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# GPU-accelerated Algorithms on solving Stochastic Shortest Path Problems

## Proposal

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### Abstract

The abstract paragraph should be indented ½ inch (3 picas) on both the left- and right-hand margins. Use 10 point type, with a vertical spacing (leading) of 11 points. The word **Abstract** must be centered, bold, and in point size 12. Two line spaces precede the abstract. The abstract must be limited to one paragraph.

## 1 Introduction

Stochastic shortest path is a problem in which we need to find the shortest path from a given start node to a goal node in a graph, where the edge weights are not deterministic but instead are random variables. This problem is commonly used in many applications, such as robotics, transportation, and network routing.

Regular shortest path algorithms, such as Dijkstra’s algorithm or the A\* algorithm, assume that the edge weights in a graph are deterministic and known in advance. These algorithms work well when the edge weights are fixed and do not change over time. However, in many real-world applications, the edge weights are not fixed but instead are subject to randomness and uncertainty. For example, in transportation networks, the travel time between two locations can vary depending on traffic conditions, weather, accidents, etc. Similarly, in robotic navigation, the cost of moving from one location to another can depend on sensor readings, terrain features, obstacles, etc. In such scenarios, regular shortest path algorithms may not be appropriate as they do not take into account the randomness and uncertainty of the edge weights. Stochastic shortest path algorithms, on the other hand, explicitly model the probabilistic nature of the edge weights and aim to find the path with the lowest expected cost.

Table 1: Milestones

Time	Tasks
Mar. 9	Proposal Investigation
Mar. 23	Review of Literature & Algorithm Design
Mar. 28	Finish up Midterm Report
Apr. 13	Mathematical proof of Algorithm
Apr. 23	Experimental Design & Validation
Apr. 25	Final Report Completion

The stochastic shortest path problem is a challenging problem in stochastic optimization, and its solution requires a combination of mathematical and computational techniques. The need for using GPUs, particularly CUDA, to speed up the stochastic shortest path algorithm depends on various factors such as the size of the graph, the complexity of the edge weight distributions, and the available computing resources. In general, stochastic shortest path algorithms involve computations with matrices and vectors, which can be computationally intensive for large graphs. GPUs, with their massively parallel architecture and high memory bandwidth, can accelerate these computations significantly, leading to faster computation times. Moreover, if the edge weight distributions are complex and involve high-dimensional probability distributions, such as multivariate normal or mixture distributions, then the computations involved in the algorithm may be even more demanding. In such cases, GPUs can provide significant speedups compared to CPU-based implementations.

## 2 Literature Review

Prior State of the Art

## 3 Proposed work

### 3.1 Tasks, Data and Testing

### 3.2 Milestones

Table 1

### 3.3 Expected results

## 4 Submission of papers to NeurIPS 2020

NeurIPS requires electronic submissions. The electronic submission site is

<https://cmt3.research.microsoft.com/NeurIPS2020/>

Please read the instructions below carefully and follow them faithfully.

### 4.1 Style

Papers to be submitted to NeurIPS 2020 must be prepared according to the instructions presented here. Papers may only be up to eight pages long, including figures. Additional pages *containing only a section on the broader impact, acknowledgments and/or cited references* are allowed. Papers that exceed eight pages of content will not be reviewed, or in any other way considered for presentation at the conference.

The margins in 2020 are the same as those in 2007, which allow for  $\sim 15\%$  more words in the paper compared to earlier years.

Authors are required to use the NeurIPS L<sup>A</sup>T<sub>E</sub>X style files obtainable at the NeurIPS website as indicated below. Please make sure you use the current files and not previous versions. Tweaking the style files may be grounds for rejection.

## 4.2 Retrieval of style files

The style files for NeurIPS and other conference information are available on the World Wide Web at

<http://www.neurips.cc/>

The file `neurips_2020.pdf` contains these instructions and illustrates the various formatting requirements your NeurIPS paper must satisfy.

The only supported style file for NeurIPS 2020 is `neurips_2020.sty`, rewritten for L<sup>A</sup>T<sub>E</sub>X 2<sub>ε</sub>. **Previous style files for L<sup>A</sup>T<sub>E</sub>X 2.09, Microsoft Word, and RTF are no longer supported!**

The L<sup>A</sup>T<sub>E</sub>X style file contains three optional arguments: `final`, which creates a camera-ready copy, `preprint`, which creates a preprint for submission to, e.g., arXiv, and `nonatbib`, which will not load the `natbib` package for you in case of package clash.

