Chapter 1

Chapter 1 - Problems

Problem 1

In axisymmetric spherical coordinates, ∇ (the gradient operator) is given by

$$\nabla = \frac{\partial}{dr} + \frac{\partial}{r\partial\theta}.$$

We also know that

$$\mathbf{H} = -\nabla\psi_m,$$

and that ψ_m is a scalar function of position:

$$\psi_m = \frac{\mathbf{m}\cdot\mathbf{r}}{4\pi r^3}.$$

Find the radial and tangential components of $\mathbf{H}$ if $\mathbf{m}$ is 80 ZAm^2, [remember that "Z" stands for Zeta which stands for 10^{21}], r is 6 x 10^6 m and θ is 45^o. What are these field values in terms of $\mathbf{B}$ (teslas)?

Write your answers in a markdown cell in a jupyter notebook using latex syntax.

Problem 2

a) In your jupyter notebook, write python functions to convert induction, moment and magnetic field quantities in cgs units to SI units. Use the conversion factors in Table 1.1. Use your function to convert the following from cgs to SI:

i) B = 3.5 x10^5 G

ii) m = 2.78 x 10^{-20} G cm^3

iii) H = 128 oe

b) In a new code block, modify your function to allow conversion from cgs => SI or SI => cgs. Rerun it to convert your answers from a) back to cgs.

HINTS: Call the functions with the values of B, m and H and have the function return the converted values. In the modified functions, you can specify whether the conversion is from cgs or SI.

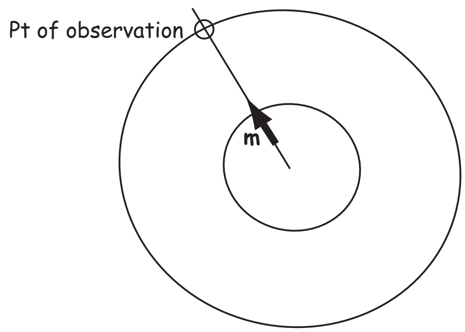

Figure 1.1: A magnetic dipole source (**m**) located 3480 km from the center of the Earth, below the point of observation at 45° latitude. The average radius of the Earth is 6370 km. The field at the point of observation is directed downward (toward the center) with a magnitude of 10 μT.

Problem 3

Figure 1.1 shows a meridional cross section through the Earth in the plane of a magnetic dipole source m. At the location directly above the dipole, the field from the dipole is directed vertically downward and has intensity 10 μT. The dipole source is placed at 3480 km from the center of the Earth. Assume a mean Earth radius of 6370 km. Adapt the geometry of Figure 1.2 and the equations describing the magnetic field of a dipole to the model dipole in Figure 1.1.

a) Calculate the magnetic dipole moment of the model dipole. Remember to keep track of your units!

b) Compare this field to the total field produced by a centered axial magnetic dipole moment (*i.e.*, one that is pointing straight up and is in the center of the circles) equivalent to that of the present geomagnetic field ($m \sim 80$ ZAm^2; Z=10^{21}). Assume a latitude for the point of observation of 60°.

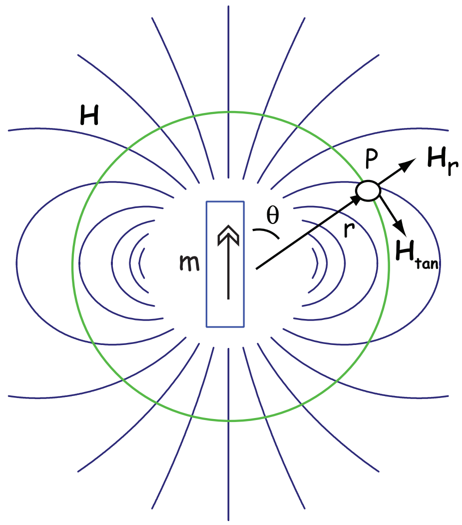

Figure 1.2: Field $\mathbf{H}$ produced at point P by a magnetic moment $\mathbf{m}$. $\mathbf{H}_r$ and $\mathbf{H}_{tan}$ are the radial and tangential fields respectively.

[HINT: the angle θ in Equation 1.1 is the co-latitude, not the latitude.]

$$\psi_m = \frac{\mathbf{m} \cdot \mathbf{r}}{4\pi r^3} = \frac{m \cos\theta}{4\pi r^2}. \tag{1.1}$$

Problem 4

Knowing that $B = \mu_o H$, work out the fundamental units of μ_o in SI units. Prepare your answer in a markdown cell in your jupyter notebook.

Table 1.1: Conversion between SI and cgs units.

Parameter	SI unit	cgs unit	Conversion
Magnetic moment ($\mathbf{m}$)	Am^2	emu	1 A m^2 = 10^3 emu
Magnetization			
by volume ($\mathbf{M}$)	Am^{-1}	emu cm^{-3}	1 Am^{-1} = 10^{-3} emu cm^{-3}
by mass (Ω)	Am^2kg^{-1}	emu gm^{-1}	1 Am^2kg^{-1} = 1 emu gm^{-1}
Magnetic Field ($\mathbf{H}$)	Am^{-1}	Oersted (oe)	1 Am^{-1} = 4π x 10^{-3} oe
Magnetic Induction ($\mathbf{B}$)	T	Gauss (G)	1 T = 10^4 G
Permeability			
of free space (μ_o)	Hm^{-1}	1	4π x 10^{-7} Hm^{-1} = 1
Susceptibility			
total (K:$\frac{\mathbf{m}}{\mathbf{H}}$)	m^3	emu oe^{-1}	1 m^3 = $\frac{10^6}{4\pi}$ emu oe^{-1}
by volume (χ: $\frac{\mathbf{M}}{\mathbf{H}}$)	-	emu cm^{-3} oe^{-1}	1 S.I. = $\frac{1}{4\pi}$ emu cm^{-3} oe^{-1}
by mass (κ:$\frac{\mathbf{m}}{m} \cdot \frac{1}{\mathbf{H}}$)	m^3kg^{-1}	emu g^{-1} oe^{-1}	1 m^3kg^{-1} = $\frac{10^3}{4\pi}$emu g^{-1} oe^{-1}

1 H = kg $m^2A^{-2}s^{-2}$, 1 emu = 1 G cm^3, $B = \mu_o H$ (in vacuum), 1 T = kg A^{-1} s^{-2}