

# Portfolio Dynamics and the Supply of Safe Securities

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## Abstract

This paper develops a model of securitized lending where financial institutions issue safe securities for cheap funding and dynamically manage the underlying loans. By committing to maintain portfolio quality, institutions can issue larger safe tranches, but in downturns, are obligated to replace deteriorated loans with less risky loans. Price pressure from these trades creates profit opportunities, incentivizing peer institutions to not securitize loans *ex ante*. The model explains why collateralized loan obligations (CLOs) include covenants that induce dynamic trades of underlying loans, why non-securitized loan funds coexist with CLOs, and how the market responds to the demand for safe securities.

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Investors in financial markets often place special value on highly-rated securities that have low default risk.<sup>1</sup> This creates an incentive for financial institutions to produce such securities, in particular by repackaging risky loans into a less risky senior tranche and riskier junior tranches. While securitization can in principle create fairly safe senior tranches, the size of these tranches is constrained by the future cash flows of the underlying loans in bad states of the world. Senior tranches exceeding this collateral constraint are exposed to a nontrivial risk of default, as witnessed in the aftermath of the 2007–09 financial crisis.

This paper sheds light on a financial innovation, dynamic collateral management, which is adopted by institutions to relax the binding constraint. The key benefit of dynamic collateral, as I show empirically and theoretically, is that institutions can manage the quality of their loan portfolios by trading in the secondary market. When collateral quality deteriorates in bad times, replacing deteriorated loans with less risky loans reduces the uncertainty of future portfolio cash flows, which helps keep the senior tranche away from default. Thus, by dynamically maintaining portfolio quality, institutions can create larger senior tranches that will be safe even after adverse systemic shocks.

A prominent example of this idea is Collateralized Loan Obligations (CLOs), which are nonbank institutions financed by securitized tranches and investing in junk-rated corporate loans. Instead of keeping portfolios static, CLOs dynamically replace the underlying loans as their quality changes, typically by trading with mutual and hedge funds that also hold corporate loans (hereafter “loan funds”). Senior tranches, or roughly 65% of the CLO capital structure, are AAA-rated and have never defaulted since the 1990s. Despite the rapid growth of CLOs and loan funds in this market, the economic mechanism remains unclear. What drives the size of safe tranches backed by dynamic portfolios? Why do institutions holding similar loans have distinct liabilities? Does trading between institutions affect the overall supply of safe debt? Will the market efficiently supply safe debt?

In this paper, I provide direct evidence of how CLOs replace deteriorated loans with less risky loans, and in doing so, maintain portfolio quality in bad times. Motivated by the evidence, I develop a micro-founded model of dynamic collateral management that endogenizes

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<sup>1</sup>Recent literature documents a special demand for highly-rated safe securities that arises from these securities’ monetary services and regulatory advantages (e.g., Krishnamurthy and Vissing-Jorgensen, 2012; Gorton, Lewellen, and Metrick, 2012; Nagel, 2016; Van Binsbergen, Diamond, and Grotteria, 2022).

the supply of safe debt and the origination of the underlying loans. My main contribution is to formalize portfolio dynamics in an equilibrium framework with flexible financing choices and explain the coexistence of, and trades between, CLOs and loan funds. My model shows that trading between institutions can raise the total supply of safe debt. However, the market supply may be still below the efficient level. This contrasts with established theories, such as Stein (2012), who finds that financial institutions tend to overproduce safe debt.

I derive these results in a three-period model. There are two types of agents: investors, who enjoy a non-pecuniary benefit from safe debt, and financial institutions, who can produce safe debt backed by loan portfolios. Because of the non-pecuniary benefit, safe debt can be issued at a premium and provides cheap funding for institutions. In the initial period, institutions issue debt and equity tranches to raise funding from investors and make loans at a private cost of effort. These loans and tranches are long-term: their payoffs are realized in the final period. Since the underlying loans are risky, the size of safe tranches is constrained by the worst realization of loan payoffs. This collateral constraint is tight, particularly because loan payoffs might be very low if a systemic shock arrives in the interim period.

My model highlights a novel dynamic link between debt safety and loans of different quality. Low-quality loans are riskier and have a lower worst-case payoff, so high-quality loans are better collateral for safe debt. However, loan quality is unknown when tranches are issued and is revealed only after the interim shock. So the size of safe tranches, if backed by static collateral, is limited by low-quality loans in the portfolio. Nevertheless, institutions may rebalance portfolios by trading in a secondary market. Their key ability is that they can credibly pre-commit to sell low-quality loans and buy high-quality loans when quality reveals. This commitment increases the portfolio's worst-case payoff beyond that of a static portfolio, thereby enabling the institution to issue a larger safe tranche *ex ante*.

Why do institutions with distinct liabilities coexist in the same lending market? My model shows that this fact arises as an equilibrium outcome: Some institutions, which resemble CLOs, specialize in creating safe debt, relying on high-quality loans sold by institutions that do not create safe debt, which resemble loan funds. This mix of institutions is driven by secondary market prices. In bad times, CLOs are obligated to sell low-quality loans and buy high-quality loans. The pressure from these trades causes the price of low-quality loans to decrease relative to the price of high-quality loans, making it profitable to trade with

CLOs. Attracted by the profit opportunities, loan funds give up issuing safe debt and provide liquidity in the secondary market.

By integrating safe debt creation and secondary market trading into an equilibrium framework, my model helps interpret a variety of empirical findings. For example, CLOs typically sell deteriorated loans to mutual and hedge funds (Giannetti and Meisenzahl, 2021), and their loan sales exert downward pressure on secondary market prices (e.g., Elkamhi and Nozawa, 2022; Kundu, 2021; Nicolai, 2020). Conversely, CLOs also buy higher-rated loans from mutual funds, improving their resilience to outflows (Emin et al., 2021). Different from these papers, which separately examine CLOs' sales and purchases, I document evidence of portfolio substitution in my model: there is a strong, nearly one-to-one relationship between CLOs' loan sales and purchases in market downturns.

My model demonstrates that dynamically reallocating loans between institutions can raise the supply of safe debt beyond a static benchmark. If portfolios were static, all institutions would pledge loans to issue safe debt. With dynamic portfolios, in contrast, only a subset of institutions, namely CLOs, issue safe debt (the *extensive* margin), and they issue beyond what static portfolios can back (the *intensive* margin). Such financing choices are more than just a reshuffling of safe debt when an exogenous debt issuance cost differs across institutions. Because of the price pressure from CLOs, institutions operating as loan funds find lending more profitable than in the static benchmark, where they would issue safe debt at relatively high costs. So they lend more, which increases high-quality loans that are reallocated in bad times, thereby allowing CLOs to produce a greater total supply of safe debt.

However, compared with a social planner, the market may not efficiently produce safe debt. This is due to an externality in dynamic collateral management: the creation of long-term safe debt relies on high-quality loans sold by other institutions, but individual institutions do not internalize their collective effects on secondary market trades. The inefficiency is two-sided. On the liability side, aggregate safe debt is bounded by the supply of high-quality loans. As more CLOs issue safe debt for cheap funding, institutions facing lower costs of creating safe debt lose debt capacity to other CLOs. On the asset side, the private profit from liquidity provision is lower than the social value of collateral. So loan funds underinvest, leading to a shortage of high-quality loans. These two effects jointly generate seemingly conflicting results: there is excessive entry into operating CLOs, and the market underproduces safe debt.

The inefficiency could be a rationale for regulatory intervention, but the market presents unique challenges: Policies targeting either safe debt or lending cannot restore efficiency. For example, one might conjecture that a policy that reduces entry into operating CLOs could improve the allocation. However, I show that such a policy exacerbates the underproduction of safe debt. This is because a reduction in CLOs mitigates price pressure and lowers the profits of liquidity provision, so loan funds decrease lending, which worsens the shortage of collateral. Such equilibrium responses are relevant to the Credit Risk Retention Rule, a US regulation. The introduction of this rule in 2014 has led to a reduction in CLO entry, a lawsuit against regulators, and resultant drastic regulatory changes. My model thus provides theoretical guidance by analyzing the channels of potential unintended effects.

This paper is related to the literature on safe debt creation by financial intermediaries (Diamond and Dybvig, 1983; Gorton and Pennacchi, 1990). The closest theory is Stein (2012), who focuses on short-term debt backed by the liquidation value of bank assets. My paper focuses on a different setting and provides novel implications. First, I study long-term debt backed by dynamic portfolios and the equilibrium mix of nonbanks. Second, instead of selling assets to exogenous buyers, institutions in my model endogenously buy and sell loans among themselves. This feature, consistent with empirical facts, leads to different efficiency results: whereas Stein (2012) shows that banks overinvest and overproduce safe debt because their fire sales inefficiently attract resources away from buyers' real investment opportunities, in my model, CLOs underproduce safe debt because loan funds underinvest and fail to supply sufficient collateral. My analysis of this inefficiency also offers new policy implications.<sup>2</sup>

More broadly, this paper adds to research on the specific ways in which intermediaries create safe debt, including risk management (DeAngelo and Stulz, 2015), early asset liquidation (Stein, 2012; Hanson et al., 2015), deposit insurance (Hanson et al., 2015), concealing asset information (Dang et al., 2017), and diversifying away idiosyncratic risk (Diamond, 2020). My paper complements these studies by examining how dynamically adjusting portfolio composition helps intermediaries make long-term debt safe.

The market equilibrium in my model is related to theories of fire sales and liquidity hoarding. In particular, the feedback from secondary market prices to ex-ante investment

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<sup>2</sup>In particular, when safe debt creation is endogenous at both the intensive and extensive margins, policies restricting nor subsidizing debt issuance, if used in isolation, cannot restore efficiency.

and financing choices is central to Shleifer and Vishny (1992), Gorton and Huang (2004), and Diamond and Rajan (2011). Unlike that short maturity or moral hazard triggers liquidation, in my model, long-term contracts obligate CLOs to rebalance portfolios. Such portfolio substitution results in a novel pressure on relative prices, which incentivizes other institutions to forgo the creation of safe debt. My paper also provides a concrete setting where individual institutions face price-dependent borrowing constraints but do not internalize their collective impact on prices. In this sense, my model is related to the literature on such “collateral externalities”, as categorized by Dávila and Korinek (2018), in the context of financial markets (e.g., Gromb and Vayanos, 2002; Stein, 2012; Neuhann, 2019).

Finally, this paper contributes to a fast-growing body of empirical research on leveraged loans and CLOs. Recent studies show that CLOs’ debt covenants based on contractible proxies induce loan trades, including the aforementioned papers by Giannetti and Meisenzahl (2021), Elkamhi and Nozawa (2022), Kundu (2021), Nicolai (2020), and Emin et al. (2021). Focusing on the liability side, Cordell, Roberts, and Schwert (2021) examine the performance of CLO securities and find that the cheap funding of senior debt leads to higher equity tranche returns. To my knowledge, my paper is the first to analyze the relationship between dynamic loan trades and nonbank institutions’ financing decisions, especially the supply of safe tranches. It contributes to this literature with new stylized facts and an equilibrium framework to interpret empirical evidence and explore policy implications.

The rest of this paper is organized as follows. Section 1 presents empirical facts that motivate my theoretical analysis. Section 2 introduces model setup. Section 3 characterizes the equilibrium and discusses its positive implications. Section 4 analyzes the equilibrium’s efficiency properties and policy implications. Section 5 concludes.

## 1 Stylized Facts

I begin with institutional background and empirical facts on how CLOs’ collateral constraints govern the dynamics of the underlying portfolios. Data details are in Appendix IA.1.

## Institutional Background: the Leveraged Loan Market

Leveraged loans are private debt extended to corporations that have a high financial leverage.<sup>3</sup> These loans are originated through syndication deals, where underwriters organize select groups of lenders to privately contract with the borrowers. Following the Federal Reserve Board (2022), I restrict attention to “institutional leveraged loans”, which are non-amortizing term loans and mostly held by nonbank institutions.

[Add Figure 1 here]

*Collateralized Loan Obligations.* CLOs are the largest group of nonbanks that hold leveraged loans. As Figure 1 shows, US leveraged loans grew from \$130 billion to \$1.2 trillion between 2001 and 2020, and CLOs consistently held about half of these loans. While other types of securitization are mostly backed by static collateral, CLOs’ portfolios, consisting of 100–300 loan shares with \$300–600 million total par values, are actively managed during a reinvestment period. CLO debt maturities are around 10 years, and the reinvestment period is typically 5 years and often extended. After this, the CLO enters its amortization period and repays debt principal gradually.<sup>4</sup> The vast majority of CLOs are “open-market CLOs”, whose managers are independent from banks. The manager’s compensation consists of size-based fixed fees and incentive fees based on equity tranche performance.

*Demand for Safe Debt.* A primary economic force behind the growth of CLOs is the demand for highly-rated securities. Senior tranches, which account for about 65% of the CLO capital structure, are rated AAA. With higher yields than safer assets (e.g., US Treasuries) and low regulatory risk weights, senior CLOs are attractive to, and mostly held by, banks. For example, Fitch (2019) reports that \$94 billion, \$113 billion, and \$35 billion of senior CLOs are held by banks in the US, Japan, and Europe, respectively. History indicates that the AAA rating was not necessarily inflated. Since the 1990s, more than two thousand senior tranches have been issued, and none of them ever defaulted.<sup>5</sup> If capital-constrained banks

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<sup>3</sup>S&P Global Market Intelligence defines a loan as leveraged if it is rated below Baa3/BBB-, or if it is secured and has a spread of at least 125 basis points.

<sup>4</sup>In the amortization period, CLO managers can buy loans using only prepaid principal of existing loans. See Fitch’s report for more details: [Reinvestment in Amortization Period of U.S. CLOs](#).

<sup>5</sup>A subset of senior CLOs were downgraded during 2007–09 but mostly recovered to original ratings (Cordell, Roberts, and Schwert, 2021). No senior tranche was downgraded in the 2020 COVID-19 crisis.

are willing to pay a premium (i.e., accept a lower risk-adjusted return) for AAA tranches, creating these safe tranches becomes a cheap source of funding for CLOs.

## **Fact 1: Coexistence of Institutions with Distinct Liabilities**

The leveraged loan market consists of two types of nonbank institutions. In addition to CLOs, the other type, primarily mutual and hedge funds, hold the majority of the rest of loans. In Appendix IA.2, I summarize the amounts of loans held by these loan funds and decompose public loan funds into open-end, close-end, and exchange-traded funds. The commonality of these funds is that they do not pledge their loans as collateral to back safe securities and hence face few restrictions on portfolio choices.

[Add Figure 2 here]

These two types of institutions are often operated by a common group of asset managers. Figure 2 illustrates that managers choose different type(s) of institutions to operate. For example, CVC Credit Partners only operates CLOs, whereas Fidelity Investments mainly manages leveraged loan mutual funds. Such choices lead to a coexistence of two types of nonbanks that invest in the same asset class but are financed by distinct liabilities.

## **Fact 2: CLOs Face Binding Collateral Constraints**

The large size of leveraged loans, typically hundreds of million or even multiple billion dollars, creates economies of scale in information production. Unlike generic business loans, leveraged loans are actively traded and have credit ratings reflecting changes in loan quality. Therefore by contracting on individual loan ratings, CLO managers can credibly commit to dynamically replace underperforming loans as their quality deteriorates.

In practice, CLO debt covenants implement this commitment with regular (e.g., monthly) collateral tests that are tied to managerial compensation. The most important test is the over-collateralization (OC) test, which calculates a ratio of quality-adjusted loan holdings to the size of a debt tranche. When the OC test fails, the manager stops receiving fees until the ratio recovers to a preset threshold. The manager can raise the ratio through either debt



acceleration (i.e., divert cash flows generated by loans to repaying the senior tranche) or portfolio substitution (i.e., replace deteriorated loans with qualified loans).

