Portfolio Dynamics and the Supply of Safe Securities

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December, 2023

Abstract

Securitization of corporate loans often uses dynamic portfolios as collateral to back safe securities. I develop an equilibrium model to understand this safety transformation, focusing on the tradeoff in operating Collateralized Loan Obligations (CLOs), whose underlying portfolios are rebalanced as loan quality evolves. By maintaining portfolio quality, CLOs create larger safe tranches, thereby enjoying lower funding costs. However, replacing deteriorated loans generates pressure on secondary loan prices, offering profitable trading opportunities to peer institutions. My model explains how this mechanism induces the coexistence of distinct institutions, reallocates loans, distorts loan prices, and increases the quantities of lending and safe securities.

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The transformation of risky assets into safe liabilities is a key function of financial intermediation, and a large share of this safety transformation is achieved through securitization. Because investors often place special value on highly-rated, low-risk securities, many financial institutions repackage risky loans to create tranches that are safer than the underlying loan portfolios.[1] Traditionally, these portfolios are static. This paper explores a relatively recent and important innovation: the creation of safe tranches backed by dynamic portfolios.

An epitome of this idea is Collateralized Loan Obligations (CLOs), which are nonbank institutions and have created over $1 trillion of securities backed by corporate loans. Notably, CLOs actively manage loan portfolios by trading with peer institutions. Senior CLO tranches, which account for roughly 65% of the capital structure, are AAA-rated with zero historical defaults.[2] The rapid growth of this market has spurred empirical studies, but fundamental questions about its mechanism remain open. What is the benefit of dynamic portfolios in safety transformation? Why don't peer institutions, typically investment funds that hold similar loans ("loan funds"), transform their holdings into safe securities? How do trading and loan prices shape the market's overall lending and supply of safe securities?

I argue that, central to these questions is CLOs' debt financing: given the initial portfolios, they create *larger-sized* safe tranches via dynamic collateral management. Unlike static collateral pools, dynamic portfolios prevent extremely bad cash flow realizations, which occur often after the quality of loans deteriorates. By replacing deteriorated loans before their cash flows are realized, CLOs make a larger tranche safe and, thus, enjoy lower funding costs. Such trades, however, may move loan prices, which affect the ex-post payoffs for CLOs and their peers and ultimately, these institutions' ex-ante balance sheet choices.

In this paper, I present a simple theoretical framework for interpreting portfolio dynamics

[1] Recent literature documents a special demand for highly-rated safe securities that arises from these securities' liquidity and regulatory advantages (e.g., Krishnamurthy and Vissing-Jorgensen, 2012; Gorton, Lewellen, and Metrick, 2012; Nagel, 2016; Van Binsbergen, Diamond, and Grotteria, 2022).

[2] Senior CLO securities, while not riskless, satisfy investor demand for safe assets and relieve regulatory capital charges (Benmelech and Dlugosz, 2009; Cordell, Roberts, and Schwert, 2023). This provides lower risk-adjusted funding costs to CLO equity investors. See Section 1 for details on the demand for senior CLOs.

in safety transformation. To establish the relevance of the mechanism, I begin with evidence that CLOs replace deteriorated loans, which substantially improves portfolio quality but exerts pressure on loan prices. I then address the research questions by developing a model that integrates safe debt and portfolio dynamics into an equilibrium framework. The model microfounds CLOs and loan funds, which endogenously emerge, coexist, and trade loans after ex-ante identical institutions make lending and financing choices. Trading helps CLOs restore portfolio quality after adverse shocks, and the resultant loan prices affect all institutions' payoffs. My analysis shows that, while this mechanism inherently distorts prices away from fundamentals, it raises lending and safe debt and improves total surplus.

Like many securitization vehicles, CLOs create long-term securities backed by long-term loans, and their securities are vulnerable to loan quality deterioration. But unlike other loans, the loans held by CLOs ("leveraged loans") are rated by credit rating agencies and traded in secondary markets. This allows CLO contracts to implement dynamic collateral management. By imposing portfolio constraints tied to time-varying loan ratings, these contracts obligate CLO managers to trade and maintain collateral quality. Yet, it is impossible for all institutions to operate as CLOs and replace deteriorated loans: the secondary market must clear. The market clears because CLOs and loan funds, operated by a common group of asset managers, coexist and trade as counterparties.

I analyze the equilibrium of these nonbank institutions in a three-period model. There are two groups of agents: investors, who enjoy a non-pecuniary benefit from safe debt, and institutions, who can produce safe debt backed by risky loans. In the initial period, institutions issue debt and equity tranches to raise funding from investors to finance lending. Given the non-pecuniary benefit, safe debt tranches may be issued at a premium and provides cheap funding to the issuers. Ex ante, the size of safe tranches is constrained by the worst-case loan payoffs, which are realized in the final period. This constraint is tight because after issuance, idiosyncratic shocks cause a random fraction of every institution's loan portfolio to deteriorate, potentially leading to very low payoffs.

My model highlights a novel dynamic link between debt safety and loans of different quality. Low-quality loans are riskier and have a lower worst-case payoff, so high-quality loans are better collateral for safe debt. But loan quality is unpredictable when tranches are issued and is revealed only after the interim shocks. So the size of safe tranches, if backed by static portfolios, is limited by the uncertainty in collateral quality. Nevertheless, institutions may rebalance portfolios: By initially incurring a fixed cost, they can adopt dynamic collateral management, which commits them to sell low-quality loans and buy high-quality loans when quality reveals. This adoption raises the portfolio's worst-case payoff beyond that of a static portfolio, thereby enabling the institution to issue a larger safe tranche.

While all institutions are ex-ante identical, if dynamic collateral management is a viable option, in equilibrium they make distinct financing choices: Some institutions, which resemble CLOs, specialize in creating safe debt, rebalancing portfolios by trading loans with other institutions that resemble loan funds. This endogenous mix of institutions is both the cause and result of distorted loan prices. Since CLOs are obligated to trade, the pressure from their trades decreases the price of low-quality loans relative to the price of high-quality loans. As such, the safety premium captured by CLOs will be shared with loan funds as a compensation for liquidity provision. Because dynamic collateral is costly to adopt, in equilibrium, the sharing of surplus will be partial — similar to that equilibrium prices partially reveal costly information (Grossman and Stiglitz, 1980) — and CLOs and loan funds will coexist.

This equilibrium framework helps interpret a variety of empirical findings. For example, CLOs typically sell deteriorated loans to mutual and hedge funds (Giannetti and Meisenzahl, 2021), and their loan sales exert downward pressure on secondary market prices (e.g., Elkamhi and Nozawa, 2022; Kundu, 2021; Nicolai, 2020). Conversely, CLOs buy higher-rated loans from mutual funds, supporting the prices of these loans (Emin et al., 2021). Different from these papers, which separately examine CLOs' sales and purchases, my model captures portfolio substitution: empirically, I find a strong, nearly one-to-one relationship between CLOs' loan sales and purchases during market downturns.

Through the lens of the model, I find that dynamic collateral management raises the total supply of safe debt through two channels: lending volume and risk sharing. First, since institutions can at least collateralize static portfolios to issue safe debt, they choose dynamic portfolios if and only if doing so improves their lending payoffs. Better payoffs lead to higher lending volumes and hence more loans to back safe debt. Second, risk sharing increases safe debt backed by each unit of loans because after idiosyncratic shocks, CLOs with deteriorated portfolios can restore quality by trading with other institutions. These two channels are complementary: risk sharing raises lending by improving the collateral value of loans, and higher lending generates more loans to share across institutions. Overall, while only a subset of institutions operate CLOs, the market produces more safe debt in total.

My analysis explains that price distortion is an inherent feature of equilibrium in the leveraged loan market. Usually, price deviations from fundamental values are interpreted as a symptom of frictions constraining liquidity provision (e.g., Coval and Stafford, 2007; Mitchell, Pedersen, and Pulvino, 2007; Ellul, Jotikasthira, and Lundblad, 2011). In contrast, price distortion occurs in my model without such frictions, and its magnitude is larger precisely when the market's total surplus is greater. This is because, when dynamic collateral management is feasible, only a subset of institutions operate loan funds, and they optimally provide imperfect liquidity, thereby sharing the safety premium with CLOs. As equilibrium prices equalize the payoffs of CLOs and loan funds, total surplus is greater when liquidity provision is better compensated, that is, when price distortion is severe.

Finally, my analysis suggests that this market's aggregate quantities are self-stabilizing: Because of the equilibrium link between loan prices and institution balance sheets, the effects of a shock to the market will have opposite signs at the extensive and intensive margins. For example, consider raising the fixed cost of dynamic collateral management, such as imposing a regulatory burden on CLOs. While this shock reduces the fraction of institutions operating as CLOs, it also mitigates the price pressure from CLOs, which relaxes price-dependent constraints on the size of CLOs' safe tranches. As a result, the reduction in institutions operating as CLOs and the increases in lending and safe debt of the remaining CLOs partially offset each other, stabilizing total loans and safe debt.

This paper extends the literature on safe debt creation in financial intermediation, which dates back to Gorton and Pennacchi (1990), by examining how dynamic portfolios facilitate the creation of safe securities. Loan trades in my context are related to asset sales in Stein (2012). In his model, liquidation price of bank assets, determined by exogenous buyers, drives banks' capacity of short-term safe debt. My paper differs by showing that dynamically replacing deteriorated loans raises long-term safe debt and leads to the endogenous rise of trading counterparties.[3] This mechanism is also distinct from existing theories of securitization, which focus on static collateral pools (e.g., DeMarzo and Duffie, 1999; DeMarzo, 2005; Hanson and Sunderam, 2013). More broadly, the literature has studied the specific ways in which intermediaries create safe debt, including risk management (DeAngelo and Stulz, 2015), early liquidation (Stein, 2012; Hanson et al., 2015), deposit insurance (Hanson et al., 2015), asset opacity (Dang et al., 2017), and diversification (Diamond, 2020). My analysis of dynamic collateral management adds a new perspective to these studies.

There is a growing body of empirical research on leveraged loans and CLOs. Recent papers, including the aforementioned by Giannetti and Meisenzahl (2021), Elkamhi and Nozawa (2022), Kundu (2021), Nicolai (2020), and Emin et al. (2021), study loan trades as a consequence of CLOs' debt covenants. Consistent with a premium for safe tranches, Cordell, Roberts, and Schwert (2023) find that CLO equity earned positive abnormal returns. Foley-Fisher, Gorton, and Verani (2021) and Griffin and Nickerson (2023) use the COVID-19 crisis as a setting to examine CLO tranches' bid-ask spreads and credit ratings, respectively.[4] To my knowledge, this paper is the first to explicitly analyze dynamic collateral management

[3] The feature that ex-post trading prices feedback to ex-ante investment and financing choices also exists in theories of fire sales (e.g., Shleifer and Vishny, 1992; Gorton and Huang, 2004; Diamond and Rajan, 2011). My model differs in that no liquidation is triggered by short maturity or moral hazard; Instead, long-term contracts obligate CLOs to rebalance portfolios, resulting in a pressure on relative prices.

[4] Griffin and Nickerson (2023) find that CLOs' loan trades improved rating agencies' collateral pool risk metrics. However, they do not study the relationship between these loan trades and CLOs' financing contracts, or its equilibrium implications.

and its role in shaping secondary market loan trades and the coexistence of CLOs and loan funds. It contributes to this literature with new stylized facts and an equilibrium framework to interpret empirical findings.

The rest of this paper is organized as follows. Section 1 presents empirical facts that motivate my theoretical analysis. Section 2 introduces model setup. Section 3 characterizes the equilibrium. Section 4 discusses the model's implications. Section 5 concludes.

1. Stylized Facts

I begin with institutional background and empirical facts on how CLOs' collateral constraints govern the dynamics of the underlying portfolios. Data details are in Appendix IA.1.

Institutional Background: the Leveraged Loan Market

Leveraged loans are broadly syndicated loans issued by corporations that have a high financial leverage.[5] These loans are originated through syndication deals, where underwriters organize select groups of lenders to privately contract with the borrowers. Following the Federal Reserve Board (2022), I restrict attention to "institutional leveraged loans", which are non-amortizing term loans and mostly held by nonbank institutions.

[Add Figure 1 here]

Collateralized Loan Obligations. CLOs are the largest group of nonbanks that hold leveraged loans. As Figure 1 shows, US leveraged loans grew from $130 billion to $1.2 trillion between 2001 and 2020, and CLOs consistently held about half of these loans. While other types of securitization are mostly backed by static collateral, CLOs' portfolios, consisting of 100–300 loan shares with $300-600 million total par values, are actively managed during a reinvestment period. CLO debt tranches mature in around 10 years, and the reinvestment

[5] S&P Global Market Intelligence defines a loan as leveraged if it is rated below Baa3/BBB-, or if it is secured and has a spread of at least 125 basis points.

period is around 5 years and often extended. After this, the CLO enters its amortization period and gradually repays debt principal.[6] The vast majority of CLOs are "open-market CLOs", whose managers are independent from banks. The manager's compensation consists of size-based fixed fees and performance fees based on equity tranche returns.

