Portfolio Dynamics and the Supply of Safe Securities

David Xiaoyu Xu†

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Abstract

This paper explores a notable development in securitization—the use of dynamic portfolios as collateral in Collateralized Loan Obligations (CLOs). I model the endogenous formation of CLOs within an industry equilibrium of nonbank lending. The model highlights CLOs' portfolio rebalancing, which maintains higher collateral quality by replacing deteriorated loans. This "self-healing" mechanism creates larger safe tranches ex ante, reducing CLOs' funding costs. In equilibrium, the capital structure of CLOs depends on secondary loan prices, which deviate from fundamentals due to price pressure caused by their own portfolio rebalancing. Overall, the mechanism facilitates risk sharing across nonbank institutions, increasing both total lending and the supply of safe securities but potentially risking financial stability.

†Southern Methodist University, Edwin L. Cox School of Business, 6212 Bishop Blvd. Dallas, TX 75275. Email: davidxu@smu.edu. I am grateful to Aydogan Alti, Daniel Neuhann, Clemens Sialm, Vasiliki Skreta, and Sheridan Titman for their guidance. For helpful comments I thank Andres Almazan, Philip Bond, John Chu, William Fuchs, Sebastian Infante, Pab Jotikasthira, Jangwoo Lee, Robert Marquez, Max Miller, Martin Oehmke, Aaron Pancost, Alessio Piccolo, Alejandro Rivera, Dimitri Vayanos, Douglas Xu, Hongda Zhong, and participants at Baruch College, BI Norwegian Business School, Cheung Kong Graduate School of Business, Chinese University of Hong Kong, Chinese University of Hong Kong at Shenzhen, Hong Kong University of Science and Technology, London School of Economics, Shanghai Advanced Institute of Finance, Southern Methodist University, Southern University of Science and Technology, University of Rochester, University of Texas at Austin, University of Washington, 2021 Young Economist Symposium, 2022 Econometric Society European Meetings, 2022 EFA Meetings, 2022 NFA Meetings, 2022 FMA Meetings, 2022 Qatar Centre for Global Banking and Finance Conference, 2022 Financial Markets and Corporate Governance Conference, 2023 OFR Rising Scholars Conference, 2023 OCC Emerging Risk in Banking Symposium, 2023 Econometric Society North American Meetings, and 2024 MFA Meetings. This paper was previously circulated under the title "Financial Market Structure and the Supply of Safe Assets: An Analysis of the Leveraged Loan Market". All errors are my own.

Investors often place special value on highly-rated securities with low default risk.[1] Catering to this preference, financial institutions have engaged in securitization, transforming risky loans into securities that are safer than the original loans. Traditionally, loan portfolios used as collateral for these securities were static. However, in recent decades, there has been a growing trend of using dynamic portfolios as collateral for safe securities. This practice is exemplified by Collateralized Loan Obligations (CLOs). By 2023, these nonbank institutions have created over $1 trillion of securities backed by corporate loans. Senior CLO tranches, accounting for 65% of these securities, are AAA-rated and have zero historical defaults.[2] Notably, CLOs dynamically rebalance portfolios by trading loans with peer institutions. The rapid growth of this market has attracted significant attention, but its core function—safety transformation—is not well understood.

This paper explores the mechanism by which CLOs create long-term safe debt tranches under uncertainty about the future quality of their underlying loans. In any securitization, the size of safe tranches is sensitive to the underlying portfolio's future cash flows, which might be very low if the quality of loans deteriorates over time. By rebalancing portfolios, CLOs replace deteriorated loans before the loans' cash flows are realized. This mechanism, referred to as "self-healing" by practitioners (e.g., Blackstone, 2020; Mellinger, 2023), enables a CLO to create a *larger* safe tranche from a given ex-ante loan portfolio, thereby reducing its funding costs. However, as CLOs' portfolio rebalancing collectively influences secondary loan prices, the benefits and costs of this mechanism should be equilibrium outcomes.

In this paper, I analyze how CLOs' self-healing mechanism drives the lending, safe debt creation, and the structure of the leveraged loan market. My analysis is motivated by new facts on CLOs' portfolio rebalancing, which substantially improves collateral quality while

[1]Recent literature documents a special demand for highly-rated safe securities, which arises from these securities' liquidity and regulatory advantages (e.g., Krishnamurthy and Vissing-Jorgensen, 2012; Gorton, Lewellen, and Metrick, 2012; Nagel, 2016; Van Binsbergen, Diamond, and Grotteria, 2022).

[2]Senior CLOs, while not riskless, satisfy investor demand for safe assets and relieve regulatory capital charges (Benmelech and Dlugosz, 2009; Cordell, Roberts, and Schwert, 2023). This provides lower risk-adjusted funding costs to CLO equity investors. See Section 1 for details on the demand for senior CLOs.

exerting pressure on loan prices. I develop a theoretical model that integrates this mechanism into an equilibrium framework. The model microfounds CLOs and other nonbank lenders, which endogenously arise and trade loans after ex-ante identical institutions make lending and financing decisions. Trading helps CLOs restore collateral quality, but it generates pressure on loan prices. I show that although this mechanism pushes prices away from fundamental values, it can raise overall nonbank lending, safe debt creation, and total surplus.

Like many securitization vehicles, CLOs create long-term securities backed by long-term loans, and the quality of these loans might deteriorate over time. But, unlike other loans, the loans held by CLOs ("leveraged loans") are rated by credit rating agencies and traded in secondary markets. This enables the self-healing mechanism. By imposing constraints tied to time-varying loan ratings, CLOs' contracts obligate CLO managers to maintain collateral quality. Since the leveraged loan market is populated by a large number of CLOs and loan funds, collateral quality can be maintained by trading with peer institutions.

I analyze the interactions of these institutions within an industry equilibrium model of nonbank lending. The model has two groups of agents: investors, who enjoy a non-pecuniary benefit from safe debt, and institutions, who can create safe debt backed by risky loans. There are three periods. Initially, institutions issue debt and equity tranches to investors and originate loans. Given the non-pecuniary benefit, safe debt tranches are issued at a premium, offering a source of cheap funding. Yet, the size of safe tranches is constrained by the payoffs of loan portfolios, which are realized in the last period. This constraint is tight because after issuance, idiosyncratic shocks will cause a random fraction of every institution's loan portfolio to deteriorate, potentially leading to very low payoffs.

My model highlights a novel dynamic link between debt safety and loans of different quality. Low-quality loans are riskier with a lower worst-case payoff, therefore high-quality loans are better collateral for safe debt. But loan quality is unpredictable and is revealed only when the idiosyncratic shocks are realized. So the size of safe tranches, if backed by static portfolios, is limited by the uncertainty in collateral quality. Nevertheless, institutions may rebalance portfolios. To capture the mechanism, I assume that by initially paying a fixed cost, they can credibly commit to sell low-quality loans and buy high-quality loans when quality reveals. This commitment raises the portfolio's worst-case payoff beyond that of a static portfolio, thereby enabling the institution to issue a larger safe tranche.

While all institutions are ex-ante identical, I show that in equilibrium they make distinct financing choices: Some institutions, which resemble CLOs, specialize in creating safe debt, whereas other institutions, like loan funds, do not create any safe debt. This endogenous mix of institutions is jointly determined with secondary market loan prices. Since CLOs are obligated to maintain collateral quality, the pressure exerted by their trades decreases the relative price of low-quality and high-quality loans. Through such non-fundamental price deviations, the safety premium captured by CLOs is shared with loan funds as a reward for liquidity provision. Because operating CLOs incurs a fixed cost, in equilibrium, the sharing of surplus will be partial—similar to that prices partially reveal costly information in Grossman and Stiglitz (1980)—and CLOs and loan funds will coexist.

This equilibrium framework helps interpret a variety of empirical findings. For example, CLOs typically sell deteriorated loans to mutual and hedge funds (Giannetti and Meisenzahl, 2021), and their loan sales exert downward pressure on secondary market prices (e.g., Elkamhi and Nozawa, 2022; Kundu, 2021; Nicolai, 2020). Conversely, CLOs buy higher-rated loans from mutual funds, supporting the prices of these loans (Emin et al., 2021). Different from these papers, which separately examine CLOs' sales and purchases, my model highlights portfolio rebalancing, a feature of CLOs' trading activities: empirically, there is a strong linear relationship between a CLO's loan sales and loan purchases.

Through the lens of the model, I find that safety transformation with dynamic portfolios raises the total supply of safe debt through two channels: risk sharing and increased lending. First, risk sharing increases total debt capacity because after idiosyncratic shocks, CLOs with deteriorated portfolios can restore quality by trading with other institutions. This ex-post reallocation of loans achieves an efficient use of scarce collateral, thereby increasing safe debt backed by each unit of loans. Second, as the collateral value of loans increases due to risk sharing, dynamic portfolios also improve the payoffs of lending. Better payoffs lead to higher lending volumes and hence more loans to back safe debt. These two channels are complementary: risk sharing raises the marginal payoffs of lending, and higher lending generates more loans to be traded among institutions. Overall, while only a subset of institutions operate CLOs, the market produces more safe debt in total.

My analysis explains that price deviation from fundamental values is an inherent feature of equilibrium in the leveraged loan market. Usually, such deviations are interpreted as a symptom of frictions constraining liquidity provision (e.g., Coval and Stafford, 2007; Mitchell, Pedersen, and Pulvino, 2007; Ellul, Jotikasthira, and Lundblad, 2011). In my model, however, loan price deviation occurs without such frictions, and the magnitude of deviation is larger precisely when the market's total surplus is greater. This is because, when safe debt is backed by dynamic portfolios, a subset of institutions give up issuing safe debt and instead profit from providing liquidity to CLOs. This liquidity provision is optimally imperfect, which allows the providers to share the safety premium with CLOs. As equilibrium prices equalize the expected payoffs of all institutions, market total surplus is greater when liquidity provision is better rewarded, or equivalently, when prices deviate to a larger extent.

Finally, I extend the model to explore the implications of CLOs' mechanism for financial stability when loan default correlations are underestimated. My extensions contrast two types of underestimated correlations–correlated shocks within a portfolio and those across institutions–and clarify their distinct impacts on the safety of senior CLO tranches. Specifically, CLOs' dynamic portfolios improve their senior tranches' resilience to underestimated correlated loan deterioration within a portfolio by facilitating an efficient use of collateral from the secondary market. In contrast, when the correlation of portfolio deterioration across institutions is underestimated, CLOs may be unable to satisfy collateral constraints through portfolio rebalancing, and their senior debt will no longer be safe, which in turn gives the CLO managers risk-shifting incentives. In this case, dynamic portfolios potentially amplify both the probability of default and the loss given default for senior tranches, exposing banks and insurance companies to losses and risking financial stability.

This paper extends the literature on safe debt creation, which dates back to Gorton and Pennacchi (1990). Focusing on banks' short-term safe debt, Stein (2012) presents a model in which the liquidation price of bank assets, set by exogenous buyers, constrains banks' debt capacity. My paper differs by studying how dynamic portfolios increase CLOs' long-term safe debt and how loan funds endogenously arise as trading counterparties.[3] This mechanism is also distinct from existing theories of securitization, which focus on static collateral pools (e.g., DeMarzo and Duffie, 1999; DeMarzo, 2005; Hanson and Sunderam, 2013). More broadly, the literature has studied the specific ways in which intermediaries create safe debt, including risk management (DeAngelo and Stulz, 2015), early liquidation (Stein, 2012; Hanson et al., 2015), deposit insurance (Hanson et al., 2015), asset opacity (Dang et al., 2017), and diversification (Diamond, 2020). My analysis adds a new perspective to these studies.

There is a growing body of empirical research on leveraged loans and CLOs. Recent papers, including the aforementioned by Giannetti and Meisenzahl (2021), Elkamhi and Nozawa (2022), Kundu (2021), Nicolai (2020), and Emin et al. (2021), study loan trades induced by CLOs' debt covenants. Consistent with a premium for safe tranches, Cordell, Roberts, and Schwert (2023) find that CLO equity earned positive abnormal returns. Foley-Fisher, Gorton, and Verani (2024) and Griffin and Nickerson (2023) examine CLO tranches' bid-ask spreads and credit ratings, respectively, in the COVID-19 crisis.[4] To the best of my knowledge, this paper is the first to analyze portfolio dynamics in safety transformation. It contributes a theoretical framework that unifies empirical findings around an important, yet understudied, mechanism that drives the two sides of CLOs' balance sheets.