[Add Figure 3 here]

Collateral constraints imposed by the covenants play a critical role in governing the dynamics of the CLO balance sheet. Figure 3 shows quarterly cross-sectional distribution of the slackness of senior OC constraints between 2010–2019. Among CLOs in reinvestment period, the average senior OC score is slightly (8%) above the threshold and stable over time.<sup>6</sup> In the cross section of CLOs, the slackness is tightly distributed around the average. These persistently binding constraints suggest that managers fully use safe debt capacity provided by their loan portfolios. By contrast, in amortization period, as CLO leverage decreases with principal repayment, the slackness becomes larger and much more dispersed.

### **Fact 3: Binding Constraints Force CLOs to Replace Loans**

Given binding collateral constraints, a shock to the quality of CLOs’ underlying loans is likely to trigger secondary market trades. In this paper, I highlight an empirical pattern that is less discussed in the literature: CLOs’ loan trades consist of both sales and purchases rather than just one of them. Hence, the portfolio’s size remains similar, but its composition changes.

[Add Figure 4 here]

Figure 4 presents CLO balance sheet dynamics before and around the onset of COVID-19 crisis in 2020. Panel (a) shows quarterly average CLO portfolio size for each age cohort. For all cohorts, portfolio size remained stable over time. This implies that overall CLOs did not shrink in size in bad times. Indeed, Panel (b) shows that accelerated repayment of senior debt actually decreased.<sup>7</sup> While the size of portfolios did not change, their composition changed drastically. In Panel (c), the average numbers of loan purchases and sales both nearly doubled upon the arrival of the negative shock.<sup>8</sup> To understand the nature of these trades, Panel (d)

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<sup>6</sup>In my data, the senior OC thresholds can be that of AAA and AA tranches, so my calculation may overstate the actual slackness of AAA OC constraints.

<sup>7</sup>Earlier cohorts repaid some of senior tranches when the non-call period ends (typically 2–3 years). Such early repayment discontinued perhaps because refinancing was difficult in 2020.

<sup>8</sup>Purchases generally exceed sales because loan holdings generate coupon and principal payments.

examines buys and sells *within* individual CLOs in the first two quarters of 2020. As the bin scatter plot shows, there is a strong positive (and nearly one-to-one) relationship between a CLO’s purchases and sales: when a CLO sells loans, it also buys loans to replace them. In other words, CLOs substitute loans in their portfolios.

## **Fact 4: Portfolio Substitution Improves Collateral Quality**

Using granular data on CLO loan holdings, I examine how such trades affect portfolio quality in 2020. Figure 5 presents the changes from February 15 (“pre”) to June 30 (“post”). Panel (a) shows senior OC slackness before and after the shock. As the pandemic caused massive downgrades of leveraged loans, the overall slackness decreased, and the dispersion across CLOs increased. When the crisis settled, however, only 1.2% of CLOs failed senior OC tests.

[Add Figure 5 here]

The reason behind rare test failure, as the previous subsection suggests, could be portfolio substitution. To quantify its causal effect, I track quality changes of individual loans and measure each CLO’s counterfactual portfolio quality in the absence of loan trades. Details of this step can be found in Appendix IA.1.3. Panel (b) shows portfolio value-weighted average ratings.<sup>9</sup> Clearly, ratings dropped overall, but managers’ trading mitigated deterioration, improving the realized ex-post distribution relative to the counterfactual distribution.

Despite similarly binding constraints *ex ante*, a larger exposure to the shock may force CLOs to respond more intensively. I measure a CLO’s exposure using the difference in rating between the pre and counterfactual portfolios. Panel (c) shows that almost all CLOs replaced downgraded loans and that the effect on quality linearly increases in exposure: on average, trading offsets 60% of deterioration caused by COVID-19. Panel (d) replaces the outcome with value-weighted average coupon rate, which measures quality based on loan pricing. In response to a 1-notch downgrading, the manager’s trades reduced average coupon by 30 basis points, or roughly one standard deviation. Panels (e) and (f) further show evidence based on the direction of loan trades by comparing ratings and coupons between the loans bought

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<sup>9</sup>A larger numeric rating corresponds to a better letter rating. See Table IA.2 in the Appendix for the conversion between letter and numeric ratings.

and sold by a CLO. Overall, these facts support the interpretation that binding collateral constraints triggered portfolio substitution that substantially improved portfolio quality.

## Fact 5: Price Pressure from CLOs

In Appendix IA.3, I document that in market downturns, transitory price changes across loans of different quality are consistent with price pressure from CLOs. Admittedly, isolating loan price changes caused by CLOs from changes in fundamentals is difficult. While the nature of this evidence is suggestive, given various findings of CLOs' price pressure in the literature, it is plausible that when a large number of CLOs substitute portfolios in the same direction, they cause the prices of bad loans to decrease relative to the prices of good loans.

## 2 The Model

Motivated by the empirical facts, I develop a model in which institutions can flexibly choose external financing and credibly commit to maintain portfolio quality. The economy has three time periods  $t \in \{0, 1, 2\}$  and two types of agents: investors and financial institutions.

**Investors.** There is a unit mass of investors, broadly interpreted as banks, insurance companies, households, and other entities that invest in CLOs and loan funds. Investors exhibit a preference for safe debt because some of them face risk-based regulatory requirements. Following the literature (e.g., Krishnamurthy and Vissing-Jorgensen, 2012; Stein, 2012; Nagel, 2016), I assume that investors maximize additively separable utility

$$U = \mathbb{E}_0 \left[ \sum_{t=0}^2 C_t \right] + \gamma A, \quad (1)$$

where  $C_t$  is consumption in period  $t$ , and  $A$  is safe debt held at  $t = 0$ : They derive a non-pecuniary benefit  $\gamma$  per unit of safe debt because of its regulatory advantage.<sup>10</sup>

At  $t = 0$ , investors are endowed with an amount  $e$  of perishable consumption goods. They cannot directly lend but can buy financial claims backed by loans. Therefore they allocate

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<sup>10</sup>In practice, senior CLOs are not riskless. Their yields are higher than the risk-free rate but may be low on a risk-adjusted basis (Cordell, Roberts, and Schwert, 2021). Given the institutional background, my model focuses on their key role as highly-rated safe securities and treats them as riskless.

between consumption and financial claims, taking claim prices as given. I assume  $e$  to be sufficiently large, so investors always choose strictly positive consumption.

**Financial Institutions.** There is a continuum of financial institutions, interpreted as asset management companies, uniformly populated on  $\mathcal{I} = [0, 1]$ . Their preference is similar to (1), except for that they do not benefit from safe debt. Each institution, indexed by  $i \in \mathcal{I}$ , can lend at  $t = 0$  to generate a risky payoff at  $t = 2$ . Institutions receive zero endowment and finance their lending by issuing senior and junior financial claims. In particular, a senior claim is referred to as *safe debt* if it is backed by loans whose payoff is enough for repayment with certainty. The key friction in this economy is that markets are incomplete: agents cannot use contracts contingent on future states. For this reason, safe debt can only be supplied by institutions as their liabilities.

**Investment Technology.** Institutions can originate loans to form diversified portfolios.<sup>11</sup> There are two types of loans, denoted by  $j \in \{h, l\}$ . Every unit of loans generates a risky payoff  $R_j$  at  $t = 2$ . In period  $t = 1$ , a public signal  $s$  realizes, which is either positive or negative. With probability  $p$ , the signal is positive, after which both types of loans will pay  $R_j = R > 1$  units of consumption goods. However, if  $s$  is negative, which occurs with probability  $1 - p$ , payoffs will depend on loan types. Whereas type  $h$  will pay  $R_h = 1$  with certainty, type  $l$  will pay  $R_l = 1$  with probability  $\pi \in (0, 1)$  and  $R_l = 0$  with probability  $1 - \pi$ .

This payoff distribution is starker than needed. For example, one can innocuously assume that loan payoffs always remain uncertain until  $t = 2$ . What is important is the existence of a strictly positive minimum payoff  $R_h$ , which makes long-term safe debt possible.<sup>12</sup>

Institutions have identical investment technology: each of them can turn  $x$  consumption goods into  $x$  units of loans at a private cost  $c(x) - x$ . This private cost captures the effort of participating in syndicated deals to put together a diversified portfolio.  $c$  is twice continuously differentiable and satisfies  $c(0) = 0$ ,  $c' > 1$ ,  $c'' > 0$  on  $\mathbb{R}_+$ . To simplify the analysis, I assume that industrial borrowers' output is fully pledgeable and that lenders extract all the rents. Hence, institutions' lending becomes as if they directly control real assets, an approach widely used in the literature (e.g., Diamond and Dybvig, 1983).

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<sup>11</sup>Since nonbanks typically pre-commit to buying loans (Taylor and Sansone, 2006), my model abstracts from the underwriting process and refers to the nonbank lending activity as "origination".

<sup>12</sup>This positive lower bound is consistent with leveraged loans' default recovery rate (typically  $\geq 50\%$ ).

**Timeline.** Events in period  $t = 0$  occur as follows. First, every institution  $i$  makes  $x_i$  units of loans without knowing loan types. Immediately after lending,  $\tilde{x}_{i,l}$  units of loans become type  $l$ , and the remaining  $\tilde{x}_{i,h} = x_i - \tilde{x}_{i,l}$  units become type  $h$ . Across institutions,  $\tilde{x}_{i,l}$  is independently drawn from a common distribution with support  $[0, \bar{x}_l]$  and mean  $x_L$ . The realization of the shock,  $x_{i,l}$ , is publicly observed because investors can assess risk at the portfolio level, but which loans are low-quality is unknown in this period. Given the loan portfolio, the institution then issues safe debt,  $a_i \geq 0$ , and external equity shares to raise a total of  $x_i$  units of consumption goods from investors.

The timing of lending and financing choices above might seem odd but is consistent with practice, where CLOs use short-term “warehouse financing” to acquire loans and form an initial portfolio before issuing securities. My assumption that the quantity of low-quality loans is an upper-bounded random variable is not crucial. For example, making low-quality loans a random fraction of the portfolio, or allowing institutions to improve the quality of lending at an extra cost, would not change the main insights of the model. Moreover, since only safe debt provides a non-pecuniary benefit, capital structure below the safe tranche is irrelevant, and it is without loss of generality to treat risky junior debt as equity.

In period  $t = 1$ , loan quality becomes publicly observable, and institutions can trade loans in a Walrasian market. By selling and buying loans, an institution can generate a different worst-case payoff than that of a static portfolio. In period  $t = 2$ , payoffs realize. As internal equity holders, institutions repay safe debt and external equity investors and collect residual portfolio payoffs. All goods are consumed, and the economy ends.

**Securitization Technology.** The ability to trade loans, if not disciplined, prevents an institution from creating safe debt. The reason is a classic agency problem (Jensen and Meckling, 1976): as equity holders, institutions privately prefer loans with riskier payoffs, which makes their debt default with a positive probability. Based on Facts 2–4 in the previous section, I assume the existence of a technology that allows institutions to pre-commit to portfolio choices at  $t = 1$ . This technology can be thought of as third-party services that perform collateral tests, monitor cash flows, and seize loans on behalf of debtholders.

The securitization technology is crucial for dynamic collateral management to be viable. In practice, CLOs hold only fairly standardized corporate loans, so their portfolios can be

effectively disciplined by enforceable contracts. With simple collateral, CLOs differ from pre-crisis collateralized debt obligations (CDOs), which held enormous complex derivatives (Cordell, Huang, and Williams, 2011). Consistent with these facts, in my model, every institution's portfolio consists exclusively of risky loans. This occurs because consumption goods are nonstorable, cross-holdings of liabilities are unprofitable, and state-contingent bilateral contracts (e.g., derivatives) are not available.

**Debt Issuance Costs.** Adopting the technology and credibly revealing it to investors (e.g., via contracting and underwriting) incur a cost specific to safe debt issuance. I assume this cost to be  $\xi_i \geq 0$  per unit of debt, which captures volume-based expenses in the issuance process.  $\xi_i$  depends on exogenous firm characteristics and may vary across institutions. For example, it is plausibly less costly for KKR to create and sell AAA-rated securities to foreign banks than Fidelity, as the latter specializes in the US public fund markets.

**The Institution's Optimization Problem.** Institutions make sequential choices to maximize their payoffs. I describe their optimization problem backwardly and consider repayment only in the final period. The option of repaying debt in period  $t = 1$ , as I discuss in Subsection 3.5, can be ignored without loss of generality under the current setup.

Let secondary market prices of the two types of loans be  $(q_l, q_h) \in \mathbb{R}_+^2$ . When signal  $s$  realizes, institution  $i$ , with balance sheet  $(x_{i,h}, x_{i,l}, a_i)$ , chooses net trades  $\Delta x_{i,h}, \Delta x_{i,l}$  to maximize conditional expected payoff to equity

$$v(x_{i,h}, x_{i,l}, a_i; s) = \max_{\Delta x_{i,h}, \Delta x_{i,l}} \sum_j (x_{i,j} + \Delta x_{i,j}) \mathbb{E}[R_j | s] - a_i. \quad (\text{P1})$$

These trades are subject to a budget constraint

$$\sum_j (x_{i,j} + \Delta x_{i,j}) q_j \leq \sum_j x_{i,j} q_j, \quad (\text{BC})$$

a maintenance collateral constraint

$$a_i \leq \sum_j (x_{i,j} + \Delta x_{i,j}) \min R_j, \quad (\text{MCC})$$

and short-sale constraints  $\Delta x_{i,h} \geq -x_{i,h}, \Delta x_{i,l} \geq -x_{i,l}$ . The budget constraint requires the trades to be self-financed by the loan portfolio. Constraint (MCC) reflects the institution's ability to commit to maintain portfolio quality: After trades, safe debt investors must receive full repayment at  $t = 2$  with probability one. This constraint keeps the institution solvent, so

equity payoff in (P1) is linear in portfolio payoff.

All market participants rationally anticipate loan trades at  $t = 1$  when institutions choose lending and financing at  $t = 0$ . Because investors are price-taking, institutions optimally price their safe debt and external equity such that investors break even in expectation. This implies a safety premium: by issuing one unit of safe debt, an institution effectively raises  $1 + \gamma - \xi_i$ . The rest of funding for  $x_i$  is raised from external equity, whose expected return will be set as zero. Taking loan prices as given, the institution chooses lending  $x_i$  and safe debt  $a_i$  to maximize the expected payoff to internal equity:

$$V_i = \max_{x_i, a_i \geq 0} \mathbb{E}_0[v(x_{i,h}, x_{i,l}, a_i; s)] - (x_i - (1 + \gamma - \xi_i)a_i) - (c(x_i) - x_i), \quad (\text{P0})$$

where  $v(x_{i,h}, x_{i,l}, a_i; s)$  is the  $t = 1$  maximized equity value in (P1) as a function of choices and the realization of  $\tilde{x}_{i,l}$  at  $t = 0$ . The second term is the expected payoff to external equity, and the last term is the institution's private cost of effort. Importantly, the maximization is subject to an endogenous initial collateral constraint:

$$a_i q_h \leq \sum_j x_{i,j} q_j. \quad (\text{ICC})$$

Since  $R_h \geq 1$ ,  $a_i q_h$  is the market value of high-quality loans that will generate at least  $a_i$  payoff. Hence, this constraint requires the portfolio's market value at  $t = 1$  to be enough for the institution to satisfy (MCC) through replacing low-quality loans with high-quality loans.