Demand for Safe Debt. A primary force behind the growth of CLOs is the demand for highly-rated securities.[7] Senior CLO tranches, which account for about 65% of the capital structure, are AAA-rated. With higher yields than safer assets (e.g., US Treasuries) and fairly low regulatory risk weights, senior CLOs are attractive to, and mostly held by, banks and insurers. For example, Fitch (2019) reports that $94 billion, $113 billion, and $35 billion of senior CLOs are held by banks in the US, Japan, and Europe, respectively. Since the 1990s, more than two thousand senior tranches have been issued, and none of them ever defaulted.[8] While senior CLOs are not necessarily riskless, they can earn a safety premium as capital-constrained investors are willing to accept a lower *risk-adjusted* return for holding highly-rated securities. This premium is a source of cheap funding for CLOs.

Fact 1: Coexistence of CLOs and Loan Funds

The leveraged loan market consists of two types of nonbank institutions: CLOs and loan funds. These loan funds, including public funds and hedge funds, hold the majority of the rest of loans. In Appendix IA.2, I summarize the amounts of loans held by different categories of loan funds. A common feature of these funds is that they do not collateralize their loans to back safe securities and face limited restrictions on portfolio choices.

[Add Figure 2 here]

[6] In the amortization period, CLO managers can buy loans using only prepaid principal of existing loans. See Fitch's report for more details: Reinvestment in Amortization Period of U.S. CLOs.

[7] The safety of senior CLOs is with respect to default risk, which determines credit ratings. While these floating-rate notes are insensitive to interest-rate risk, they can be exposed to liquidity and inflation risks.

[8] A subset of senior CLOs were downgraded during 2007–09 but mostly recovered to original ratings (Cordell, Roberts, and Schwert, 2023). No AAA tranche was downgraded in the 2020 COVID-19 crisis.

Notably, CLOs and loan funds are often operated by a common group of asset managers. Figure 2 presents some of the largest managers that operate these nonbanks. For example, CVC Credit Partners only operates CLOs, whereas Fidelity Investments mainly manages leveraged loan mutual funds. These managers' choices lead to a coexistence of two types of nonbanks that both invest in leveraged loans but are financed by distinct liabilities.

Fact 2: CLOs' Binding Collateral Constraints

The large size of leveraged loans, typically hundreds of million or even multiple billion dollars, creates economies of scale in information production. Unlike smaller business loans, leveraged loans have credit ratings reflecting changes in loan quality and are traded in secondary markets. Therefore by contracting on loan ratings, CLO managers can credibly commit to dynamically replace underperforming loans as their quality deteriorates.

CLO contracts implement this commitment with regular (e.g., monthly) collateral tests that are tied to managerial compensation. The most important is the over-collateralization (OC) test, which calculates a ratio of quality-adjusted loan holdings to the total size of tranches senior to a particular tranche, including itself. If an OC test fails, the manager stops receiving fees for that tranche until the ratio recovers to a preset threshold. The manager can raise the OC ratio through either debt acceleration (i.e., repay the principal of senior tranches) or portfolio substitution (i.e., replace deteriorated loans with qualified loans).

[Add Figure 3 here]

Collateral constraints imposed by these contracts play a critical role in governing the dynamics of the CLO balance sheet. Figure 3 shows quarterly cross-sectional distribution of the slackness of senior OC constraints between 2010–2019. Among CLOs in reinvestment period, the average senior OC score is slightly (8%) above the threshold and stable over time.[9]

[9] In my data, the senior OC thresholds can be that of AAA and AA tranches, so my calculation may overstate the actual slackness of AAA OC constraints.

In the cross section of CLOs, the slackness is tightly distributed around the average. These persistently binding constraints suggest that managers fully use safe debt capacity provided by their loan portfolios. By contrast, in amortization period, as CLO leverage decreases with principal repayment, the slackness becomes larger and much more dispersed.

Fact 3: Binding Constraints Force CLOs to Replace Loans

Given that the collateral constraints are binding, shocks to the quality of CLOs' underlying loans are likely to trigger secondary market trades. Here I emphasize an empirical pattern that is less discussed in the literature: CLOs' loan trades consist of both sales and purchases rather than just one of them. Hence, the portfolio's size remains similar, but its composition changes over time.

[Add Figure 4 here]

Figure 4 presents CLO balance sheet dynamics before and around the onset of COVID-19 crisis in 2020. Panel (a) shows quarterly average CLO portfolio size for each age cohort. For all cohorts, portfolio size remained stable over time. This implies that overall CLOs did not shrink in size. Indeed, Panel (b) shows that accelerated repayment of senior debt actually decreased.[10] While the size of portfolios did not change, their composition changed drastically. In Panel (c), the average numbers of loan purchases and sales both nearly doubled upon the arrival of the adverse shock.[11] To understand the nature of these trades, Panel (d) examines buys and sells *within* individual CLOs in the first two quarters of 2020. As the scatterplot shows, there is a strong positive (and nearly one-to-one) relationship between a CLO's purchases and sales: when a CLO sells loans, it also buys loans to replace them. In other words, CLOs substitute loans in their portfolios.

[10] Earlier cohorts repaid more of their senior tranches when the non-call period ends (typically 2–3 years). Such early repayment discontinued in 2020.

[11] Purchases generally exceed sales because loan holdings generate coupon and principal payments.

Fact 4: Portfolio Substitution Improves Collateral Quality

Using granular data on CLO loan holdings, I examine how loan trades affect portfolio quality in 2020. Figure 5 presents the changes from February 15 ("pre") to June 30 ("post"). Panel (a) shows senior OC slackness before and after the shock. As the pandemic caused massive downgrades of leveraged loans, the overall slackness decreased, and the dispersion across CLOs increased. When the crisis settled, however, only 1.2% of CLOs failed senior OC tests.

[Add Figure 5 here]

The reason that test failures are rare, as Fact 3 suggests, could be portfolio substitution. To quantify its causal effect, I track individual loans' quality changes and measure each CLO's counterfactual portfolio quality in the absence of loan trades. Details of this step can be found in Appendix IA.1.3. Panel (b) shows portfolio value-weighted average ratings.[12] Overall, portfolio ratings worsened, but managers' trading mitigated deterioration, improving the realized ex-post distribution relative to the counterfactuals.

Despite similarly binding ex-ante constraints, idiosyncratic exposures to the shock may force CLOs to respond differently. I measure a CLO's exposure using the difference in rating between the pre and counterfactual portfolios. Panel (c) shows that almost all CLOs replaced downgraded loans and that the effect on quality linearly increases in exposure: on average, trading offsets 60% of deterioration caused by COVID-19. Panel (d) replaces the outcome with value-weighted average loan spread, which measures quality based on priced risk. In response to a 1-notch downgrading, the manager's trades reduced average spread by 30 basis points, or roughly one standard deviation. Panels (e) and (f) show further evidence based on the direction of loan trades by comparing ratings and spreads between the loans bought and sold by a CLO. Overall, these facts support the interpretation that binding collateral constraints triggered portfolio substitution that substantially improved portfolio quality.

[12] A larger numeric rating corresponds to a better letter rating. Table IA.2 details the conversion between letter and numeric ratings.

Fact 5: Price Pressure from CLOs

Portfolio substitution documented above is costly to CLO managers and equity holders because these trades not only reduce portfolio payoff uncertainty, but also exert pressure on loan prices. In Appendix IA.3, I document that during market downturns, the magnitude of transitory price drops is decreasing in loan quality, ranging from nearly 15% for "B-" to only 5% for "BB+". Consistent with price pressure from CLOs, this monotonic pattern is observed among leveraged loans but not high yield bonds. Admittedly, isolating loan price changes caused by CLOs from changes in fundamentals is difficult. While the nature of this evidence is suggestive, given various findings of price pressure in the literature, it is plausible that when a large number of CLOs substitute portfolios in the same direction, they cause the prices of bad loans to decrease relative to the prices of good loans.

2. Model

The empirical facts above suggest a tradeoff between the ex-ante benefit of issuing more safe debt and the ex-post cost of replacing deteriorated loans. It is still unclear how this tradeoff affects the choices of individual institutions operating as CLOs and loan funds, as well as the equilibrium prices and quantities of risky loans and safe debt. Such an equilibrium view is important for understanding the market and its response to policy interventions. To analyze these issues, I develop a model in which lending institutions can flexibly choose external financing and rebalance portfolios. The economy has three periods, $t \in \{0, 1, 2\}$, and two types of agents: investors and financial institutions.

Investors. There is a unit mass of investors. They can be banks, insurance companies, and other entities that invest in CLOs and loan funds. Some of these investors face risk-based regulatory requirements and prefer securities with sufficiently low default risks (e.g., AAA rated). To capture this important preference, I abstract away the default risk of senior CLOs and follow the safe asset literature (e.g., Krishnamurthy and Vissing-Jorgensen, 2012; Stein, 2012; Nagel, 2016) by assuming that investors maximize additively separable utility[13]

$$U = \mathbb{E}_0\Big[\sum_{t=0}^{2} C_t\Big] + \gamma D, \tag{1}$$

where C_t is consumption in period t, and D is safe debt held at $t = 0$. Parameter $\gamma \geq 0$ is a non-pecuniary benefit from holding safe debt. Its value is exogenous and determined by forces outside of this model.

At $t = 0$, investors are endowed with an amount e of perishable consumption goods. They cannot directly lend out resources but can buy financial claims backed by loans. Hence, they allocate between consumption and financial claims, taking claim prices as given. I assume e to be sufficiently large, so investors always choose strictly positive consumption.

Financial Institutions. There is a continuum of identical financial institutions, which can be interpreted as asset managers, uniformly populated on $\mathcal{I} = [0, 1]$. Their preference is similar to (1), except for that they do not derive any non-pecuniary benefit from safe debt. Each institution, indexed by $i \in \mathcal{I}$, can lend at $t = 0$ to generate a risky payoff at $t = 2$. Institutions receive zero endowment but can finance their lending by issuing senior and junior financial claims. In particular, a senior claim is referred to as *safe debt* if it is backed by loans whose payoff is enough for repayment with certainty. Since investors cannot store or lend, safe debt may only be supplied by institutions as their senior liabilities.

Investment Technology. Institutions make loans as follows. Each of them can turn x consumption goods into x units of loans at a private cost $c(x) - x$. This private cost captures the effort of participating in syndication deals and forming a diversified loan portfolio. c is twice continuously differentiable and satisfies $c(0) = 0$, $c' > 1$, $c'' > 0$ on $\mathbb{R}_+$. Every unit

[13] Therefore, γ should be interpreted as the risk-adjusted return that investors are willing to give up for holding highly-rated low-risk securities. Empirically, senior CLO securities indeed offer lower risk-adjusted returns (Cordell, Roberts, and Schwert, 2023).

of loans generates a risky payoff that depends on state $s \in \{g, b\}$ at $t = 2$.[14] The loans have two quality types, denoted by $j \in \{h, l\}$. In state g, which realizes with probability $p \in (0, 1)$, both types of loans pay $R_j = R > 1$. In state b, which realizes with probability $1 - p$, high-quality loans (h) pay $R_h = 1$, and low-quality loans (l) pay $R_l = 0$.

Timeline. All institutions simultaneously choose lending and financing in period $t = 0$. Specifically, each institution i raises x_i units of consumption goods from investors by issuing safe debt $d_i \geq 0$ and external equity shares. Meanwhile, the institution makes x_i units of loans without knowing loan types. In this period, the institutions may opt into keeping its portfolio static until $t = 2$. This choice is denoted by a binary variable $\mathbb{s}_i \in \{0, 1\}$.

In period $t = 1$, an idiosyncratic shock determines loan quality: α_i fraction of institution i's loans deteriorate to low-quality, and $1 - \alpha_i$ fraction are high-quality. Across institutions, α_i is independently drawn from a common distribution with support $[0, \bar{\alpha}] \subseteq [0, 1]$ and mean $\alpha \in (0, \bar{\alpha})$. Loan quality is publicly observable and contractible, and institutions with dynamic portfolios (i.e., $\mathbb{s}_i = 0$) can trade loans in a Walrasian market. In period $t = 2$, payoffs realize. As internal equity holders, institutions repay safe debt and external equity and collect residual portfolio payoffs. All goods are consumed, and the economy ends.