[3]The feedback between ex-post trading prices and ex-ante investment and financing choices also exists in theories of fire sales (e.g., Shleifer and Vishny, 1992; Gorton and Huang, 2004; Diamond and Rajan, 2011). My model differs in that no liquidation is triggered by short maturity or moral hazard; Instead, long-term contracts obligate CLOs to rebalance portfolios, resulting in a pressure on relative prices.

[4]Griffin and Nickerson (2023) find that CLOs' loan trades improved rating agencies' collateral pool risk metrics. However, they do not study the relationship between these loan trades and CLOs' financing contracts, or its equilibrium implications.

The rest of this paper is organized as follows. Section 1 presents empirical facts that motivate my theoretical analysis. Section 2 introduces model setup. Section 3 characterizes the equilibrium. Section 4 discusses the model's implications, and Section 5 concludes.

1. Stylized Facts

This section introduces institutional background and empirical facts on leveraged loans and CLOs. Details on data can be found in Appendix IA.1.

The Leveraged Loan Market

Leveraged loans are broadly syndicated loans issued by corporations with a high financial leverage.[5] These loans are originated through syndication deals, where underwriters organize select groups of lenders to privately contract with the borrowers. In this paper, I restrict attention to term loans, which are mostly held by nonbank institutional investors, and exclude loan commitments held by banks (Federal Reserve Board, 2022).

[Add Figure 1 here]

Collateralized Loan Obligations. CLOs are the largest group of nonbanks that hold leveraged loans. As Figure 1 shows, US leveraged loans grew from $130 billion to $1.2 trillion between 2001–2020, and CLOs consistently held at least half of these loans. While other types of securitization are mostly backed by static collateral, CLOs' portfolios, consisting of 100–300 loan shares with $300-600 million total par values, are actively managed during a reinvestment period. CLO debt tranches mature in around 10 years, and the reinvestment period is around 5 years and often extended. After this, the CLO enters its amortization period and gradually repays debt principal.[6] The vast majority of CLOs are "open-market

[5] S&P Global Market Intelligence defines a loan as leveraged if it is rated below Baa3/BBB-, or if it is secured and has a spread of at least 125 basis points.

[6] In the amortization period, CLO managers can buy loans using only prepaid principal of existing loans. See Fitch's report for more details: Reinvestment in Amortization Period of U.S. CLOs.

CLOs", whose managers are independent from banks. The manager's compensation consists of size-based fixed fees and performance fees based on equity tranche returns.

Demand for Safe Debt. A primary force behind the growth of CLOs is the demand for highly-rated securities.[7] Senior CLO tranches, which account for about 65% of the capital structure, are AAA-rated. Since the 1990s, more than two thousand senior tranches have been issued, and none of them ever defaulted.[8] With higher yields than typical safe assets (e.g., US Treasuries) and fairly low regulatory risk weights, senior CLOs are attractive to, and mostly held by, banks and insurers. For example, Fitch (2019) reports that $94 billion, $113 billion, and $35 billion of senior CLOs are held by banks in the US, Japan, and Europe, respectively. While senior CLOs are not riskless, they may earn a "safety premium" as capital-constrained investors are willing to accept a lower *risk-adjusted* return for holding highly-rated securities (Cordell, Roberts, and Schwert, 2023). This premium is a source of cheap funding for CLOs.

Fact 1: Asset Managers Operate Both CLOs and Loan Funds The leveraged loan market consists of two types of nonbank institutions: CLOs and loan funds. Most of the loan funds are mutual funds and hedge funds, and Appendix IA.2 provides a detailed summary for the amount of loans held by these funds. Loan funds generally do not use their loans to back safe securities and face limited restrictions on portfolio choices.

[Add Figure 2 here]

Notably, CLOs and loan funds are often operated by a common group of asset managers. Figure 2 displays the largest asset managers that operate these nonbanks. For example, CVC Credit Partners only operates CLOs, whereas Fidelity Investments mainly manages leveraged loan mutual funds. These managers' choices lead to a coexistence of two types of nonbanks that both invest in leveraged loans but are financed by distinct liabilities.

[7]The safety of senior CLOs is with respect to default risk, as reflected by credit ratings. While these floating-rate notes are insensitive to interest-rate risk, they can be exposed to liquidity and inflation risks.

[8]A subset of senior CLOs were downgraded during 2007–09 but mostly recovered to original ratings. No AAA tranche was downgraded in the 2020 COVID-19 crisis.

Fact 2: CLOs' Binding Collateral Constraints

The size of leveraged loans, typically hundreds of million or even multiple billion dollars, creates economies of scale in information production. As a result, leveraged loans have credit ratings reflecting changes in loan quality and are traded in the secondary market. By contracting on loan ratings, CLO managers can credibly commit to dynamically replacing deteriorated loans as their quality evolves.

CLO contracts implement this commitment with regular (e.g., monthly) collateral tests that are tied to managerial compensation. The most important is the over-collateralization (OC) test, which calculates a ratio of quality-adjusted loan holdings to the total size of tranches that are senior or equal to a specific tranche. If an OC test fails, the manager stops receiving fees for that tranche until the ratio recovers to a preset threshold. The manager can raise the OC ratio through either debt acceleration (i.e., repay the principal of senior tranches) or portfolio rebalancing (i.e., replace deteriorated loans with qualified loans).

[Add Figure 3 here]

Collateral constraints imposed by these contracts play a critical role in governing the dynamics of the CLO balance sheet. Figure 3 shows quarterly cross-sectional distribution of the slackness of senior OC constraints between 2010–2019. Among CLOs in reinvestment period, the average senior OC score is slightly (8%) above the threshold and stable over time.[9] In the cross section of CLOs, the slackness is tightly distributed around the average. These persistently binding constraints suggest that managers fully use safe debt capacity provided by their loan portfolios. By contrast, in amortization period, as CLO leverage decreases with principal repayment, the slackness becomes larger and much more dispersed.

[9] In my data, the senior OC thresholds can be that of AAA and AA tranches, so my calculation may overstate the actual slackness of AAA OC constraints.

Fact 3: CLOs Rebalance Portfolios In Response to Adverse Shocks

Given that the collateral constraints are binding, shocks to the quality of CLOs' underlying loans are likely to trigger secondary market trades. This paper highlights an empirical pattern that is less discussed in the literature: CLOs' loan trades consist of both sales and purchases. Hence, the portfolio's size remains similar, but its composition changes over time.

[Add Figure 4 here]

Figure 4 presents CLO balance sheet dynamics before and around the onset of COVID-19 crisis in 2020. Panel (a) shows quarterly average CLO portfolio size for each age cohort. For all cohorts, portfolio size remained stable over time. This implies that overall CLOs did not shrink in size. Indeed, Panel (b) shows that accelerated repayment of senior debt actually decreased.[10] While the size of portfolios did not change, their composition changed drastically. In Panel (c), the average numbers of loan purchases and sales both nearly doubled upon the arrival of the adverse shock.[11] To understand the nature of these trades, Panel (d) examines buys and sells *within* individual CLOs in the first two quarters of 2020. As the scatterplot shows, there is a strong positive, nearly one-to-one relationship between a CLO's purchases and sales: when a CLO sells loans, it also buys loans to replace them. In other words, CLOs rebalance portfolios instead of liquidating loans.

Fact 4: Portfolio Rebalancing Improves Collateral Quality

Using granular data on CLO loan holdings, I examine how loan trades affect collateral quality in 2020. Figure 5 presents the changes from February 15 ("pre") to June 30 ("post"). Panel (a) shows senior OC slackness before and after the shock. As the pandemic caused massive downgrades of leveraged loans, the overall slackness decreased, and the dispersion across CLOs increased. When the crisis settled, however, only 1.2% of CLOs failed senior OC tests.

[10] Earlier cohorts repaid more of their senior tranches when the non-call period ends (typically 2–3 years). Such early repayment discontinued in 2020.

[11] Purchases generally exceed sales because loan holdings generate coupon and principal payments.

[Add Figure 5 here]

The reason that test failures are rare, as Fact 3 suggests, could be portfolio rebalancing. To quantify its causal effect, I track individual loans' quality changes and measure each CLO's counterfactual collateral quality in the absence of loan trades. Details of this step can be found in Appendix IA.1.3. Panel (b) shows portfolio value-weighted average ratings.[12] Overall, portfolio ratings worsened, but managers' trading mitigated deterioration, improving the realized ex-post distribution relative to the counterfactuals.

Despite similarly binding ex-ante constraints, CLOs had idiosyncratic exposures to the shock. I measure a CLO's exposure using the difference in rating between the pre and counterfactual portfolios. Panel (c) shows that almost all CLOs replaced downgraded loans and that the effect on quality linearly increases in exposure: on average, trading offsets 60% of deterioration caused by COVID-19. Panel (d) replaces the outcome with value-weighted average loan spread, which measures quality based on priced risk. In response to a 1-notch downgrading, the manager's trades reduced average spread by 30 basis points, or roughly one standard deviation. Panels (e) and (f) show further evidence based on the direction of loan trades by comparing ratings and spreads between the loans bought and sold by a CLO.

Overall, CLO contracts triggered portfolio rebalancing that substantially improved collateral quality. By reducing the uncertainty of portfolio cash flows, this mechanism helps create larger safe tranches for any given initial portfolio of risky loans.

Fact 5: CLOs Exert Pressure on Secondary Loan Prices

Replacing deteriorated loans is costly to CLOs' equity (and managers) because these trades not only reduce portfolio payoff uncertainty, but also exert pressure on loan prices. In Appendix IA.3, I document that during market downturns, the magnitude of transitory price drops is decreasing in loan quality, ranging from nearly 15% for "B-" to only 5% for

[12] A larger numeric rating corresponds to a better letter rating, as detailed in Table IA.2. Note that portfolio average ratings differ from OC test results, as the OC score is nonlinear in loan ratings.

"BB+". Consistent with price pressure from CLOs, this monotonic pattern is observed among leveraged loans but not high yield bonds. Admittedly, isolating loan price changes caused by CLOs from changes in fundamentals is difficult. While the nature of this evidence is suggestive, given various findings of price pressure in the literature, it is plausible that when a large number of CLOs rebalance portfolios in the same direction, they cause the prices of bad loans to decrease relative to the prices of good loans.

2. Model

The empirical facts above suggest a tradeoff between the ex-ante benefit of issuing more safe debt and the ex-post cost of replacing deteriorated loans. It is still unclear how this tradeoff affects the choices of individual institutions that operate CLOs and loan funds, as well as the equilibrium prices and quantities of risky loans and safe debt. To analyze these issues, I develop a model in which nonbank lending institutions can flexibly choose external financing and rebalance portfolios. The economy has three periods, $t \in \{0, 1, 2\}$, and two types of agents: investors and nonbank institutions.

Investors. There is a unit mass of investors. They can be banks, insurance companies, and other entities that invest in CLOs and loan funds. Some of these investors face risk-based regulatory requirements and prefer securities with sufficiently low default risks (e.g., AAA rated). To capture this preference, I abstract away the default risk of senior CLOs and follow the safe asset literature (e.g., Krishnamurthy and Vissing-Jorgensen, 2012; Stein, 2012; Nagel, 2016) by assuming that investors maximize additively separable utility[13]

$$U = \mathbb{E}_0\Big[\sum_{t=0}^{2} C_t\Big] + \gamma D, \tag{1}$$

where C_t is consumption in period t, and D is safe debt held at $t = 0$. Parameter $\gamma \geq 0$ is a non-pecuniary benefit from holding safe debt. Its value is exogenous and determined by

[13]Therefore, γ should be interpreted as the risk-adjusted return that investors are willing to give up for holding highly-rated low-risk securities.

forces outside of this model.

At $t = 0$, investors are endowed with an amount e of perishable consumption goods. They cannot directly lend out resources but can buy financial claims backed by loans. Hence, they allocate between consumption and financial claims, taking claim prices as given. I assume e to be sufficiently large, so investors always choose strictly positive consumption.

