2 to node 5), assume that the capacity is infinity.

- Formulate the multi-commodity flow problem as an LP.
- Find the optimal solution (using any method – by hand/a computer program).
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