Bridging Psychometric and Cognitive Models of (General) Intelligence

A Simulation Approach to Investigate Process Overlap Theory

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Agenda

- The Conventional Theories of Intelligence
  - Psychometric theories
  - Cognitive theories
  - Problems with the conventional theories
- Process Overlap Theory (POT) to Bridge the Gaps
  - Study 1: A Simulation Study of POT
  - Study 2: A Network of POT Simulation
  - Study 3: The Latent Network of Kane et al. (2004): An Inspection from POT perspective

Intelligence

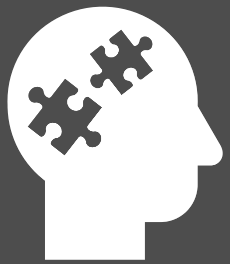
- Defined to explain the patterns and variations of human intellectual behaviors
  - On different occasions, in different domains, as judged by different criteria.
- Most definitions rely on observations of human performance in specific intellectual activities

Boring (1923): Intelligence is what the intelligence tests test

Intelligence

- Various definitions, robust patterns
  - Positive manifold of mental ability tests

- Two broad branches of conventional theories



Psychometric theories

Cognitive theories

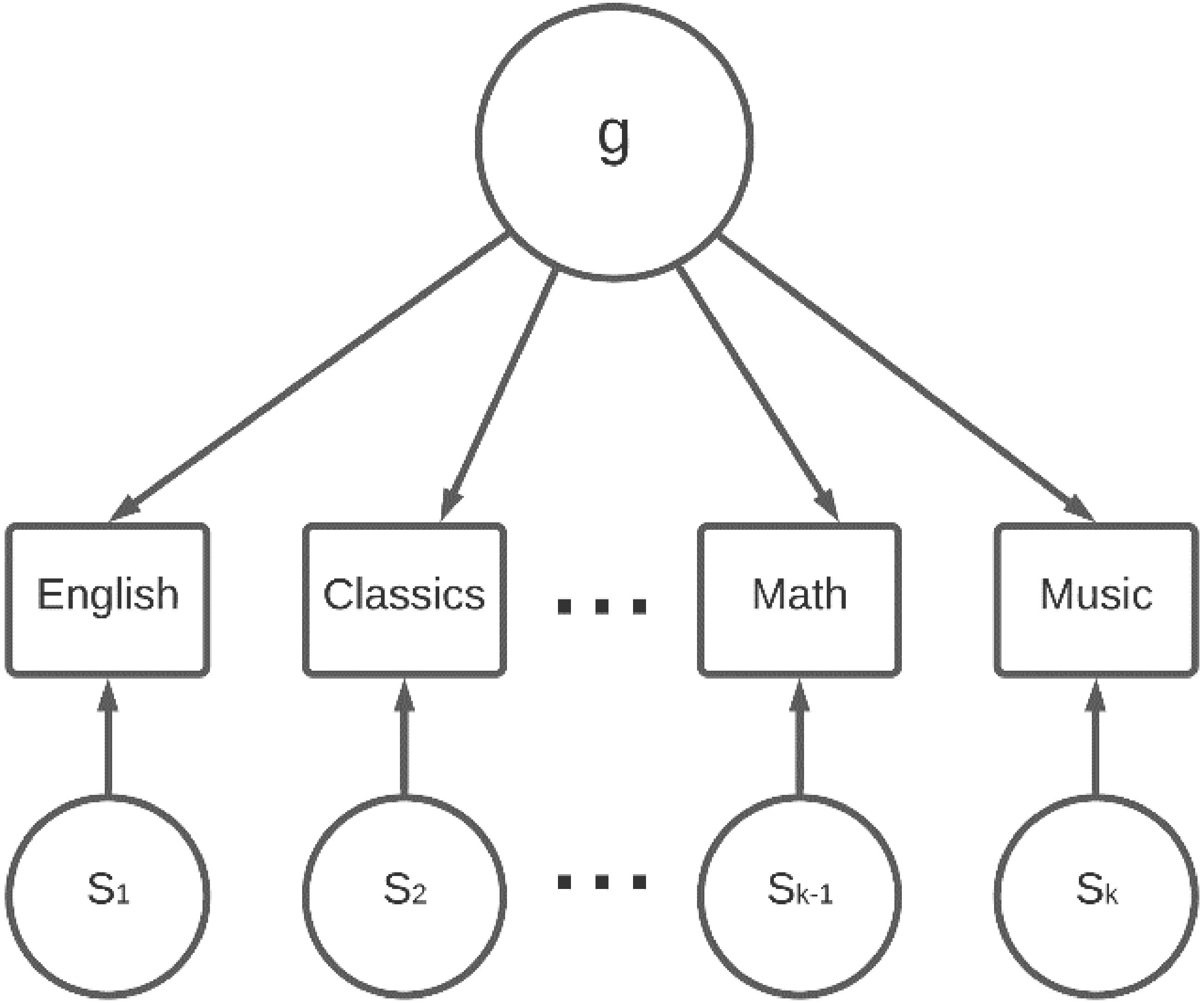
Psychometric theories

- Investigate correlational relationships and individual differences in performance of (cognitive) tests that require cognitive activities to understand the "map of mind" (Sternberg, 2012, p.19)

- Correlational data, latent variable models

Psychometric theories

- **Spearman's theory of general Intelligence**
  - **One-factor model of Intelligence**
- **Thurstone's Primary Mental Abilities**
  - **Multi-factor models of Intelligence**
- **The Cattell-Horn-Carroll Theory**
  - **Carroll's Three-Stratum Theory**
  - **Cattell-Horn's Gf-Gc Theory**



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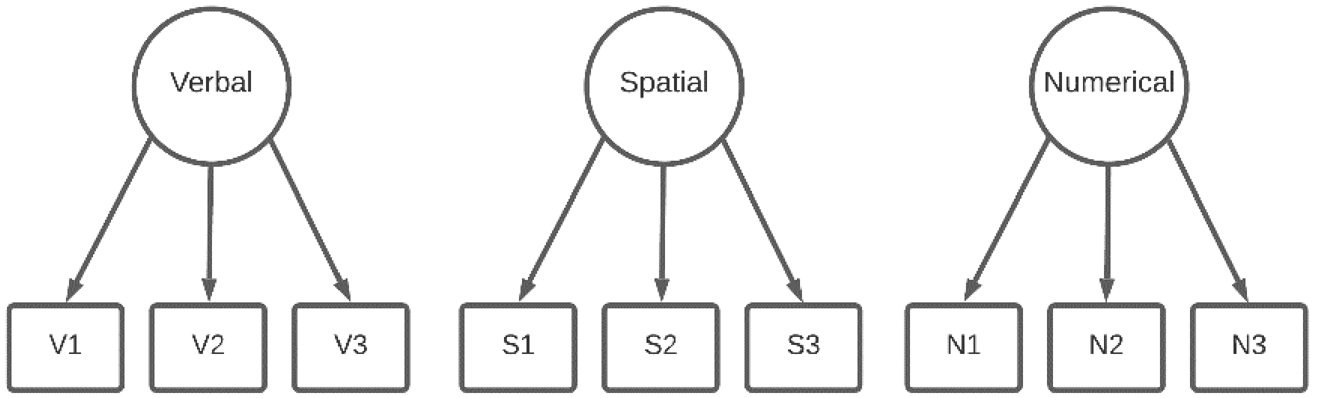