Dynamic Collateral Management. The distribution of loan payoffs implies that, if the portfolio is static, each unit of loans can back no more than $\rho_s = 1 - \bar{\alpha}$ safe debt. To increase its debt capacity beyond ρ_s, an institution may choose a dynamic portfolio. But the ability to trade loans, if not disciplined, prevents an institution from creating any safe debt. The reason is a classic agency problem (Jensen and Meckling, 1976): as equity holders, institutions privately prefer loans with riskier payoffs, which makes their debt default with a positive probability. Given empirical facts in the last section, I assume the existence of a technology that pre-commits institutions to replace low-quality loans at $t = 1$. This technology can be thought of as third-party services that continuously update loan ratings,

[14] For simplicity, industrial borrowers' output is fully pledgeable, and lenders extract all the rents. Hence, institutions' lending becomes as if they directly control real assets, an approach often used in the literature (e.g., Diamond and Dybvig, 1983).

perform collateral tests, and seize assets on behalf of debtholders. Adopting the technology and credibly revealing it to investors at $t = 0$ incur a fixed cost $\xi \geq 0$.

The Institution's Optimization Problem. Institutions with dynamic portfolios make sequential choices to maximize their payoffs. I describe their optimization problem backwardly and consider repayment only in the final period.[15]

Let secondary market prices of the two types of loans be $\mathbb{q} = (q_l, q_h) \in \mathbb{R}^2_+$. When α_i realizes, institution i, with balance sheet (x_i, d_i, α_i), chooses trades $\Delta\mathbb{x}_i = (\Delta x_{i,h}, \Delta x_{i,l})$ to maximize conditional expected payoff to equity

$$v(x_i, d_i, \alpha_i) = \max_{\Delta\mathbb{x}_i} \; ((1-\alpha_i)x_i + \Delta x_{i,h})\mathbb{E}[R_h] + (\alpha_i x_i + \Delta x_{i,l})\mathbb{E}[R_l] - d_i. \tag{2}$$

These trades are subject to a budget constraint

$$((1-\alpha_i)x_i + \Delta x_{i,h})q_h + (\alpha_i x_i + \Delta x_{i,l})q_l \leq (1-\alpha_i)x_i q_h + \alpha_i x_i q_l, \tag{3}$$

a maintenance collateral constraint

$$d_i \leq (1-\alpha_i)x_i + \Delta x_{i,h}, \tag{4}$$

and short-sale constraints $\Delta x_{i,h} \geq -(1-\alpha_i)x_i, \Delta x_{i,l} \geq -\alpha_i x_i$. Constraint (3) requires the trades to be self-financed by the loan portfolio. Constraint (4) reflects the ability to credibly commit to replacing deteriorated loans: After trades, safe debt investors must receive full repayment at $t = 2$ with probability one. This constraint keeps the institution solvent, which is why equity payoff in (2) is linear in portfolio payoff.

All market participants rationally anticipate loan trades at $t = 1$ when institutions choose lending and financing at $t = 0$. Because investors are price-taking, institutions optimally price their safe debt and external equity such that investors break even in expectation. This implies a safety premium: an institution can raise $1 + \gamma$ from issuing each unit of safe debt. The rest of funding is raised from external equity, whose expected return will be set to zero.

[15]The option of repaying debt in period $t = 1$ will be discussed in Section 4.

Taking loan prices as given, the institution chooses lending x_i and safe debt d_i to maximize the expected payoff to internal equity:

$$V_i = \max_{x_i, d_i \geq 0} \quad \mathbb{E}_0[v(x_i, d_i, \alpha_i)] - (x_i - (1+\gamma)d_i) - (c(x_i) - x_i) - \mathbb{1}\{d_i > 0\}\xi, \tag{5}$$

$$s.t.\ 0 \leq d_i \leq \rho x_i, \tag{6}$$

where $v(x_i, d_i, \alpha_i)$ is the $t = 1$ maximized total equity value in (2). The second term is the funding raised from (i.e., expected payoff of) external equity, and the third term is the institution's private cost of effort. The last term is the fixed cost of adopting dynamic collateral management: ξ is incurred if and only if safe debt $d_i > 0$ is backed by a dynamic portfolio.

Importantly, the maximization is subject to a price-dependent collateral constraint (6), where $\rho := \rho_s + \bar{\alpha}\frac{q_l}{q_h}$ is the debt capacity provided by each unit of loans. By selling and buying loans, an institution can generate a higher worst-case payoff than that of a static portfolio. This allows for more safe debt than ρ_s, and $\bar{\alpha}\frac{q_l}{q_h}$ is the maximum incremental debt capacity provided by dynamic collateral management.

Different from the above, if an institution opts into a static portfolio (i.e., $\mathbb{s}_i = 1$), it only chooses lending and financing at $t = 0$ and cannot trade loans at $t = 1$.

2.1. Equilibrium Definition

In equilibrium, all institutions take loan prices as given and optimally choose lending, financing, trading, and the adoption of dynamic collateral management. Formally, a market equilibrium in this economy is a collection $\{(\mathbb{s}_i, x_i, d_i, \Delta\mathbb{x}_i)_{i\in\mathcal{I}}, \mathbb{q}\}$ such that given $\mathbb{q}$, choice $\mathbb{s}_i$ maximizes institution i's expected payoff, taking into account that (x_i, d_i) solves the its lending and financing problem at $t = 0$ and that if $\mathbb{s}_i = 0$, $\Delta\mathbb{x}_i$ solves its trading problem at $t = 1$; Moreover, the secondary market clears:

$$\int_i \Delta x_{i,j}\,\mathrm{d}i = 0 \quad \text{for } j \in \{h, l\}. \tag{7}$$

Because of the collateral constraints, institutions' ex-ante lending and financing choices affect their ex-post trades, which in turn affect the lending and financing problem through endogenous loan prices. As such, the equilibrium features a feedback loop between primary and secondary markets.

Before analyzing the model, I introduce two intuitive notations for exposition. First, let $y_0 = pR + (1-p)(1-\alpha)$ be the average hold-to-maturity payoff per unit of lending. Second, I define a function $f(y) := y \cdot c'^{-1}(y) - c(c'^{-1}(y))$, where $c'^{-1}(\cdot)$ is the inverse function of the first-order derivative of c. As will be clear in Section 3, this function maps the marginal payoff of lending to an institution's maximized expected payoff V_i.

I impose three parametric conditions to restrict my analysis to interesting cases. The first condition ensures that lending has a positive NPV for at least some positive quantity:

Condition 1. $y_0 \geq c'(0)$.

Next, the cost function c will never lead to negative expected payoffs, which ensures that institutions always participate in lending:

Condition 2. $f(y_0) \geq 0$.

Last, the mean of portfolio quality deterioration is sufficiently large:

Condition 3. $\frac{\alpha}{\bar{\alpha}} > \frac{1-p+\gamma}{pR+1-p+\gamma}$.

This condition rules out equilibrium where all institutions adopt dynamic collateral management to create safe debt, which is possible if the fraction of low-quality loans in a portfolio is never significant.

2.2. Discussion of Model Setup

The key feature of my model is that safe debt provides cheap funding, but the underlying loans are overall scarce because of a convex cost of lending. Therefore, committing to replace low-quality loans with high-quality loans allows institutions to create more safe debt from a given quantity of lending. The loan payoff distribution is starker than necessary. What is crucial is the existence of a strictly positive minimum payoff R_h, which makes long-term safe debt possible.[16] Moreover, since only safe debt provides a non-pecuniary benefit, capital structure below the safe tranche is irrelevant, and it is without loss of generality to treat risky junior debt as equity.

The commitment technology is crucial for dynamic collateral management to be viable. In practice, CLOs hold only fairly standardized corporate loans that have credit ratings, so their portfolios can be disciplined by enforceable contracts. With simple collateral, CLOs differ from pre-crisis collateralized debt obligations (CDOs), which held enormous complex derivatives (Cordell, Huang, and Williams, 2011). Consistent with these facts, in my model, every institution's portfolio consists exclusively of risky loans. This occurs because consumption goods are nonstorable, cross-holdings of liabilities are unprofitable, and state-contingent bilateral contracts (e.g., derivatives) are not available.

3. Equilibrium Analysis

3.1. Static Benchmark

To provide a basic benchmark, suppose the technology of dynamic collateral management or the secondary loan market does not exist, and as a result, safe debt must be backed by static portfolios.

Lemma 1. *If portfolios must be static, all institutions would issue safe debt and fully use debt capacity: $x_i = x_s$ and $d_i = d_s$ for all $i \in \mathcal{I}$, where $x_s := c'^{-1}(y_0 + \gamma\rho_s)$ and $d_s := \rho_s x_s$.*

Proof. See the Appendix. □

[16] This positive lower bound is consistent with the leveraged loan market, where senior secured first lien term loans' default recovery rate is typically greater than 50%.

Since safe debt provides cheap funding, every institution collateralizes its loans to issue safe debt and fully uses its debt capacity.[17] Lending x_s increases in γ because loans not only generate monetary payoffs but also can back safe debt, and debt capacity is more valuable when the safety premium is greater. As institutions make identical choices, aggregate lending $X_s = x_s$, and the supply of safe debt $D_s = \rho_s X_s$.

In the rest of this section, I show how the lending and financing choices differ from this static benchmark when institutions can adopt dynamic collateral management and trade loans in a secondary market. I first analyze individual institutions' lending, financing, and trading choices for given secondary market prices. I then study balance sheets and loan prices that clear the secondary market. At last, institutions' choices between static and dynamic portfolios will be determined in equilibrium.

3.2. Secondary Market Trades

The lending and financing choices at $t = 0$ depend on continuation value v. To derive v, in this subsection I analyze the institution's secondary market problem (2) in period $t = 1$.

In this period, budget constraint (3) binds, and since $d_i \geq 0$, constraint (4) implies that $\Delta x_{i,h} \geq -(1 - \alpha_i)x_i$ is slack. Omitting predetermined terms, the problem simplifies to

$$\max_{\Delta x_{i,l}} \quad \left(\mathbb{E}[R_l] - \frac{q_l}{q_h}\mathbb{E}[R_h]\right)\Delta x_{i,l}, \tag{8}$$

subject to constraints $\Delta x_{i,l}\frac{q_l}{q_h} + d_i \leq (1 - \alpha_i)x_i$ and $\Delta x_{i,l} \geq -\alpha_i x_i$.

Essentially, the institution substitutes between high-quality and low-quality loans. This substitution is constrained by safe debt outstanding d_i and a short-sale constraint on $\Delta x_{i,l}$. I proceed to solve problem (8) based on the following observation.

Lemma 2. *If an equilibrium with dynamic portfolios exists, loan prices deviate away from their fundamental values:* $\frac{q_l}{q_h} < \frac{\mathbb{E}[R_l]}{\mathbb{E}[R_h]}$.

[17] This market structure resembles traditional banking, where every bank creates deposits, and loans stay on bank balance sheets.

Proof. See the Appendix. □

Lemma 2 reflects an inevitable consequence of dynamic collateral management: portfolio substitution exerts pressure on loan prices. Intuitively, after idiosyncratic shocks cause the quality of loans to deteriorate, some institutions are obligated to buy high-quality loans and sell low-quality loans, which puts them in demand for liquidity. The natural providers of liquidity are institutions that hold similar loans but face no binding collateral constraints. But for the latter to be willing to provide liquidity, low-quality loans, which are inferior as collateral for safe debt, must offer a higher expected return. As a result, low-quality loans must have a lower price-to-fundamental ratio relative to high-quality loans.

The solution to (8) below indicates that, consistent with Fact 3, the institution's optimal trades lead to portfolio substitution:

$$\Delta x_{i,h} = d_i - (1-\alpha_i)x_i, \;\; \Delta x_{i,l} = \frac{q_h}{q_l}((1-\alpha_i)x_i - d_i) \tag{9}$$

for any i. These trades reallocate loans among institutions. An institution with $d_i > (1-\alpha_i)x_i$ optimally sells just enough low-quality loans to increase its holding of high-quality loans and keep its debt safe. Such portfolio substitution is costly to equity holders because it not only decreases the portfolio's payoff uncertainty, but also moves prices in unfavorable directions. By contrast, an institution with $d_i < (1-\alpha_i)x_i$ sells its high-quality loans and buys low-quality loans to profit from the deviation of loan prices from fundamentals.

3.3. Lending and Financing Choices

Next, I characterize the institution's optimal lending and financing choices at $t = 0$ for given loan prices. Optimal secondary market trades in (9) imply that for any given (x_i, d_i, α_i), equity continuation value v at $t = 1$ is

$$v(x_i, d_i, \alpha_i) = x_i p R\Big(\alpha_i + \frac{q_h}{q_l}(1-\alpha_i)\Big) - d_i p\Big(R\Big(\frac{q_h}{q_l} - 1\Big) + 1\Big). \tag{10}$$

Substitute v into (5) and take expectation over α_i, the institution's lending and financing problem becomes

$$\max_{x_i, d_i} \; x_i pR\Big(\alpha + \frac{q_h}{q_l}(1-\alpha)\Big) - d_i p\Big(R\Big(\frac{q_h}{q_l} - 1\Big) + 1\Big) + (1+\gamma)d_i - c(x_i) - \mathbb{1}\{d_i > 0\}\xi \quad (11)$$

subject to constraint (6). Because the objective function is discontinuous at $d_i = 0$, in what follows, I consider two cases separately.