**Preprint option** If you wish to post a preprint of your work online, e.g., on arXiv, using the NeurIPS style, please use the `preprint` option. This will create a nonanonymized version of your work with the text “Preprint. Work in progress.” in the footer. This version may be distributed as you see fit. Please **do not** use the `final` option, which should **only** be used for papers accepted to NeurIPS.

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The file `neurips_2020.tex` may be used as a “shell” for writing your paper. All you have to do is replace the author, title, abstract, and text of the paper with your own.

The formatting instructions contained in these style files are summarized in Sections 5, 6, and 7 below.

## 5 General formatting instructions

The text must be confined within a rectangle 5.5 inches (33 picas) wide and 9 inches (54 picas) long. The left margin is 1.5 inch (9 picas). Use 10 point type with a vertical spacing (leading) of 11 points. Times New Roman is the preferred typeface throughout, and will be selected for you by default. Paragraphs are separated by 1/2 line space (5.5 points), with no indentation.

The paper title should be 17 point, initial caps/lower case, bold, centered between two horizontal rules. The top rule should be 4 points thick and the bottom rule should be 1 point thick. Allow 1/4 inch space above and below the title to rules. All pages should start at 1 inch (6 picas) from the top of the page.

For the final version, authors’ names are set in boldface, and each name is centered above the corresponding address. The lead author’s name is to be listed first (left-most), and the co-authors’ names (if different address) are set to follow. If there is only one co-author, list both author and co-author side by side.

Please pay special attention to the instructions in Section 7 regarding figures, tables, acknowledgments, and references.

## 6 Headings: first level

All headings should be lower case (except for first word and proper nouns), flush left, and bold.

First-level headings should be in 12-point type.

### 6.1 Headings: second level

Second-level headings should be in 10-point type.

### 6.1.1 Headings: third level

Third-level headings should be in 10-point type.

**Paragraphs** There is also a `\paragraph` command available, which sets the heading in bold, flush left, and inline with the text, with the heading followed by 1 em of space.

## 7 Citations, figures, tables, references

These instructions apply to everyone.

### 7.1 Citations within the text

The `natbib` package will be loaded for you by default. Citations may be author/year or numeric, as long as you maintain internal consistency. As to the format of the references themselves, any style is acceptable as long as it is used consistently.

The documentation for `natbib` may be found at

<http://mirrors.ctan.org/macros/latex/contrib/natbib/natnotes.pdf>

Of note is the command `\citet`, which produces citations appropriate for use in inline text. For example,

```
\citet{hasselmo} investigated\dots
```

produces

Hasselmo, et al. (1995) investigated...

If you wish to load the `natbib` package with options, you may add the following before loading the `neurips_2020` package:

```
\PassOptionsToPackage{options}{natbib}
```

If `natbib` clashes with another package you load, you can add the optional argument `nonatbib` when loading the style file:

```
\usepackage[nonatbib]{neurips_2020}
```

As submission is double blind, refer to your own published work in the third person. That is, use “In the previous work of Jones et al. [4],” not “In our previous work [4].” If you cite your other papers that are not widely available (e.g., a journal paper under review), use anonymous author names in the citation, e.g., an author of the form “A. Anonymous.”

### 7.2 Footnotes

Footnotes should be used sparingly. If you do require a footnote, indicate footnotes with a number<sup>1</sup> in the text. Place the footnotes at the bottom of the page on which they appear. Precede the footnote with a horizontal rule of 2 inches (12 picas).

Note that footnotes are properly typeset *after* punctuation marks.<sup>2</sup>

### 7.3 Figures

All artwork must be neat, clean, and legible. Lines should be dark enough for purposes of reproduction. The figure number and caption always appear after the figure. Place one line space before the figure caption and one line space after the figure. The figure caption should be lower case (except for first word and proper nouns); figures are numbered consecutively.

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<sup>1</sup>Sample of the first footnote.

<sup>2</sup>As in this example.

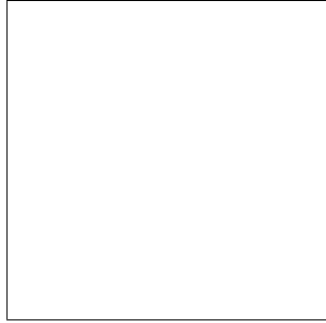


Figure 1: Sample figure caption.