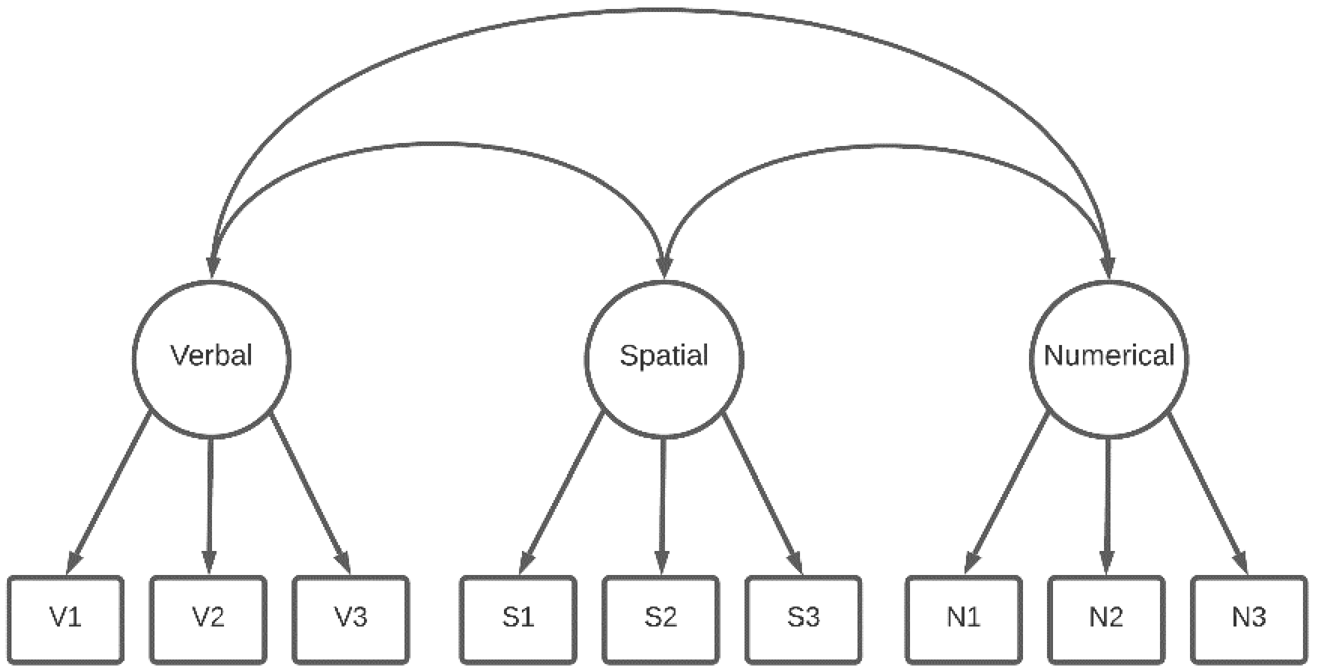

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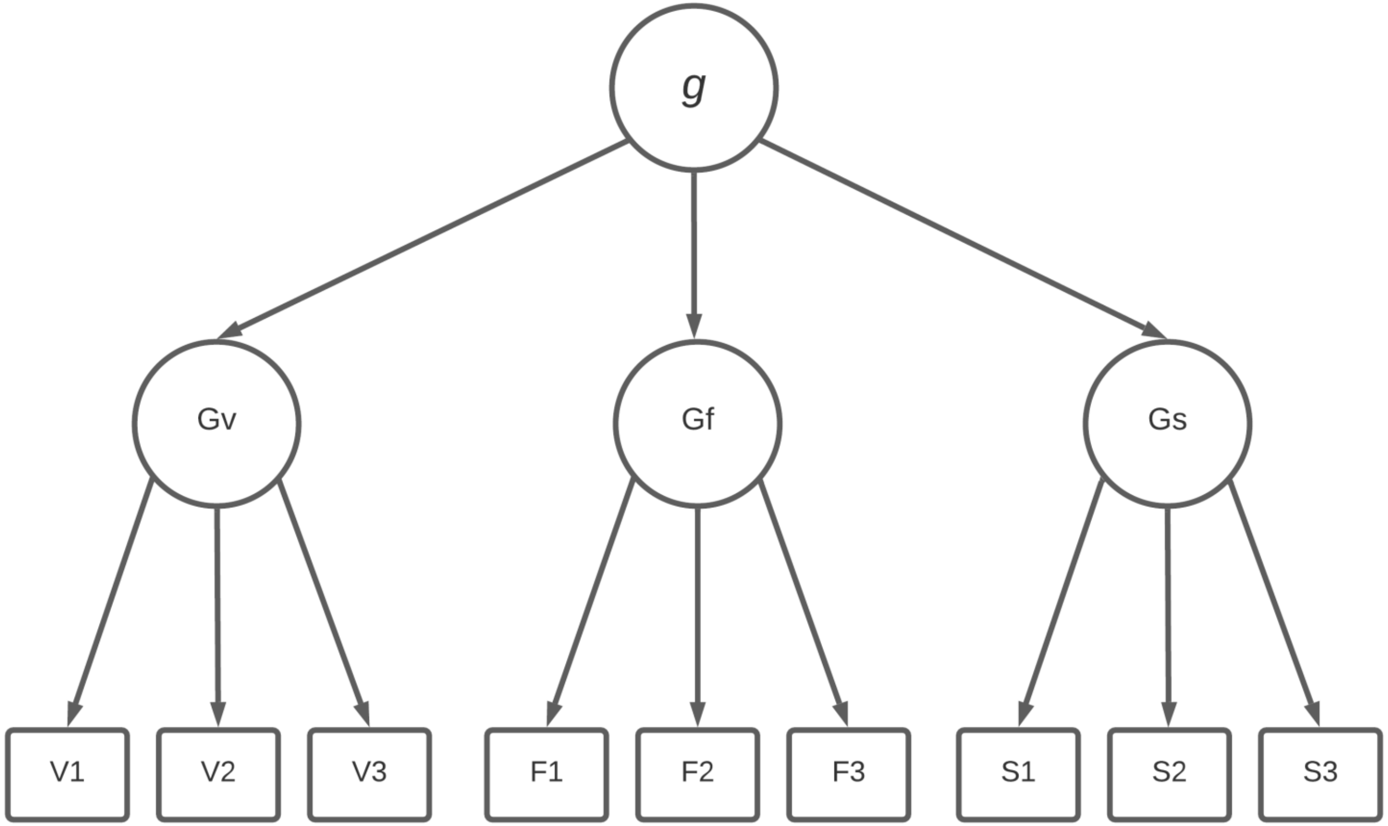


Cognitive theories

- Investigate the specific roles of **cognitive processes** in intelligent activities as basic components in information processing
- Reductionism: What is the most influential cognitive process in intelligence?
- Experimental data from cognitive tasks, (computational) cognitive models

Cognitive theories

- What is general intelligence?

  - Processing Speed
    - Moderately correlated with Intelligence

  - Working Memory
    - Strongly correlated with intelligence

  - Executive Functions
    - correlated with intelligence, but evidence is mixed

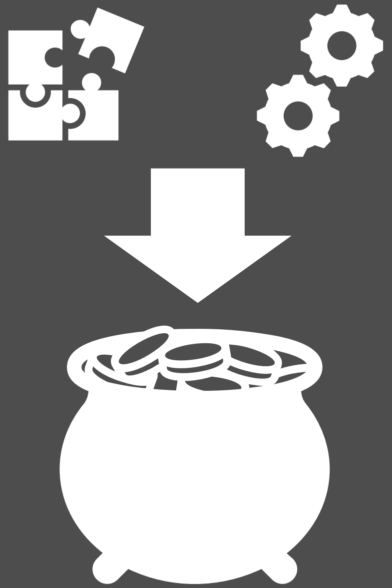
Problems with The Conventional Theories

- Psychometric Theories: “The Common-Cause premise Problem”
  - The common variance among the measures is regarded as a general source that causes the communality of the individual differences
  - But a latent factor != a cognitive process/function

- Cognitive Theories: “The Measurement Problem”
  - In empirical research, it is usually challenging to associate the differences in cognitive tasks to specific cognitive processes as most cognitive tasks are complex in nature

Process Overlap Theory (POT)

- Kovacs & Conway (2016; 2019)
  - A unified theory of intelligence based on the sampling theories (Thomson, 1916)
  - Attempts to explain inter-individual differences in intelligence in terms of intra-individual psychological processes
  - Proposes an alternative cognitive foundation of positive manifold of intelligence (formative *g*)



Process Overlap Theory (POT)

- A Sampling framework of Cognitive Processes

Processes

Tests

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	+	+	+						+	+							
2		+	+	+					+		+						
3	+			+	+		+	+				+					
4	+	+		+	+	+						+	+				
5					+	+	+			+					+		+
6						+	+	+		+					+		

Process Overlap Theory (POT)

- The Sampling Mechanism of POT:
  - Domain-specific and domain-general processes are sampled in **an overlapping manner** across a battery of tests
  - Executive attention processes (as domain-general processes) are sampled more often than domain-specific processes across different tasks
  - EF processes are also sampled more often in fluid reasoning tasks than in domain-specific tasks (e.g., math knowledge, grammar sensitivity, etc.)