Equity Financing. In the first case, institution i issues only equity and gives up safe debt: $d_i = 0$. The optimal lending choice, x_e, is given by first-order condition

$$y = c'(x_e), \quad (12)$$

where $y := pR\Big(\alpha + (1-\alpha)\frac{q_h}{q_l}\Big)$ is the expected payoff per unit of lending when portfolios are dynamic.[18] The choice x_e is determined by a tradeoff between this payoff and the marginal cost of lending. Different from the hold-to-maturity payoff y_0, here y depends on loan prices at $t = 1$: It is decreasing in price ratio q_l/q_h because price distortion in Lemma 2 generates an expected profit from trading as compensation for liquidity provision.

Debt Financing. In the second case, institution i adopts dynamic collateral management and issues both safe debt and equity. Let η be the shadow price of debt capacity constraint $d_i \leq \rho x_i$. The conditions for optimality are

$$y + \eta\rho = c'(x_d), \quad (13)$$

$$\eta = \gamma - \Big(pR\Big(\frac{q_h}{q_l} - 1\Big) - (1-p)\Big), \quad (14)$$

$$\eta \geq 0, \eta(d_i - \rho x_d) = 0. \quad (15)$$

When $\eta > 0$, the collateral constraint binds ($d_i = \rho x_d$): the institution fully uses debt capacity to exploit cheap funding. On the asset side, as characterized by equation (13), optimal lending x_d exceeds x_e. The additional investment is due to $\eta\rho$, the collateral value of loans. Since

[18] $x_e > 0$ is guaranteed by Condition 1 and Lemma 5 below: $z \leq \bar{z} < \frac{pR}{pR+1-p}$, which implies $y > y_0$.

both debt capacity (ρ) and its per-unit value (η) decrease in price ratio q_l/q_h, price pressure reduces the collateral value that can be extracted from lending.

3.4. Market Equilibrium of Financial Institutions

Optimal choices in the two cases above determine the balance sheets of institutions that hold dynamic portfolios. Substitute these choices into objective (11), we can write these institutions' payoffs as $V_e = f(y)$ and $V_d = f(y + \eta\rho) - \xi$ for equity financing and debt financing, respectively, where function f was defined in Subsection 2.1. Similarly, using Lemma 1, the payoff of an institution with a static portfolio can be written as $V_s = f(y_0 + \gamma\rho_s)$. In equilibrium, every institution achieves its highest possible expected payoff: it obtains $\max\{V_e, V_d, V_s\}$ from its optimal choices.

The primary interest of this section is an equilibrium with dynamic loan portfolios. If such an equilibrium exists, a key endogenous variable would be price ratio $\frac{q_l}{q_h}$, or equivalently, the rate of portfolio substitution in the secondary market. As analyzed earlier, this ratio affects V_e and V_d through the optimal lending and financing choices. Since only the ratio (rather than the level) of loan prices is relevant, I use notation $z = q_l/q_h$ hereafter for convenience. The lemmas below collectively characterize the properties of the equilibrium.

Lemma 3. *The lower bound of price ratio is $\underline{z} := \frac{pR}{pR+1-p+\gamma}$, at which the collateral value of lending vanishes to zero (i.e., $\eta = 0$).*

Proof. See the Appendix. □

While the safety premium is directly captured by institutions that create safe debt, the price pressure of their trades transfers part of the surplus to peer institutions serving as liquidity providers. The price ratio determines how the safety premium is shared through loan trades. If the price ratio is as low as $\underline{z}$, the sharing would be perfect, which implies $V_d = V_e - \xi < V_e$. Similar to the Grossman and Stiglitz (1980) paradox, here if the benefit of creating safe debt is fully shared through prices, then no institution would adopt dynamic collateral management because doing so is privately costly. Therefore in equilibrium, price ratio must be higher that $\underline{z}$ whenever $\xi > 0$.

Lemma 4. *When there exist institutions that choose dynamic portfolios ($\mathbb{s}_i = 0$ for some $i \in \mathcal{I}$), the market clears only if equity financing and debt financing coexist among these institutions.*

Proof. See the Appendix. □

Obviously, an equilibrium in which all institutions use equity financing cannot exist: Equation (9) suggests that these institutions would trade in the same direction. It is also impossible for all institutions with dynamic portfolios to use debt financing. While some institutions using debt financing can also provide liquidity at $t = 1$ as long as they do not experience severe portfolio quality deterioration (i.e., $\alpha_i < 1 - \rho$), the quantity of high-quality loans they sell is never enough for meeting the demand from institutions that are forced to replace deteriorated loans. Thus for dynamic portfolios to appear in equilibrium, the two financing choices must coexist, which in turn implies $V_e = V_d$ in such an equilibrium.

Lemma 5. *An upper bound of price ratio is $\bar{z} := \frac{pR}{pR+1-p+\gamma\frac{1-\bar{\alpha}}{1-\alpha}}$. $V_e > V_s$ if and only if $z < \bar{z}$. Suppose $z > \bar{z}$, all institutions choose static portfolios.*

Proof. See the Appendix. □

Institutions using equity financing indirectly profit from the safety premium by providing liquidity, and their profit is higher when the price pressure is more severe. Suppose the price ratio is higher than $\bar{z}$, these institutions would prefer static portfolios: they would be better off issuing their own safe debt by collateralizing loans. Given Lemma 4, this in turn prevents any institution to adopt dynamic collateral management, so all institutions would end up holding static portfolios.

The next lemma shows that institution payoffs are monotone in z over $[\underline{z}, \bar{z}]$, the range of price ratio identified by Lemma 3 and Lemma 5.

Lemma 6. *V_e is strictly decreasing in z, and V_d is strictly increasing in z.*

Proof. See the Appendix. $\square$

If price ratio z is lower, liquidity provision is more profitable, and replacing low-quality loans is more costly. This makes equity financing more attractive relative to debt financing. By contrast, if z is higher, maintaining portfolio quality is less costly, and providing liquidity to others is less profitable. Hence, in an equilibrium with dynamic portfolios, the price ratio adjusts until $V_e = V_d$, and institutions' lending and financing choices collectively equalize the demand and supply for the two types of loans in the secondary market.

Proposition 1. *When $\xi > 0$ is not too large, there exists a unique equilibrium. All institutions hold dynamic portfolios, price ratio $z^* \in (\underline{z}, \bar{z})$, and two distinct financing choices coexist: a fraction $\lambda \in (0, 1)$ of institutions adopt dynamic collateral management and fully use their debt capacity (i.e., operate as CLOs), and $1 - \lambda$ of institutions do not issue any safe debt (i.e., operate as loan funds). In the secondary market, CLOs on average sell low-quality loans and buy high-quality loans, and loan funds sell high-quality loans and buy low-quality loans.*

Proof. See the Appendix. $\square$

Proposition 1 characterizes the unique equilibrium with dynamic loan portfolios. Notably, while all institutions are ex-ante identical, there is an endogenous mix of two distinct financing choices: Consistent with Fact 1, CLOs and loan funds emerge and coexist. The CLOs fully use debt capacity, which they maximize by adopting dynamic collateral management at a fixed cost. Binding collateral constraints generated by this adoption and the resulting portfolio substitution are consistent with Fact 2, Fact 3, and Fact 4. Institutions operating as CLOs enjoy cheap funding from larger safe tranches and receive high payoffs if realized portfolio deterioration is relatively low.

By contrast, institutions operating as loan funds completely give up issuing safe debt. They do so because market-clearing loan prices deviate from the loans' fundamental values (i.e., $z^* < \mathbb{E}[R_l]/\mathbb{E}[R_h]$), which is consistent with Fact 5. Such price pressure makes providing liquidity in the secondary market as profitable as operating CLOs. After loan quality realizes, loan funds sell their high-quality loans and absorb low-quality loans sold by CLOs. The loans they sell are used by CLOs as collateral to keep senior debt tranches safe.

[Add Figure 6 here]

Figure 6 illustrates how this equilibrium is determined. As the price ratio increases over $[\underline{z}, \bar{z}]$, V_e decreases because liquidity provision becomes less profitable, whereas V_d increases because replacing deteriorated loans becomes less costly. The unique equilibrium with dynamic portfolios exists if z^*, the single-crossing point between V_e and V_d, is below $\bar{z}$. If $z^* \in (\bar{z}, \bar{\bar{z}})$ instead, where $\bar{\bar{z}} := \mathbb{E}[R_l]/\mathbb{E}[R_h]$, then $V_d = V_e < V_s$ at z^*, and all institutions would choose static portfolios.

4. Implications

4.1. The Supply of Safe Debt

How does the presence of dynamic loan portfolios and an interior mix of institutions with distinct financing choices affect the market's total supply of safe debt? Through the lens of the model, this subsection analyzes the effects of dynamic collateral management on individual institutions and total quantities.

Lending Volume. Ex ante, every unit of lending generates an uncertain fraction α_i of high-quality loans, which always pay off $R_h \geq 1$ regardless of which state realizes at $t = 2$. Hence, others equal, a higher lending volume always provides greater debt capacity. My model indicates that in the equilibrium with dynamic portfolios, both CLOs and loan funds lend more than what they would had they chosen static portfolios.

Lemma 7. *When institutions hold dynamic portfolios, they choose higher lending volumes than with static portfolios:* $\mathbb{s}_i = 0$ *implies* $x_i > x_s$ *for all* $i \in \mathcal{I}$.

Proof. See the Appendix. □

Lending increases because dynamic collateral management helps institutions extract surplus from the safety premium. Specifically, trading interactions mitigate the impact of uncertain portfolio deterioration on individual debt capacity (see below), which allows for more cheap funding to be raised from investors for any given quantity of lending. While loan funds do not use their debt capacity, they share part of the safety premium captured by CLOs through equilibrium loan prices. Therefore, when holding dynamic portfolios, all institutions face a higher marginal payoff of lending (i.e., $y + \eta\rho > y > y_0 + \gamma\rho$) and optimally raise their lending volumes.

Risk Sharing. Ex post, dynamic collateral management facilitates risk sharing across institutions. While institutions are risk-neutral, sharing risk is valuable because the interim shocks causing portfolio quality deterioration, and hence limiting ex-ante debt capacity, are unpredictable and idiosyncratic. After these shocks realize, CLOs that experienced severe deterioration can restore portfolio quality by trading with institutions whose collateral constraints are slack. This reallocation of loans among institutions achieves a more efficient use of overall collateral.

The value of risk sharing is reflected in total debt capacity. The market-clearing condition (7) and loan trades in (9) jointly imply that with dynamic portfolios, total safe debt

$$D = \int_{i \in \mathcal{I}} d_i \,\mathrm{d}i = (1 - \alpha)X. \tag{16}$$

Hence in aggregate, debt capacity per unit of loans in this market is determined by the mean (α), rather than the minimum ($\bar{\alpha}$), of realized individual portfolio deterioration. So by relaxing collateral constraints, risk sharing increases total debt capacity for any given total quantity of lending.

Overall, despite that only a subset of institutions, namely the CLOs, create safe debt, they produce more safe debt in total because of larger lending volumes and the benefit of risk sharing. The next proposition summarizes these effects of dynamic collateral management on the total supply of safe debt.

Proposition 2. *When dynamic collateral management is adopted, the market produces a greater supply of safe debt than the static benchmark: $D > D_s$. This increase in supply comes from two complementary channels: (i) Dynamic portfolios increase the payoff, and therefore the quantity, of lending: $X > X_s$. (ii) Risk sharing across institutions allows for greater total debt capacity for any given quantity of loans: $(1-\alpha)X > \rho_s X$ for any X.*

In the presence of dynamic collateral management, the market is more responsive to an increased demand for safe debt. Figure 7 illustrates the comparative statics with respect to non-pecuniary benefit γ. When γ is very small, all institutions choose to hold static portfolios. Once γ is above a threshold level γ_{min}, a fraction λ of institutions incur the fixed cost ξ to adopt dynamic collateral management. At this threshold, risk sharing leads to a discrete jump in both debt capacity ρ and CLO lending x_d. Accordingly, there is a discontinuous increase in aggregate lending X and safe debt D. As γ increases further, the lending of both CLOs and loan funds grows. Meanwhile, the equilibrium price ratio z^* drops, thereby keeping the safety premium shared between CLOs and loan funds.[19]

The aggregate quantities generated by dynamic portfolios can be compared against equilibrium with static portfolios. The equilibrium total lending X exceeds X_s, which translates into greater total debt capacity, even if individual debt capacity per unit of loans remained at ρ_s. The actual total safe debt $D = (1-\alpha)X$ is even greater than $\rho_s X$ because of risk sharing.