Nonbank Institutions. There is a continuum of identical nonbank institutions, uniformly populated on $\mathcal{I} = [0, 1]$. Their preference is similar to (1), except for that they do not derive any non-pecuniary benefit from safe debt. Each institution, indexed by $i \in \mathcal{I}$, can make loans to generate a risky payoff.[14] Institutions receive zero endowment and finance their lending by issuing senior and junior financial claims. In particular, a senior claim is referred to as *safe debt* if it is backed by loans whose payoff is enough for repaying the claim with certainty. Since investors cannot store or lend, safe debt can only be supplied by institutions as their senior liabilities.

Investment Technology. Each institution can convert x units of consumption goods into x units of loans at a private cost $c(x) - x$. This cost captures the efforts of participating in syndication deals, conducting credit analysis, and managing diversified portfolios. c is twice continuously differentiable and satisfies $c(0) = 0$, $c' > 1$, $c'' > 0$ on $\mathbb{R}_+$.[15] Every unit of loans generates a risky payoff that depends on state $s \in \{g, b\}$ at $t = 2$.[16] The loans have two quality types, denoted by $j \in \{h, l\}$. In state g, which realizes with probability $p \in (0, 1)$, both types of loans pay $R_j = R > 1$. In state b, which realizes with probability $1 - p$, high-quality loans (h) pay $R_h = 1$, and low-quality loans (l) pay $R_l = 0$.

Timeline. All institutions simultaneously choose lending and financing in period $t = 0$.

[14] In practice, nonbanks including CLOs actively participate in leveraged loan origination, but they often commit to "secondary market" purchases from banks that occur shortly after syndication (Taylor and Sansone, 2006; Kohn et al., 2008). Investor demand revealed during the syndication process influences the issuance and pricing of loans (Bruche, Malherbe, and Meisenzahl, 2020; Bruche, Meisenzahl, and Xu, 2023).

[15] The convexity of c captures a decreasing return to scale at the institution level. This is consistent with the large number of CLOs and loan funds and the fact that their loan portfolios are generally very small relative to the size of this trillion-dollar market.

[16] For simplicity, I assume that corporate borrowers' output is fully pledgeable and that lenders extract all the rents, an approach used in the literature (e.g., Diamond and Dybvig, 1983).

Specifically, each institution i raises x_i units of consumption goods from investors by issuing safe debt $d_i \geq 0$ and external equity shares. Meanwhile, the institution makes x_i units of loans without knowing loan types. In this period, the institutions may opt into keeping its portfolio static until $t = 2$. This choice is denoted by a binary variable $\mathbb{s}_i \in \{0, 1\}$.

In period $t = 1$, an idiosyncratic shock determines loan quality: α_i fraction of institution i's loans deteriorate to low-quality, and $1 - \alpha_i$ fraction are high-quality. Across institutions, α_i is independently drawn from a common distribution with support $[0, \bar{\alpha}] \subseteq [0, 1]$ and mean $\alpha \in (0, \bar{\alpha})$. Loan quality is publicly observable and contractible, and institutions with dynamic portfolios (i.e., $\mathbb{s}_i = 0$) can trade loans in a Walrasian market. In period $t = 2$, payoffs realize. As internal equity holders, institutions repay safe debt and external equity and collect residual portfolio payoffs. All goods are consumed, and the economy ends.

Commitment. The distribution of loan payoffs implies that, if the portfolio is static, each unit of loans can back no more than $\rho_s = 1 - \bar{\alpha}$ of safe debt. To increase its debt capacity beyond ρ_s, an institution may opt for a dynamic portfolio. However, the ability to trade loans, if not disciplined, may compromise the creation of safe debt. The reason is a classic agency problem (Jensen and Meckling, 1976): as equity holders, institutions privately prefer loans with riskier payoffs, which makes their debt default with a positive probability.

Given empirical facts in the last section, I assume the existence of a technology that enables institutions to credibly commit to maintain collateral quality. This *commitment technology* can be thought of as third-party trustees that regularly perform collateral tests, monitor cash flows, and enforce contracts on behalf of investors. Adopting the technology at $t = 0$ incurs a fixed cost $\xi \geq 0$, which captures the unique costs of operating CLOs, including the expenses of drafting indentures, performing regular collateral tests, and administering the cash flows of complex structures.

The Institution's Optimization Problem. Institutions with dynamic portfolios make sequential choices to maximize their payoffs. I describe their optimization problem backwardly and consider repayment only in the final period.[17]

Let secondary market prices of the two types of loans be $\mathbb{q} = (q_l, q_h) \in \mathbb{R}^2_+$. When α_i realizes, institution i, with balance sheet (x_i, d_i, α_i), chooses trades $\Delta\mathbb{x}_i = (\Delta x_{i,h}, \Delta x_{i,l})$ to maximize conditional expected payoff to equity

$$v(x_i, d_i, \alpha_i) = \max_{\Delta\mathbb{x}_i} \; ((1-\alpha_i)x_i + \Delta x_{i,h})\mathbb{E}[R_h] + (\alpha_i x_i + \Delta x_{i,l})\mathbb{E}[R_l] - d_i. \tag{2}$$

These trades are subject to a budget constraint

$$((1-\alpha_i)x_i + \Delta x_{i,h})q_h + (\alpha_i x_i + \Delta x_{i,l})q_l \le (1-\alpha_i)x_i q_h + \alpha_i x_i q_l, \tag{3}$$

a maintenance collateral constraint

$$d_i \le (1-\alpha_i)x_i + \Delta x_{i,h}, \tag{4}$$

and short-sale constraints $\Delta x_{i,h} \ge -(1-\alpha_i)x_i, \Delta x_{i,l} \ge -\alpha_i x_i$. Constraint (3) requires the trades to be self-financed by the loan portfolio. Constraint (4) reflects the ability to credibly commit to replacing deteriorated loans: After portfolio rebalancing, safe debt investors must receive full repayment at $t = 2$ with probability one. This constraint keeps the institution solvent, making equity payoff in (2) a linear function of portfolio payoff.

All market participants rationally anticipate loan trades at $t = 1$ when institutions choose lending and financing at $t = 0$. Because investors are price-taking, institutions optimally price their safe debt and external equity such that investors break even in expectation. This implies a safety premium: an institution can raise $1 + \gamma$ from issuing each unit of safe debt. The rest of funding is raised by issuing external equity, whose expected return will be set to zero. Taking loan prices as given, the institution chooses lending x_i and safe debt d_i to

[17] The option of repaying debt in period $t = 1$ will be discussed in Section 4.

maximize the expected payoff to internal equity:

$$V_i = \max_{x_i, d_i \geq 0} \quad \mathbb{E}_0[v(x_i, d_i, \alpha_i)] - (x_i - (1+\gamma)d_i) - (c(x_i) - x_i) - \mathbb{1}\{d_i > 0\}\xi, \tag{5}$$

$$s.t.\ 0 \leq d_i \leq \rho x_i, \tag{6}$$

where $v(x_i, d_i, \alpha_i)$ is the $t = 1$ maximized total equity value in (2). The second term is the funding raised from (and equals the expected payoff of) external equity, and the third term is the institution's private cost of effort. The last term is the fixed cost of commitment: ξ is incurred if and only if safe debt $d_i > 0$ is backed by a dynamic portfolio.

Importantly, the maximization is subject to a price-dependent collateral constraint (6), where $\rho := \rho_s + \bar{\alpha}\frac{q_l}{q_h}$ is the debt capacity provided by each unit of loans. By selling and buying loans, an institution can generate a higher worst-case payoff than that of a static portfolio. This allows for more safe debt than ρ_s, and $\bar{\alpha}\frac{q_l}{q_h}$ is the maximum incremental debt capacity provided by the use of a dynamic portfolio as collateral.

Different from the above, if an institution chooses a static portfolio (i.e., $\mathbb{s}_i = 1$), it optimizes lending and financing at $t = 0$ without the ability to trade loans at $t = 1$.

2.1. Equilibrium Definition

In equilibrium, all institutions take loan prices as given and optimally choose lending, financing, trading, and whether to use dynamic collateral for safe debt. Formally, a market equilibrium in this economy is a collection $\{(\mathbb{s}_i, x_i, d_i, \Delta\mathbb{x}_i)_{i\in\mathcal{I}}, \mathbb{q}\}$ such that given $\mathbb{q}$, institution i's choice of $\mathbb{s}_i$ maximizes its expected payoff, taking into account that (x_i, d_i) solves the its lending and financing problem at $t = 0$ and that if $\mathbb{s}_i = 0$, $\Delta\mathbb{x}_i$ solves its trading problem at $t = 1$; Moreover, the secondary market clears:

$$\int_i \Delta x_{i,j}\,\mathrm{d}i = 0 \quad \text{for } j \in \{h, l\}. \tag{7}$$

Because of the collateral constraints, institutions' ex-ante lending and financing choices affect their ex-post trades, which in turn affect the lending and financing problem through endogenous loan prices.

Before analyzing the model, I introduce two notations for expositional convenience. First, let $y_0 = pR + (1-p)(1-\alpha)$ be the expected hold-to-maturity payoff per unit of lending. Second, I define a function $f(y) := y \cdot c'^{-1}(y) - c(c'^{-1}(y))$, where $c'^{-1}(\cdot)$ is the inverse function of the first-order derivative of c. As will be clear in Section 3, this function has an intuitive interpretation: it maps per-unit lending payoff under a given financing choice to the institution's maximized expected payoff, V_i.

I impose two parametric conditions to restrict my analysis to interesting cases. First, I require lending to have a positive NPV for some positive quantity, which ensures that institutions always participate in lending:

Condition 1. $y_0 \geq c'(0)$.

Second, the average fraction of low-quality loans in an institution's loan portfolio is sufficiently large:

Condition 2. $\frac{\alpha}{\bar{\alpha}} > \frac{1-p+\gamma}{pR+1-p+\gamma}$.

Under this condition, CLOs collectively demand liquidity in the secondary market as they to maintain collateral quality by replacing low-quality loans with high-quality loans.[18]

2.2. Discussion of Model Setup

The key feature of my model is that safe debt provides cheap funding, but the underlying loans are overall scarce because of a convex cost of lending. Therefore, committing to replace low-quality loans with high-quality loans allows institutions to create more safe debt from a given quantity of lending. The loan payoff distribution is starker than necessary. What is necessary is a good recovery rate in default (captured by the strictly positive minimum payoff

[18]This condition rules out equilibria in which all institutions simultaneously face binding collateral constraints at $t = 0$, which I discuss in Appendix A.

R_h), which makes long-term safe debt possible.[19] Moreover, since only safe debt provides a non-pecuniary benefit, capital structure below the safe tranche is irrelevant (Modigliani and Miller, 1958), and it is without loss of generality to treat risky junior debt as equity.

The commitment technology is also essential for the self-healing mechanism to be viable. This technology requires verifiable signals of time-varying asset quality to ensure the enforceability of contracts. In practice, CLOs typically hold large-sized, standardized corporate loans that have credit ratings. As a result, their dynamic portfolios can be disciplined by long-term contracts. This distinguishes CLOs from mortgage-backed securities (MBS), which hold many small mortgages that are difficult to evaluate individually, and pre-crisis collateralized debt obligations (CDOs), which held enormous complex derivatives (Cordell, Huang, and Williams, 2011). Consistent with these facts, in my model, every institution's portfolio consists exclusively of risky loans with contractible quality.[20]

3. Equilibrium Analysis

3.1. Equilibrium with Static Portfolios

For now, suppose the commitment technology (or the secondary loan market) does not exist, and as a result, safe debt can only be backed by static portfolios. The lemma below gives institutions' lending and financing choices in this benchmark case.

Lemma 1. *If safe debt can only be backed by static portfolios, all institutions fully use their debt capacity: $x_i = x_s$ and $d_i = d_s$ for all $i \in \mathcal{I}$, where $x_s := c'^{-1}(y_0 + \gamma\rho_s)$ and $d_s := \rho_s x_s$.*

Proof. See Appendix B. $\square$

Since safe debt offers a source of cheap funding, every institution collateralizes its loans

[19] This positive lower bound is consistent with the fact that in the leveraged loan market, senior secured first lien term loans' recovery rate in default is typically greater than 50%.