POT– IRT Model (POT-I)

$$P(U_{pi} = 1 \mid \Theta_{plm}, a_{il}, b_{il}) = \boxed{\prod_{l=1}^{D}} \frac{\boxed{e^{\sum_{m=1}^{C}}} a_{il}(\Theta_{plm} - b_{il})}{1 + e^{\sum_{m=1}^{C} a_{il}(\Theta_{plm} - b_{il})}}$$

where:

Θ_{plm} = the process score for the p^{th} person on the m^{th} process of the l^{th} domain

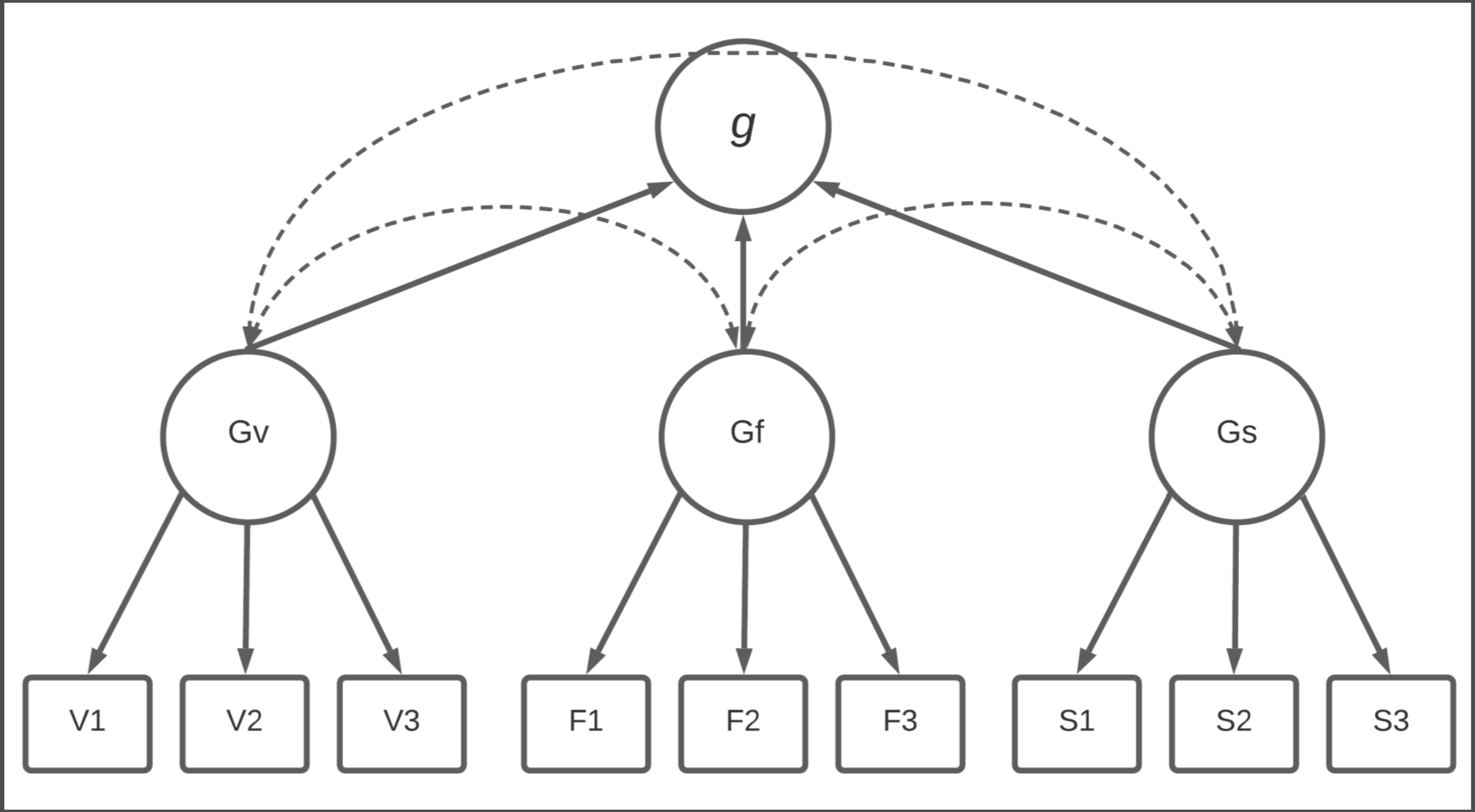
a_{il} = the discrimination parameter for the l^{th} domain on the i_{th} item

b_{il} = the difficulty parameter for the l^{th} domain on the i_{th} item

D = number of domains tapped by the item

C = number of processes in the given domain tapped by the item

POT– Structural Model (POT-S)



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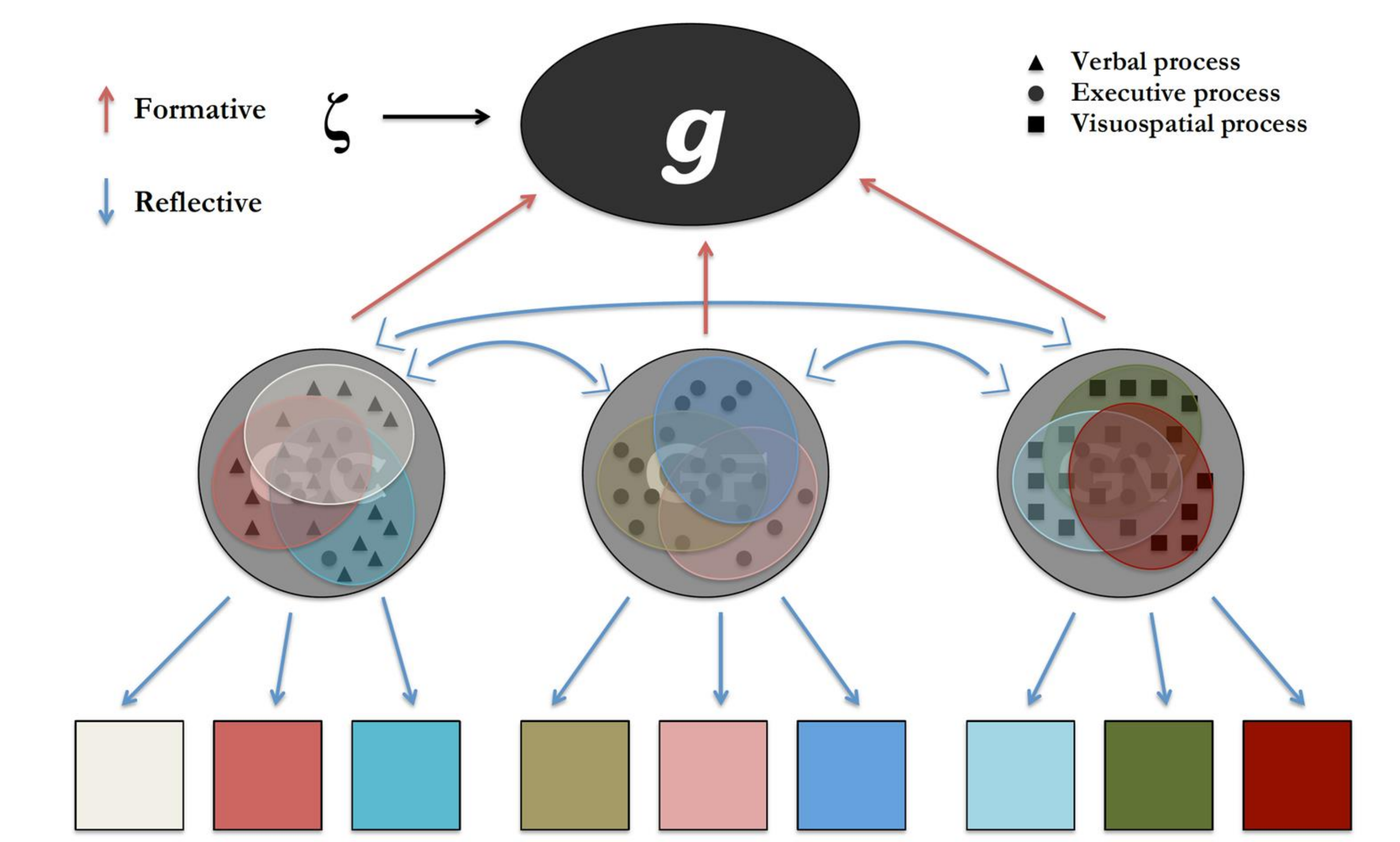


Study I: A Simulation of POT

Simulating the Sampling Mechanisms (POT-I)

Investigate the Psychometric Model of the Simulation (POT-S)

Study I: A Simulation Study of POT

- Purpose: **Proposing a simulation approach based on POT algorithm** of process sampling and demonstrating:

  - A) “*g*” can emerge from the simulated test scores in the absence of a general cognitive ability

  - B) the broad ability factors can emerge by introducing a distinction between domain-general and domain-specific processes, and how they are sampled by different types of tests