[19] Although a declining price ratio tightens up CLO collateral constraints and lowers ρ, total safe debt increases because the increases in lending volumes and the fraction of institution operating CLOs.

4.2. Price Distortion and Total Surplus

A salient feature of dynamic collateral management is that when CLOs rebalance portfolios, they exert a pressure on relative loan prices. As a result, the price of low-quality loans will decrease relative to the price of high-quality loans. My model shows that this price pressure is an inherent equilibrium result, which arises as institutions facing external demand for safe debt optimize their balance sheets without financing frictions.

Interestingly, the magnitude of this price pressure is informative about the market-wide total surplus achieved with the help of dynamic collateral management. Given that price-taking investors break even in expectation, this total surplus equals the sum of expected payoffs across all institutions: $TS := \int_{i \in \mathcal{I}} V_i \, \mathrm{d}i$. In equilibrium $V_d = V_e$, so $TS = V_e = f(y)$, where $y = pR(\alpha + \frac{1-\alpha}{z^*})$. Thus, given the exogenous investment technology (e.g., p, R, α), total surplus TS is captured by a single endogenous variable: equilibrium price ratio z^*. Moreover, since f is strictly increasing, TS and z^* are negatively associated with each other.

Proposition 3. *When dynamic collateral management is adopted, equilibrium market-wide total surplus is greater when loan prices deviate more from fundamental values.*

Intuitively, as individual lending and financing choices are functions of loan prices, the equilibrium price ratio is sufficient to summarize all the gains from lending and safe debt creation. The positive relationship between price distortion and total surplus arises from the mechanism of this market's equilibrium. Because the equilibrium loan prices always adjust to equalize CLOs' and loan funds' expected payoffs, the total surplus is greater whenever institutions operating loan funds are better off. This occurs precisely when the price pressure is more severe and liquidity provision is more profitable. Therefore, my model suggests that in this market, price distortion is a reflection of value creation rather than a symptom of frictions that constrain liquidity provision.

4.3. Self-Stabilizing Aggregate Quantities

My model features a feedback loop between institutions' balance sheets and loan prices, which operates through CLOs' price-dependent collateral constraints. This feedback gives rise to a self-stabilizing effect on aggregate quantities. Specifically, the direct effect of a shock to this market tends to be accompanied by an opposite indirect effect, which attenuates the net effect of the shock on aggregate quantities in equilibrium.

[Add Figure 8 here]

I illustrate this effect with two examples. Figure 8 presents the equilibrium under different values of ξ, the fixed cost of adopting dynamic collateral management.[20] The direct effect of a higher ξ is that, at the extensive margin, a smaller fraction of institutions operate CLOs, so the total supply of safe debt D may decrease. But meanwhile, an indirect effect exists at the intensive margin. As fewer CLOs demand liquidity in the secondary market, the price pressure eases. As a result, equilibrium price ratio z^* relaxes the remaining CLOs' collateral constraints, which in turn induces these CLOs to lend more than before: both ρ and x_d increase. Overall, the net impact of the increase in ξ on aggregate lending X and safe debt D depends on these two counteracting effects. The impact could be limited, as long as ξ does not exceed the maximum level ξ_{max}, beyond which all institutions choose static portfolios.

[Add Figure 9 here]

In the second example, I consider a change in $\bar{\alpha}$, the worst-case realization for the fraction of low-quality loans in a portfolio. This may be thought of as a regime shift that systematically weakens loan covenants, which lowers default recovery rates in bad times. Figure 9 shows that if $\bar{\alpha}$ is very close to α, institutions would hold static portfolios because the benefit of risk sharing is limited. When $\bar{\alpha} > \bar{\alpha}_{min}$, the direct effect of an increase in $\bar{\alpha}$ is at the

[20] The increase in ξ can be viewed as a shock to the cost of operating CLOs, such as the introduction of a regulatory burden on CLO managers. In Appendix IA.4.2, I discuss a real-world regulation that increased the cost of operating CLOs.

intensive margin: CLO debt capacity $\rho = 1 - \bar{\alpha} + \bar{\alpha} z^*$ declines as the worst-case portfolio payoff falls. This reduces the safe premium captured by CLOs, and equilibrium price ratio z^* increases to equalize institutions' payoffs. As a result, lending by both CLOs (x_d) and loan funds (x_e) decrease. However, at the extensive margin, more institutions may operate CLOs since replacing deteriorated loans becomes less costly. As such, the shock's impact on total quantities of lending X and safe debt D is mitigated by an increase in λ.

4.4. Discussions

4.4.1. Why Do CLO Securities Have Long Maturities?

My focus on long-term debt leaves the question open as to why CLOs do not issue short-term debt, which can rollover in normal times and trigger liquidation in bad times. I argue that the observed CLO debt maturity is an equilibrium outcome: Given that the leveraged loan market is segmented from public securities markets and that it is difficult for outsiders to buy liquidated loans, issuing long-term safe debt is optimal for CLOs.

This argument can be formalized by introducing a costly storage technology at $t = 0$, which allows investors to purchase loans and institutions to repay debt at $t = 1$. When investors' storage cost is high, so is their required return from liquidated loans. As a result, liquidating loans and repaying debt will be costly to CLO equity holders. Thus, long-term contract, which helps CLOs maximize and maintain cheap leverage, will be preferred. This argument explains the maturity of CLO debt based on the high participation costs faced by potential buyers who are not active lenders in this market.

4.4.2. Will Institutions Internalize Loan Trades?

My model allows institutions to flexibly choose external financing. It is in principle possible that an institution operates two entities with very different liabilities (e.g., a CLO and a loan fund), which seems appealing because the institution can then internalize loan trades after the quality of its portfolio deteriorates. That is, instead of buying and selling loans in the secondary market, the manger could reallocate loans between the entities it operates.

However, doing so is suboptimal for the institution. This is because without trading in the secondary market, the quantity of safe debt an institution can create is constrained by its initial loan portfolio ($d_i \leq \rho_s$), which is dominated by its choices in equilibrium ($V_d = V_e > V_s$). Therefore in equilibrium, institutions tend to specialize, which is consistent with Fact 1.

4.4.3. Contractual Frictions

Dynamic collateral management relies on long-term contracts to discipline portfolio choices. These contracts are enforced with publicly observable risk metrics, primarily credit ratings, which do not perfectly measure loan quality.[21] To simplify the analysis, my model has assumed that institutions can fully commit to replacing low-quality loans, even if such trades reduce their own payoffs. Nonetheless, as long as imperfectly contractible risk metrics are sufficiently informative about loan quality, the contract should still be calibrated and adjusted to implement desired outcomes (e.g., over-collateralization provisions).[22]

4.4.4. Valuation of CLO Securities and Loan Fund Shares

Because all agents have linear preferences, my model is silent about the pricing of nonbank liabilities when marginal investors are risk-averse. That said, the intuition provided by my analysis suggests that it is important to account for the effects of CLO contracts on portfolio dynamics in the valuation of CLO securities and loan fund shares.

Evaluating the risk of CLO securities solely based on current balance sheets understates the safety of senior tranches and the risk of junior (equity) tranches. Because CLO managers commit to maintain portfolio quality, junior debt and equity tranches tend to lose more value

[21]Secondary loan prices are often available and more informative for distressed borrowers, so CLO contracts typically use par values except for severely deteriorated loans. As a strategic response, CLO managers may engage in par-building trades that inflate their OC test scores (Loumioti and Vasvari, 2019).

[22]In a dynamic environment with repeated interactions, the asset manager's reputation can also provide incentives for maintaining portfolio quality. Analyzing this channel is beyond the scope of the current paper.

in bad times than they would with static portfolios. Therefore, when portfolios are actively managed, junior tranche investors may require additional compensation for riskier returns. Meanwhile, loan funds' shares are less risky than their current portfolios, because these funds' losses in bad times will be mitigated by their trades with CLOs. These endogenous pro-cyclical and counter-cyclical payoffs should be considered by investors and analysts.

4.4.5. Dynamic Collateral Management in Other Markets

While my paper focuses its attention on the leveraged loan market, dynamic collateral management has been also adopted in other financial markets. For example, securitization vehicles called CRE CLOs create securitized tranches backed by actively-managed commercial real estate mortgage portfolios.[23] In the market of securities backed by credit card receivables, the sponsors often purchase high-quality accounts and remove delinquent accounts in the collateral pool under the monitoring of a rating agency. Similarly, to enlarge debt capacity and keep loans solvent, a cryptocurrency-backed lending platform named ALEX developed a Collateral Rebalancing Pool ("CRP") technology. The CRP uses algorithms to dynamically rebalance collateral pools between riskier digital assets (e.g., Bitcoins) and less risky tokens as market conditions evolve. Therefore, the framework in my paper can potentially help interpret empirical findings and inform policy designs in these markets.

5. Conclusion

This paper analyzes a lending market where securitized tranches are backed by dynamic loan portfolios. Before the 2007–09 financial crisis, the securitization industry manufactured large quantities of senior tranches backed by static loan portfolios. Many of these tranches defaulted because their underlying loans deteriorated and failed to generate sufficient cash flows for repayment. By contrast, in the leveraged loan market, CLOs have been creating

[23] According to BofA Global Research, during the COVID pandemic, active portfolio management helped CRE CLOs achieve considerably lower delinquency rates than the conduit and SASB deal.

AAA-rated securities for more than three decades without any default record.

The financial innovation of CLOs is dynamic collateral management whereby a long-term contract obligates the managers to dynamically maintain the quality of the underlying loan portfolios. This contract generates an intertemporal tradeoff: it helps CLOs create larger safe tranches ex ante but triggers ex-post portfolio substitution, which exerts price pressure in the secondary market. This paper develops a model to understand how this mechanism drives loan prices and the quantities of lending and safe debt. My model provides an equilibrium framework to interpret the coexistence of CLOs and loan funds and the trades that reallocate loans across these institutions. It also sheds light on on how portfolio dynamics raises the supply of safe debt, improves total surplus, and drives the market's responses to shocks.

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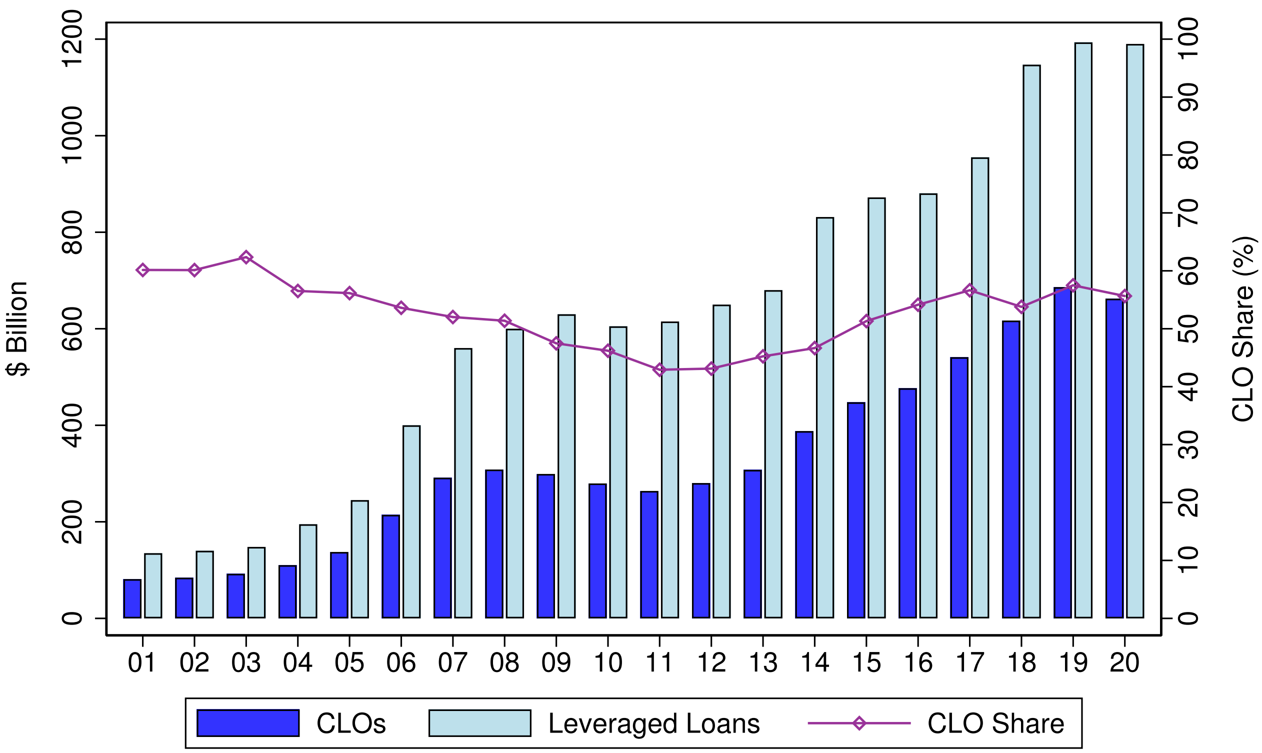


Figure 1: **Leveraged Loans and CLOs Outstanding in the US, 2001–2020.**
This figure plots annual total par values outstanding for leveraged loans (i.e., institutional term loan facilities) and CLOs in the US market. Data source: SIFMA.