I impose two parametric assumptions to simplify the analysis.

**Assumption 1.** *The cost function  $c$  satisfies  $c'(\bar{x}_l) < pR + 1 - p$  and  $c(\bar{x}_l) \leq (pR + (1 - p)\pi)\bar{x}_l$ .*

The first inequality ensures that institutions, if participating in lending, always raise and lend more than  $\bar{x}_l$ . This simplifies the optimization problem because the marginal unit of loans is high-quality, and the lending choice does not depend on the realization of  $\tilde{x}_l$ . The second inequality makes lending sufficiently profitable, so institutions always participate.

**Assumption 2.** *Investors' non-pecuniary benefit is greater than any institution's safe debt issuance cost:  $\gamma > \xi_i$  for all  $i \in \mathcal{I}$ .*

Under Assumption 2, every institution can benefit from cheap funding by issuing safe debt. However, I will show that in equilibrium, some institutions choose not to do so.

An equilibrium in this economy is defined as follows.

**Definition 1** (Competitive Equilibrium). *An equilibrium consists of lending and financing choices  $(x_i, a_i)$ , secondary market trades  $(\Delta x_{i,h}, \Delta x_{i,l})$ , and secondary market loan prices  $(q_h, q_l)$  such that (i) given loan prices,  $(x_i, a_i)$  solves the institution's lending and financing problem (P0), (ii) given loan prices,  $(\Delta x_{i,h}, \Delta x_{i,l})$  solves the institution's trading problem (P1), and (iii) the secondary market clears, that is,*

$$\int_i \Delta x_{i,j} di = 0 \quad \text{for } j \in \{h, l\}. \quad (2)$$

Because of the collateral constraints, institutions' ex-ante lending and financing choices affect their ex-post trades, which in turn affect the lending and financing problem through endogenous loan prices. As such, the equilibrium features a feedback loop between primary and secondary markets.

### 3 Equilibrium Characterization

#### 3.1 Benchmark: Static Securitization

To provide a basic benchmark, suppose institutions cannot credibly commit, or the secondary market for loans does not exist, and as a result, securitization has to be backed by static portfolios. Let  $c'^{-1}(\cdot)$  be the inverse function of  $c'(\cdot)$ , the first-order derivative of  $c$ .

**Lemma 1.** *If portfolios are restricted to be static, every institution issues safe debt and fully uses debt capacity:  $a_i^{STA} = x_{i,h}^{STA}$ . Lending decreases in  $\xi_i$ , the cost of issuing safe debt:  $x_i^{STA} = c'^{-1}(pR + 1 - p + \gamma - \xi_i)$  for all  $i \in \mathcal{I}$ .*

*Proof.* See the Appendix. □

Since safe debt provides cheap funding, every institution pledges its loans as collateral and fully uses debt capacity. Lending decreases in  $\xi_i$  because loans not only generate payoffs but can also back safe debt, and debt capacity is more valuable when debt issuance is less costly. This market structure resembles traditional banking, where risky loans stay on bank balance sheets, and a bank's productivity in raising deposits plays a key role in its value creation (Egan, Lewellen, and Sunderam, 2022).



In the rest of this section, I show how the lending and financing choices differ from this benchmark when institutions can trade loans in a secondary market and credibly commit to future portfolio choices. I first analyze individual institutions' lending, financing, and trading choices for given secondary market prices. I then study balance sheets and loan prices that clear the secondary market.

### 3.2 Secondary Market Trades

The lending and financing choices at  $t = 0$  depend on continuation value  $v$ . To derive  $v$ , in this subsection I analyze the institution's secondary market problem (P1) in period  $t = 1$ .

If the public signal is positive, maintenance collateral constraint (MCC) will be slack for all institutions. Hence, non-trivial trades occur only if the signal is negative. In this stage, budget constraint (BC) binds, and since  $a_i \geq 0$ , constraint (MCC) implies that  $\Delta x_{i,h} \geq -x_{i,h}$  is slack. Omitting terms predetermined at  $t = 1$ , the problem is equivalent to

$$\max_{\Delta x_{i,l}} \Delta x_{i,l} \left( \pi - \frac{q_l}{q_h} \right), \quad (\text{P1a})$$

subject to constraints  $\Delta x_{i,l} \frac{q_l}{q_h} + a_i \leq x_{i,h}$  and  $\Delta x_{i,l} \geq -x_{i,l}$ .

Essentially, the institution exchanges between the two types of loans. This exchange is constrained by safe debt outstanding and a short-sale constraint. Note that a negative signal updates the fundamental values of high-quality and low-quality loans to 1 and  $\pi$ , respectively. I proceed to solve this problem based on the following observation.

**Lemma 2.** *In the secondary market, the ratio of the prices of low-quality and high-quality loans is lower than the ratio of their fundamental values:  $\frac{q_l}{q_h} \leq \pi$ .*

*Proof.* See the Appendix. □

Lemma 2 shows that secondary market trades triggered by binding collateral constraints exert pressure on relative prices. Intuitively, in bad times, a subset of institutions are obligated to buy high-quality loans and sell low-quality loans, which puts them in demand for liquidity. Their natural counterparties are institutions who are holding similar loans but not constrained by liabilities. For the latter to be willing to provide liquidity, low-quality loans, which are

inferior as collateral for safe debt, must offer a higher expected return and hence a lower price-to-fundamental ratio relative to high-quality loans.<sup>13</sup>

The solution to (P1a) indicates that, consistent with Fact 3, the institution's optimal trades lead to portfolio substitution:

$$\Delta x_{i,h} = a_i - x_{i,h}, \quad \Delta x_{i,l} = -(a_i - x_{i,h}) \frac{q_h}{q_l} \quad (3)$$

for any given  $x_{i,h}$  and  $a_i$ . These trades reallocate loans among institutions. An institution with  $a_i > x_{i,h}$  optimally sells just enough bad loans to increase the holding of good loans and keep its debt safe. Such portfolio substitution is costly to equity holders (including the institution) because it not only decreases portfolio volatility, but also moves prices in unfavorable directions. By contrast, an institution with  $a_i < x_{i,h}$  sells its good loans and buys bad loans to profit from the deviation of loan prices from fundamentals.

### 3.3 Lending and Financing Choices

Next, I characterize the institution's optimal lending and financing choices at  $t = 0$  for given loan prices. Optimal secondary market trades in (3) imply that equity continuation value  $v$  after positive and negative signals are  $x_i R - a_i$  and  $\pi(x_{i,l} + (x_{i,h} - a_i) \frac{q_h}{q_l})$ , respectively. By no arbitrage,  $0 < q_l < q_h$ , and initial collateral constraint (ICC) is equivalent to

$$a_i \leq x_i - x_{i,l} + x_{i,l} \frac{q_l}{q_h}. \quad (\text{ICCa})$$

Substitute  $v$  into (P0), the institution's lending and financing problem becomes

$$\max_{x_i, a_i} p(x_i R - a_i) + (1 - p)\pi\left((x_i - x_{i,l} - a_i) \frac{q_h}{q_l} + x_{i,l}\right) - (c(x_i) - (1 + \gamma - \xi_i)a_i) \quad (\text{P0a})$$

subject to constraints (ICCa) and  $a_i \geq 0$ .<sup>14</sup> Let  $\eta_i$  and  $\mu_i$  respectively be the Lagrangian multipliers of these constraints. The Kuhn-Tucker conditions for optimality are

$$pR + (1 - p)\pi \frac{q_h}{q_l} - c'(x_i) + \eta_i = 0, \quad (4)$$

$$\gamma - \xi_i - (1 - p)\left(\pi \frac{q_h}{q_l} - 1\right) - \eta_i + \mu_i = 0, \quad (5)$$

<sup>13</sup>The inequality will be shown to be generally strict in equilibrium, so I ignore the corner case (i.e.,  $\frac{q_l}{q_h} = \pi$ ) throughout this section.

<sup>14</sup>Given Assumption 1, the realized quantity can be used in the optimization problem.

and

$$\eta_i \geq 0, \eta_i \left( a_i - \left( x_{i,h} + x_{i,l} \frac{q_l}{q_h} \right) \right) = 0, \mu_i \geq 0, \mu_i a_i = 0. \quad (6)$$

Because of the price pressure, replacing low-quality loans is costly, and providing liquidity is profitable. This gives rise to an endogenous intertemporal tradeoff, captured by Equation (5): the institution's financing choice is based on a comparison between the funding benefit of safe debt,  $\gamma - \xi_i$ , and the expected profit from liquidity provision,  $(1 - p)(\pi \frac{q_h}{q_l} - 1)$ . It follows that two cases are possible. In the first case, the benefit is less than the profit, hence no safe debt is issued ( $\mu_i > 0$ ), and the collateral constraint is slack ( $\eta_i = 0$ ). Accordingly, the lending choice in (4) is solely based on an asset-side tradeoff between the expected payoff and the cost of investment.

In the second case, the funding benefit exceeds the profit, and collateral constraint (ICCa) binds. On the liability side, the institution fully uses safe debt capacity to exploit cheap funding. On the asset side, as characterized by Equation (4), lending goes beyond maximizing returns. The additional investment, captured by  $\eta_i = \gamma - \xi_i - (1 - p)(\pi \frac{q_h}{q_l} - 1) > 0$ , reflects the collateral value of loans. As  $\eta_i$  decreases in  $\xi_i$ , institutions facing lower issuance costs originate more loans to back larger safe tranches.

### 3.4 Market Equilibrium of Financial Institutions

Optimal financing choices in the two cases above endogenously determine the mix of financial institutions. A key metric in the equilibrium's feedback loop is price ratio  $\frac{q_l}{q_h}$ , which is the marginal rate of portfolio substitution. When this ratio is higher, replacing low-quality loans is less costly, and providing liquidity to others is less profitable, so issuing safe debt is more attractive. However, safe debt issuance increases secondary market demand for high-quality loans and supply of low-quality loans, and the market cannot clear unless the price ratio drops sufficiently. In equilibrium, the price ratio adjusts until institutions' lending and financing choices equilibrate demand and supply in the secondary market.

The market-clearing condition (2) and optimal trades in (3) jointly imply a relationship between the quantities of safe debt and risky loans:

$$\int_i a_i \, di = \int_i x_{i,h} \, di. \quad (7)$$

Intuitively, the market's total safe debt capacity is bounded by worst-case aggregate loan payoffs, and because safe debt is priced at a premium, institutions always collectively use up this capacity. Equation (7) reflects this aggregate relationship.

I characterize the equilibrium in two steps. First, I analyze the equilibrium when institutions face generally different costs of debt issuance. I then consider a knife-edge case in which all institutions are homogeneous. This special case provides intuition useful for understanding the competitive allocation and its efficiency properties.

### 3.4.1 The Intensive and Extensive Margins of Safe Debt Supply

My equilibrium framework addresses not only how much safe debt is created (i.e., intensive margin), but also which institutions create safe debt (i.e., extensive margin). For ease of exposition, my main analysis is based on a setting where institution cost types are continuous. This setting allows for a parsimonious characterization that preserves the intuition of results based on discrete cost types, which I analyze in Appendix IA.4.

Without loss of generality, let institution  $i$ 's cost of issuing safe debt be  $\xi_i = 2\xi i$  for constant  $\xi \in (0, \gamma/2)$ . Given loan prices fixed, an institution facing a lower issuance cost benefits strictly more from issuing safe debt than an institution facing a higher issuance cost, so the constraints on the choice of  $a_i$  in problem (P0a) binds for (almost) every institution. Hence, financing choices at the extensive margin can be summarized by a cutoff  $\lambda \in [0, 1]$ : institution  $i \leq \lambda$  issues safe debt, and institution  $i > \lambda$  issues only equity. The cutoff type is indifferent between issuing safe debt and providing liquidity to others:

$$\gamma - \xi_\lambda = (1 - p) \left( \pi \frac{q_h}{q_l} - 1 \right). \quad (8)$$

Equilibrium is reached when the price ratio adjusts to satisfy this indifference condition and clear the secondary market.

**Proposition 1** (Market Equilibrium). *There exists a unique equilibrium.<sup>15</sup> The equilibrium features an interior mix of two distinct financing choices: there is a cutoff  $\lambda^{CE} \in (0, 1)$  such that (i) institutions below the cutoff commit to maintain portfolio quality and fully use safe debt capacity, and (ii) institutions above the cutoff do not issue any safe debt. Formally,*

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<sup>15</sup>The uniqueness is about the quantities and loan price ratio, not the level of prices.

$$x_i^{CE} = \begin{cases} c'^{-1}(pR + 1 - p + \gamma - \xi_i), & \text{if } i \leq \lambda^{CE} \\ c'^{-1}(pR + 1 - p + \gamma - \xi_{\lambda^{CE}}), & \text{if } i > \lambda^{CE} \end{cases}, \quad (9)$$

$$a_i^{CE} = \begin{cases} x_i^{CE} - x_{i,l} + x_{i,l} \frac{q_l}{q_h}, & \text{if } i \leq \lambda^{CE} \\ 0, & \text{if } i > \lambda^{CE} \end{cases}, \quad (10)$$

and

$$\frac{q_l}{q_h} = \frac{(1-p)\pi}{1-p+\gamma-\xi_{\lambda^{CE}}}. \quad (11)$$

*Proof.* See the Appendix. □

Proposition 1 characterizes the equilibrium lending and financing choices, demonstrating that the observed mix of nonbank institutions arises as an endogenous outcome. Consistent with Fact 1, two distinct types of institutions, resembling CLOs and loan funds, emerge and coexist. The CLOs fully use safe debt capacity, which they maximize by committing to replace low-quality loans with high-quality loans in bad times. Binding collateral constraints generated by this commitment, and the resulting portfolio substitution, are consistent with Fact 2, Fact 3, and Fact 4. Institutions operating as CLOs enjoy cheap funding from larger safe tranches and receive high equity tranche payoffs in good times.

By contrast, institutions operating as loan funds completely give up issuing safe debt. They do so because in bad times, market-clearing loan prices deviate from the loans' fundamental values (i.e.,  $q_l/q_h < \pi$ ), which is consistent with Fact 5. Such price pressure makes providing liquidity in the secondary market more profitable for these institutions. In market downturns, loan funds sell their high-quality loans and absorb low-quality loans sold by CLOs. The loans they sell are used by CLOs as collateral to keep senior debt tranches safe.

**Corollary 1.1.** *Dynamic collateral management increases the total supply of safe debt.*

One immediate corollary of Proposition 1 is that this market produces more safe debt than the static benchmark. Although only a subset of institutions create safe debt at the extensive margin, each CLO creates more at the intensive margin:  $a_i^{CE} > a_i^{STA}$  for  $i \leq \lambda^{CE}$ . Overall, safe debt supply is greater thanks to a larger quantity of high-quality loans, which comes from the lending of loan funds: Equation (9) shows that  $x_i^{CE} = x_i^{STA}$  for  $i \leq \lambda^{CE}$ .

and  $x_i^{CE} > x_i^{STA}$  for  $i > \lambda^{CE}$ . Despite having no need for collateral, these funds lend more than they would in static securitization because providing liquidity is more profitable than issuing safe debt at relatively high costs. Their increased lending adds to aggregate safe debt capacity in Equation (7), which allows CLOs to produce a greater total supply of safe debt.

**Corollary 1.2.** *Every institution is better off than in static securitization.*

*Proof.* See the Appendix. □

Institutions can always choose static securitization. So by revealed preference, each of them is better off with dynamic portfolios. Given that investors break even, this implies a welfare improvement from specialization. Intuitively, dynamic collateral management transfers debt capacity from loan funds to CLOs. This transfer adds value because institutions facing lower costs of creating safe debt can create more, while institutions facing higher costs can instead profit from providing liquidity.