Study I: Sampling Algorithm

- Compensatory processes (within a domain):

$$\theta = a \cdot x^T = \sum_{\mathrm{k=1}}^{\mathrm{n}} a_k \, \mathrm{x}_k$$

- Non-compensatory processes (across domains):

$$\mathrm{P} = \prod_{k=1}^{n} p_k$$

Study I: Procedure – 1/5

- **Step 1: Specify the cognitive processes and tests**
- Simulate a sample of **1000** subjects performing **9** tests, each has **100** items
  - **3** fluid reasoning tests, **3** verbal tests, **3** spatial tests

- Each subject has a set of **60** cognitive processes
  - **15** Executive Function Processes
  - **15** Fluid Reasoning Processes, **15** Verbal Processes, and **15** Spatial Processes

- Each individual subject has an ability level on each process (orthogonal and normally distributed)
  - A 1000 x 60 Matrix

Study I: Procedure – 2/5

- **Step 2: Apply 2 specific sampling algorithms to the simulated processes**

- We compared two algorithms on the same set of simulated processes: a POT sampling algorithm (POT) and a general sampling algorithm (GSM)

- In the GSM algorithm,
- All **60** processes are sampled with equal probability (p = **.10**) across every task and item
  - For a specific item, about **60*0.10 = 6** processes are expected to be sampled

Study I: Procedure -2/5 (continued)

- In the POT algorithm,
- For an item in Gf tests, domain-general (EF) processes are sampled with greater probability (p = **.28**) than domain-specific (Fluid Reasoning) processes (p = **.12**)
  - On average, **4** EF (15*0.28) + **2** Fluid (15*0.12) processes are expected to be sampled for an item
- For an item in verbal/spatial tests, domain-general (EF) processes are sampled with smaller probability (p = **.12**) than domain-specific (Verbal/Spatial) processes (p = **.28**)
  - On average, **2** EF (15*0.28) + **4** specific (15*0.12) processes are expected to be sampled for an item

Study I: Procedure – 3 & 4/5

- **Step 3: Achieve latent traits that are reflected in specific tasks**
  - For GSM, all 6 processes are summed and standardized to calculated the corresponding "latent trait" required for an item
  - For POT, the processes within a domain (e.g., verbal processes sampled for a verbal item) are summed and standardized as the dimensional "latent trait"
- **Step 4: Generate the item/task level responses**
  - "Latent traits" are converted to probabilities by IRT functions and are used to generate binary responses of items (0s and 1s)
  - "1PL" for the IRT functions of simulated items, ***a* = 1, *b* ~ N(0, I)**

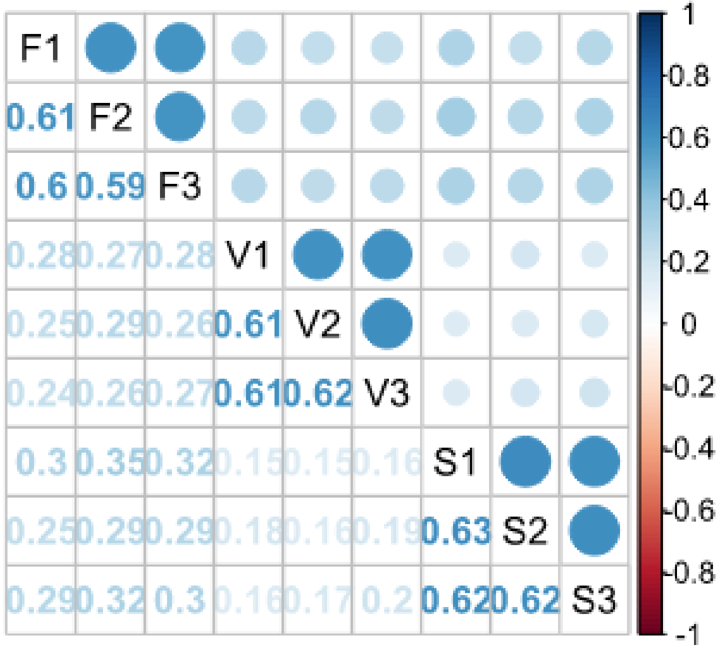
Study I: Procedures – 5/5

- **Step 5: Fit the psychometric models**
- Two simulated Datasets (GSM & POT):
  - Two 1000 x 9 matrices with total scores of the 9 tests based on the simulated responses of 9*100 items

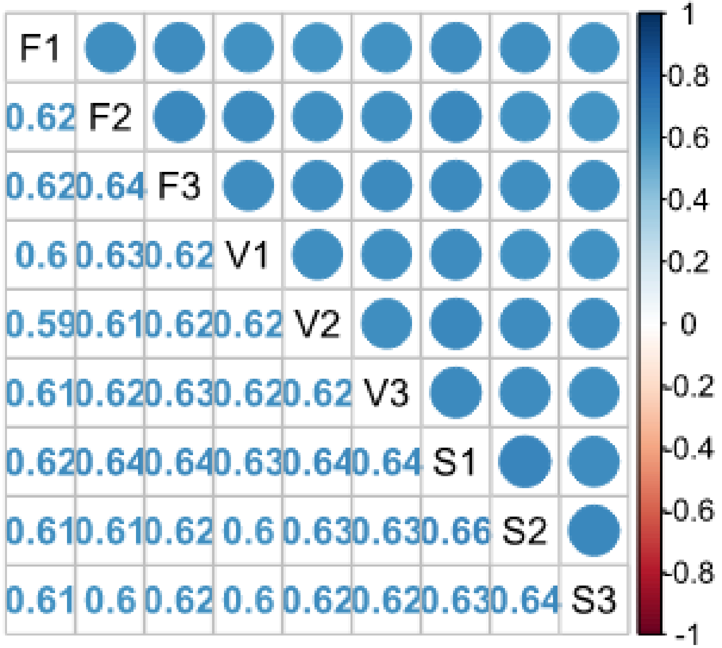
- Two Types of Models:
  - General factor model: 9 items loading onto a general factor
  - Higher-order factor model: 3 subfactors loading onto a higher-order factor