[20] In the model, institutions hold only loans because consumption goods are nonstorable, cross-holdings of liabilities are unprofitable, and state-contingent bilateral contracts (e.g., derivatives) are not available.

to issue safe debt and fully uses its debt capacity.[21] Lending x_s increases in γ because loans not only generate monetary payoffs but also can back safe debt, and debt capacity is more valuable when the safety premium is greater. As institutions make identical choices, aggregate lending $X_s = x_s$, and the supply of safe debt $D_s = \rho_s X_s$.

In the rest of this section, I study lending and financing choices when institutions can use dynamic portfolios as collateral for safe debt. I first analyze individual institutions' lending, financing, and trading choices for given secondary market prices. I then characterize balance sheets and loan prices that clear the secondary market. At last, institutions' choices between static and dynamic portfolios will be determined in equilibrium.

3.2. Secondary Market Trades

The lending and financing choices at $t = 0$ depend on continuation value v. To derive v, in this subsection I analyze the institution's secondary market problem (2) in period $t = 1$.

In this period, budget constraint (3) binds, and since $d_i \geq 0$, constraint (4) implies that $\Delta x_{i,h} \geq -(1-\alpha_i)x_i$ is slack. Omitting predetermined terms, the problem simplifies to

$$\max_{\Delta x_{i,l}} \quad \left(\mathbb{E}[R_l] - \frac{q_l}{q_h}\mathbb{E}[R_h]\right)\Delta x_{i,l}, \tag{8}$$

subject to constraints $\Delta x_{i,l}\frac{q_l}{q_h} + d_i \leq (1-\alpha_i)x_i$ and $\Delta x_{i,l} \geq -\alpha_i x_i$.

Essentially, the institution substitutes between high-quality and low-quality loans. This substitution is constrained by safe debt outstanding d_i and a short-sale constraint on $\Delta x_{i,l}$. I proceed to solve problem (8) based on the following observation.

Lemma 2. *If an equilibrium with dynamic portfolios exists, secondary loan prices deviate away from their fundamental values:* $\frac{q_l}{q_h} < \frac{\mathbb{E}[R_l]}{\mathbb{E}[R_h]}$.

Proof. See Appendix B. □

[21] This market structure resembles traditional banking, where every bank creates safe debt (deposits), and loans stay on bank balance sheets.

Lemma 2 shows the self-healing mechanism's impact on secondary loan prices. After idiosyncratic shocks are realized, some institutions' collateral constraints bind. This creates a demand for liquidity, as such institutions are obligated to buy high-quality loans and sell low-quality loans. The natural providers of liquidity are institutions that hold similar loans but do not face binding collateral constraints. But for the latter to be willing to provide liquidity, low-quality loans, which are inferior as collateral for safe debt, must offer a higher expected return. Hence, the relative price of low-quality loans and high-quality loans must decrease, deviating away from the ratio of their fundamental values.

The solution to (8) below indicates that, consistent with Fact 3, the institution's optimal trades lead to portfolio rebalancing:

$$\Delta x_{i,h} = d_i - (1-\alpha_i)x_i, \;\; \Delta x_{i,l} = \frac{q_h}{q_l}((1-\alpha_i)x_i - d_i) \tag{9}$$

for any i. These trades reallocate loans among institutions. An institution with $d_i > (1-\alpha_i)x_i$ optimally sells just enough low-quality loans to increase its holding of high-quality loans and keep its debt safe. Such portfolio rebalancing is costly to equity holders because it not only decreases the portfolio's payoff uncertainty, but also moves prices in unfavorable directions. By contrast, an institution with $d_i < (1-\alpha_i)x_i$ sells its high-quality loans and buys low-quality loans to profit from the deviation of loan prices from fundamentals.

3.3. Lending and Financing Choices

Next, I characterize the institution's optimal lending and financing choices at $t = 0$ for given loan prices. Optimal secondary market trades in (9) imply that for any given (x_i, d_i, α_i), equity continuation value v at $t = 1$ is

$$v(x_i, d_i, \alpha_i) = x_i p R\Big(\alpha_i + \frac{q_h}{q_l}(1-\alpha_i)\Big) - d_i p\Big(R\Big(\frac{q_h}{q_l} - 1\Big) + 1\Big). \tag{10}$$

Substitute v into (5) and take expectation over α_i, the institution's lending and financing problem becomes

$$\max_{x_i, d_i} \; x_i pR\Big(\alpha + \frac{q_h}{q_l}(1-\alpha)\Big) - d_i p\Big(R\Big(\frac{q_h}{q_l} - 1\Big) + 1\Big) + (1+\gamma)d_i - c(x_i) - \mathbb{1}\{d_i > 0\}\xi \quad (11)$$

subject to constraint (6). Because the objective function is discontinuous at $d_i = 0$, in what follows, I consider two cases separately.

Equity Financing. In the first case, institution i issues only equity and gives up safe debt: $d_i = 0$. The optimal lending choice, x_e, is given by first-order condition

$$y = c'(x_e), \quad (12)$$

where $y := pR\Big(\alpha + (1-\alpha)\frac{q_h}{q_l}\Big)$ is the expected payoff per unit of lending when portfolios are dynamic.[22] The choice x_e is determined by a tradeoff between this payoff and the marginal cost of lending. Different from the hold-to-maturity payoff y_0, here y depends on loan prices at $t = 1$: It is decreasing in price ratio q_l/q_h since the prices' deviation from fundamentals (Lemma 2) generates an expected profit that rewards liquidity provision.

Debt Financing. In the second case, institution i issues both safe debt and equity and commits to maintain collateral quality. Let η be the shadow price of debt capacity constraint $d_i \leq \rho x_i$. The conditions for optimality are

$$y + \eta\rho = c'(x_d), \quad (13)$$

$$\eta = \gamma - \Big(pR\Big(\frac{q_h}{q_l} - 1\Big) - (1-p)\Big), \quad (14)$$

$$\eta \geq 0, \eta(d_i - \rho x_d) = 0. \quad (15)$$

When $\eta > 0$, the collateral constraint binds ($d_i = \rho x_d$): the institution fully uses debt capacity to exploit cheap funding. On the asset side, as characterized by equation (13), optimal lending x_d exceeds x_e. The additional investment is due to $\eta\rho$, the collateral value of loans. Since

[22] $x_e > 0$ is guaranteed by Condition 1 and Lemma 5 below: $z \leq \bar{z} < \frac{pR}{pR+1-p}$, which implies $y > y_0$.

both debt capacity (ρ) and its per-unit value (η) decrease in price ratio q_l/q_h, price pressure reduces the collateral value that can be extracted from lending.

3.4. Industry Equilibrium of Nonbank Institutions

Optimal choices in the two cases above determine the balance sheets of institutions that hold dynamic portfolios. Ex ante, whether institutions chooses dynamic portfolios depends on their expected payoffs at $t = 0$. Substitute optimal choices in Subsection 3.3 into objective (11), we can write these institutions' payoffs as $V_e = f(y)$ and $V_d = f(y + \eta\rho) - \xi$ for equity financing and debt financing, respectively, where function f was defined in Subsection 2.1. Similarly, using Lemma 1, the payoff of an institution with a static portfolio can be written as $V_s = f(y_0 + \gamma\rho_s)$. In equilibrium, every institution achieves its highest possible expected payoff: it obtains $\max\{V_e, V_d, V_s\}$ from its optimal choices.

The primary interest of this section is an equilibrium with dynamic portfolios. In such an equilibrium, a key endogenous variable is price ratio $\frac{q_l}{q_h}$, or equivalently, the rate of portfolio substitution via secondary market trading. As analyzed earlier, this ratio affects institutions' optimal lending and financing choices and hence V_e and V_d. Since only the ratio, rather than the levels, of loan prices is relevant, I use notation $z = q_l/q_h$ hereafter for convenience. The lemmas below characterize the equilibrium's properties.

Lemma 3. *The lower bound of price ratio is* $\underline{z} := \frac{pR}{pR+1-p+\gamma}$.

Proof. See Appendix B. □

While the safety premium is directly captured by institutions creating safe debt, part of the surplus is transferred to peer institutions that provide liquidity in the secondary market. The price ratio determines this surplus sharing. If the price ratio is as low as $\underline{z}$, the surplus would be fully shared, and equity financing would strictly dominate debt financing: $V_d = V_e - \xi < V_e$. Similar to the Grossman and Stiglitz (1980) paradox, here if the benefit of creating safe debt is fully shared through prices, then no institution would use dynamic collateral because adopting the commitment technology is privately costly. Therefore in equilibrium, price ratio must be higher that $\underline{z}$ whenever $\xi > 0$.

Lemma 4. *Equity financing and debt financing must coexist among institutions that choose dynamic portfolios.*

Proof. See Appendix B. □

Obviously, an equilibrium in which institutions with dynamic portfolios all use equity financing cannot exist: facing non-fundamental price deviations, they would all attempt to trade loans in the same direction. It is also impossible for all institutions with dynamic portfolios to use debt financing. While some institutions using debt financing can also provide liquidity at $t = 1$ as long as their portfolios do not experience severe quality deterioration (i.e., $\alpha_i < 1 - \rho$), Condition 2 guarantees that the quantity of high-quality loans they sell is always less than the demand from institutions that are forced to replace low-quality loans. Therefore, the two financing choices must coexist among institutions that hold dynamic portfolios, which in turn implies $V_e = V_d$ in such an equilibrium.

Lemma 5. *The upper bound of price ratio is* $\bar{z} := \frac{pR}{pR+1-p+\gamma\frac{1-\bar{\alpha}}{1-\alpha}}$.

Proof. See Appendix B. □

Institutions using equity financing are rewarded for liquidity provision. If the price ratio exceeds $\bar{z}$, the reward would be too low, and these institutions would prefer static portfolios. That is, they would be better off by collateralizing loans and issuing their own safe debt. Given Lemma 4, this in turn prevents any institution from benefiting from backing safe debt with dynamic portfolios. As a result, all institutions would end up holding static portfolios.

The next lemma shows that institution payoffs are monotone in z over $[\underline{z}, \bar{z}]$, the range of price ratio identified by Lemma 3 and Lemma 5.

Lemma 6. *V_e is strictly decreasing in z, and V_d is strictly increasing in z.*

Proof. See Appendix B. □

If price ratio z is lower, liquidity provision is more profitable, and replacing low-quality loans is more costly. This makes equity financing more attractive relative to debt financing. By contrast, if z is higher, maintaining collateral quality is less costly, and providing liquidity to others is less profitable. In an equilibrium with dynamic portfolios, the price ratio adjusts until $V_e = V_d$, and institutions' lending and financing choices collectively equalize the demand and supply for the two types of loans in the secondary market.

Proposition 1. *When ξ is relatively small, there exists a unique equilibrium. All institutions hold dynamic portfolios, price ratio $z^* \in (\underline{z}, \bar{z})$, and two distinct financing choices coexist: a fraction $\lambda \in (0,1)$ of institutions fully use their debt capacity (CLOs), and $1-\lambda$ of institutions do not issue any safe debt (loan funds). In the secondary market, CLOs sell low-quality loans and buy high-quality loans, and loan funds sell high-quality loans and buy low-quality loans.*

Proof. See Appendix B. □

Proposition 1 characterizes the unique equilibrium with dynamic loan portfolios. Notably, while all institutions are ex-ante identical, there is an endogenous mix of two distinct financing choices: Consistent with Fact 1, CLOs and loan funds coexist. The CLOs fully use debt capacity, which they maximize by committing to hold sufficient high-quality loans. Binding collateral constraints generated by this commitment and the resulting portfolio rebalancing are consistent with Fact 2, Fact 3, and Fact 4. Institutions operating CLOs enjoy cheap funding from larger safe tranches and receive high payoffs if realized portfolio deterioration is relatively low.

By contrast, institutions operating loan funds completely give up issuing safe debt. They do so because market-clearing loan prices deviate from the loans' fundamental values (i.e., $z^* < \mathbb{E}[R_l]/\mathbb{E}[R_h]$), which is consistent with Fact 5. Such price pressure makes providing liquidity in the secondary market as profitable as operating CLOs. After loan quality realizes, loan funds sell their high-quality loans and buy low-quality loans sold by CLOs. The loans they sell are acquired by CLOs and used as collateral to keep senior debt tranches safe.