[Add Figure 6 here]

Figure 6 illustrates the equilibrium and compares the allocations with and without dynamic portfolios. Unlike that everyone issues safe debt in static securitization, the market has an interior mix of two distinct types of institutions. Institutions facing lower costs of debt issuance ( $i \leq \lambda^{CE}$ ) operate as CLOs. Their increased safe debt capacity from dynamic collateral management can be seen in the wedge between  $\mathbb{E}[a_i^{CE}]$  and  $\mathbb{E}[a_i^{STA}]$ . From an equilibrium perspective, the increase in safe debt supply comes from the increased lending of the loan funds ( $i > \lambda^{CE}$ ), which is reflected by the wedge between  $x_i^{CE}$  and  $x_i^{STA}$ .

### 3.4.2 Comparative Statics

The key driving force behind the equilibrium is that the non-pecuniary benefit of safe debt,  $\gamma$ , generates a safety premium. In practice, this premium might change for various reasons, such as a tightening of bank capital regulation or the growth of government debt supply. The following corollary summarizes how changes in  $\gamma$  affect equilibrium outcomes.

**Corollary 1.3.** *When the non-pecuniary benefit of safe debt ( $\gamma$ ) is greater,*

- (a) A larger fraction of institutions issue safe debt:  $\lambda^{CE}$  increases,
- (b) Lending  $x_i$  increases for every institution, and the market produces more safe debt,
- (c) Secondary market experiences larger price pressure in bad times:  $q_l/q_h$  decreases.

*Proof.* See the Appendix. □

When the non-pecuniary benefit of safe debt increases, the market reacts as follows. At the extensive margin, more institutions issue safe debt, and fewer institutions provide liquidity. In bad times, the price pressure makes secondary market prices deviate more from the fundamentals. At the intensive margin, lending grows for each institution and in aggregate, which leads to a greater supply of safe debt.

### 3.4.3 Special Case: Homogeneous Institutions

While it is reasonable to believe that institutions face heterogeneous costs of creating safe debt, it is worth understanding the role of this liability-side heterogeneity in equilibrium. Here I consider a special case where institutions are ex ante identical:  $\xi_i = \xi^* \in [0, \gamma)$  for all  $i \in \mathcal{I}$ .

**Corollary 1.4.** *If institutions are ex ante identical, equilibria are multiple and share the same price ratio and aggregate quantities. There exists an equilibrium in which institutions make two distinct financing choices, as they do in Proposition 1. In every equilibrium, the total supply of safe debt is the same as in the static benchmark.*

*Proof.* See the Appendix. □

Corollary 1.4 shows that the coexistence of two types of institutions with distinct liabilities may arise even if institutions are homogenous. However, this is no longer the unique equilibrium. The intuition behind the multiplicity follows from Equation (5). When managers are homogenous, secondary market prices adjust until everyone is indifferent about financing choice. That is, the marginal benefit of issuing safe debt exactly equals the marginal profit of providing liquidity: otherwise the secondary market would not clear. Since everyone is indifferent, the total debt capacity can be arbitrarily allocated among institutions (within their collateral constraints), and each allocation features a different equilibrium.

In this special case, dynamic collateral does not add any value.<sup>16</sup> To see this, recognize that in every equilibrium, the marginal payoff of lending does not depend on an institution's financing choice. Hence, lending choices will coincide with an equilibrium where no institution trades loans, namely, the static benchmark (Lemma 1). Then by Equation (7), the supply of safe debt will also coincide, so dynamic portfolios do not lead to any difference in aggregate quantities. My analysis of this special case suggests that a liability-side heterogeneity across institutions could be important for the adoption of dynamic collateral management.

## 3.5 Discussion

Now I discuss several theoretical aspects that are related to the model.

### 3.5.1 Why Do CLOs Create Long-Term Securities?

My focus on long-term debt leaves the question open as to why CLOs do not issue short-term debt, which can roll over in good times and enforce liquidation before losses fully materialize in bad times. I argue that the observed maturity is also an equilibrium outcome: Given that the leveraged loan market is segmented from public markets and that it is costly for outsiders to buy loans in bad times, issuing long-term safe debt is optimal for CLOs.

To formalize this argument and analyze debt maturity choices, I extend the model in Appendix IA.5. This extension allows for asset liquidation and debt repayment at  $t = 1$ , which endogenously affects institutions' lending and financing choices as well as investors' storage decision at  $t = 0$  and loan purchases at  $t = 1$ . In equilibrium, the issuance of long-term debt, short-term debt, and equity are jointly determined with investor purchases and the levels of secondary market prices.

The extended model provides the following intuition. When investors' storage cost is high, so is their required return. As a result, liquidating loans and repaying debt in bad times will be costly to CLO equity holders. Thus, long-term contract, which helps CLOs maximize and maintain cheap leverage, will be preferred. Knowing this, investors will not attempt to store consumption goods for participating in the secondary market. As a result, in bad times,

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<sup>16</sup>My model abstracts from economic forces such as fixed issuance costs and idiosyncratic asset shocks, which can mechanically make dynamic collateral management valuable, even if institutions are homogenous.



loans will be traded between institutions rather than sold to outside investors. This result explains that the maturity of CLO debt tranches is related to the high participation costs faced by potential buyers who are not active lenders in this market.

### **3.5.2 How Limiting are the Limitations of Credit Ratings?**

Dynamic collateral management relies on long-term contracts to discipline institutions. These contracts are enforced with observable and verifiable risk measures, primarily credit ratings, which in practice do not perfectly measure loan quality. To simplify the analysis, my model has assumed that institutions can fully commit to replacing low-quality loans, even if doing so reduces their own payoffs.

Can contracting on noisy risk measures properly discipline institutions? In Appendix IA.6, I consider imperfectly contractible loan types. This friction makes it possible for institutions to hold fewer high-quality loans than required without violating the contract.<sup>17</sup> I illustrate that, when contractible risk measures are sufficiently informative about loan quality, long-term contracts can still induce institutions to hold enough high-quality loans.

It is worth noting that in my model, contracts are perfectly calibrated to future possible states of the world. In practice, these contracts must be adjusted for parameter and model uncertainties for senior tranches to be sufficiently safe. Such adjustments can be carried out as, for example, over-collateralization provisions. However, when uncertainty surges during market distress, the difficulty in recalibration and a decline in investor demand might make dynamic collateral uneconomical.

### **3.5.3 Will Institutions Internalize Loan Trades?**

My model allows institutions to flexibly choose external financing. It is in principle possible that an institution operates two entities with very different liabilities (e.g., a CLO and a mutual fund), which seems appealing because the institution can then internalize loan trades in bad times. That is, instead of being forced to buy and sell loans in the secondary market, the manager could reallocate loans between the entities it operates.

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<sup>17</sup>Such agency friction is not just a theoretical possibility, as evidence based on normal time periods shows that CLOs strategically sell loans traded at higher prices (e.g., Loumiotis and Vasvari, 2019).

However, doing so is suboptimal for institutions. This is because without trading in the secondary market, the quantity of safe debt an institution can create is constrained by its initial portfolio:  $a_i \leq x_{i,h}$ . By Corollary 1.2, the institution will be better off by pushing its financing choice to the limit, i.e., choosing either  $a_i > x_{i,h}$  or  $a_i = 0$ . Therefore in equilibrium, institutions tend to specialize, as illustrated by Fact 1. A similar argument applies to the case where two institutions merge into one entity and face the same issuance cost after the merger. Moreover, since loans are tradable, a heterogeneity on the asset side (e.g., investment technology) would not affect institutions' optimal financing choices.

### 3.5.4 Nonbank Liabilities: Implications on Valuation

Because all agents have linear preferences, my model is silent about the pricing of nonbank liabilities when marginal investors are risk-averse. That said, the intuition provided by my analysis has pointed to the importance of accounting for the effects of CLO contracts on portfolio dynamics in the valuation of CLO securities and loan fund shares.

Evaluating the risk of CLO tranches solely based on current balance sheets understates both the safety of senior tranches and the risk of junior tranches. Because CLO managers are committed to replace deteriorated loans, junior debt and equity tranches tend to lose more value in bad times than they would with static portfolios. Therefore, when portfolios are actively managed, junior tranche investors may require additional compensation for riskier returns and a higher positive correlation with aggregate states. Meanwhile, loan funds' shares are less risky than buying and holding their current portfolios, because these funds' losses in bad times will be mitigated by their trades with CLOs. These endogenous pro-cyclical and counter-cyclical payoffs should be considered by investors and analysts.

## 4 Efficiency and Policy Implications

My analytical framework is positive in that I take the non-pecuniary benefit of safe debt as given and study prices and quantities when individual institutions maximize their payoffs. In this section, I continue the analysis by focusing on whether the market as a whole optimally responds to the demand for safe debt and how a particular policy affects the equilibrium.

To do so, I consider a planner who controls every institution's lending and financing choices in period  $t = 0$ . The planner respects all the constraints faced by individual institutions but does not take secondary market prices as given. Instead, by choosing quantities, he can target any specific prices that admit his choices at  $t = 0$  and clear the secondary market at  $t = 1$ . After uncertainty resolves at  $t = 2$ , if payoffs permit, he can redistribute consumption goods to ensure that no agent is worse off than in the competitive allocation.

The planner's optimization problem is as follows. Let the total quantity of loans be  $X = \int_i x_i di$ . By law of large numbers, total low-quality loans  $\int_i \tilde{x}_{i,l} di = x_L$ . Given that all agents have linear preferences, the planner maximizes the sum of expected payoffs and non-pecuniary benefits, minus total investment and securitization costs:

$$\max_{\{x_i, a_i\}_{i \in \mathcal{I}}} pXR + (1-p)(X - x_L + \pi x_L) + \gamma A - \int_i (c(x_i) + \xi_i a_i) di \quad (\text{SP})$$

subject to individual collateral constraint

$$a_i \leq x_i - x_{i,l} + x_{i,l} \frac{q_l}{q_h}, \quad \forall i \in \mathcal{I} \quad (\text{ICC})$$

and nonnegativity constraint  $a_i \geq 0$ ,  $\forall i \in \mathcal{I}$ . The market-clearing condition (2) imposes an additional constraint on the planner. After a negative signal, institutions trade loans as in Equation (3), so the secondary market clears if and only if  $\int_i (a_i - x_{i,h}) di \leq 0$ , which gives rise to an aggregate collateral constraint:<sup>18</sup>

$$A \leq X - x_L, \quad (\text{ACC})$$

Note that constraint (ACC) binds at the optimum: otherwise, there would be some  $i$  such that  $a_i \in [0, x_{i,h})$ , and since  $\gamma > \xi_i$ , increasing  $a_i$  would improve the objective, a contradiction to optimality. Also, the slackness of constraint (ICC) strictly increases in price ratio  $\frac{q_l}{q_h}$ , and loan prices do not affect the planner's objective or any other constraint. Therefore, a higher price ratio at least weakly improves the maximized total surplus, and the planner targets the highest market-clearing price ratio, namely,  $q_l/q_h = \pi$ .

Let  $\eta_i^{SP}$ ,  $\mu_i^{SP}$ , and  $\psi^{SP}$  be the Lagrangian multipliers for the three (sets of) constraints.

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<sup>18</sup>When this inequality is strict, the market clears if  $q_l/q_h = \pi$ , a corner case in which unconstrained institutions are indifferent between the two types of loans.

For each  $i \in \mathcal{I}$ , the Kuhn-Tucker conditions for optimality are

$$pR + 1 - p - c'(x_i) + \psi^{SP} + \eta_i^{SP} = 0, \quad (12)$$

$$\gamma - \xi_i - \psi^{SP} - \eta_i^{SP} + \mu_i^{SP} = 0, \quad (13)$$

and

$$\eta_i^{SP} \geq 0, \eta_i^{SP}(a_i - x_{i,h} - x_{i,l}\pi) = 0, \mu_i^{SP} \geq 0, \mu_i^{SP}a_i = 0. \quad (14)$$

The planner internalizes the externalities of institutions' lending and financing. His choice of lending, as characterized by (12), accounts for both individual ( $\eta_i^{SP}$ ) and social ( $\psi^{SP}$ ) collateral values. The social collateral value captures that an institution's lending increases collateral available to others because loans can be reallocated in the secondary market. For financing choices characterized by (13), the planner trades off between the net benefit from creating safe debt and the opportunity cost of using safe debt capacity. This social cost differs from an institution's private cost, which is calculated based on loan prices.

**Proposition 2** (Efficiency Properties). *The competitive equilibrium is constrained inefficient. The planner's choices lead to price ratio  $\frac{q_l}{q_h} = \pi$  and a unique cutoff  $\lambda^{SP} \in (0, 1)$  such that (i) institutions below the cutoff commit to maintain portfolio quality and fully use safe debt capacity, and (ii) institutions above the cutoff do not issue any safe debt. The allocations satisfy  $\lambda^{CE} > \lambda^{SP}$ ,  $x_i^{CE} = x_i^{SP}$  for  $i \in [0, \lambda^{SP}]$ ,  $x_i^{CE} < x_i^{SP}$  for  $i \in (\lambda^{SP}, 1]$ , and  $A^{CE} < A^{SP}$ .*

*Proof.* See the Appendix. □

Under the planner's allocation, there are two types of institutions with distinct liabilities, which is qualitatively similar to the competitive allocation. Cutoff  $\lambda^{SP}$  reflects the market's optimal mix of institutions, or equivalently, the efficient entry into safe debt creation. The social value of collateral,  $\psi^{SP} = \gamma - \xi_{\lambda^{SP}}$ , equals the net benefit of creating safe debt by the cutoff type. Proposition 2 indicates that in the competitive market, there is excessive entry into operating CLOs, underinvestment by loan funds, and an underproduction of safe debt.

[Add Figure 7 here]

Figure 7 illustrates the differences between the competitive and planner's allocations. The planner assigns institutions  $i \in [0, \lambda^{CE}]$  to issue safe debt, and each of them on average

issues more than their competitive levels:  $\mathbb{E}[a_i^{SP}] > \mathbb{E}[a_i^{CE}]$ . Meanwhile, the planner forces the rest of institutions to not issue any safe debt and lend more than their competitive levels:  $x_i^{SP} > x_i^{CE}$ . The area of the shaded region measures aggregate underinvestment, which, by Equation (7), equals the underproduction of safe debt.

## 4.1 Source of Inefficiency

In my model economy, institutions face price-dependent collateral constraints because they cannot allocate current and future quantities with state-contingent contracts. Under this friction, a pecuniary externality arises from dynamic collateral management: CLOs' secondary market trades move loan prices, which in turn affect both the collateral constraints faced by all CLOs and the profit of liquidity provision faced by all loan funds. Individual institutions take loan prices as given when maximizing their own payoffs and do not internalize this externality.

Given a liability-side heterogeneity across institutions, efficiency requires specialized safe debt creation at both the intensive and extensive margins. However, competitive prices, and the corresponding individually optimal lending and financing choices, prevent the market from efficiently producing safe debt. The discrepancy between individual and planner tradeoffs, which gives rise to the inefficiency, is twofold.