Study I: Results

Correlation Matrix for POT Algorithm



Correlation Matrix for GSM Algorithm



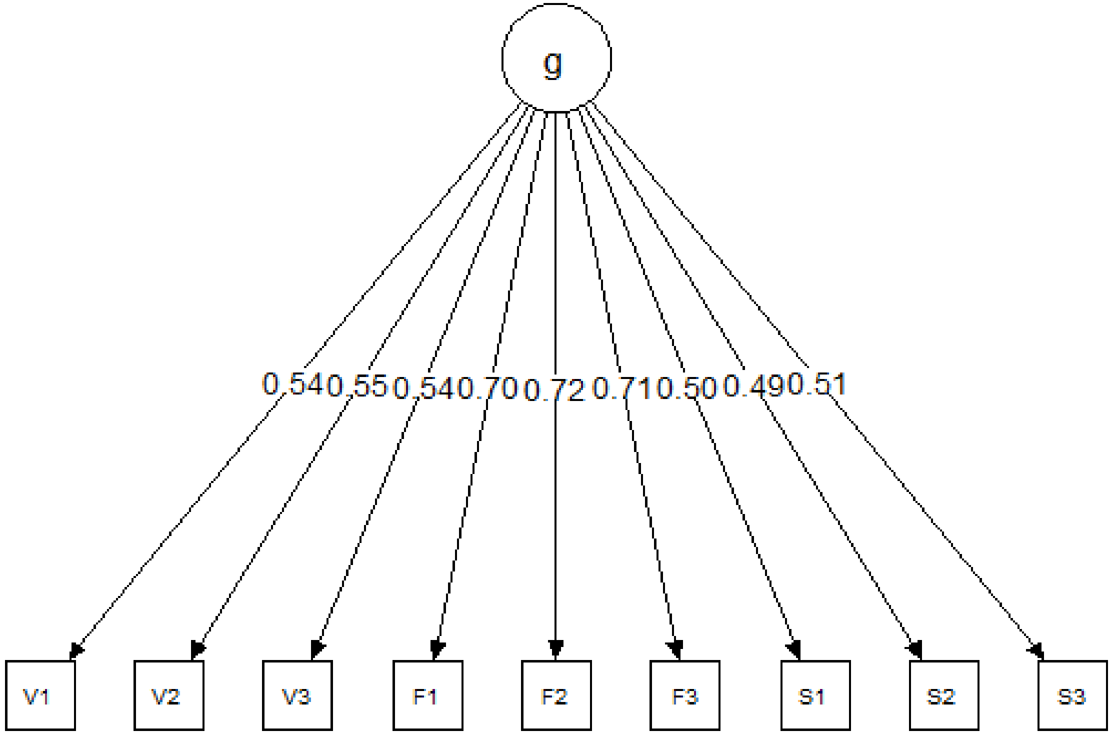
Study I: Results

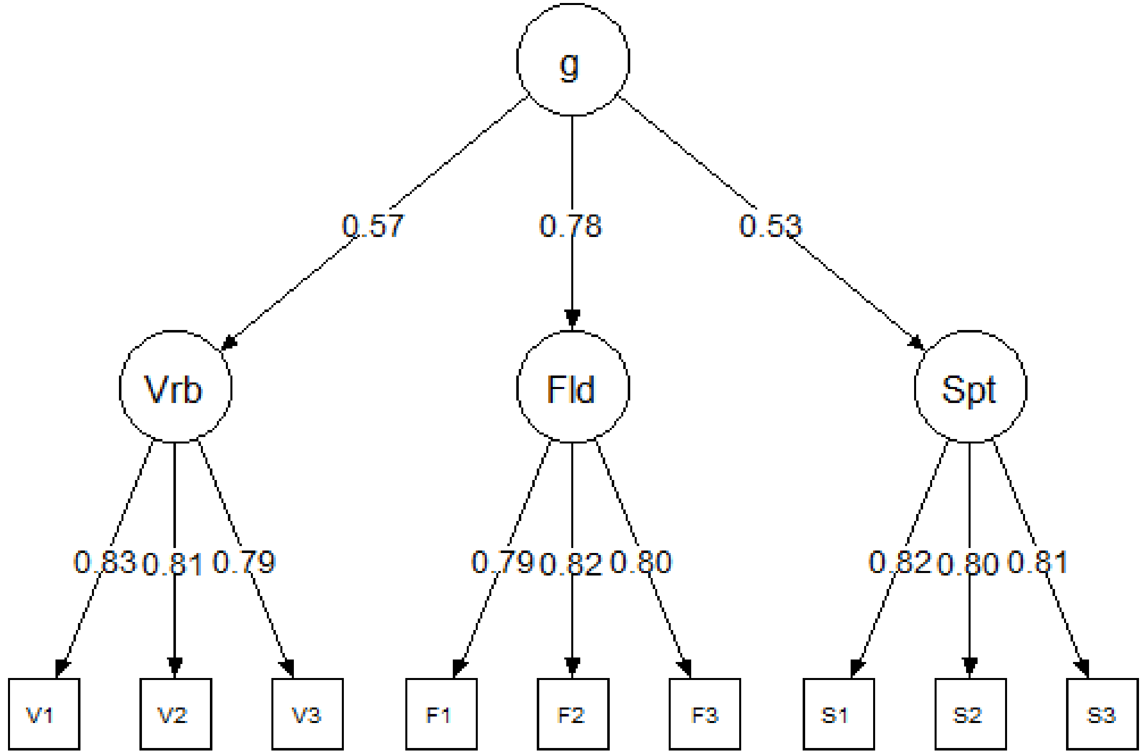
Fit indices for the one factor model and the hierarchical three factor model based on the POT and GSM algorithms

	χ^2 (*df*)	*p-value*	*CFI*	*RMSEA*	*SRMR*
POT One Factor	1985.39 (*27*)	<.001	.54	.27	.16
POT Three Factor	19.15 (*24*)	.740	1.00	.01	.01
GSM One Factor	9.58 (*27*)	.999	1.00	.00	.01
GSM Three Factor	9.58 (*24*)	.999	1.00	.00	.01

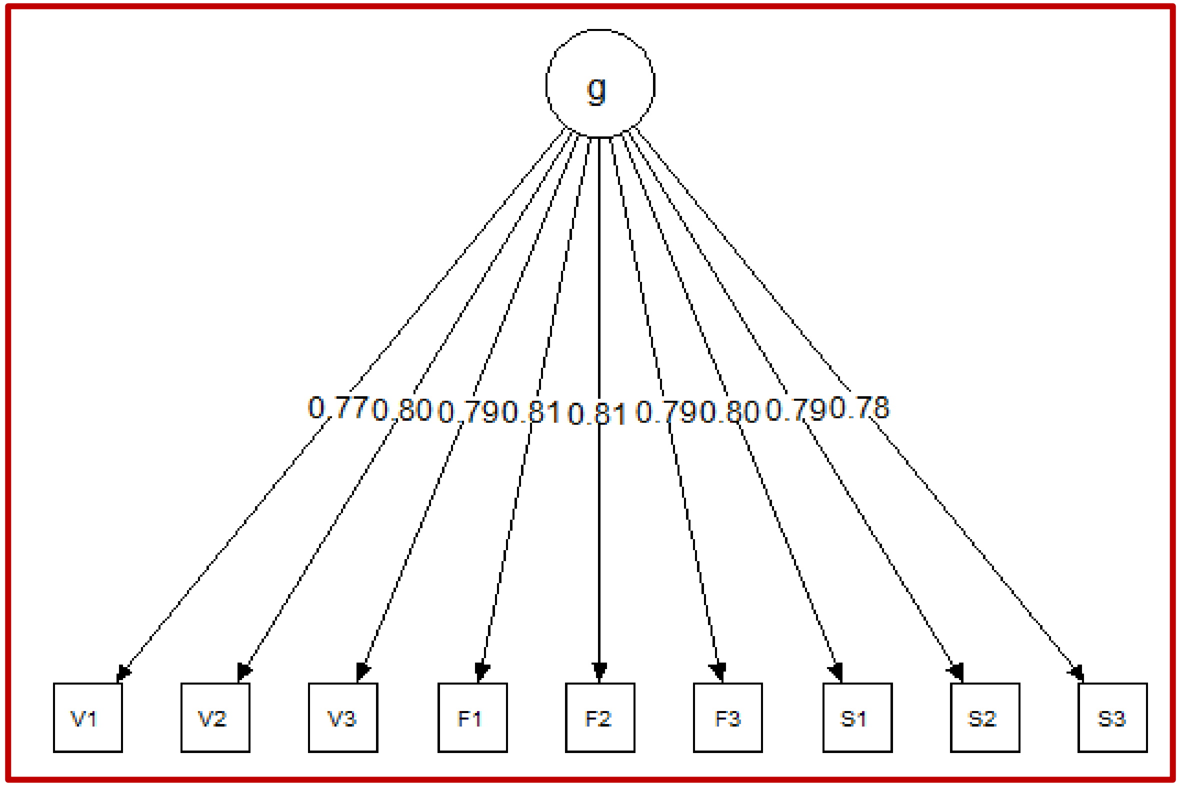
Note: χ^2 (*df*) = Chi-Square (degree of freedom). CFI = comparative fit index. RMSEA = root mean square error of approximation. SRMR = standardized root mean square residual.

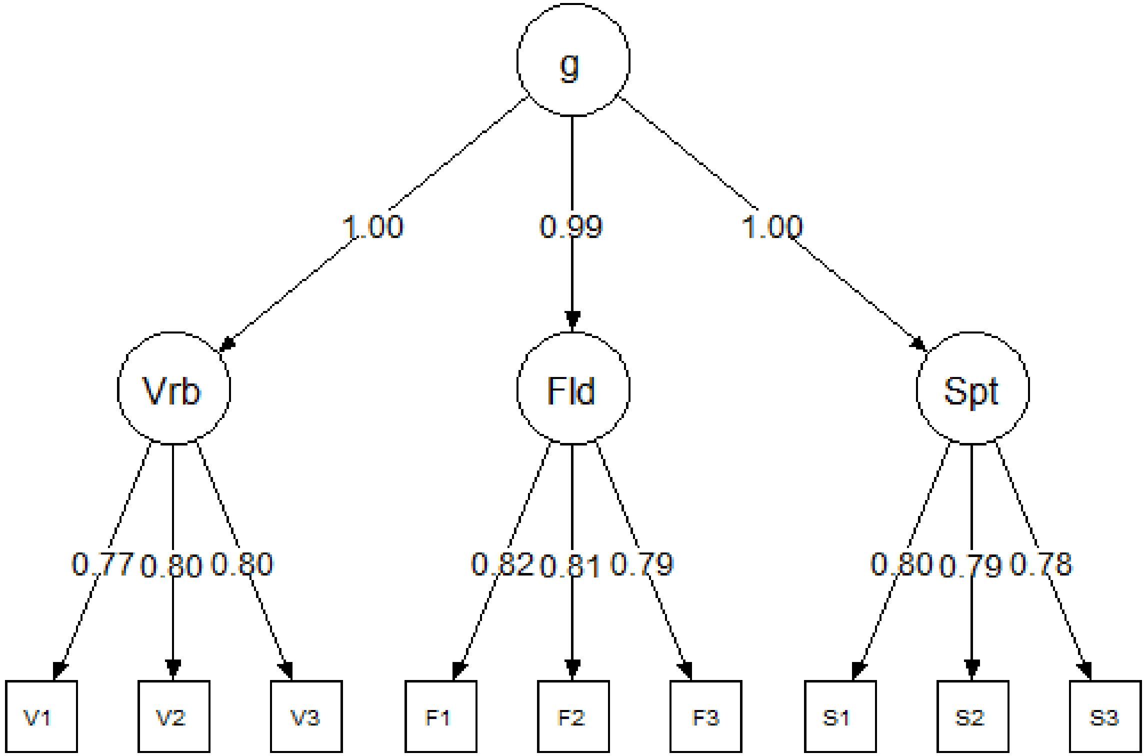
Study I: Results (POT Algorithm)