3.5. Determination of Equilibrium with Dynamic Portfolios

Figure 6 illustrates how the equilibrium in Proposition 1 is determined. As the price ratio increases over $[\underline{z}, \bar{z}]$, V_e decreases because liquidity provision becomes less profitable, whereas V_d increases because replacing deteriorated loans becomes less costly.[23] An equilibrium with dynamic portfolios exists and is unique if and only if the single-crossing point of V_e and V_d is below $\bar{z}$. This crossing point gives equilibrium price ratio z^*, at which $V_d = V_e > V_s$, and indeed all institutions choose dynamic portfolios ($\mathbb{s}_i = 0$). Otherwise, if V_e and V_d cross at above $\bar{z}$, all institutions would choose static portfolios ($\mathbb{s}_i = 1$), resulting in an equilibrium in Subsection 3.1.

[Add Figure 6 here]

Given the investment technology, whether an equilibrium with dynamic portfolios arises (i.e., whether V_d and V_e cross below $\bar{z}$) depends on parameters γ and ξ. Intuitively, the mechanism of CLOs' dynamic portfolios for creating safe debt is more attractive when the safety premium is higher. However, to use this mechanism, CLO managers also incur a fixed cost to credibly commit to rebalancing portfolios in a direction that favors debtholders. These two parameters thus jointly determine whether $V_d = V_e$, after loan prices adjust endogenously, is better or worse than V_s. Others equal, a sufficiently high γ and a relatively small ξ ensure that $V_d = V_e > V_s$, and vice versa.[24]

[Add Figure 7 here]

[23] In the figure, $\bar{\bar{z}} := \mathbb{E}[R_l]/\mathbb{E}[R_h]$ is the ratio of low-quality and high-quality loans' fundamental values.

[24] In the leveraged loan market, fixed cost ξ is relatively small than other lending markets. This is because syndicated term loans are large and fairly standardized, which allows contracts based on the loans' credit ratings to dynamically discipline CLOs' portfolios.

Numerical solutions in Figure 7 illustrate how these parameters affect the $t = 0$ equilibrium. In Panel (a), all parameters other than safety premium γ are held constant. There exists a threshold γ_{min} above which $V_d = V_e > V_s$. Hence, equilibrium with dynamic portfolios arises if $\gamma > \gamma_{min}$. Similarly, in Panel (b), where only fixed cost ξ varies, there is a threshold ξ_{max} below which all institutions choose dynamic portfolios in equilibrium.

4. Implications

4.1. The Supply of Safe Debt

How does the CLOs' self-healing mechanism and secondary market trading between different institutions affect the market's total supply of safe debt? Through the lens of the model, this subsection analyzes the effects of safety transformation based on dynamic portfolios on individual institutions and total quantities.

Risk Sharing. At $t = 1$, the commitment to replacing deteriorated loans facilitates risk sharing across institutions. While institutions are risk-neutral, sharing risk is valuable because the interim shocks, which cause loan quality deterioration and limit ex-ante debt capacity, are unpredictable and idiosyncratic. After these shocks are realized, CLOs that experienced severe deterioration can restore collateral quality by trading with institutions whose collateral constraints are slack. This reallocation of loans among institutions achieves a more efficient use of overall collateral.

At the market level, the benefit of risk sharing is reflected in total debt capacity. The market-clearing condition (7) and loan trades in (9) jointly imply that with dynamic portfolios, total safe debt is

$$D = \int_{i \in \mathcal{I}} d_i \, \mathrm{d}i = (1 - \alpha)X. \tag{16}$$

Clearly, in aggregate, debt capacity per unit of loans in this market is determined by the average (α), rather than the worst ($\bar{\alpha}$), realization of individual portfolio deterioration. Thus by relaxing collateral constraints, risk sharing increases total debt capacity for any given total quantity of lending.

Lending Volume. Ex ante, every unit of lending generates an uncertain quantity α_i of high-quality loans. Each unit of such loans pays off $R_h \geq 1$ regardless of which state realizes at $t = 2$. Hence, others equal, a higher lending volume always provides greater debt capacity. Given a decreasing return to scale in lending, loans (i.e., collateral for safe debt) are overall scarce in this economy, which limits total debt capacity. The next lemma shows that with dynamic portfolios, both CLOs and loan funds always have higher lending volumes than what they would lend had they chosen static portfolios.

Lemma 7. *When institutions hold dynamic portfolios, their lending volumes are higher than in an equilibrium with static portfolios: $x_i > x_s$ for all $i \in \mathcal{I}$ if $\mathbb{s}_i = 0$.*

Proof. See Appendix B. □

Lending increases because the self-healing mechanism helps institutions extract surplus from the safety premium. Specifically, risk sharing among institutions relaxes individual debt capacity constraints, which allows for more cheap funding to be raised for a given quantity of lending. While loan funds do not use their debt capacity, they share part of the safety premium captured by CLOs through equilibrium loan prices. Therefore, when holding dynamic portfolios, all institutions face a higher payoff per unit of lending (i.e., $y + \eta\rho > y > y_0 + \gamma\rho$) and optimally raise their lending volumes.

Overall, despite that only a subset of institutions, namely the CLOs, issue safe debt, they produce more safe debt in total because of risk sharing and increased lending volumes. The next proposition summarizes these effects.

Proposition 2. *When safe debt is backed by dynamic portfolios, the market's total supply of safe debt exceeds that in a counterfactual market with static portfolios: $D > D_s$. This increase in supply comes from two complementary channels: (i) Risk sharing across institutions allows* *for greater debt capacity per unit of aggregate loans:* $1-\alpha > \rho_s = 1-\bar{\alpha}$. *(ii) Dynamic portfolios increase the payoffs of lending, and therefore the aggregate quantity of loans:* $X > X_s$.

The aggregate quantities generated by dynamic portfolios can be compared with those in the equilibrium with static portfolios. With dynamic portfolios, the market's total supply of safe debt is greater for two reasons. First, even if the quantity of loans is the same X, total safe debt $D = (1-\alpha)X$ would be greater than $\rho_s X$ because risk sharing achieves a more efficient use of collateral. Second, total lending X exceeds X_s, which mechanically increases total debt capacity, even if individual debt capacity per unit of loans remained at ρ_s. These two forces jointly contribute to a greater total supply of safe debt, $D > D_s$, than the equilibrium with static portfolios.

The difference in the quantities of total lending and safe debt are shown in Panels (a)-(b) of Figures 7. In Panel (a), equilibrium with dynamic portfolios exists whenever $\gamma > \gamma_{min}$. In this equilibrium, total lending X is larger than X_s, the total lending in a counterfactual equilibrium with static portfolios, and total safe debt D exceeds its counterfactual D_s to an even greater extent. A similar comparison can be seen in Panel (b), where changes in ξ affects total quantities only through the choices between dynamic and static portfolios.

4.2. Price Deviations and Total Surplus

An important consequence of the self-healing mechanism is that when CLOs rebalance portfolios, they exert a pressure on relative loan prices. As a result, the price of low-quality loans will decrease relative to the price of high-quality loans. My model shows that this price pressure is an inherent equilibrium result, which arises as institutions facing external demand for safe debt optimize their balance sheets without financing frictions.

Interestingly, the magnitude of this price pressure is informative about the market-wide total surplus achieved with the self-healing mechanism. Given that price-taking investors break even in expectation, this total surplus equals the sum of expected payoffs across all institutions: $TS := \int_{i \in \mathcal{I}} V_i \, \mathrm{d}i$. In equilibrium $V_d = V_e$, so $TS = V_e = f(y)$, where $y = pR(\alpha + \frac{1-\alpha}{z^*})$. Thus, given the exogenous investment technology (p, R, α), total surplus TS is summarized by a single endogenous variable: equilibrium price ratio z^*. Moreover, since f is strictly increasing, TS and z^* are negatively associated with each other.

Proposition 3. *When safe debt is backed by dynamic portfolios, the market's total surplus is greater if and only if loan prices deviate more significantly from fundamental values.*

This positive relationship between price deviations and total surplus is intuitive. Given that individual lending and financing choices are functions of loan prices, the equilibrium price ratio is sufficient to summarize all the gains from lending and safe debt creation. Because the equilibrium loan prices always adjust to equalize CLOs' and loan funds' expected payoffs, the total surplus is greater whenever institutions operating loan funds enjoy better payoffs. This occurs precisely when the price pressure is more severe and liquidity provision is more profitable. Therefore, my model suggests that in this market, price deviation is a sign of value creation rather than a symptom of frictions that constrain liquidity provision.

[Add Figures 8 here]

Figure 8 illustrates this intuition. In Panel (a), the equilibrium switches from static portfolios to dynamic portfolios once safety premium γ exceeds γ_{min}. Total surplus increases with γ at a faster rate when $\gamma > \gamma_{min}$ because CLOs' mechanism improves safe debt creation. Meanwhile, as γ increases, there is more price pressure from CLOs. As a result, equilibrium ratio z^* declines, indicating more severe loan price deviations from fundamentals. Panel (b) presents the relationship between total surplus and z^* by varying fixed cost ξ. As using the self-healing mechanism to create safe debt becomes increasingly costly, total surplus decreases, while loan price deviations are mitigated due to relaxed price pressure from CLOs. These changes continue until ξ exceeds ξ_{max}, after which the market switches to the static equilibrium, and total surplus flattens.

4.3. Implications on Financial Stability

So far, all institutions and investors are fully rational with perfect foresight on the distribution of future shocks. Under this complete-information setup, CLOs' safe debt never default, and dynamic portfolios strictly improve total surplus relative to a static counterfactual market. While this framework offers novel insights on the mechanism of CLOs and the equilibrium of nonbanks, it is silent on potential risks associated with this mechanism. In this subsection, I extend the model to analyze how dynamic portfolios affect CLOs' probability of default and loss given default, and their implications on financial stability.

My model extensions aim to capture the risks that stem from underestimated correlations in loan default, both within a portfolio and across institutions.[25] Loan default correlation is a key source of systematic risk in securitization and difficult to estimate in practice (Coval, Jurek, and Stafford, 2009). An underestimation of this correlation could be particularly risky given an increasing similarity of loan portfolios across CLOs (Elkamhi and Nozawa, 2022). To clarify how potentially underestimated of loan default correlations affect senior CLO tranches, I consider two separate extensions.

Extension 1: Correlated Deterioration Within A Portfolio. My first extension analyzes correlated loan quality deterioration within a CLO's loan portfolio. To isolate the impact of underestimating this correlation, I introduce a mean-preserving spread (MPS) in the distribution of idiosyncratic loan quality shocks (α_i). This MPS captures underestimated correlation among loans within an institution's portfolio by allowing the true distribution of α_i to have fatter tails than what market participants believed, while keeping its mean and the correlation of its realizations between institutions unchanged.

Formally, in period $t = 0$, all market participants believe that α_i has a distribution with full support over $[\underline{\alpha}, \bar{\alpha}]$, but its true distribution has full support over $[\underset{\sim}{\alpha}, \tilde{\alpha}] \subseteq [0, 1]$.[26] This

[25]Incorrect beliefs are critical to risks in securitization (e.g., neglected states in Gennaioli, Shleifer, and Vishny (2012, 2013)), as any rationally anticipated possible states of the world would have been reflected in the size and pricing of senior tranches.

[26]My model's results in previous sections do not require α_i's distribution to have a zero lower bound.

true support is a MPS of the perceived support: $\underset{\sim}{\alpha} < \underline{\alpha} < \bar{\alpha} < \tilde{\alpha}$, and $\mathbb{E}[\alpha_i] = \alpha$. Under the incorrect belief, all institutions' lending and financing choices at $t = 0$ are the same as their equilibrium levels in previous sections. That is, (x_d, x_e, d_i, λ) are jointly determined with price ratio z^*, all based on a belief that α_i is drawn from $[\underline{\alpha}, \bar{\alpha}]$.

In this extension, the size of CLOs' debt tranches is determined when the tails of the distribution of α_i are underestimated. As a result, some CLOs will experience unexpectedly large realizations of loan quality deterioration (i.e., $\alpha_i > \bar{\alpha}$) at $t = 1$. Will these CLOs fail to keep their debt safe? The next proposition shows that the self-healing mechanism can still help CLOs restore collateral quality to ensure debt safety.