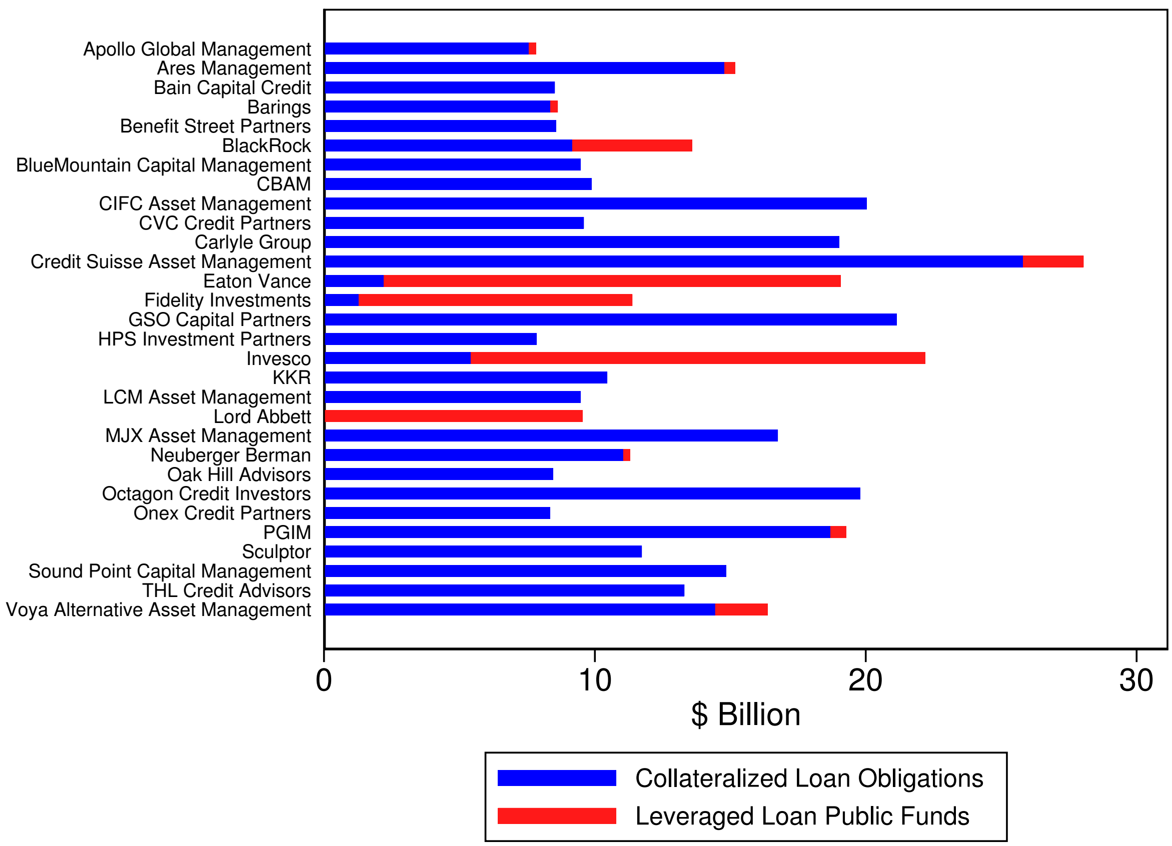
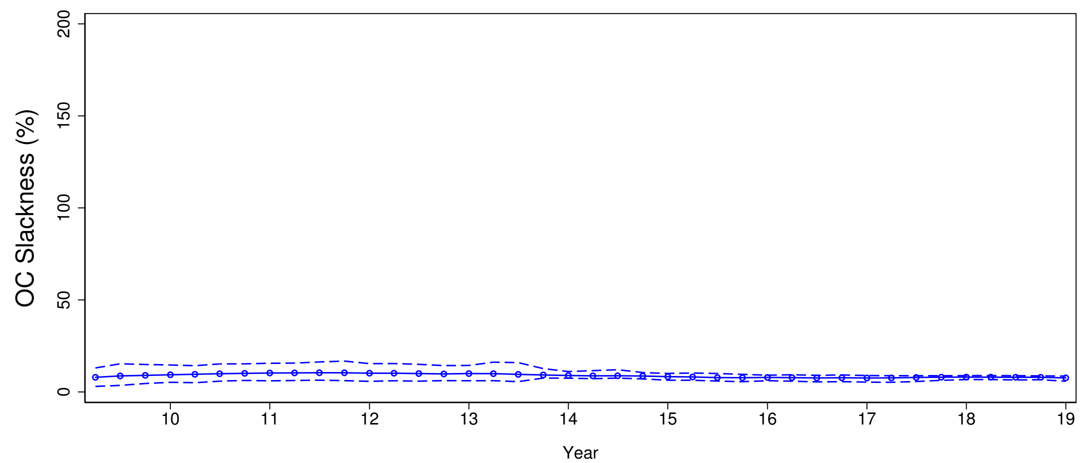
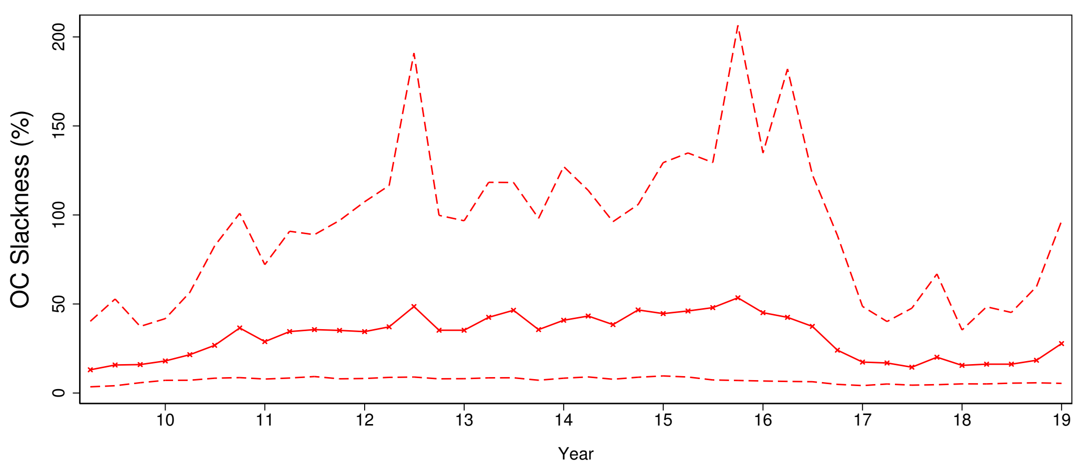


Figure 2: **Asset Managers and Nonbank Institutions.**
This figure presents assets under management for US CLOs and public loan funds (the sum of open-end mutual funds, closed-end mutual funds, and exchange-traded funds) operated by the 30 largest asset managers at the end of 2019. Data come from Creditflux CLO-i, Morningstar, and the SEC's Form ADV databases.

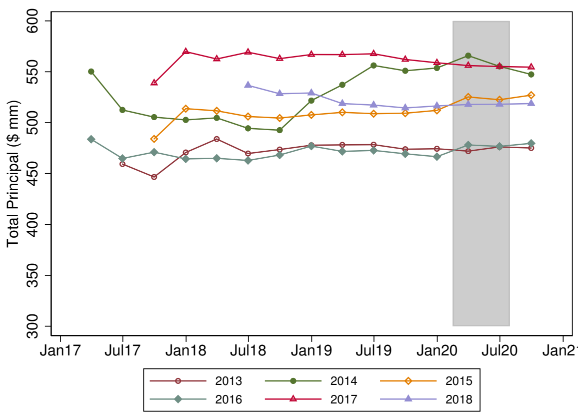


(a) CLOs in Reinvestment Period

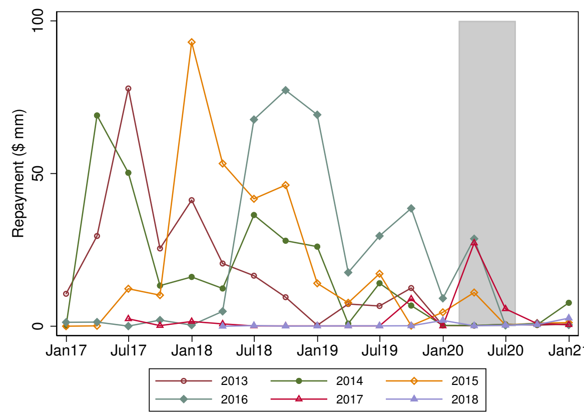


(b) CLOs in Amortization Period

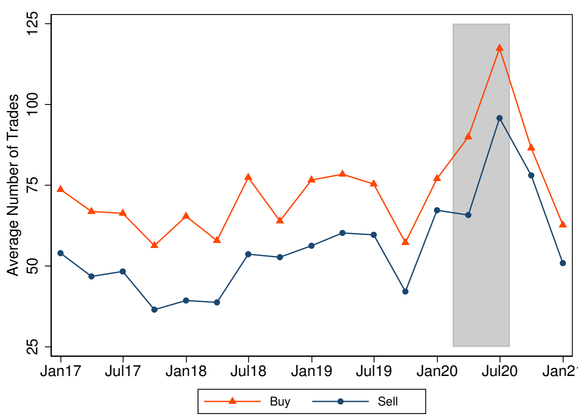
Figure 3: **Slackness of Senior Tranche Over-Collateralization Constraint.**
This Figure presents quarterly time series of cross-sectional dispersion in the slackness of CLO senior tranche over-collateralization (OC) constraints between 2010–2019. The slackness is defined as extra OC score scaled by the OC test's predetermined threshold level. Dashed lines indicate 5th and 95th percentiles in each cross section. Panel (a) reports CLOs in reinvestment period, and panel (b) reports CLOs in amortization period.



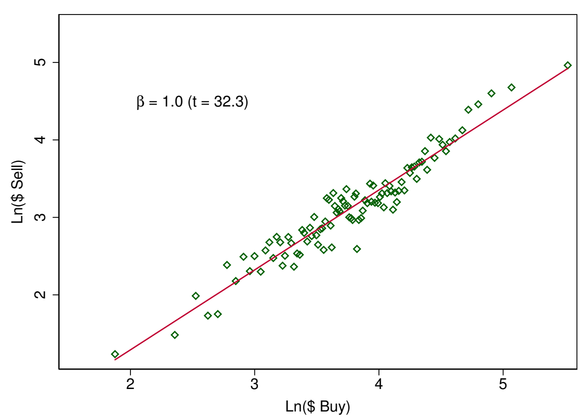
(a) Portfolio Total Loan Holdings



(b) Accelerated Debt Repayment

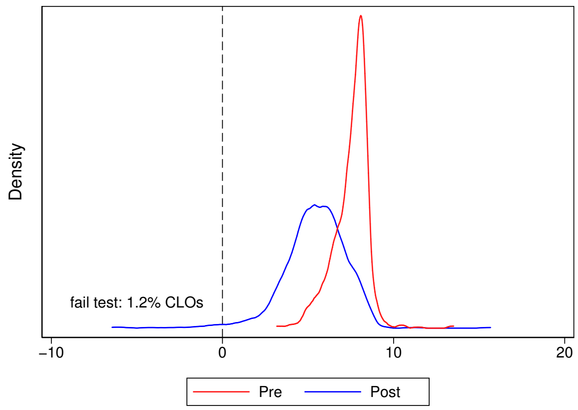


(c) Quarterly Loan Trades

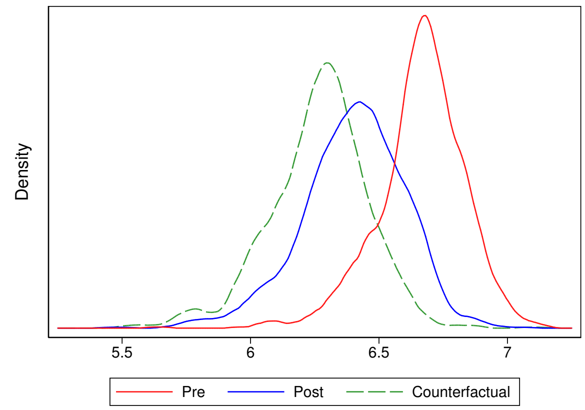


(d) Individual CLOs' Purchases and Sales

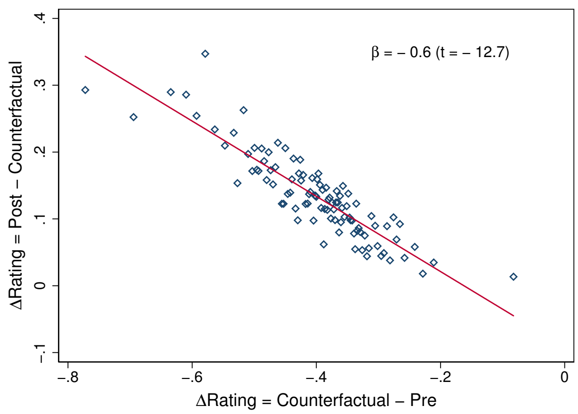
Figure 4: **Balance Sheet Dynamics Around the Onset of COVID-19 Pandemic.** This Figure shows quarterly changes in CLOs' assets and liabilities before and during the COVID-19 shock in 2020. Panel (a) plots average portfolio size by CLO age cohort. Panel (b) plots average accelerated repayment of AAA tranches by CLO age cohort. Panel (c) plots quarterly average numbers of loan purchases and sales. Panel (d) is a scatter plot that groups CLOs into 100 bins based on the dollar volumes of individual CLOs' loan purchases and sales during the first two quarters of 2020. Only CLOs in reinvestment period are included.