**Corollary 2.1.** *For institutions  $i \in (\lambda^{SP}, \lambda^{CE})$ , the private benefit of issuing safe debt is lower than the social value of collateral:  $\gamma - \xi_i < \psi^{SP}$ .*

On the liability side, an excessive fraction of institutions create safe debt. With dynamic collateral, issuing safe debt drives up the demand for high-quality loans in the secondary market. Since high-quality loans are overall scarce, CLOs' commitment to replace low-quality loans crowds out each other's debt capacity by depressing the price ratio  $q_l/q_h$ . But price-taking institutions only care about their own cost of financing. Hence, despite a relatively lower private benefit from issuing safe debt, institutions facing higher issuance costs may still prefer to operate as CLOs. The result of this excessive issuance at the extensive margin is that institutions facing lower costs cannot create more safe debt at the intensive margin.

**Corollary 2.2.** *The private profit of liquidity provision is lower than the social value of collateral:  $(1 - p)(\pi_{q_i}^{q_h} - 1) < \psi^{SP}$ .*

On the asset side, institutions facing higher costs of creating safe debt,  $i \in (\lambda^{SP}, 1]$ , underinvest.<sup>19</sup> These institutions, operating as loan funds, ignore that their loans can be bought and used as collateral by CLOs. While providing liquidity is profitable, at competitive prices, the private profit is still lower than the social value of collateral.<sup>20</sup> So these funds' optimal lending choices limit the secondary market supply of high-quality loans, which results in a shortage of overall collateral and thus the underproduction of safe debt.

The analysis above suggests that the inefficiency is associated with the heterogeneity on the liability side. Indeed, this can be verified in the special case of Corollary 1.4.

**Corollary 2.3.** *If institutions are ex ante identical, the equilibrium is constrained efficient.*

*Proof.* See the Appendix. □

In the special case, the planner cannot do better than the competitive market. Although by internalizing the pecuniary externality, he can implement different individual financing choices, these choices would not affect efficiency because no institution is better at securitizing loans than others. Also, competitive and planner's lending choices coincide because the prices adjust to equate social collateral value and private payoffs. Therefore, if institutions are ex ante identical, there is no justification for intervention.

## 4.2 The Impact of a Regulation

This regulation, generally referred to as Credit Risk Retention Rule, was initially proposed by 6 federal agencies (collectively, “regulators”) in 2011 to implement the credit risk retention requirements of the Dodd-Frank Act. The rule requires securitizers to retain at least 5% of un-hedged credit risk of collateral assets for any asset-backed securities (ABS). Securitizers can choose to retain 5% of every tranche (“vertical retention”), the bottom tranche with a

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<sup>19</sup>For institutions in  $[0, \lambda^{SP}]$ , individually and socially optimal lending choices coincide, because they directly benefit from and hence fully internalize the collateral value of their loans.

<sup>20</sup>The social value of collateral,  $\psi^{SP}$ , equals the net benefit of creating safe debt by the efficient cutoff type,  $\gamma - \lambda^{SP}$ , whereas the profit of liquidity provision equals the net benefit of creating safe debt by the competitive cutoff type,  $\gamma - \lambda^{CE}$ , which is smaller than the former.

fair value of 5% of all tranches (“horizontal retention”), or any convex combination of these two.<sup>21</sup> The final rule became effective for residential mortgage-backed securities (RMBS) in December 2015 and for other ABS, including CLOs, in December 2016.

The regulation’s inclusion of CLOs provoked resistance from practitioners. The main complaint was that the rule imposes substantial operational and capital costs on the portfolio managers, rather than the owners (“securitizers”, as defined by the Dodd-Frank Act), of the underlying loans and might drive smaller managers out of the CLO business. In November 2014, the Loan Syndications and Trading Association (LSTA), representing CLO managers, filed a lawsuit against the Federal Reserve and the SEC. In February 2018, the US Court of Appeals for the D.C. Circuit concluded that managers of open-market CLOs are not “securitizers” under the Dodd-Frank Act and are not subject to the requirements of the Risk Retention Rule. Consequently, CLO managers became exempted from the rule in May 2018.

[Add Figure 8 here]

Figure 8 presents the timing of the regulatory events and annual CLO entry rate in the US and European markets between 2000–2019. Before 2008, CLO entry rates in the US and Europe had similar time trends. Probably because a similar risk retention rule was introduced in Europe in 2010, the European CLO market recovered slowly compared to the US market. However, after the finalization of the US risk retention rule in late 2014, there was a salient drop in CLO entry. This drop quickly reversed when the policy got revoked in early 2018. In Appendix IA.7.2, I further examine the regulation’s effect on CLO entry, which was predicted to be devastating by the LSTA and CLO managers.<sup>22</sup>

#### **4.2.1 Equilibrium under an Entry Cost Policy**

While there might be a lack of theoretical guidance behind these regulatory changes, my analysis in this section has provided a rationale for policy intervention. As Proposition 2 shows, in equilibrium, the market has excessive entry into operating CLOs. Hence, imposing an entry cost to deter a subset of institutions from creating safe debt, which is what the

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<sup>21</sup>See SEC Final Rules 34-73407 for more details.

<sup>22</sup>Consistent with the fact that before the regulation, CLO managers were already exposed to the equity tranche through performance-based compensation, I do not find robust evidence that the regulation changed CLO portfolio choices.

regulation effectively did, might improve the allocation. I apply the model to examine how such a policy affects equilibrium outcomes.

Suppose the policy imposes a fixed cost  $\zeta_i > 0$  in the beginning of period  $t = 0$  on institutions that issue safe debt  $a_i > 0$ . The cost can be an arbitrary increasing function of index  $i \in \mathcal{I}$ . This allows for any monotonic heterogeneity: a less resourceful institution (i.e., having a higher safe debt issuance cost  $\xi_i$ ) may also face a higher policy-induced entry cost.

Under this policy, the institution's optimization problem in period  $t = 0$  is discontinuous at  $a_i = 0$ . I refer to the solution to (P0a) conditional on a binary choice between  $a_i = 0$  and  $a_i > 0$  as *locally optimal* choices, which are characterized by conditions (4)–(6).

The policy distorts institutions' financing choices, which in turn affect their lending choices. If an institution issues zero safe debt, its payoff is

$$V_i^e = y_i^e c'^{-1}(y_i^e) - c(c'^{-1}(y_i^e)) - (1-p)\pi x_L \left( \frac{q_h}{q_l} - 1 \right), \quad (15)$$

where  $y_i^e := pR + (1-p)\pi \frac{q_h}{q_l}$  is the marginal payoff of lending. Alternatively, if the institution issues a locally optimal quantity of safe debt, its payoff is

$$V_i^d = y_i^d c'^{-1}(y_i^d) - c(c'^{-1}(y_i^d)) - (1-p)\pi x_L \left( \frac{q_h}{q_l} - 1 \right) - x_L \eta_i \left( 1 - \frac{q_l}{q_h} \right) - \zeta_i, \quad (16)$$

where  $y_i^d := y_i^e + \eta_i$  is the institution's marginal payoff from lending, which includes collateral value  $\eta_i$ . Note that  $V_i^d$  is strictly increasing in  $\eta_i$  and that  $\eta_i$  decreases in index  $i$ .<sup>23</sup> This implies that  $V_i^d$  is strictly greater for a smaller  $i$ . Since  $V_i^e$  is identical across  $i$ , others equal, only institutions facing lower issuance costs issue safe debt.

Similar as before, I use  $\lambda$  to denote the institution that is *locally indifferent* between issuing safe debt and issuing only equity, so this type satisfies Equation (8). Since the indifference is local (i.e., it is conditional on  $a_i > 0$ ) and does not reflect globally optimal choices,  $\lambda \leq 1$  no longer has to hold; Instead, Lemma 2 and Equation (8) imply that  $\lambda$  is now upper bounded by  $\frac{\gamma}{2\xi} > 1$ . I denote the new cutoff type  $\iota : [0, \frac{\gamma}{2\xi}] \mapsto [0, 1]$  as a function of  $\lambda$ . This type satisfies a global indifference condition:  $V_{\iota(\lambda)}^d = V_i^e$ .

Given loan prices and  $\lambda$ , there is a unique cutoff type  $\iota(\lambda) < \lambda$  because  $\zeta_i > 0$ , and  $V_i^d$  is monotonic in  $i$ . When the entry cost approaches zero, the new cutoff converges to  $\lambda$ :

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<sup>23</sup>The monotonicity in  $\eta_i$  can be seen from  $\frac{\partial V_i^d}{\partial \eta_i} = c'^{-1}(y_i^d) - x_{i,l}(1 - \frac{q_l}{q_h}) > c'^{-1}(y_i^d) - x_{i,l} > 0$ , where the last inequality follows from Assumption 1 because  $y_d > pR + 1 - p$  by Lemma 2.



$\lim_{\bar{\zeta} \rightarrow 0+} \iota(\lambda) = \lambda$ , where  $\bar{\zeta} := \max_{i \in \mathcal{I}} \zeta_i$ .

Equilibrium under the policy can be defined similarly as Definition 1, except for that the institution's  $t = 0$  problem takes the entry cost into consideration. The limiting property of  $\iota(\lambda)$  indicates that, by continuity of the equilibrium, an interior cutoff exists when  $\bar{\zeta}$  is relatively small. Let  $\lambda^{ECP}$  and  $\iota(\lambda^{ECP})$  respectively denote the locally indifferent type and the actual cutoff type in equilibrium.

**Proposition 3** (Equilibrium under an Entry Cost Policy). *The entry cost policy reduces the fraction of CLOs, increases price ratio  $q_l/q_h$ , allows the remaining CLOs to issue more safe debt, but worsens the underproduction of safe debt:  $\iota(\lambda^{ECP}) < \lambda^{CE}$ ,  $\mathbb{E}[a_i^{ECP}] > \mathbb{E}[a_i^{CE}]$  for  $i \in [0, \iota(\lambda^{ECP})]$ ,  $A^{ECP} < A^{CE}$ .*

Proposition 3 shows that, while the policy corrects the excessive entry into safe debt creation, it fails to move the equilibrium towards constrained efficiency. This is because, as the entry cost deters a subset of institutions from creating safe debt, there is less pressure on secondary market prices. On the one hand, a higher price ratio relaxes collateral constraints and allows the remaining CLOs' to create more safe debt. On the other hand, providing liquidity in the secondary market becomes less profitable, which discourages loan funds' lending. As a larger fraction of institutions operate as loan funds, and each of them lends less than before, the policy leads to a reduction in overall collateral. In aggregate, the increase in safe debt at the intensive margin is overwhelmed by the decrease in collateral, and the market ends up producing even less safe debt after the policy intervention.

[Add Figure 9 here]

Figure 9 compares the competitive allocation (same as in Figure 6) and the allocation under the entry cost policy. Whereas institutions  $i \in [0, \iota(\lambda^{ECP})]$  do not change their lending choices, institutions currently operating as loan lenders ( $i \in [\iota(\lambda^{ECP}), 1]$ ) all lower their levels of lending. This leads to a reduction in high-quality loans, the quantity of which equals the area of the shaded region. Despite that every remaining CLO creates more safe debt than before ( $\mathbb{E}[a_i^{ECP}] > \mathbb{E}[a_i^{CE}]$ ), the market underproduces safe debt to an even greater extent because of a decrease in collateral.

One practical implication of Proposition 3 is that the Credit Risk Retention Rule could

cause unintended consequences. By reducing CLO entry, the rule worsens the underproduction of safe debt and exacerbates the inefficiency of the leveraged loan market. As the debate over whether the rule should be reapplied to the US market continues, policymakers should take such equilibrium effects into consideration.

### 4.3 Policy Challenges

The market presents unique policy challenges because all institutions' assets and liabilities are jointly determined with secondary market prices. My analysis has shown that a policy that targets only the liability side of the balance sheet cannot implement constrained efficiency. Similarly, policies forcing loan funds to lend at the efficient level will worsen the equilibrium mix of institutions. This is because lending beyond privately optimal levels reduces the payoff to these funds, who may respond by operating CLOs.

To move the equilibrium towards constrained efficiency, an ideal policy has to correct both sides of balance sheets: reduce entry into safe debt creation and increase lending by loan funds. If the regulator's information set includes the model and all of its parameters, such a policy can be implemented as, for instance, a combination of lump sum taxes on CLO managers and subsidies on loan funds' lending. In practice, however, model and parameter uncertainties can make policy design challenging.

## 5 Conclusion

This paper analyzes a lending market where securitized tranches are backed by actively-managed loan portfolios. Before the 2007–09 financial crisis, the securitization industry manufactured large quantities of senior tranches. Many of these tranches defaulted because their underlying loans deteriorated and failed to generate sufficient cash flows for repayment. In sharp contrast, in the leveraged loan market, CLOs have been creating AAA-rated securities for more than three decades without any default record.

The key financial innovation of CLOs is that the underlying loans are governed by a contract that obligates the managers to dynamically maintain portfolio quality. This contract generates an intertemporal tradeoff: it helps CLOs create larger safe tranches ex ante but

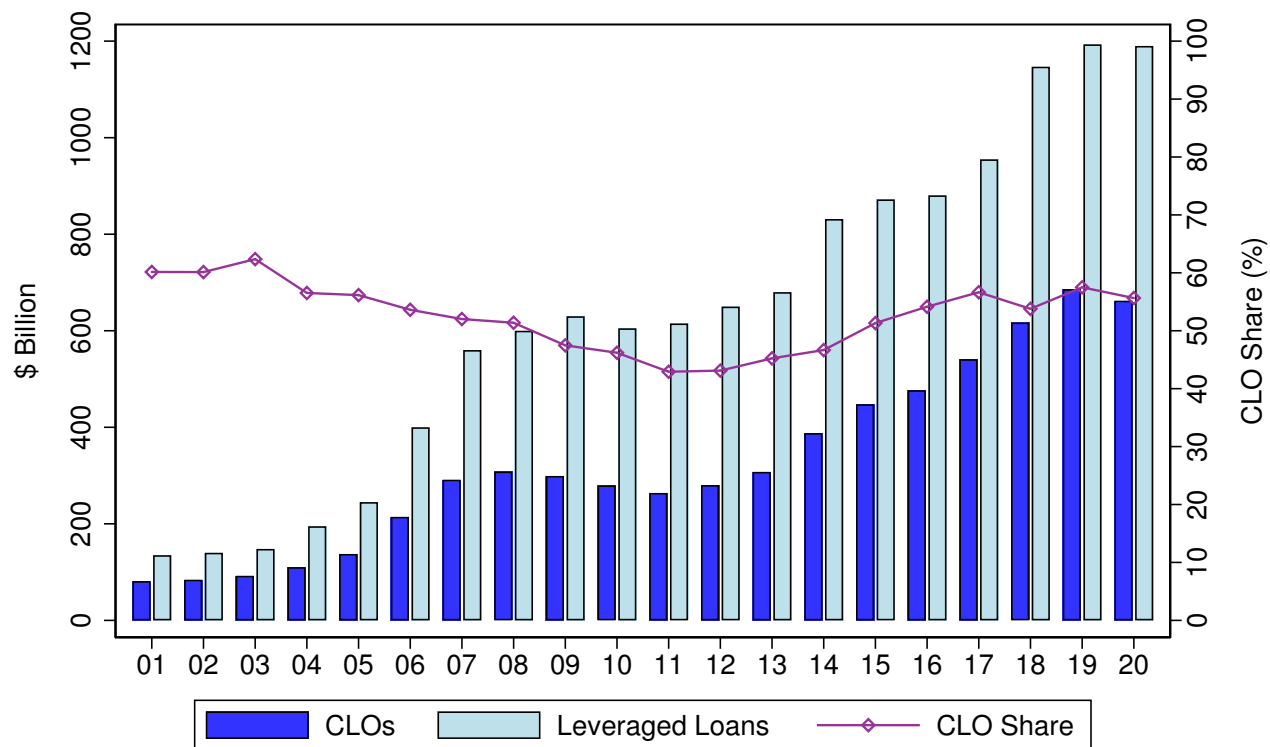
triggers costly portfolio substitution in bad times, which exerts price pressure in the secondary market. To understand how dynamic collateral management affects the supply of safe debt at the individual and market levels, this paper develops an equilibrium model in which institutions flexibly choose external financing and can commit to future portfolio choices. My model explains the coexistence of CLOs and non-securitized loan funds, the trades between these two types of institutions, and the economic mechanism of this market. As the idea of dynamic collateral management has expanded into other financial markets, such as commercial real estate and cryptocurrency-backed lending platforms, the framework presented in this paper can be useful to interpret more empirical facts and inform new policy designs.