Proposition 4. *When the correlation in loan quality deterioration within portfolios is underestimated, CLOs' mechanism mitigates senior tranches' default risk relative to static portfolios: as long as* $\frac{\tilde{\alpha}}{\bar{\alpha}} > 1$ *is relatively small, there exists a market-clearing price ratio* $\tilde{z}_1$ *at* $t = 1$ *such that after secondary market trades, all CLO debt remains safe.*

Proof. See Appendix B. □

Compared to static portfolios, CLOs' self-healing mechanism improves their senior tranches' resilience to underestimated correlation in loan deterioration within portfolios. Clearly, if portfolios were static, all institutions would have $d_i = (1 - \bar{\alpha})x_s$, and their debt would not be safe because of a positive probability of default after realized $\alpha_i \in (\bar{\alpha}, \tilde{\alpha}]$ at $t = 1$. In contrast, CLOs with dynamic portfolios can still satisfy the maintenance collateral constraint (4), even after experiencing unexpectedly large collateral deterioration.

CLOs experiencing unexpectedly large realized deterioration can remain safe because, by construction, this extension captures only correlated shocks within portfolios. Since α_i is independent across institutions, there also exist institutions that experience unexpectedly low realized deterioration. As the mean of α_i is correctly estimated, the total quantity of high-quality loans will still be enough for the total quantity of debt. Thus, secondary market trades can reallocate loans among institutions, allowing all CLOs to keep their debt safe.

Before the Great Financial Crisis, the underestimation of default correlation led to the mispricing and over-production of senior CDO tranches, particularly those backed by subprime mortgage-backed securities. The subsequent defaults of these CDOs caused severe losses on investors. In contrast, none of senior CLOs ever defaulted. This distinct performance is partially attributable to the CLOs' self-healing mechanism, which makes senior tranches resilient to underestimated correlations in loan defaults within portfolios.[27]

Extension 2: Correlated Shocks Across Institutions. My second extension analyzes correlated shocks to loan quality across institutions. To capture institutions' similar portfolio exposures to aggregate shocks, I introduce a small-probability state at $t = 1$. Only in this state, loan quality deterioration is worse on average and less dispersed across institutions than previously believed. I call this aggregate state as a "disaster state". Specifically, in the disaster state, an institution's α_i is independently drawn from a distribution over $[\underset{\sim}{\alpha}, \bar{\alpha}]$, where $\underset{\sim}{\alpha} > \underline{\alpha}$, with a higher mean $\mathbb{E}[\alpha_i | disaster] > \alpha$. The possibility of this state is neglected at $t = 0$ by all market participants, resulting in underestimated risks.

Because the possibility of this disaster state is neglected, all institutions' lending and financing choices at $t = 0$ are the same as their equilibrium levels in previous sections. However, as the size of CLOs' debt tranches is determined by incorrect beliefs, in the disaster state, some CLOs might fail to satisfy the maintenance collateral constraint, regardless of how they rebalance portfolios. These CLOs' debt tranches will no longer be safe debt, as the debt would default if the bad state realizes at $t = 2$. Moreover, with their debt tranches becoming defaultable, these CLOs' optimal trading choices at $t = 1$ change as well: they may shift risk to debtholders and gamble for resurrection at $t = 2$.

Now I analyze outcomes in the disaster state. Let $\tilde{z}_2$ be the loan price ratio in this state, which generally differs from z^*. Before solving for trading and default outcomes, I make two assumptions for tractability. First, CLOs give up the maintenance collateral constraint

[27] The contrasting senior tranche performance of CLOs and CDOs during the crisis have other reasons. For example, corporate loans higher default recovery rates, and the collateral pools of CDOs also included enormous complex derivatives.

(4) only when it is impossible to satisfy, as violations lead to lost management fees and reputational damage. Second, the distribution of α_i over its support is uniform. Under these assumptions, the following proposition characterizes market outcomes in the disaster state.

Proposition 5. *When the correlation in loan quality deterioration across institutions is underestimated, CLOs' mechanism worsens senior tranches' default risk and their loss given default: In the disaster state, loan price ratio $\tilde{z}_2 < z^*$, and there exists a unique cutoff realization $\delta \in (\underset{\sim}{\alpha}, \bar{\alpha})$ such that CLOs with $\alpha_i > \delta$ sell high-quality loans and buy low-quality loans; After the disaster state, with probability $1 - p$, these risk-shifting CLOs' debt tranches default at $t = 2$ and have a zero recovery rate.*

Proof. See Appendix B. □

This proposition shows that the underestimation of correlated shocks between institutions is particularly risky to senior CLOs. If portfolios were static, by construction, the disaster state would be irrelevant, as market participants have correct beliefs on the upper bound of α_i's distribution. However, with dynamic portfolios, debt tranches may default if the disaster state occurs, and risk shifting further worsens the consequences following this state.

CLOs experiencing relatively higher portfolio deterioration fail to keep their debt safe in the disaster state because the worse overall deterioration generates an expectedly low total quantity of high-quality loans. Despite that idiosyncratic components of shocks are shared between institutions, these CLOs cannot have enough high-quality loans regardless of how they rebalance portfolios. The resulting failure of maintenance collateral constraints gives them incentives to gamble for resurrection by shifting risk to debtholders. Therefore, if the correlation in loan quality deterioration across institutions is underestimated, dynamic portfolios may compromise the safety of senior tranches and amplify loss given default, reducing investors' (e.g., banks and insurers) capital in bad states of the world.

4.4. Discussions

The Roles of Banks in Leveraged Lending. As explained earlier, this paper focuses on syndicated term loans, most of which are provided by nonbank lenders, and has abstracted from banks.[28] While banks are no longer major lenders of syndicated term loans, they still play at least three roles in the leveraged loan market. First, as lenders, banks provide most of the revolving credit lines (Pitchbook, 2022; SNC, 2023). Second, as underwriters, banks arrange syndication deals and help discover nonbank investors' loan demand via a bookbuilding procedure (Bruche, Malherbe, and Meisenzahl, 2020; Bruche, Meisenzahl, and Xu, 2023). Finally, as broker-dealers, banks serve as market makers in the over-the-counter secondary loan market (Phillips, 2023). Understanding the linkage between these roles played by banks and CLOs' dynamic portfolios is left to future research.

Why Do CLO Securities Have Long Maturities? My model leaves open the question of why CLOs do not issue short-term debt, which can rollover in normal times and trigger liquidation in bad times. I argue that the observed debt maturity is an equilibrium outcome: Given the segmentation between the leveraged loan market and public securities markets and the difficulty for outsiders to buy liquidated loans, issuing long-term safe debt is optimal for CLOs. This argument can be formalized by introducing a costly storage technology that allows investors to purchase loans at $t = 1$ and institutions to repay debt early. When investors' storage cost is high, so is their required return from liquidated loans. As a result, liquidating loans and repaying debt will be costly to CLOs. Thus, long-term contract, which helps CLOs maximize cheap leverage, will be optimal in equilibrium.

Will Institutions Internalize Loan Trades? My model allows institutions to flexibly choose external financing. In principle, an institution can operate two entities with very different liabilities (e.g., a CLO and a loan fund), which might seem appealing as the institution can then internalize loan trades. That is, instead of buying and selling loans in the secondary

[28] For example, \$1,037.6 billion (more than 80%) of non-investment-grade term loans identified by the Federal Reserve Board's Shared National Credit (SNC) Program are held by nonbanks in 2023.

market, the asset manger could reallocate loans between the entities it operates. However, doing so is suboptimal for the institution. This is because without trading in the secondary market, the quantity of safe debt the institution can create is constrained by its initial loan portfolio ($d_i \leq \rho_s$), which is dominated by its choices in equilibrium ($V_d = V_e > V_s$). Therefore in equilibrium, institutions tend to specialize, consistent with empirical Fact 1.

Heterogeneity in Ex-Ante Loan Quality. In my model, loans are homogeneous ex ante, and CLOs create larger safe tranches by replacing deteriorated loans after loan quality reveals. Introducing heterogeneity in ex-ante loan quality would not change the model's implications. This is because in aggregate, safe tranches are always constrained by the cash flows generated by loans. One might think that CLOs could create larger safe tranches by choosing relatively safer loans ex ante. However, to the extent that safer lending opportunities are scarce, this would not increase the aggregate quantity of safe tranches, even if it enlarges safe tranches at the individual CLO level. In contrast, with dynamic portfolios, CLOs can economize on scarce collateral, thereby creating more safe debt relative to a counterfactual economy that has only static portfolios.

Valuation of CLO Securities. The intuition provided by my analysis suggests that it is important to consider portfolio dynamics in the valuation of CLO securities. In the presence of the self-healing mechanism, assessing the risk of a CLO's securities solely based on its current balance sheet potentially understates the safety of senior tranches and the risk of junior tranches. Since CLO managers are obligated to maintain collateral quality, their portfolio rebalancing transfers value from junior tranches to senior tranches in bad states of the world.[29] As a result, senior (junior) tranches tend to lose less (more) value than they would with static portfolios. A forward-looking approach that incorporates such pro-cyclical and counter-cyclical value transfers may further improve recent developments in CLO valuation (e.g., Cordell, Roberts, and Schwert, 2023; Elkamhi, Li, and Nozawa, 2024).

[29]The manager's loan trades reduce the uncertainty in the portfolio's future cash flows, which benefits senior tranches at the cost of junior tranches.

Dynamic Collateral Pools in Other Markets. While my paper focuses on the leveraged loan market, dynamic collateral pools have been also used in other financial markets. For example, in commercial real estate market, CRE CLOs create tranches backed by actively-managed mortgage portfolios.[30] In the securitization of credit card receivables, the sponsors often purchase high-quality accounts and remove delinquent accounts in the collateral pool under the monitoring of a rating agency. Similarly, to improve debt capacity and keep loans solvent, a cryptocurrency-backed lending platform named ALEX developed a Collateral Rebalancing Pool ("CRP") technology. The CRP uses algorithms to dynamically rebalance collateral pools between riskier digital assets (e.g., Bitcoins) and less risky tokens as market conditions evolve. Therefore, the framework in this paper can also help interpret findings and inform policies in other markets.

5. Conclusion

This paper analyzes safety transformation with dynamic collateral pools. Before the 2007–09 financial crisis, the securitization industry manufactured large quantities of senior tranches backed by static loan portfolios. Many of these tranches defaulted because their underlying loans deteriorated and failed to generate sufficient cash flows for repayment. By contrast, in the leveraged loan market, CLOs have been creating AAA-rated securities for more than three decades without any default record.

The distinct feature of CLOs is a long-term contract that obligates CLO managers to dynamically maintain collateral quality by replacing deteriorated loans. This contract generates an intertemporal tradeoff: it helps CLOs create larger safe tranches ex ante and triggers costly portfolio rebalancing, which exerts price pressure in the secondary market. I develop an industry equilibrium model to understand how this self-healing mechanism

[30] According to BofA Global Research, during the COVID pandemic, active portfolio management helped CRE CLOs achieve considerably lower delinquency rates than the conduit and SASB deal.

affects loan prices and the quantities of nonbank lending and safe debt. My model provides a framework that rationalizes the coexistence of CLOs and loan funds and the trades that reallocate loans across these institutions. It also sheds light on how portfolio dynamics raises the supply of safe debt, improves total surplus, and poses risks to financial stability.