Study I: Results (GSM Algorithm)





Study I: Summary

- The positive manifold emerged from the simulated test scores **in the absence of a general cognitive ability**, as if there were a general intelligence at play
- By specifying domain-specific/domain-general processes, **a higher-order structure** can be achieved (**POT-S**)

Study I: Summary

- However,
  - POT interprets the higher-order *g* as a formative factor instead of a reflective one
  - The g factor is **the outcome, not the resource**, of the common variances in test scores
  - Thus, a conventional latent factor model is not fully compatible with POT

POT-S is not enough.

Study 2: A Network of POT Simulation

Apply a Psychometric Network to the Simulation (POT-N)

Combine Latent Factor Modeling and Network Modeling

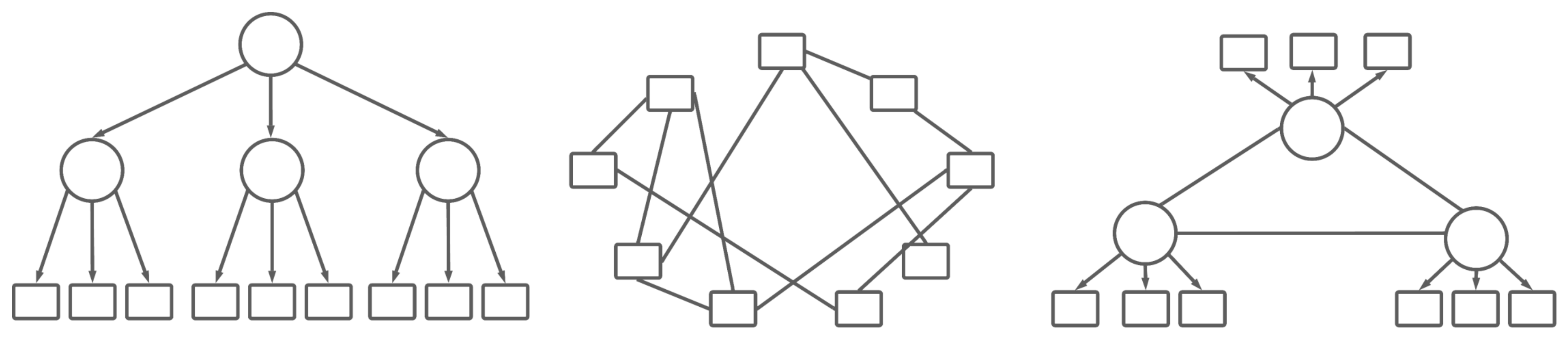
Study 2: A Network of POT Simulation

- Purpose: Extending the original simulation results by applying a network structure to the psychometric model of POT (**POT-N**)

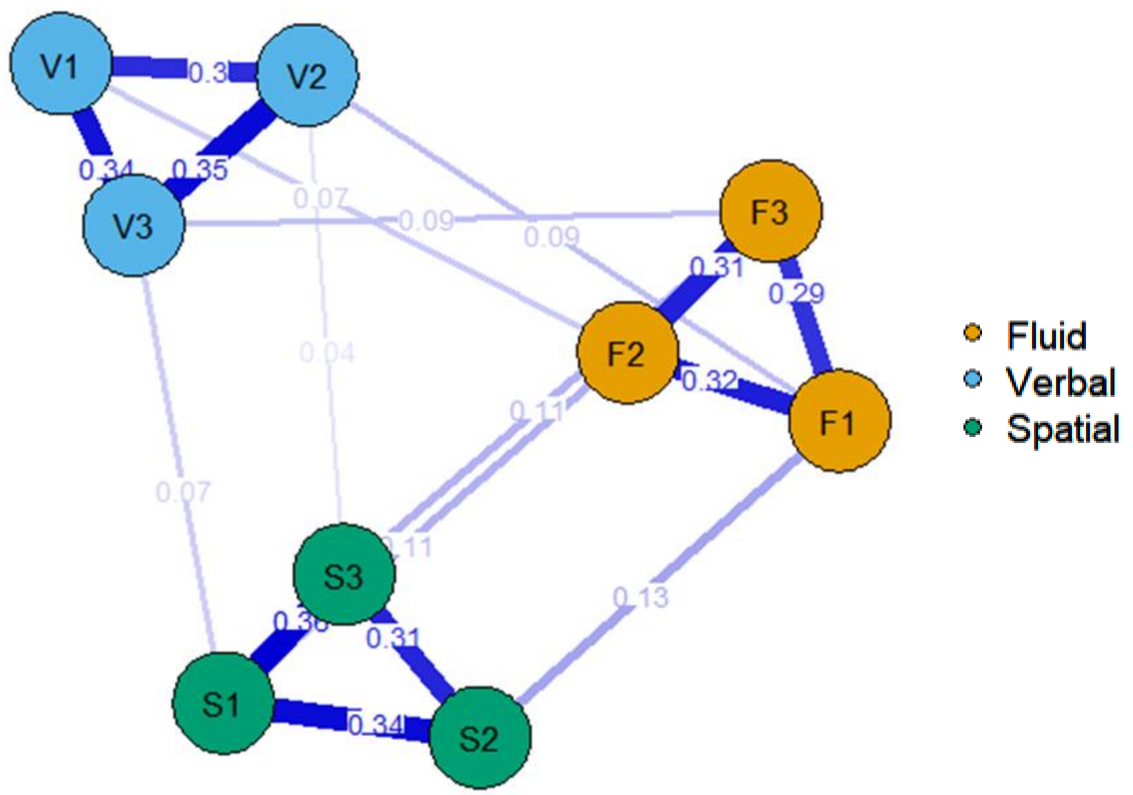
- Why POT-N?
  - An alternative representation to the positive manifold
  - Shifts the main emphasis from a common cause to the direct mutual associations among specific cognitive measures

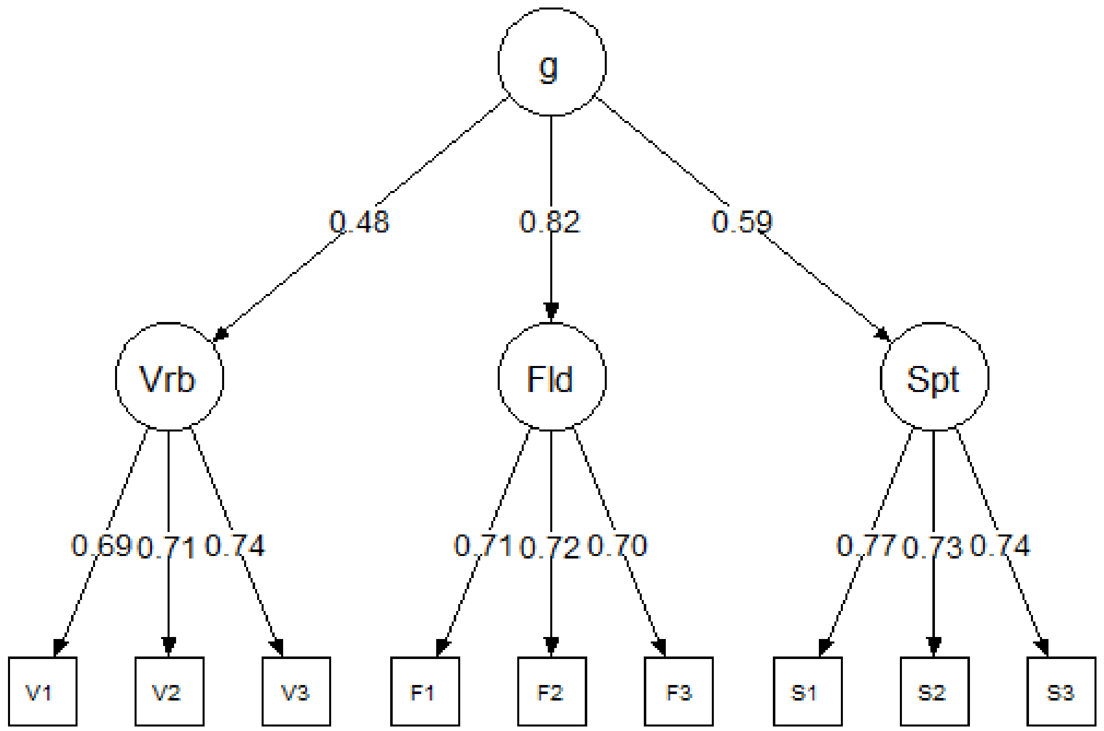
Study 2: A Network of POT Simulation

- We apply two types of network models:
  - Psychometric network of the 9 tests (regular graphic model)
  - Latent network model of the 3 types of tests (latent factors + graphic network)