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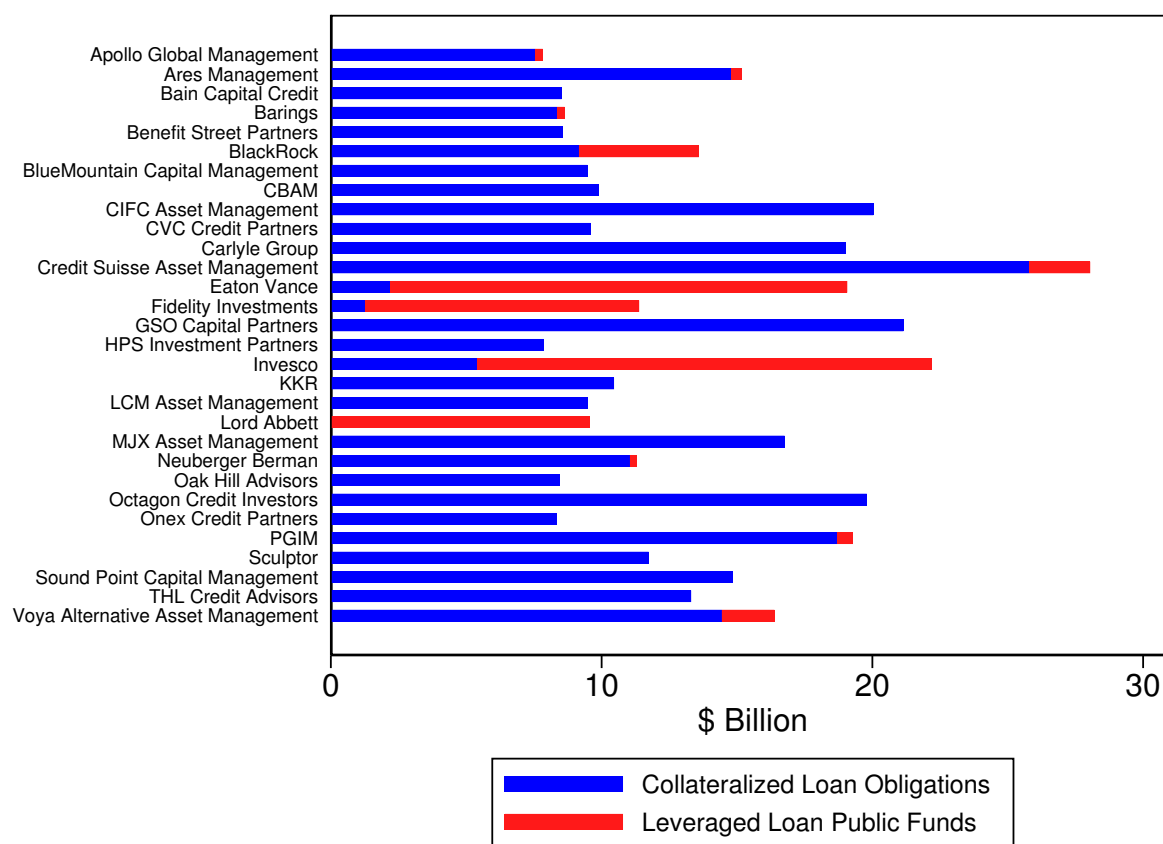
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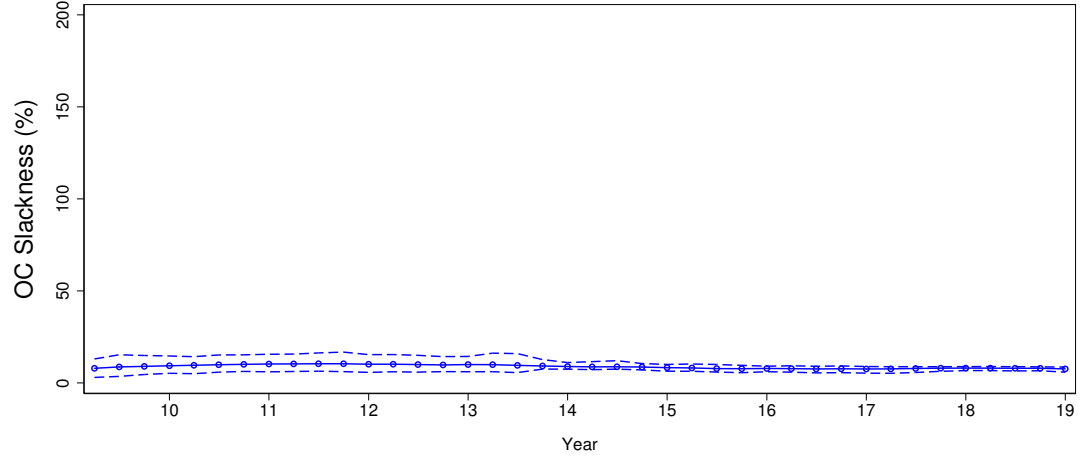
**Figure 1: Leveraged Loans and CLOs Outstanding, 2001–2020.**

This figure plots annual total par values outstanding for leveraged loans (i.e., institutional term loan facilities) and CLOs in the US market. Data source: SIFMA.

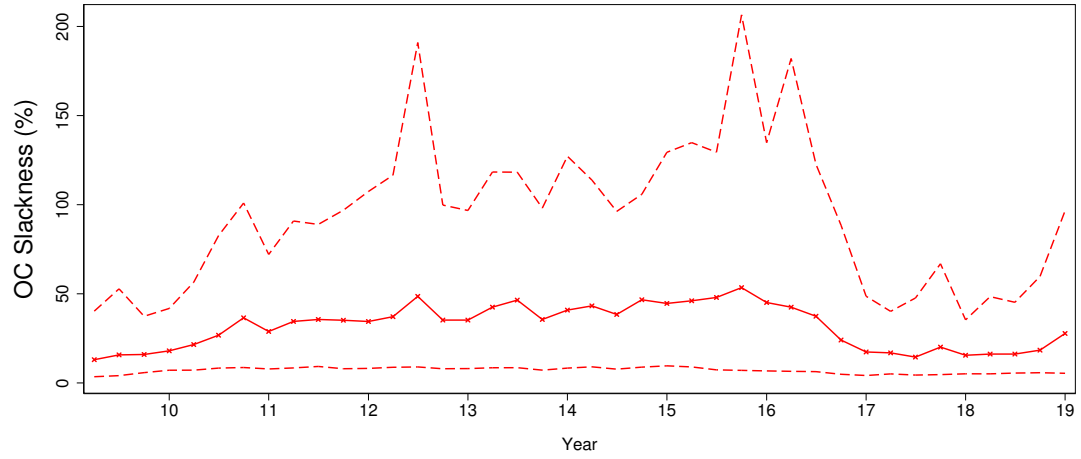


**Figure 2: Asset Managers and Nonbank Institutions.**

This figure presents assets under management for US CLOs and public loan funds (the sum of open-end mutual funds, closed-end mutual funds, and exchange-traded funds) operated by the 30 largest asset managers at the end of 2019. Data come from Creditflux CLO-i, Morningstar, and the SEC’s Form ADV databases.



(a) CLOs in Reinvestment Period

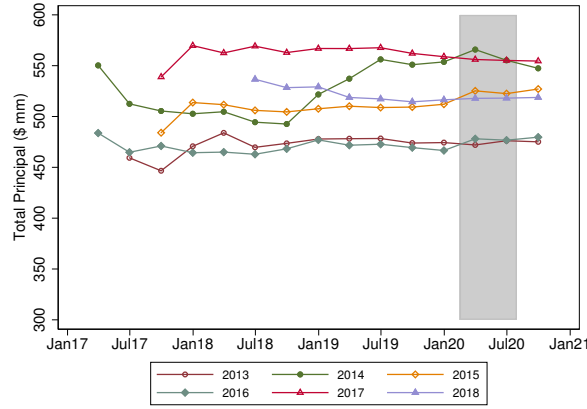


(b) CLOs in Amortization Period

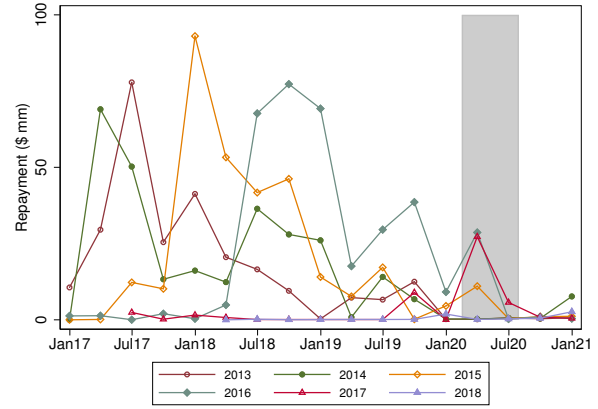
**Figure 3: Slackness of Senior Tranche Over-Collateralization Constraint.**

This Figure presents quarterly time series of cross-sectional dispersion in the slackness of CLO senior tranche over-collateralization (OC) constraints between 2010–2019. The slackness is defined as extra OC score scaled by the OC test’s predetermined threshold level. Dashed lines indicate 5th and 95th percentiles in each cross section. Panel (a) reports CLOs in reinvestment period, and panel (b) reports CLOs in amortization period.

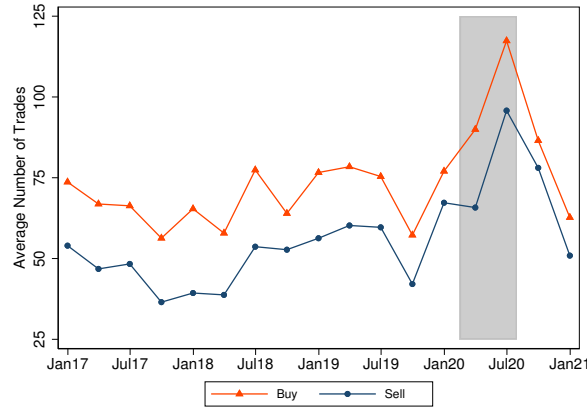




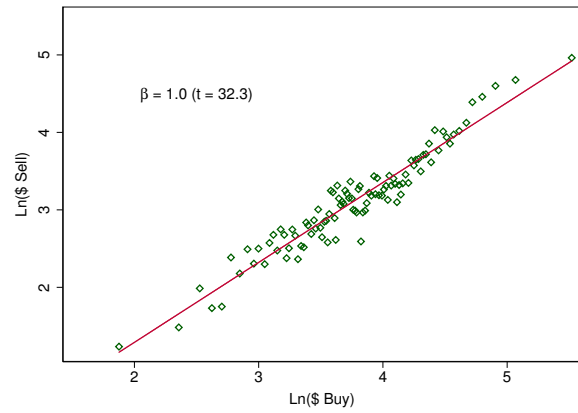
(a) Portfolio Total Loan Holdings



(b) Accelerated Debt Repayment



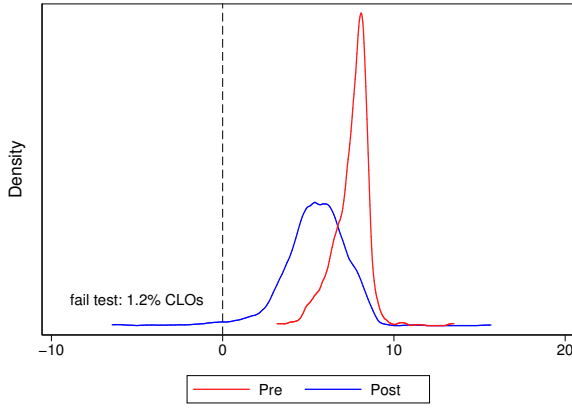
(c) Quarterly Loan Trades



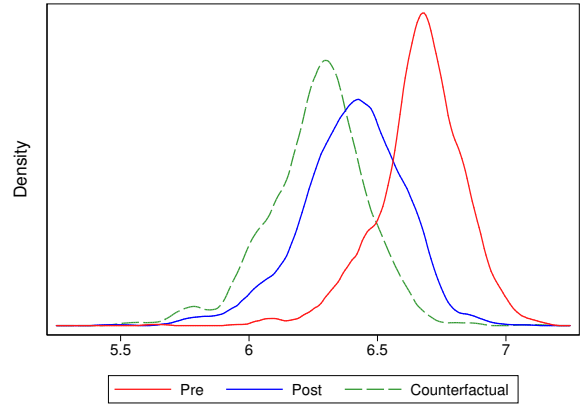
(d) Individual CLOs' Purchases and Sales

**Figure 4: Balance Sheet Dynamics Around the Onset of COVID-19 Pandemic.**

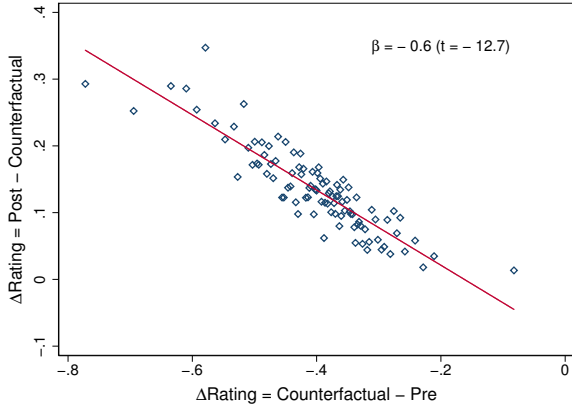
This Figure shows quarterly changes in CLOs' assets and liabilities before and during the COVID-19 shock in 2020. Panel (a) plots average portfolio size by CLO age cohort. Panel (b) plots average accelerated repayment of AAA tranches by CLO age cohort. Panel (c) plots quarterly average numbers of loan purchases and sales. Panel (d) is a scatter plot that groups CLOs into 100 bins based on natural logarithms of individual CLOs' loan buy and sell dollar volumes during the first two quarters of 2020. Only CLOs in reinvestment period are included.