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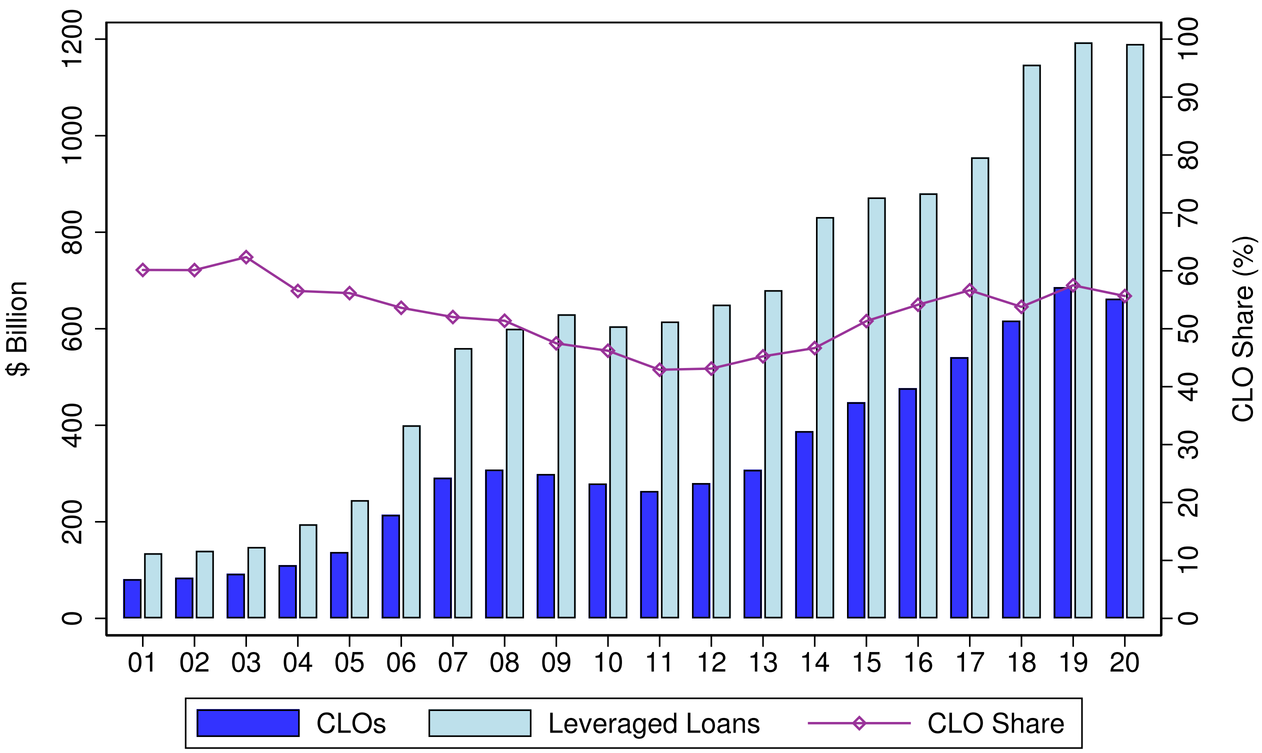


Figure 1: **Leveraged Loans and CLOs Outstanding in the US, 2001–2020.**
This figure plots annual total par values outstanding for leveraged loans (i.e., institutional term loan facilities) and CLOs in the US market. Data source: SIFMA.

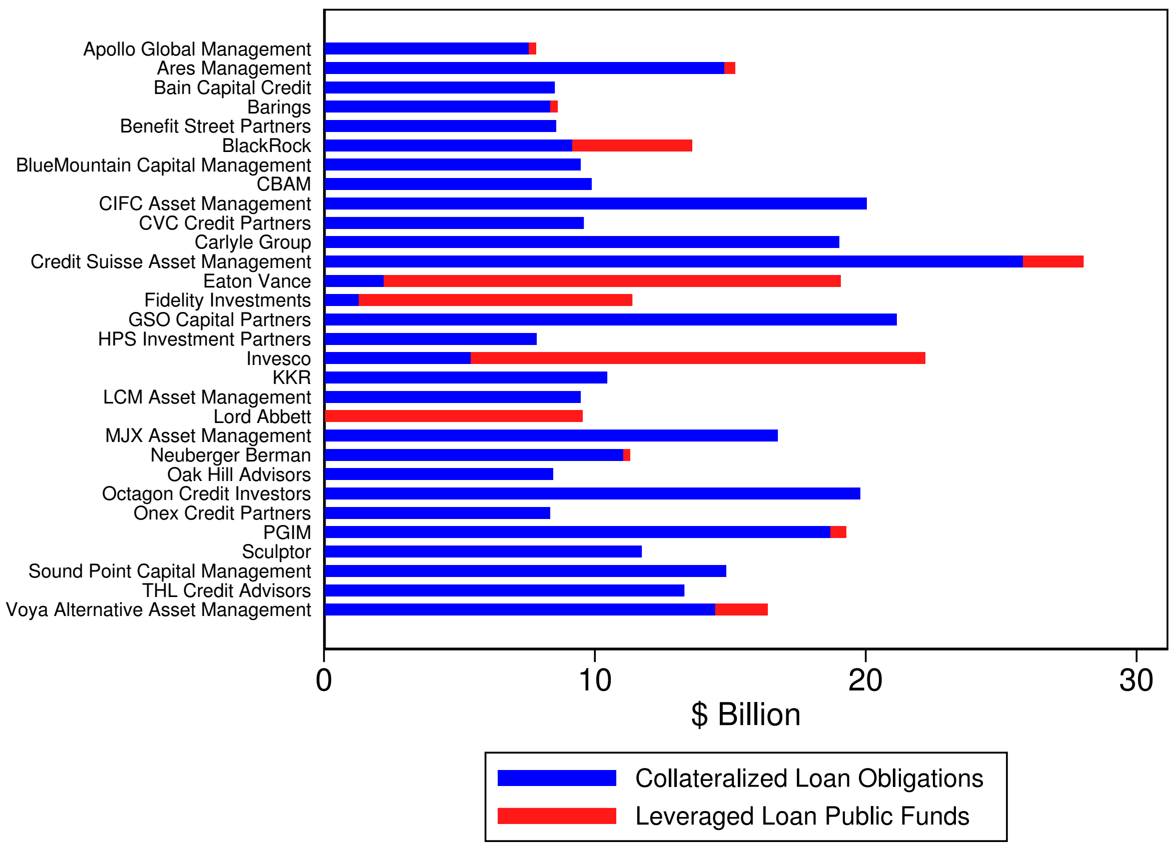
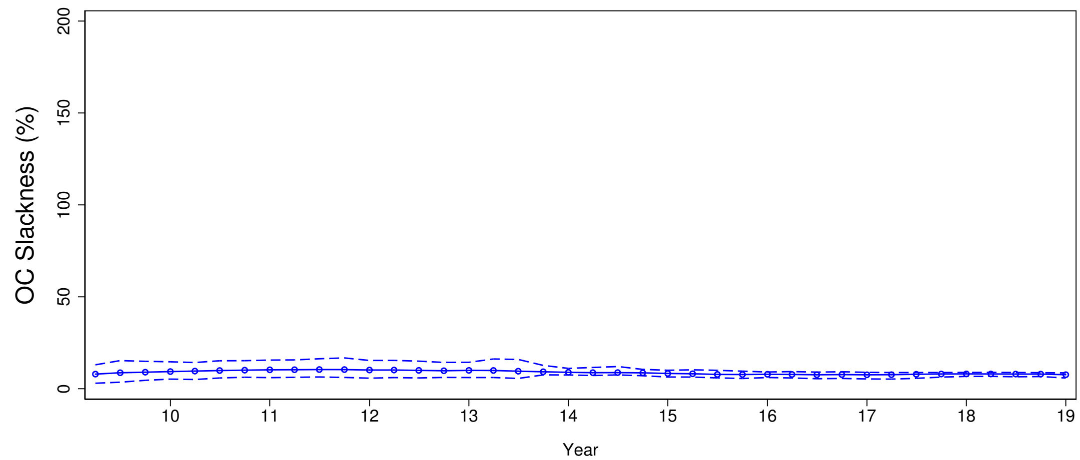


Figure 2: **Asset Managers and the Mix of Nonbank Institutions.**
This figure presents assets under management for US CLOs and public loan funds (the sum of open-end mutual funds, closed-end mutual funds, and exchange-traded funds) operated by the 30 largest asset managers at the end of 2019. Data come from Creditflux CLO-i, Morningstar, and the SEC's Form ADV databases.

(a) CLOs in Reinvestment Period



(b) CLOs in Amortization Period

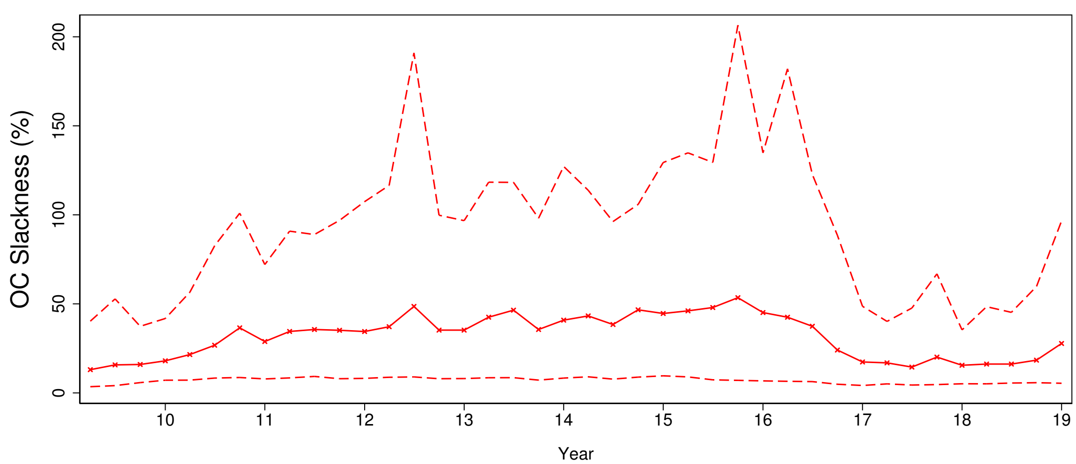


Figure 3: **CLOs' Binding Collateral Constraints.**
This Figure presents quarterly time series of cross-sectional dispersion in the slackness of CLO senior tranche over-collateralization (OC) constraints between 2010–2019. The slackness is defined as extra OC score scaled by the OC test's predetermined threshold level. Dashed lines indicate 5th and 95th percentiles in each cross section. Panel (a) reports CLOs in reinvestment period, and panel (b) reports CLOs in amortization period.

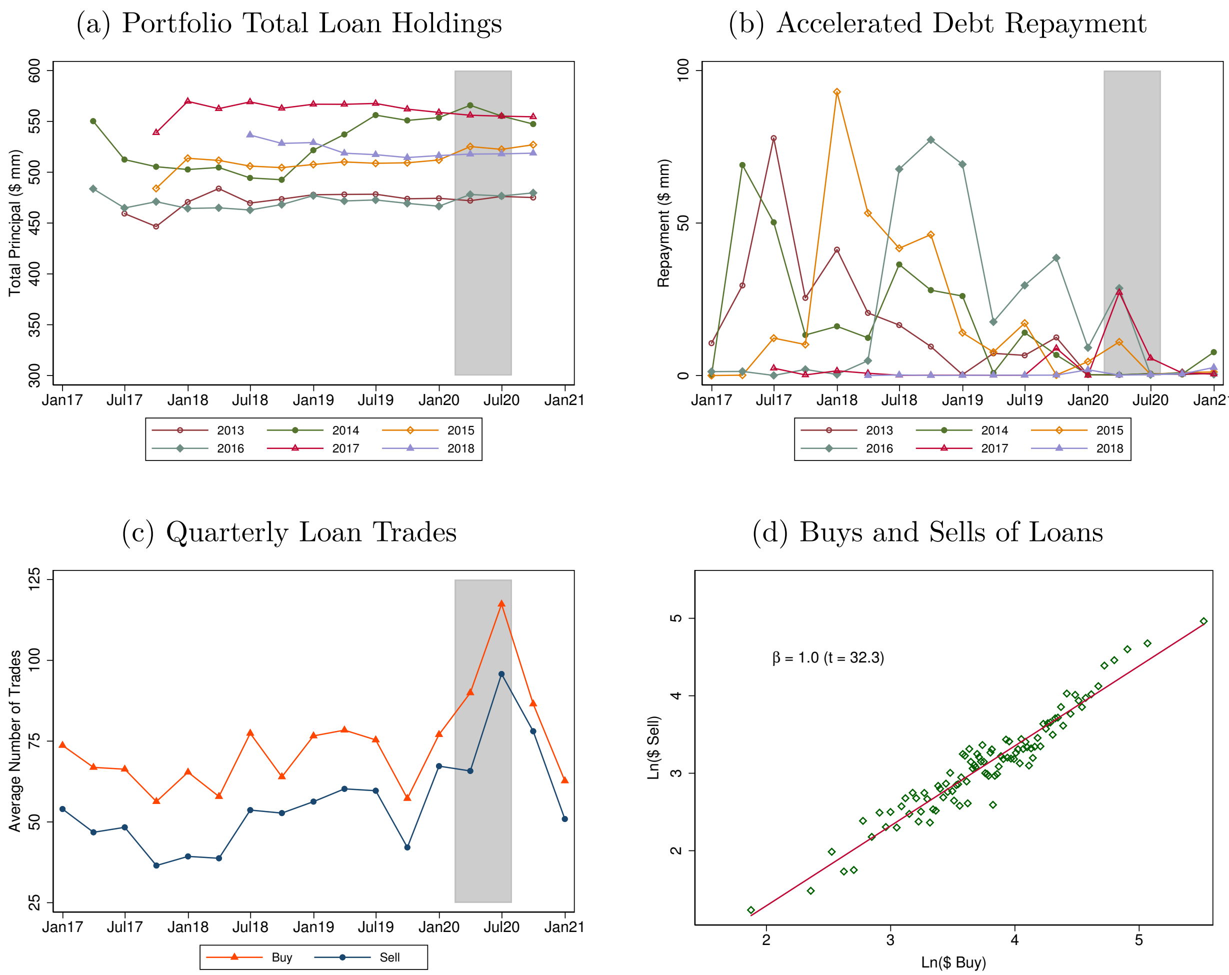


Figure 4: **CLO Balance Sheets Around the Onset of COVID-19 Pandemic.**
This Figure shows quarterly changes in CLOs' assets and liabilities before and during the COVID-19 shock in 2020. Panel (a) plots average portfolio size by CLO age cohort. Panel (b) plots average accelerated repayment of AAA tranches by CLO age cohort. Panel (c) plots quarterly average numbers of loan purchases and sales. Panel (d) is a scatter plot that groups CLOs into 100 bins based on the dollar volumes of individual CLOs' loan purchases and sales during the first two quarters of 2020. Only CLOs in reinvestment period are included.