Study 2: A Network of POT Simulation (a = 0.70)





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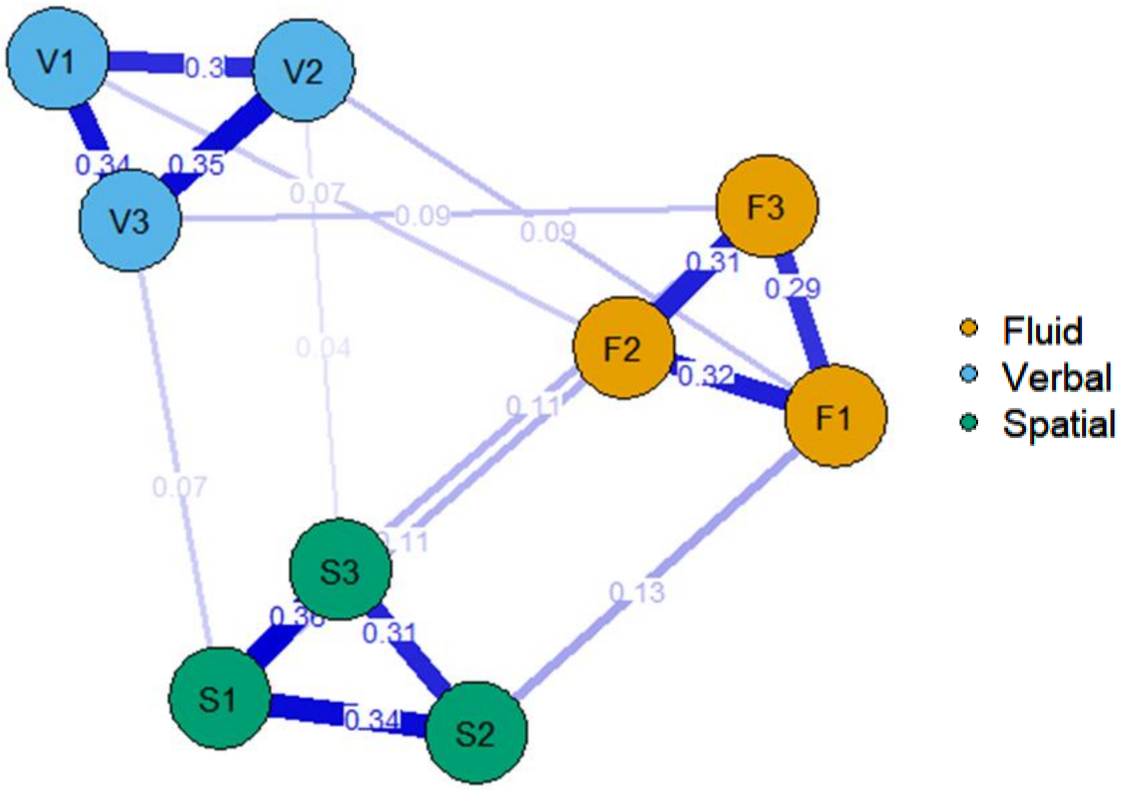
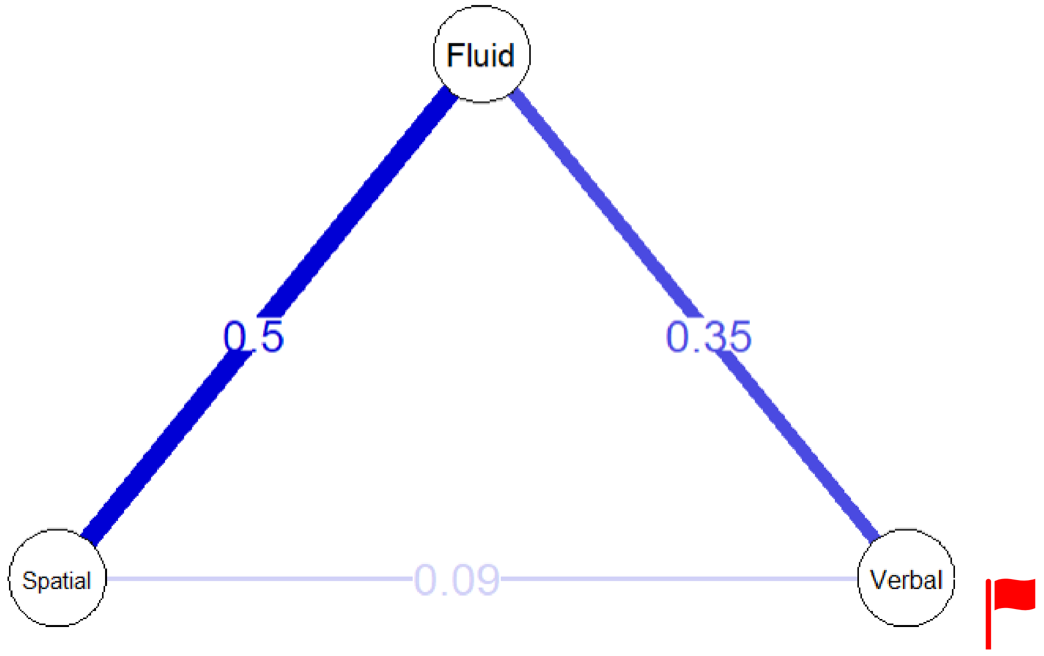




Study 2: Summary

- The psychometric network models offer **an alternative perspective** to investigate the correlational structure of individual differences in cognitive abilities without a common cause assumption

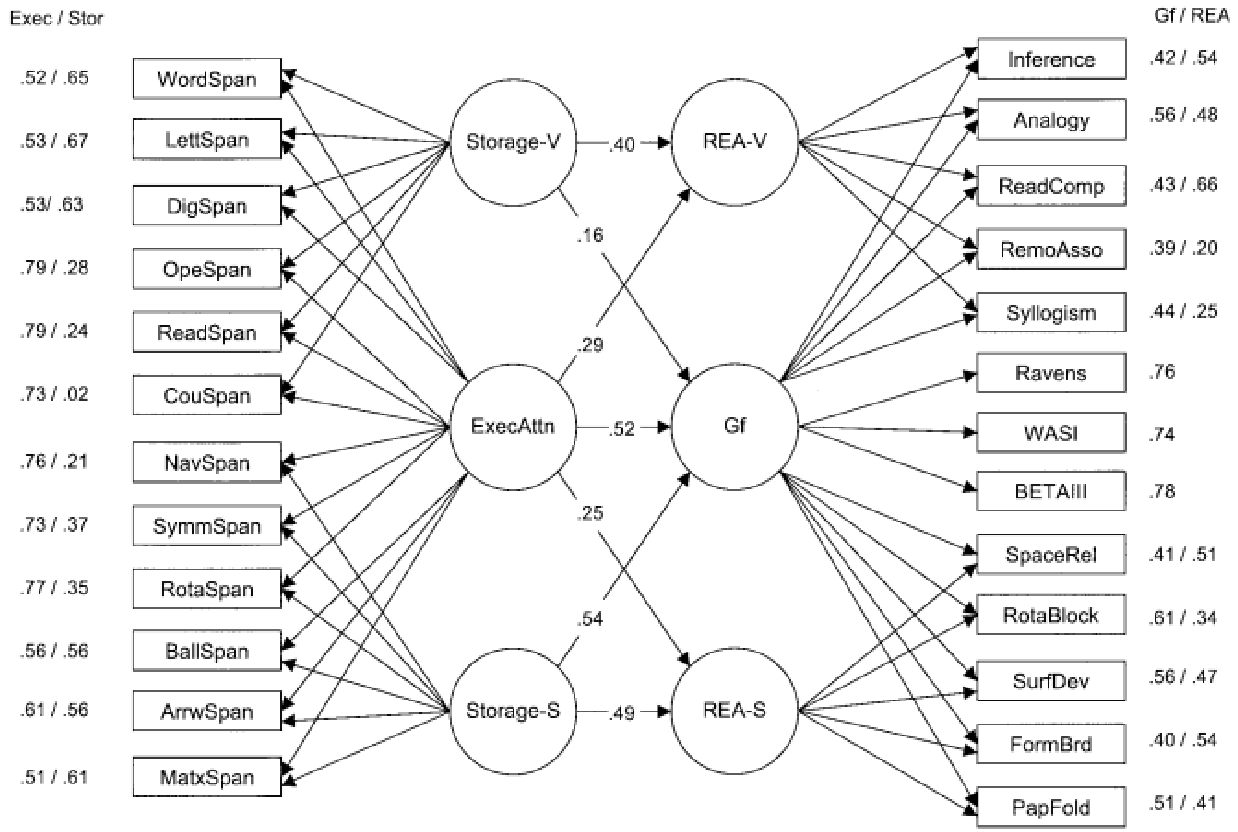
- The latent network model **combines the benefits** of both latent factor models and psychometric networks
  - The latent broad ability factors retain the benefits of latent factor models to account for measurement errors of tasks
  - The network structure among broad abilities rejects the notion of general intelligence as a common cause of positive manifold