Table 2: Sample table title

Part		
Name	Description	Size ( $\mu\text{m}$ )
Dendrite	Input terminal	$\sim 100$
Axon	Output terminal	$\sim 10$
Soma	Cell body	up to $10^6$

You may use color figures. However, it is best for the figure captions and the paper body to be legible if the paper is printed in either black/white or in color.

## 7.4 Tables

All tables must be centered, neat, clean and legible. The table number and title always appear before the table. See Table 2.

Place one line space before the table title, one line space after the table title, and one line space after the table. The table title must be lower case (except for first word and proper nouns); tables are numbered consecutively.

Note that publication-quality tables *do not contain vertical rules*. We strongly suggest the use of the `booktabs` package, which allows for typesetting high-quality, professional tables:

<https://www.ctan.org/pkg/booktabs>

This package was used to typeset Table 2.

## 8 Final instructions

Do not change any aspects of the formatting parameters in the style files. In particular, do not modify the width or length of the rectangle the text should fit into, and do not change font sizes (except perhaps in the **References** section; see below). Please note that pages should be numbered.

## 9 Preparing PDF files

Please prepare submission files with paper size “US Letter,” and not, for example, “A4.”

Fonts were the main cause of problems in the past years. Your PDF file must only contain Type 1 or Embedded TrueType fonts. Here are a few instructions to achieve this.

- You should directly generate PDF files using `pdflatex`.
- You can check which fonts a PDF files uses. In Acrobat Reader, select the menu Files>Document Properties>Fonts and select Show All Fonts. You can also use the program `pdf fonts` which comes with `xpdf` and is available out-of-the-box on most Linux machines.

- The IEEE has recommendations for generating PDF files whose fonts are also acceptable for NeurIPS. Please see <http://www.emfield.org/icuwb2010/downloads/IEEE-PDF-SpecV32.pdf>
- xfig "patterned" shapes are implemented with bitmap fonts. Use "solid" shapes instead.
- The `\bbold` package almost always uses bitmap fonts. You should use the equivalent AMS Fonts:

```
\usepackage{amsfonts}
```

followed by, e.g., `\mathbb{R}`, `\mathbb{N}`, or `\mathbb{C}` for  $\mathbb{R}$ ,  $\mathbb{N}$  or  $\mathbb{C}$ . You can also use the following workaround for reals, natural and complex:

```
\newcommand{\RR}{I\!\!R} %real numbers
\newcommand{\Nat}{I\!\!N} %natural numbers
\newcommand{\CC}{I\!\!C} %complex numbers
```

Note that `amsfonts` is automatically loaded by the `amssymb` package.

If your file contains type 3 fonts or non embedded TrueType fonts, we will ask you to fix it.

## 9.1 Margins in L<sup>A</sup>T<sub>E</sub>X

Most of the margin problems come from figures positioned by hand using `\special` or other commands. We suggest using the command `\includegraphics` from the `graphicx` package. Always specify the figure width as a multiple of the line width as in the example below:

```
\usepackage[pdftex]{graphicx} ...
\includegraphics[width=0.8\linewidth]{myfile.pdf}
```

See Section 4.4 in the graphics bundle documentation (<http://mirrors.ctan.org/macros/latex/required/graphics/grfguide.pdf>)

A number of width problems arise when L<sup>A</sup>T<sub>E</sub>X cannot properly hyphenate a line. Please give LaTeX hyphenation hints using the `\-` command when necessary.

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Authors are required to include a statement of the broader impact of their work, including its ethical aspects and future societal consequences. Authors should discuss both positive and negative outcomes, if any. For instance, authors should discuss a) who may benefit from this research, b) who may be put at disadvantage from this research, c) what are the consequences of failure of the system, and d) whether the task/method leverages biases in the data. If authors believe this is not applicable to them, authors can simply state this.

Use unnumbered first level headings for this section, which should go at the end of the paper. **Note that this section does not count towards the eight pages of content that are allowed.**

## Acknowledgments and Disclosure of Funding

Use unnumbered first level headings for the acknowledgments. All acknowledgments go at the end of the paper before the list of references. Moreover, you are required to declare funding (financial activities supporting the submitted work) and competing interests (related financial activities outside the submitted work). More information about this disclosure can be found at: <https://neurips.cc/Conferences/2020/PaperInformation/FundingDisclosure>.

Do **not** include this section in the anonymized submission, only in the final paper. You can use the `ack` environment provided in the style file to automatically hide this section in the anonymized submission.

## References

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