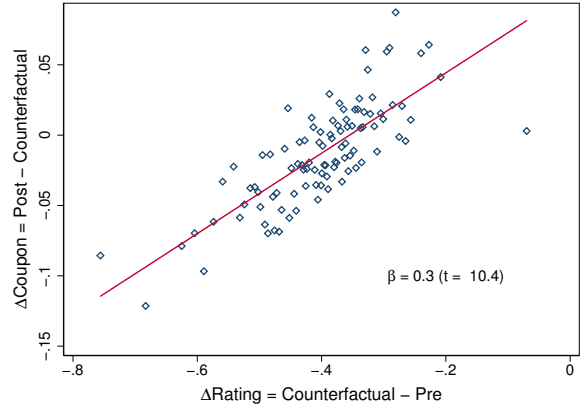
(a) Slackness of OC Constraint (%)



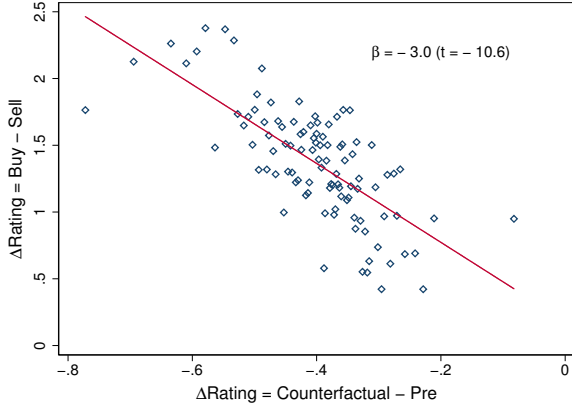
(b) Portfolio Value-Weighted Average Rating



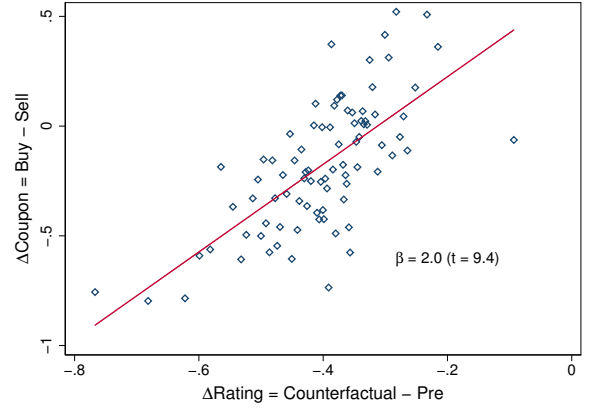
(c) Quality Improvement: Rating



(d) Quality Improvement: Coupon



(e) Trading Direction: Rating



(f) Trading Direction: Coupon

Figure 5: **Portfolio Substitution Improves Collateral Quality.**

This Figure shows the effect of portfolio substitution on CLOs' collateral quality between February 15 and June 30 of 2020. Panel (a) plots kernel density estimates for the distribution of senior OC constraint slackness before and after the onset of COVID-19 pandemic. Panel (b) plots kernel density estimates for the distribution of value-weighted average credit rating for portfolios before and after the shock as well as counterfactual static portfolios. Panels (c)-(f) are scatter plots that group CLOs into 100 bins by counterfactual collateral deterioration and depict the average effect of loan trading within each bin. The fitted lines represent OLS estimates, and t-statistics are based on heteroskedasticity-robust standard errors. Only CLOs in reinvestment period are included.

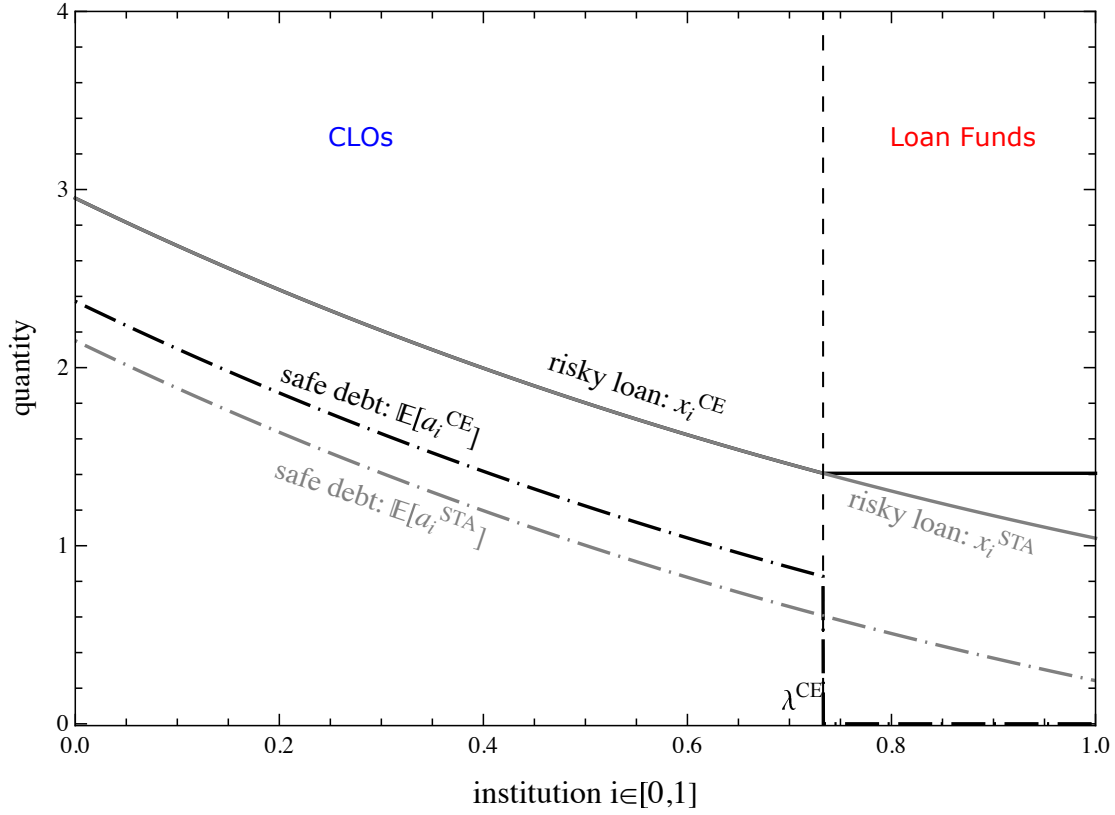


Figure 6: **Competitive Equilibrium versus Static Benchmark.**

This figure illustrates the lending and financing choices in competitive equilibrium. Superscripts CE and STA indicate the equilibrium with dynamic portfolios and the static benchmark, and  $x_i$  and  $\mathbb{E}[a_i]$  denote institution  $i$ 's quantities of lending and expected safe debt issuance, respectively. Functional form and parameter values:  $c(x) = x^{1.2}$ ,  $p = 0.95$ ,  $R = 1.2$ ,  $\pi = 0.8$ ,  $\gamma = 0.3$ ,  $\xi = 0.14$ ,  $x_L = 0.8$ .

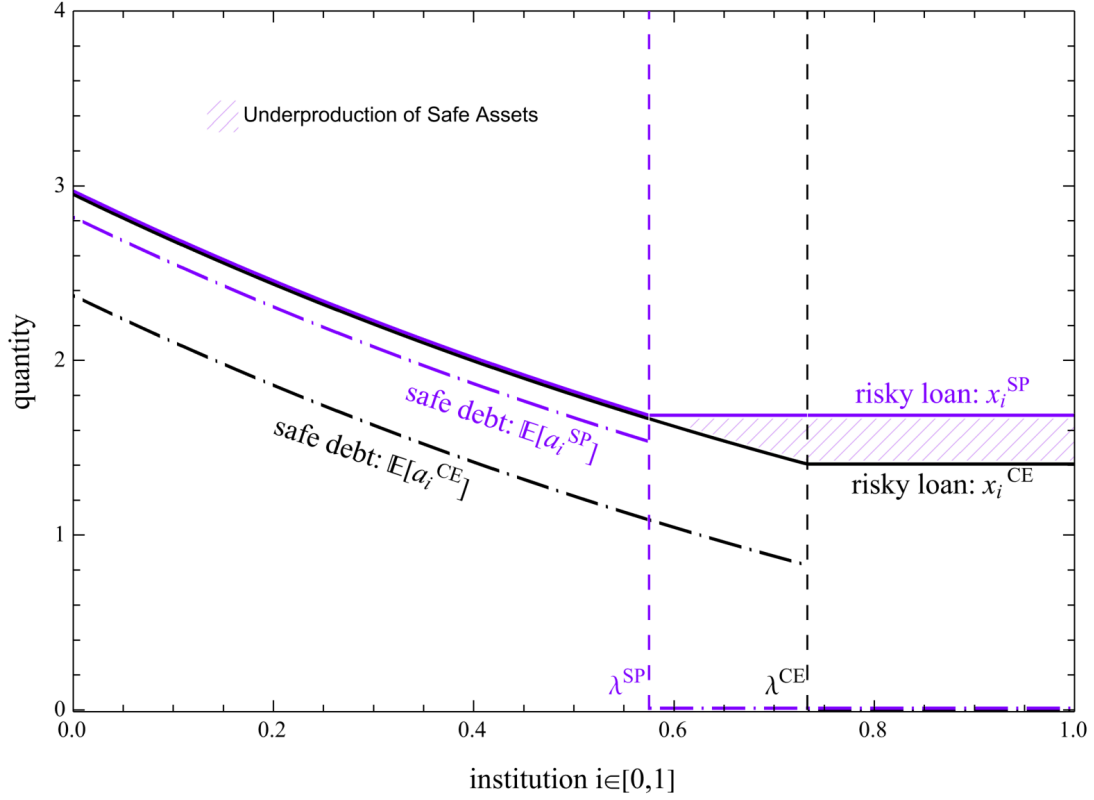


Figure 7: **Constrained Inefficiency.**

This figure illustrates the constrained inefficiency of the equilibrium. Superscripts CE and SP indicate the competitive and the social planner's allocations, and  $x_i$  and  $\mathbb{E}[a_i]$  denote institution  $i$ 's quantities of lending and expected safe debt issuance, respectively. The area of the shaded region represents the quantity of safe debt underproduction. Functional form and parameter values are the same as in Figure 6.

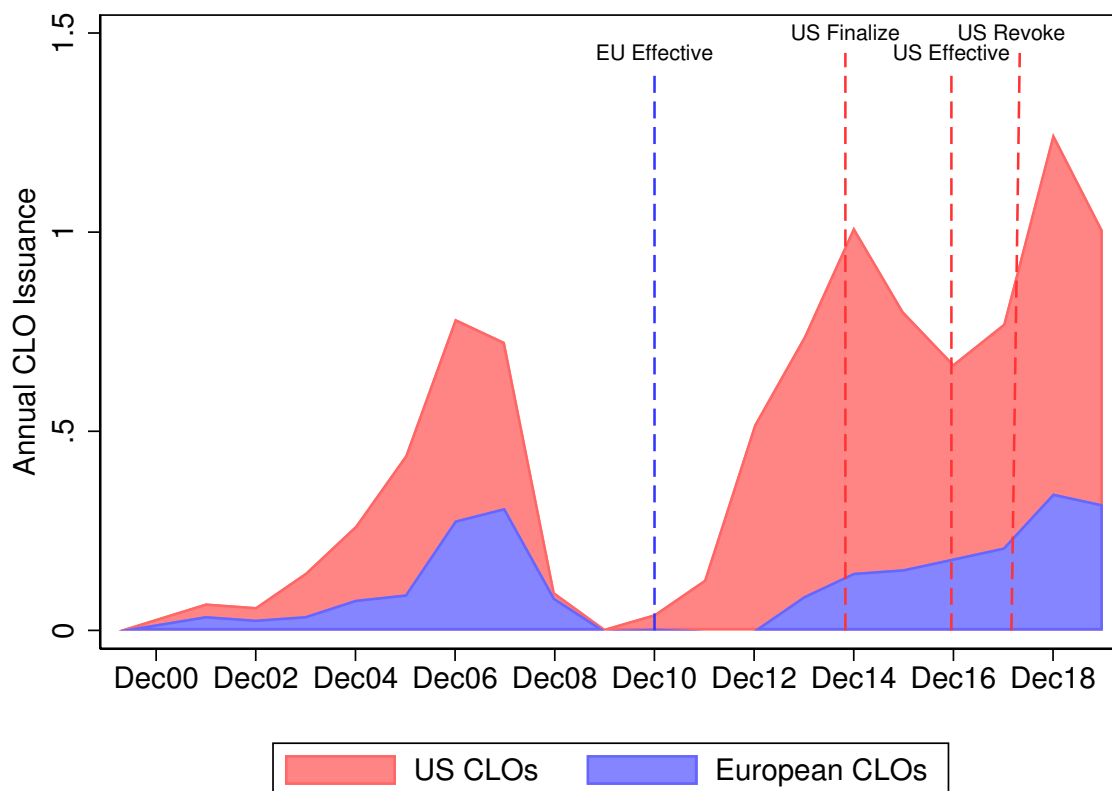


Figure 8: **Risk Retention and CLO Entry in the US and European Markets.**

This figure plots the timing of regulatory events and annual average number of an asset manager's CLO deals issued in the US and European markets. The Capital Requirements Directive II introduced in Europe requires 5% risk retention for all new securitization deals issued after January 2011. These provisions were superseded by an equivalent requirement in Capital Requirements Regulation in January 2014. In the US, the Credit Risk Retention Rule, finalized in October 2014 to require a 5% risk retention, became effective for CLOs in December 2016 and got revoked in February 2018.

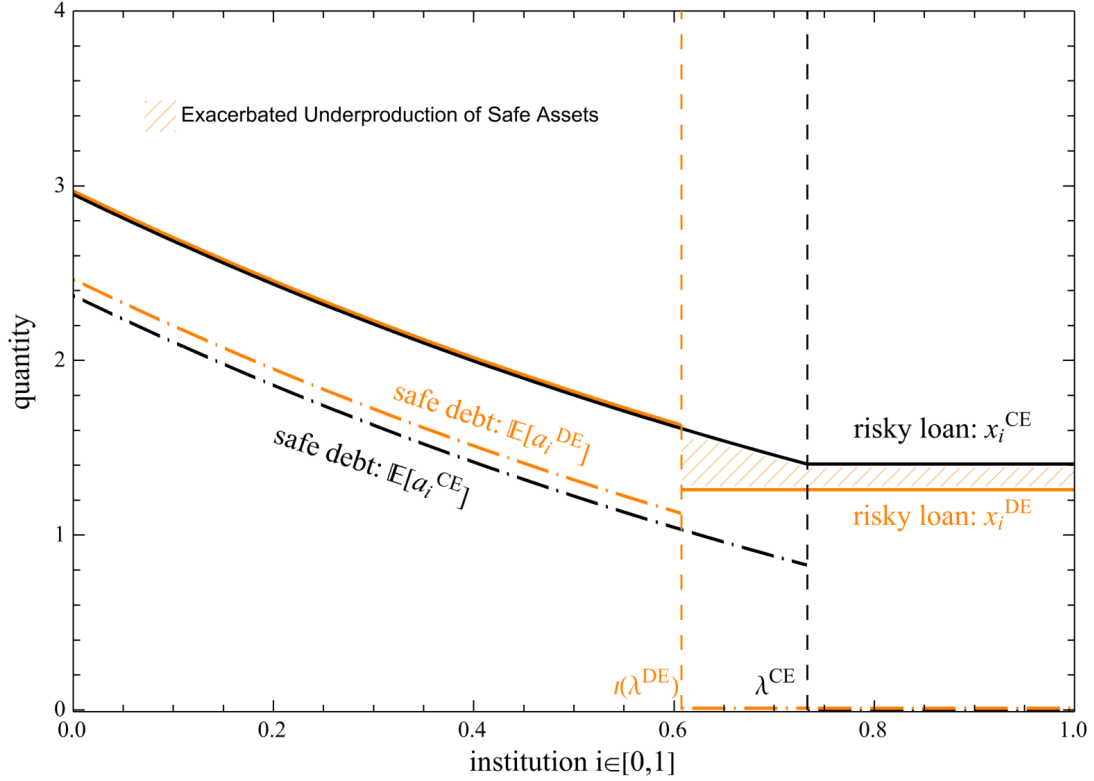


Figure 9: **Equilibrium under An Entry Cost Policy.**

This figure illustrates the equilibrium when an entry cost is imposed on institutions that issue safe debt. Superscripts CE and ECP indicate the unregulated competitive equilibrium and the equilibrium under the entry cost policy, and  $x_i$  and  $\mathbb{E}[a_i]$  denote institution  $i$ 's quantities of lending and expected safe debt issuance, respectively. The area of the shaded region represents the incremental underproduction of safe debt. Entry cost  $\zeta_i = \zeta i$ ,  $\zeta = 0.1$ , and other functional form and parameter values are the same as in Figure 6.