Study 3: A Network Model of Working Memory and Reasoning

Re-analyze Kane et al. (2004) WM and Gf data from a Network Perspective Based on POT

Study 3: The Network Model of Kane et al. (2004)

- Applying this (latent) network framework to an empirical dataset with working memory and reasoning tasks
- Explain the network patterns observed in the framework from an exploratory perspective based on POT



Study 3: The Network Model of Kane et al. (2004)

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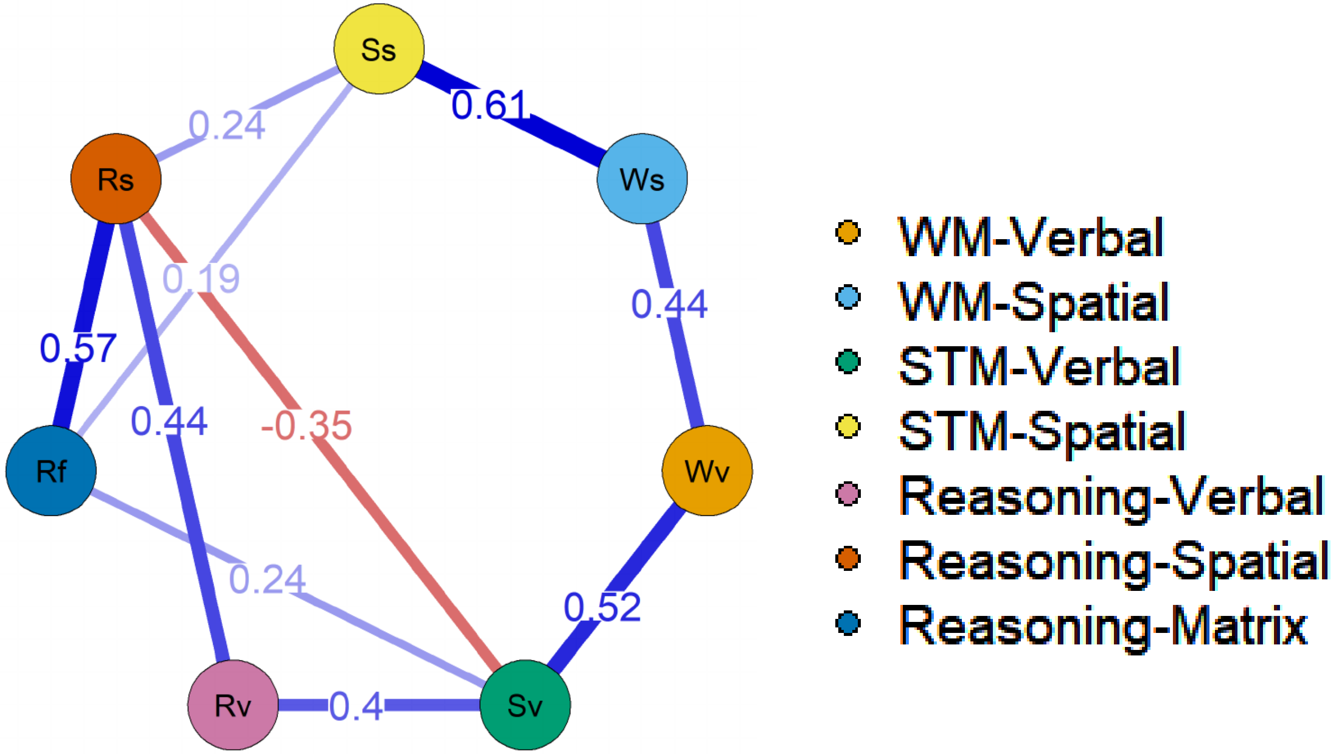
Task	Label	M	SD	Skewness	Kurtosis
Word Span	wor	0.78	0.11	-0.46	0.22
Letter Span	let	0.78	0.10	-0.29	-0.31
Digit Span	dig	0.80	0.10	-0.39	0.26
Ball Span	bal	0.58	0.12	-0.12	0.82
Array Span	arr	0.65	0.16	-0.76	1.26
Matrix Span	mat	0.51	0.15	-0.14	-0.15
Operation Span	ope	0.65	0.17	-0.47	0.05
Counting Span	cou	0.69	0.15	-0.26	-0.39
Reading Span	rea	0.67	0.16	-0.32	-0.13
Navigation Span	nav	0.48	0.18	0.10	-0.44
Symmetry Span	sym	0.47	0.20	0.26	-0.58
Rotation Span	rot	0.61	0.18	-0.29	-0.45
ETS Inference	etsin	0.61	0.22	-0.35	-0.58
AFOQT Analogies	afqan	0.56	0.22	-0.12	-0.54
Remote Associates	remass	0.46	0.17	-0.21	-0.18
ETS Nonsense Syllogisms	etssy	0.54	0.17	0.26	-0.22
AFOQT Reading Comprehension	afqrc	0.51	0.26	0.16	-1.03
ETS Paper Folding	papfo	0.60	0.24	-0.25	-0.63
DAT Space Relations	datsr	0.57	0.21	-0.23	-0.52
AFOQT Rotated Blocks	afqrb	0.37	0.23	0.48	-0.37
ETS Surface Development	etssd	0.60	0.25	-0.10	-1.02
ETS Form Board	etsfb	0.39	0.24	0.37	-0.72
WASI	wasim	0.69	0.14	-0.53	0.75
RAPM	raven	0.55	0.15	-0.58	0.66
BETA III	beta3	0.86	0.13	-1.80	5.05

Study 3: The Network Model of Kane et al. (2004)

- Applying this (latent) network framework to an empirical dataset with working memory and reasoning tasks
- Explain the network patterns observed in the framework from an exploratory perspective based on POT



model	DF	AIC	BIC	RMSEA	Chisq	Chisq_diff	DF_diff	p_value
Initial Network Model	0	-7645.38	-6434.52		~ 0			
Final Network Model	248	-7855.23	-7502.36	0.026	286.14	286.14	248	0.048
Initial Latent Network	254	-7799.65	-7467.53	0.035	328.68	42.53	6	< 0.0001
Final Latent Network	265	-7786.39	-7492.32	0.040	363.94	35.26	11	0.00022

Note: Chi-square difference test assumes models are nested.

Takeaways

- Study 1: A Simulation of POT
  - Simulated a cognitive mechanism based on POT and reflected it in psychometric models
  - Demonstrated that the positive manifold and a higher-order structure of psychometric intelligence can be achieved without a common resource
- Study 2: A network of POT Simulation
  - Proposed a network perspective of psychometric intelligence
  - Combined latent factor modeling and network modeling
- Study 3: The Latent Network of Kane et al. (2004)
  - Revisited a correlational dataset of cognitive abilities from an exploratory, network perspective
  - Emphasized more on the roles of specific abilities

Going Forward…

- Update the sampling mechanisms (associating the mechanisms to more behavioral and neural observations)
- Test different assumptions of cognitive theories using the sampling mechanisms and IRT parameters
- Network modeling on different samples of tests and subjects
- Integrate the drift-diffusion model

Thank you!

Download slides at:
https://hanhao23.github.io/files/HHCogForum04152022.pdf