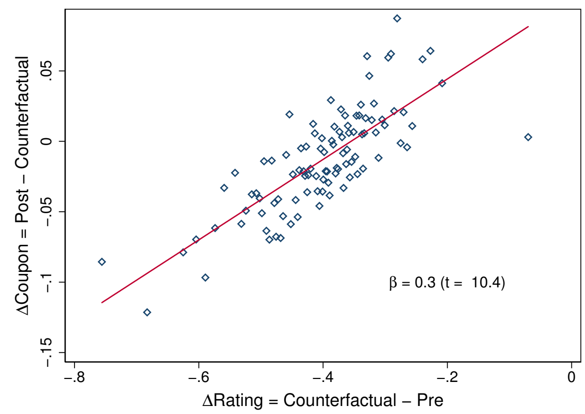
(a) Slackness of OC Constraint (%)



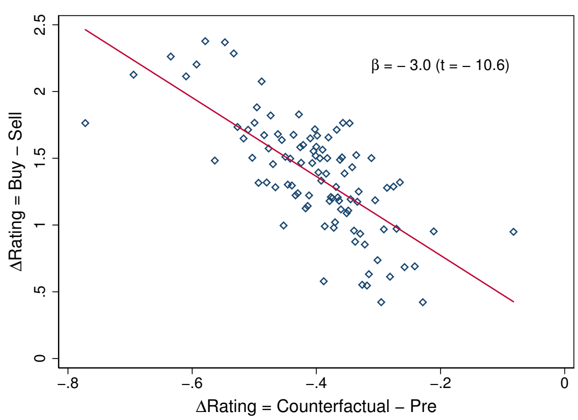
(b) Portfolio Value-Weighted Average Rating



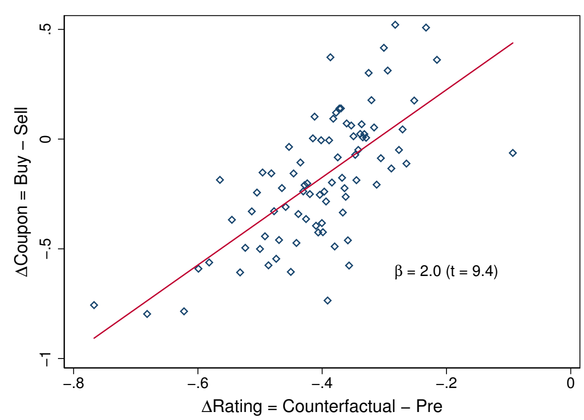
(c) Quality Improvement: Rating



(d) Quality Improvement: Spread



(e) Trading Direction: Rating



(f) Trading Direction: Spread

Figure 5: **Portfolio Substitution Improves Collateral Quality.**
This Figure shows the effect of portfolio substitution on CLOs' collateral quality between February 15 and June 30 of 2020. Panel (a) plots kernel density estimates for the distribution of senior OC constraint slackness before and after the onset of COVID-19 pandemic. Panel (b) plots kernel density estimates for the distribution of value-weighted average credit rating for portfolios before and after the shock as well as counterfactual static portfolios. Panels (c)–(f) are scatter plots that group CLOs into 100 bins by counterfactual collateral deterioration and depict the average effect of loan trading within each bin. The fitted lines represent OLS estimates, and t-statistics are based on heteroskedasticity-robust standard errors. Only CLOs in reinvestment period are included.

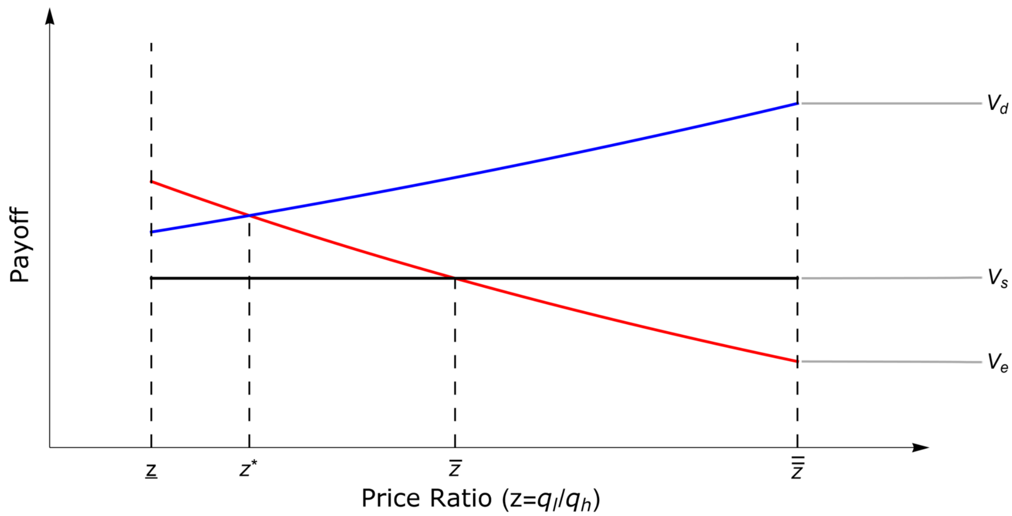


Figure 6: **Determination of Equilibrium.**
This figure illustrates how the equilibrium is determined. Solid lines V_d, V_e, and V_s indicate the payoffs of CLOs, loan funds, and institutions issuing safe debt backed by static portfolios, respectively, as functions of secondary market loan price ratio $z = q_l/q_h$. A unique equilibrium with dynamic portfolios exists and has price ratio z^*. Parameter values and functional forms: $p = 0.9$, $R = 1.2$, $\alpha = 0.4$, $\gamma = 0.15$, $\bar{\alpha} = 0.7$, $\xi = 0.05$, and $c(x) = \frac{4}{5}x^{\frac{5}{4}}$.

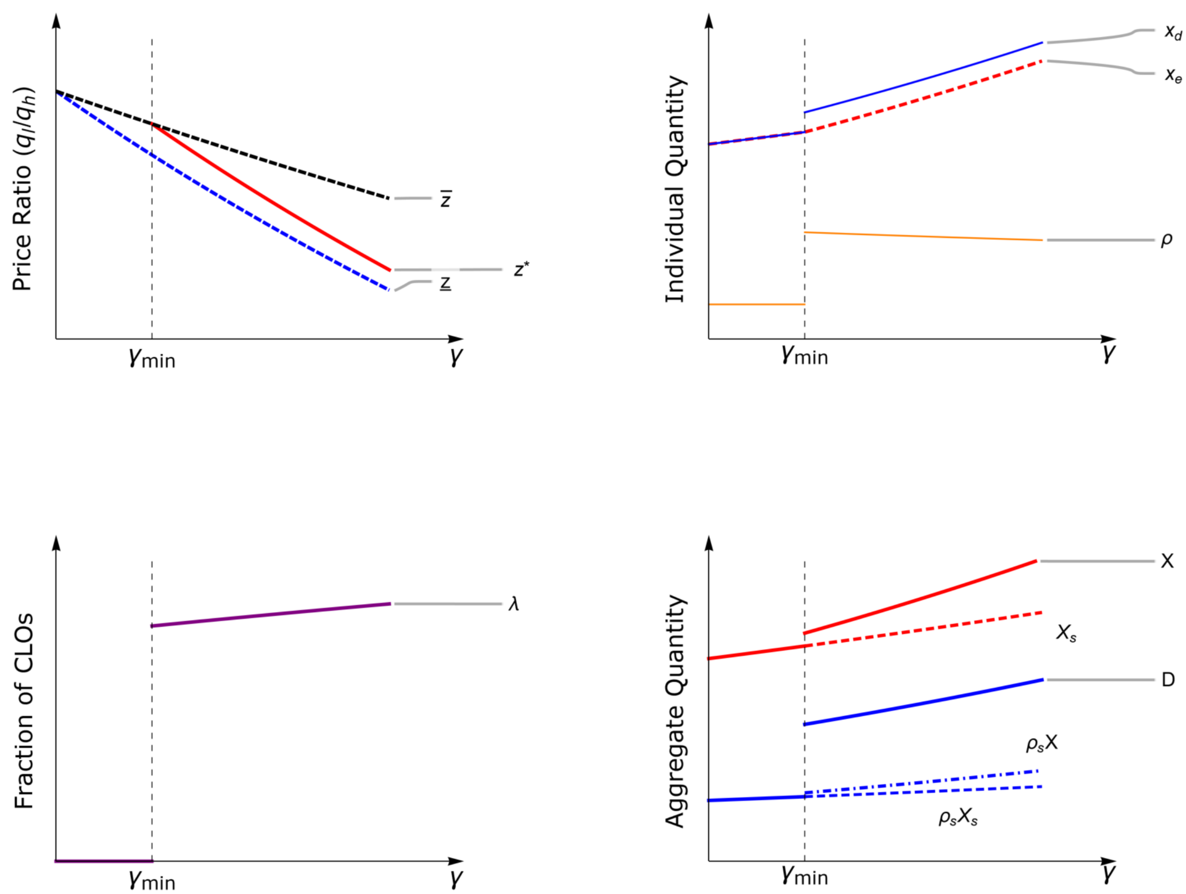


Figure 7: **The Market's Responses to Changes in Safety Premium.**
This figure illustrates equilibrium prices and quantities when the safety premium γ changes. Parameter values: $p = 0.9$, $R = 1.2$, $\alpha = 0.4$, $\bar{\alpha} = 0.7$, $\xi = 0.05$, and functional form $c(x) = \frac{4}{5}x^{5/4}$.

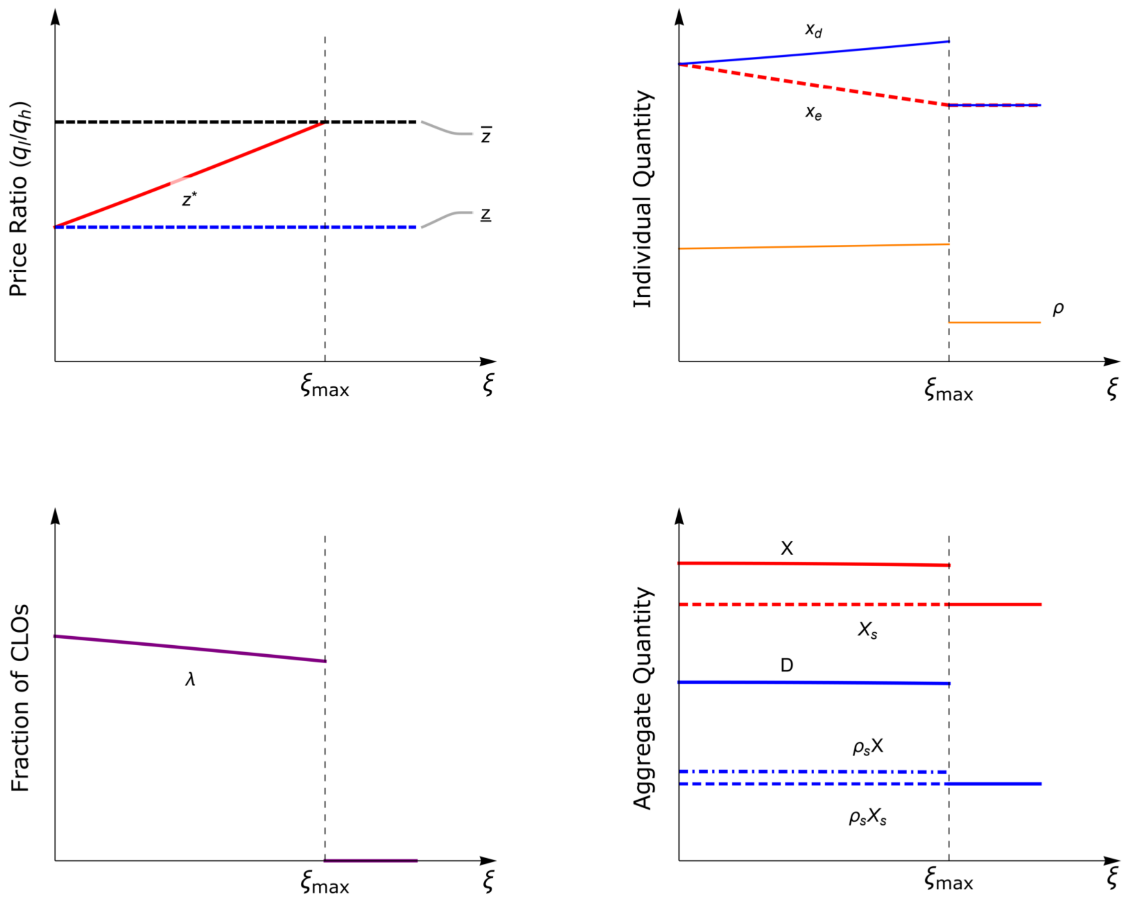


Figure 8: **The Fixed Cost of Dynamic Collateral Management.**
This figure illustrates equilibrium prices and quantities when the fixed cost of dynamic collateral management, ξ, changes. Parameter values: $p = 0.9$, $R = 1.2$, $\alpha = 0.4$, $\bar{\alpha} = 0.7$, $\gamma = 0.15$, and functional form $c(x) = \frac{4}{5}x^{5/4}$.