## Appendix: Proofs

**Proof of Lemma 1.** If  $\Delta x_{i,h} = \Delta x_{i,l} = 0$  for all  $i$ , the two constraints (ICC) and (MCC) reduce to a single constraint  $a_i \leq x_{i,h}$ . By Assumption 2, the objective in (P0) is strictly increasing in  $a_i$ , so this constraint binds at  $a_i^{STA}$ . The first-order condition with respect to  $x_i$  is  $pR + 1 - p - c'(x_i) + \gamma - \xi_i = 0$ , which gives the lending choice  $x_i^{STA}$ .

**Proof of Lemma 2.** Suppose  $\frac{q_l}{q_h} > \pi$ , the objective in program (P1a) would be strictly decreasing in  $\Delta x_{i,l}$ , and the optimal choice would be  $\Delta x_{i,l} = -x_{i,l}$  for all  $i \in \mathcal{I}$ . This contradicts the low-quality loan's market clearing condition (2).

**Proof of Proposition 1.** If a competitive equilibrium exists, the cutoff type's indifference condition (8) implies that

$$\frac{q_l}{q_h} = \frac{(1-p)\pi}{1-p+\gamma-\xi_\lambda}, \quad (\text{A.1})$$

which is well-defined and strictly positive by Assumption 2. The optimal lending choices follow from substituting  $\eta_i$  and (A.1) into (4). Given the financing choices and optimal secondary market trades in (3), the market clearing condition (2) can be rewritten as

$$\frac{q_l}{q_h} \int_0^\lambda x_{i,l} di = \int_\lambda^1 x_{i,h} di. \quad (\text{A.2})$$

By law of large numbers,  $\int_0^\lambda x_{i,l} di = \lambda x_L$ , and  $\int_\lambda^1 x_{i,h} di = (1-\lambda)(x_i - x_L)$ . Both  $\frac{q_l}{q_h}$  and  $x_i$  are functions of  $\lambda$ , so Equations (A.1) and (A.2) are equivalent to an aggregate excess demand condition  $\chi^{CE}(\lambda) = 0$ , where  $\chi^{CE} : [0, 1] \mapsto \mathbb{R}$  is defined as:

$$\chi^{CE}(\lambda) = \frac{\lambda(1-p)\pi x_L}{1-p+\gamma-2\xi\lambda} - (1-\lambda)(c'^{-1}(pR+1-p+\gamma-2\xi\lambda) - x_L). \quad (\text{A.3})$$

This excess demand satisfies  $\chi^{CE}(0) = x_L - c'^{-1}(pR+1-p+\gamma) < 0$  by Assumption 1 and  $\chi^{CE}(1) = \frac{(1-p)\pi x_L}{1-p+\gamma-2\xi} > 0$ , so the existence of a real root follows from intermediate value theorem. Moreover, by the properties of  $c$ ,  $\chi^{CE}$  is continuous and strictly increasing on  $[0, 1]$ , so the root is unique.

**Proof of Corollary 1.2.** The institution's payoff in static securitization is

$$V_i^{STA} = p x_i^{STA} R + (1-p)(x_i^{STA} - x_{i,l} + \pi x_{i,l}) + (\gamma - \xi_i) a_i^{STA} - c(x_i^{STA}). \quad (\text{A.4})$$

For  $i \in [0, \lambda^{CE})$ ,  $x_i^{CE} = x_i^{STA}$  and  $a_i^{CE} > a_i^{STA} = x_i^{STA} - x_{i,l}$ . Substitute  $x_i^{CE}, a_i^{CE}$  into

(P0a) and collect terms, it follows that  $V_i^{CE} = V_i^{STA} + (\gamma - \xi_i)(a_i^{CE} - a_i^{STA}) > V_i^{STA}$ .

For  $i \in (\lambda^{CE}, 1]$ ,  $x_i^{CE} > x_i^{STA}$ ,  $a_i^{CE} = 0$ , and  $\gamma - \xi_i < (1-p)(\pi \frac{q_h}{q_l} - 1)$ . Define  $\phi^{CE} = (1-p)(\pi \frac{q_h}{q_l} - 1)$  and  $\phi^{STA} = \gamma - \xi_i$ . Recognize that  $x_i^{CE}$  and  $x_i^{STA}$  are solutions to

$$V_i(\phi_i) = \max_{x_i} p x_i R + (1-p)(x_i - x_{i,l} + \pi x_{i,l}) + \phi_i(x_i - x_{i,l}) - c(x_i). \quad (\text{A.5})$$

By the envelope theorem,  $\frac{\partial V_i}{\partial \phi_i} > 0$ , therefore  $\phi^{CE} > \phi^{STA}$  implies  $V_i^{CE} > V_i^{STA}$ .

**Proof of Corollary 1.3.** By properties of  $c$ , the excess demand  $\chi^{CE}(\lambda)$  is continuously differentiable and strictly decreasing in  $\gamma$  for a given  $\lambda$ . Given that  $\lambda^{CE}$  solves  $\chi^{CE}(\lambda) = 0$ , by the implicit function theorem,  $\frac{\partial \lambda^{CE}}{\partial \gamma} = -\frac{\partial \chi^{CE}}{\partial \gamma} / \frac{d\chi^{CE}}{d\lambda} > 0$ , so  $\lambda^{CE}$  is strictly increasing in  $\gamma$ .

Next, note that  $\gamma - \xi_\lambda$  strictly increases in  $\gamma$ : otherwise,  $\chi^{CE}(\lambda^{CE}) = 0$  would imply that  $\lambda^{CE}$  decreases in  $\gamma$ , a contradiction. It then follows that the quantity of lending,  $x_i$ , strictly increases in  $\gamma$  by Equation (9) and that  $q_l/q_h$  strictly decreases in  $\gamma$  by Equation (A.1).

**Proof of Corollary 1.4.** The complementary slackness condition (6) requires  $\eta_i, \mu_i \geq 0$  to not be both positive for any  $i \in \mathcal{I}$ . Suppose  $\xi_i = \xi^*$  for all  $i$ , the institution's first-order condition (5) implies that  $\eta_i - \mu_i$  is a constant across all  $i$ . If  $\eta_i > 0$  for all  $i$ , or if  $\mu_i > 0$  for all  $i$ , Equation (7) would be violated, so  $\eta_i = \mu_i = 0$  for all  $i \in \mathcal{I}$ . This implies that  $\frac{q_l}{q_h} = \frac{(1-p)\pi}{1-p+\gamma-\xi^*}$ ,  $x_i = c^{-1}(pR + 1 - p + \gamma - \xi^*)$ , and any  $\{a_i : a_i \leq x_{i,h} + x_{i,l} \frac{q_l}{q_h}\}_{i \in \mathcal{I}}$  that satisfies Equation (7) is an equilibrium. Also, by Equation (7), the supply of safe debt is the same as in the static benchmark because here  $x_i$  equals  $x_i^{STA}$  in Lemma 1 for all  $i \in \mathcal{I}$ .

**Proof of Proposition 2.** Individual collateral constraint (ICC) faced by the planner must be slack for a proper subset of institutions, otherwise aggregate collateral constraint (ACC) would be violated. By monotonicity of  $\xi_i$  in  $i$ , Equation (13) implies that there exists some  $\lambda \in (0, 1)$ , such that  $\eta_i^{SP} = \gamma - \xi_i - \psi^{SP} > 0$ ,  $\mu_i^{SP} = 0$  for each  $i \in [0, \lambda)$ , and  $\eta_i^{SP} = 0$ ,  $\mu_i^{SP} > 0$  for each  $i \in (\lambda, 1]$ . The planner is indifferent about the financing choice of the cutoff type  $i = \lambda$ , which satisfies  $\psi^{SP} = \gamma - \xi_\lambda$ . This implies the planner's financing choices

$$a_i^{SP} = \begin{cases} x_i^{SP} - x_{i,l} + x_{i,l}\pi, & \text{if } i \leq \lambda^{SP} \\ 0, & \text{if } i > \lambda^{SP} \end{cases}. \quad (\text{A.6})$$

The planner's lending choices follow from substituting  $\eta_i^{SP} = \max\{\xi_\lambda - \xi_i, 0\}$  and  $\psi^{SP} = \gamma - \xi_\lambda$  into (12):



$$x_i^{SP} = \begin{cases} c'^{-1}(pR + 1 - p + \gamma - \xi_i), & \text{if } i \leq \lambda^{SP} \\ c'^{-1}(pR + 1 - p + \gamma - \xi_{\lambda^{SP}}), & \text{if } i > \lambda^{SP} \end{cases}, \quad (\text{A.7})$$

Given the cutoff property, the binding constraint (ACC) is equivalent to

$$\pi \int_0^\lambda x_{i,l} di = \int_\lambda^1 (x_i - x_{i,l}) di, \quad (\text{A.8})$$

and the cutoff type  $\lambda$  solves  $\chi^{SP}(\lambda) = 0$ , where

$$\chi^{SP}(\lambda) = \pi \lambda x_L - (1 - \lambda) \left( c'^{-1}(pR + 1 - p + \gamma - 2\xi\lambda) - x_L \right). \quad (\text{A.9})$$

Similar to  $\chi^{CE}$  defined in (A.3),  $\chi^{SP} : [0, 1] \mapsto \mathbb{R}$  is continuous, strictly increasing, and satisfies  $\chi^{SP}(0) < 0$ ,  $\chi^{SP}(1) > 0$ . So cutoff  $\lambda^{SP} \in (0, 1)$  exists and is unique.

By construction,  $\chi^{SP}(0) = \chi^{CE}(0)$  and  $\chi^{SP}(\lambda) > \chi^{CE}(\lambda), \forall \lambda \in (0, 1]$ . This implies  $\chi^{SP}(\lambda^{CE}) > \chi^{CE}(\lambda^{CE}) = 0$ , and hence  $\lambda^{SP} \in (0, \lambda^{CE})$  by properties of  $\chi^{SP}$ . Using aggregate relationship  $A = X - x_L$ , it follows that

$$A^{SP} - A^{CE} = X^{SP} - X^{CE} = \int_{\lambda^{SP}}^1 (x_i^{SP} - x_i^{CE}) di > 0 \quad (\text{A.10})$$

because  $x_i^{SP} > x_i^{CE}$  for any  $i \in (\lambda^{SP}, 1]$  by Equation (9) and Equation (A.7).

**Proof of Corollary 2.3.** Substitute the equilibrium price ratio in Corollary 1.4 into problem (P0a), it follows that the objective is independent to  $a_i$ . Apply similar arguments as in the proof of Corollary 1.4 to the planner's optimality conditions (12)-(14), it follows that  $\eta_i^{SP} = \mu_i^{SP} = 0$ ,  $\psi^{SP} = \gamma - \xi^*$ ,  $x_i = c'^{-1}(pR + 1 - p + \gamma - \xi^*)$ , and any  $\{a_i : a_i \leq x_{i,h} + x_{i,l}\pi\}_{i \in \mathcal{I}}$  that satisfies the binding aggregate collateral constraint (ACC) is constrained efficient. Note for any realization of  $\{\tilde{x}_{i,l}\}_{i \in \mathcal{I}}$ , the set of competitive allocation is a subset of the planner's allocation, so every competitive allocation is constrained efficient.

**Proof of Proposition 3.** If an equilibrium exists, the secondary market clearing condition (2) requires

$$\frac{q_l}{q_h} \int_0^{\iota(\lambda)} x_{i,l} di = \int_{\iota(\lambda)}^1 x_{i,h} di. \quad (\text{A.11})$$

The corresponding aggregate excess demand equation under the entry cost policy is

$$\chi^{ECP}(\lambda) = \frac{q_l}{q_h} \int_0^{\iota(\lambda)} x_{i,l} di - \int_{\iota(\lambda)}^1 (x_i - x_{i,l}) di. \quad (\text{A.12})$$

The proof is based on an auxiliary lemma on the relationship among equilibrium cutoff types. Given this lemma below, the proposition follows immediately from the lending choices as functions of  $\lambda$  in Proposition 1 and the aggregate relationship in Equation (7).

**Lemma A.1.**  $\iota(\lambda^{ECP}) < \lambda^{CE} < \lambda^{ECP}$ .

I prove Lemma A.1 by contradiction in two steps, both of which are constructed using the cutoff condition (8), the market clearing condition (A.11), and individually optimal lending choices (9) in Proposition 1. For exposition, I use superscript  $CE$  to label variables in competitive equilibrium and  $ECP$  to label variables in the equilibrium under the policy.

*Step 1:* Suppose  $\lambda^{ECP} < \lambda^{CE}$ , and hence  $\iota(\lambda^{ECP}) < \lambda^{ECP} < \lambda^{CE}$ . By Equation (8), this implies  $(\frac{q_l}{q_h})^{ECP} < (\frac{q_l}{q_h})^{CE}$ , and hence

$$\left(\frac{q_l}{q_h}\right)^{ECP} \int_0^{\iota(\lambda^{ECP})} x_{i,l} di < \left(\frac{q_l}{q_h}\right)^{ECP} \int_0^{\lambda^{CE}} x_{i,l} di < \left(\frac{q_l}{q_h}\right)^{CE} \int_0^{\lambda^{CE}} x_{i,l} di. \quad (\text{A.13})$$

By Equation (9), the conjectured inequality also implies  $x_i^{ECP} > x_i^{CE}$  for any  $i > \lambda^{CE}$ , which further implies

$$\int_{\iota(\lambda^{ECP})}^1 (x_i^{ECP} - x_{i,l}) di > \int_{\lambda^{CE}}^1 (x_i^{ECP} - x_{i,l}) di > \int_{\lambda^{CE}}^1 (x_i^{CE} - x_{i,l}) di. \quad (\text{A.14})$$

Given Equation (A.2),

$$\left(\frac{q_l}{q_h}\right)^{CE} \int_0^{\lambda^{CE}} x_{i,l} di = \int_{\lambda^{CE}}^1 (x_i^{CE} - x_{i,l}) di, \quad (\text{A.15})$$

so inequalities (A.13) and (A.14) jointly imply

$$\left(\frac{q_l}{q_h}\right)^{ECP} \int_0^{\iota(\lambda^{ECP})} x_{i,l} di < \int_{\iota(\lambda^{ECP})}^1 (x_i^{ECP} - x_{i,l}) di. \quad (\text{A.16})$$

This contradicts that  $\lambda^{ECP}$  solves the zero aggregate excess demand equation  $\chi^{ECP}(\lambda) = 0$ . Clearly,  $\lambda^{ECP} \neq \lambda^{CE}$  as  $\iota(\lambda^{ECP}) < \lambda^{ECP}$ , therefore  $\lambda^{ECP} > \lambda^{CE}$  if an equilibrium exists.

*Step 2:* Suppose  $\lambda^{CE} < \iota(\lambda^{ECP}) < \lambda^{ECP}$ . Using similar arguments as in Step 1, these inequalities imply

$$\left(\frac{q_l}{q_h}\right)^{ECP} \int_0^{\iota(\lambda^{ECP})} x_{i,l} di > \int_{\iota(\lambda^{ECP})}^1 (x_i^{ECP} - x_{i,l}) di, \quad (\text{A.17})$$

which is a contradiction, too. This completes the proof.