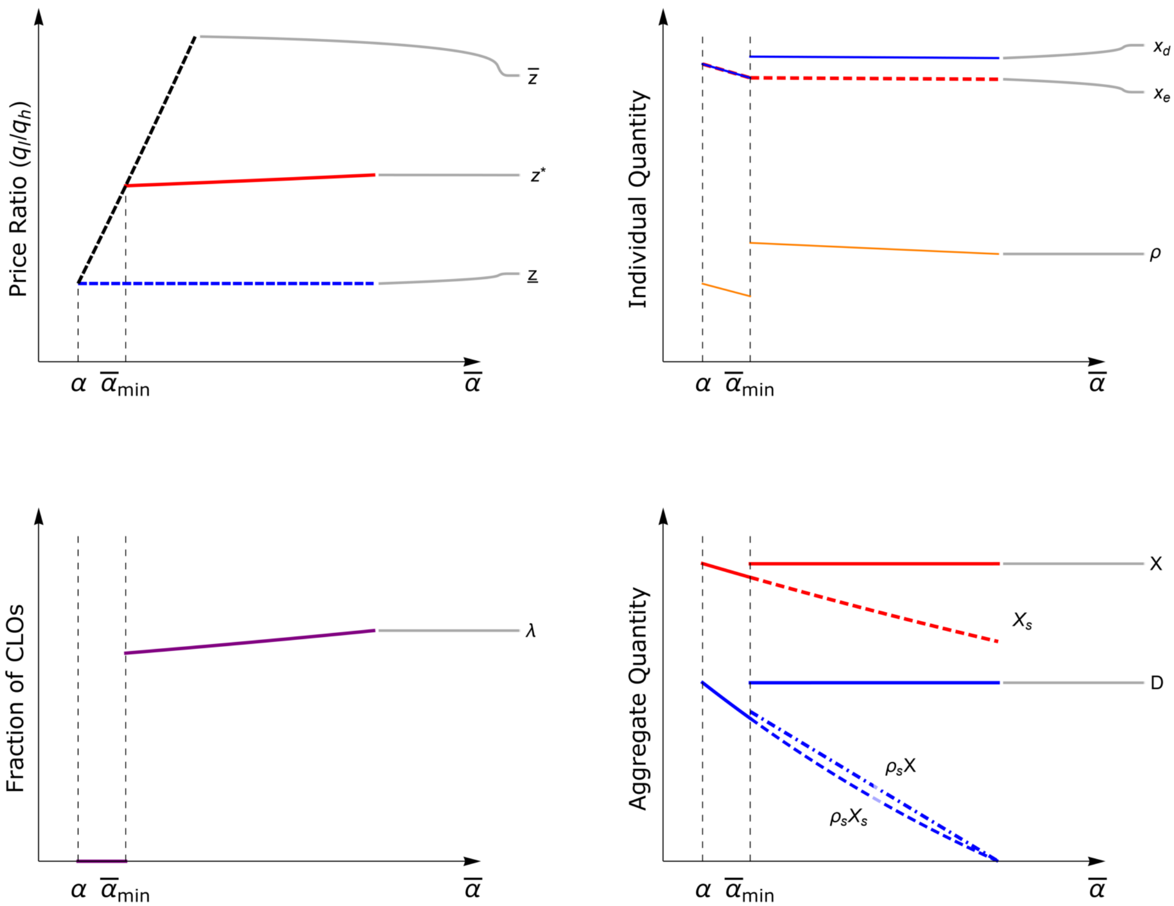


Figure 9: **The Worst-Case Fraction of Low-Quality Loans in Portfolio.**
This figure illustrates equilibrium prices and quantities when the worst-case realization of portfolio deterioration, $\bar{\alpha}$, varies over $(\alpha, 1]$. Parameter values: $p = 0.9$, $R = 1.2$, $\alpha = 0.4$, $\gamma = 0.15$, $\xi = 0.05$, and functional form $c(x) = \frac{4}{5}x^{5/4}$.

Appendix: Model Proofs

Proof of Lemma 1. Given $\Delta \mathbb{x}_i = 0$, $v(x_i, d_i, \alpha_i) = x_i(pR + (1-\alpha_i)(1-p)) - d_i$, and the institution's $t = 0$ problem reduces to

$$V_s = \max_{x_i, d_i \geq 0} \; x_i y_0 + \gamma d_i - c(x_i) \tag{A.1}$$

$$s.t. \; 0 \leq d_i \leq \rho_s x_i \tag{A.2}$$

Since the objective strictly increases in d_i, constraint $d_i \leq \rho_s x_i$ binds. The first-order condition with respect to x_i is $y_0 - c'(x_i) + \eta_s \rho_s = 0$, where $\eta_s = \gamma$ is the shadow price of the binding collateral constraint. This gives $x_s = c'^{-1}(y_0 + \gamma\rho_s)$ and $d_s = \rho_s x_s$. Note that $c' > 0$ and Condition 1 guarantee that $x_s > 0$, and that Condition 2 ensures all institutions participate.

Proof of Lemma 2. Suppose $\frac{q_l}{q_h} > \frac{\mathbb{E}[R_l]}{\mathbb{E}[R_h]}$, the objective in program (8) would be strictly decreasing in $\Delta x_{i,l}$, and the optimal choice would be $\Delta x_{i,l} = -x_{i,l}$ for all $i \in \mathcal{I}$. This contradicts the market clearing condition (7). It is also impossible that $\frac{q_l}{q_h} = \frac{\mathbb{E}[R_l]}{\mathbb{E}[R_h]}$. This is because otherwise, optimal choice in (5) would be $d_i = \rho x_i$ for all i, where $\rho = 1 - \bar{\alpha} + \bar{\alpha}\frac{\mathbb{E}[R_l]}{\mathbb{E}[R_h]}$. The maintenance collateral constraint in (8) then implies $\int \Delta x_{i,l} \, \mathrm{d}i \leq \frac{q_h}{q_l} x_i(\bar{\alpha} - \alpha - \bar{\alpha}\frac{\mathbb{E}[R_l]}{\mathbb{E}[R_h]})$. Given Condition 3, $\bar{\alpha} - \alpha < \bar{\alpha}\frac{\mathbb{E}[R_l]}{\mathbb{E}[R_h]}$, so $\int \Delta x_{i,l} \, \mathrm{d}i < 0$, a contradiction to market clearing.

Proof of Lemma 3. Substitute $\frac{q_l}{q_h} = \underline{z}$ into equation (14).

Proof of Lemma 4. Let $\mathcal{I}' \subseteq \mathcal{I}$ denote the set of institutions that choose dynamic portfolios. Given (9), it is obvious that if all institutions in $\mathcal{I}'$ choose equity financing ($d_i = 0$), then total demand for high-quality loans is $\int_{i \in \mathcal{I}'} \Delta x_{i,h} \, \mathrm{d}i \propto -(1-\alpha)x_e < 0$, hence the market cannot clear. If instead all institutions choose debt financing ($d_i = \rho x_d$), the total demand is $\int_{i \in \mathcal{I}'} \Delta x_{i,h} \, \mathrm{d}i \propto (\alpha - \bar{\alpha} + \bar{\alpha} z)x_d$. By Lemma 3 and Condition 3, $\alpha - \bar{\alpha} + \bar{\alpha} z \geq \alpha - \bar{\alpha} + \bar{\alpha}\underline{z} > 0$, hence the market cannot clear.

Proof of Lemma 5. Recognize that $y > y_0 + \gamma\rho_s$ if and only if $z < \bar{z}$. Given the monotonicity of f, this implies that $V_e > V_s$ if and only if $z < \bar{z}$. If $z > \bar{z}$, $V_e < V_s$, so no institution would choose equity financing, and hence by Lemma 4, all institutions choose static portfolios.

Proof of Lemma 6. By construction, f is continuously differentiable, and

$$f'(y) = c'^{-1}(y) + y \cdot \frac{\mathrm{d}}{\mathrm{d}y} c'^{-1}(y) - c'(c'^{-1}(y)) \cdot \frac{\mathrm{d}}{\mathrm{d}y} c'^{-1}(y) = c'^{-1}(y) > 0. \tag{A.3}$$

Hence f is strictly increasing. Given that $y = pR(\alpha + z^{-1}(1-\alpha))$, $V_e = f(y)$ is strictly decreasing in z. Also, by Condition 3, $\underline{z} > 1 - \frac{\alpha}{\bar{\alpha}}$, so

$$\begin{aligned} \frac{\mathrm{d}(y + \eta\rho)}{\mathrm{d}z} &= \bar{\alpha}(pR + 1 - p + \gamma) - pR\frac{\bar{\alpha} - \alpha}{z^2} && \text{(A.4)} \\ &> \bar{\alpha}\Big(pR + 1 - p + \gamma - pR\frac{\underline{z}}{z^2}\Big) && \text{(A.5)} \\ &\geq \bar{\alpha}\Big(pR + 1 - p + \gamma - pR\frac{1}{\underline{z}}\Big) = 0, && \text{(A.6)} \end{aligned}$$

where the last equation follows by the definition of $\underline{z}$. Since $V_d = f(y + \eta\rho) - \xi$, this implies that V_d is strictly increasing in z.

Proof of Proposition 1. Define the utility differential between debt financing (V_d) and equity financing (V_e) as a function of the price ratio, $\Delta v : [\underline{z}, \bar{z}] \mapsto \mathbb{R}$. By Lemma 6, $\Delta v(z) = V_d - V_e$ is strictly increasing. At $z = \underline{z}$, $\eta = 0$ and $V_d = V_e - \xi$, so $\Delta v(\underline{z}) < 0$. Moreover, given that f is strictly increasing, for any $z > \underline{z}$, $V_d + \xi = f(y + \eta\rho) > f(y) = V_e$, so $\Delta v(\bar{z}) > 0$ whenever ξ is not too large. Since Δv is continuously differentiable by construction, by intermediate value theorem, equation $\Delta v(z) = 0$ has a unique solution $z^* \in (\underline{z}, \bar{z})$. By Lemma 5, at $z = z^*$, $V_d = V_e > V_s$, hence all institutions hold dynamic portfolios.

Given the optimal lending and financing choices, as well as trades in (9), market clearing requires $\lambda(\rho - (1-\alpha))x_d = (1-\lambda)(1-\alpha)x_e$, where λ is the fraction of institutions choosing $d_i = (1-\rho)x_d$. By Lemma 3 and Condition 3, $\rho - (1-\alpha) = \alpha - \bar{\alpha} + \bar{\alpha}z \geq \alpha - \bar{\alpha} + \bar{\alpha}\underline{z} > 0$, therefore $\lambda = \frac{(1-\alpha)x_e}{(1-\alpha)x_e + (\rho-(1-\alpha))x_d} \in (0,1)$.

Proof of Lemma 7. In equilibrium $V_d = V_e > V_s$, or equivalently, $f(y+\eta\rho) > f(y+\eta\rho) - \xi = f(y) > f(y_0 + \gamma\rho_s)$. Since f is strictly increasing, this implies $y + \eta\rho > y > y_0 + \gamma\rho_s$. Therefore, $x_d = c'^{-1}(y + \eta\rho) > x_e = c'^{-1}(y) > x_s = c'^{-1}(y_0 + \gamma\rho_s)$.