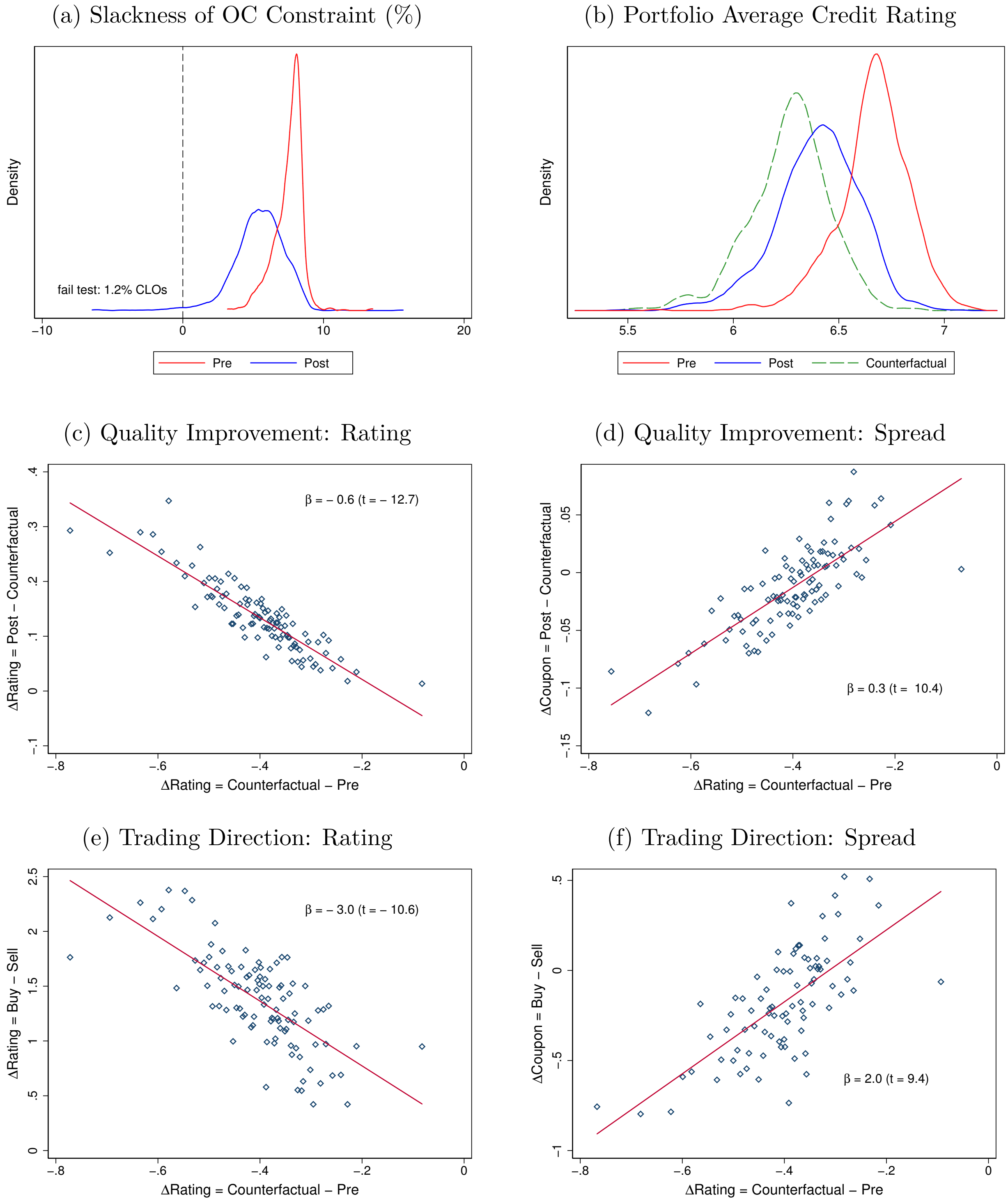


Figure 5: **Portfolio Rebalancing Improves Collateral Quality.**
This Figure shows the effect of portfolio rebalancing on CLOs' collateral quality between February 15 and June 30 of 2020. Panel (a) plots kernel density estimates for the distribution of senior OC constraint slackness before and after the onset of COVID-19 pandemic. Panel (b) plots kernel density estimates for the distribution of value-weighted average credit rating for portfolios before and after the shock as well as counterfactual static portfolios. Panels (c)–(f) are scatter plots that group CLOs into 100 bins by counterfactual collateral deterioration and depict the average effect of loan trading within each bin. The fitted lines represent OLS estimates, and t-statistics are based on heteroskedasticity-robust standard errors. Only CLOs in reinvestment period are included.

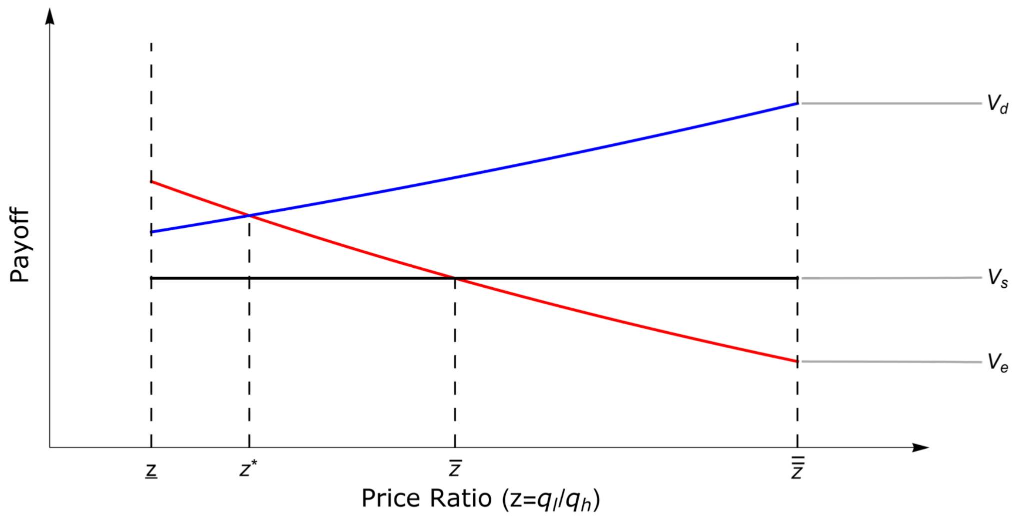
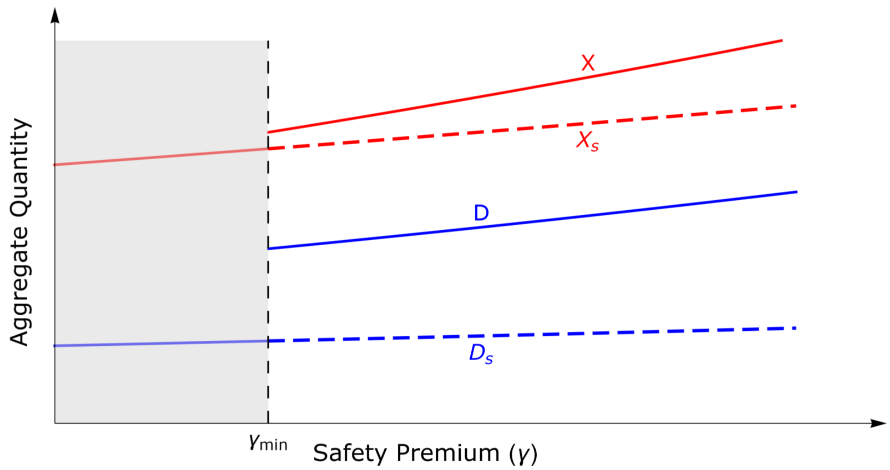


Figure 6: **Determination of Equilibrium.**
This figure illustrates how the equilibrium is determined. Solid lines V_d, V_e, and V_s indicate the payoffs of CLOs, loan funds, and institutions issuing safe debt backed by static portfolios, respectively, as functions of secondary market loan price ratio $z = q_l/q_h$. An equilibrium with dynamic portfolios exists and is unique if and only if price ratio $z^* \in [\underline{z}, \bar{z}]$. Parameter values and functional forms: $p = 0.9$, $R = 1.2$, $\alpha = 0.4$, $\gamma = 0.15$, $\bar{\alpha} = 0.7$, $\xi = 0.05$, and $c(x) = \frac{4}{5}x^{\frac{5}{4}}$.

(a) Changes in Safety Premium



(b) Changes in Fixed Cost of Commitment

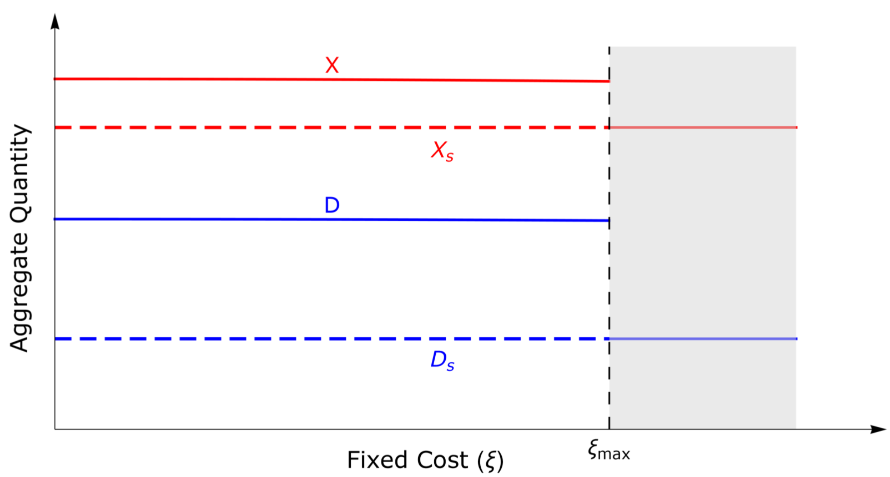


Figure 7: **Comparative Statics: Total Lending and Safe Debt in Equilibrium.** This figure presents comparative statics of aggregate quantities of lending (X) and safe debt (D). Panel (a) displays comparative statics with respect to safety premium γ. Panel (b) displays comparative statics with respect to the fixed cost of commitment ξ in adopting the self-healing mechanism. Shaded gray areas correspond to equilibrium with static portfolios, and dashed lines and subscript s indicate conterfactual values with static portfolios. Parameter values: $p = 0.9$, $R = 1.2$, $\alpha = 0.4$, $\bar{\alpha} = 0.7$, $\xi = 0.05$, and functional form $c(x) = \frac{4}{5}x^{5/4}$.

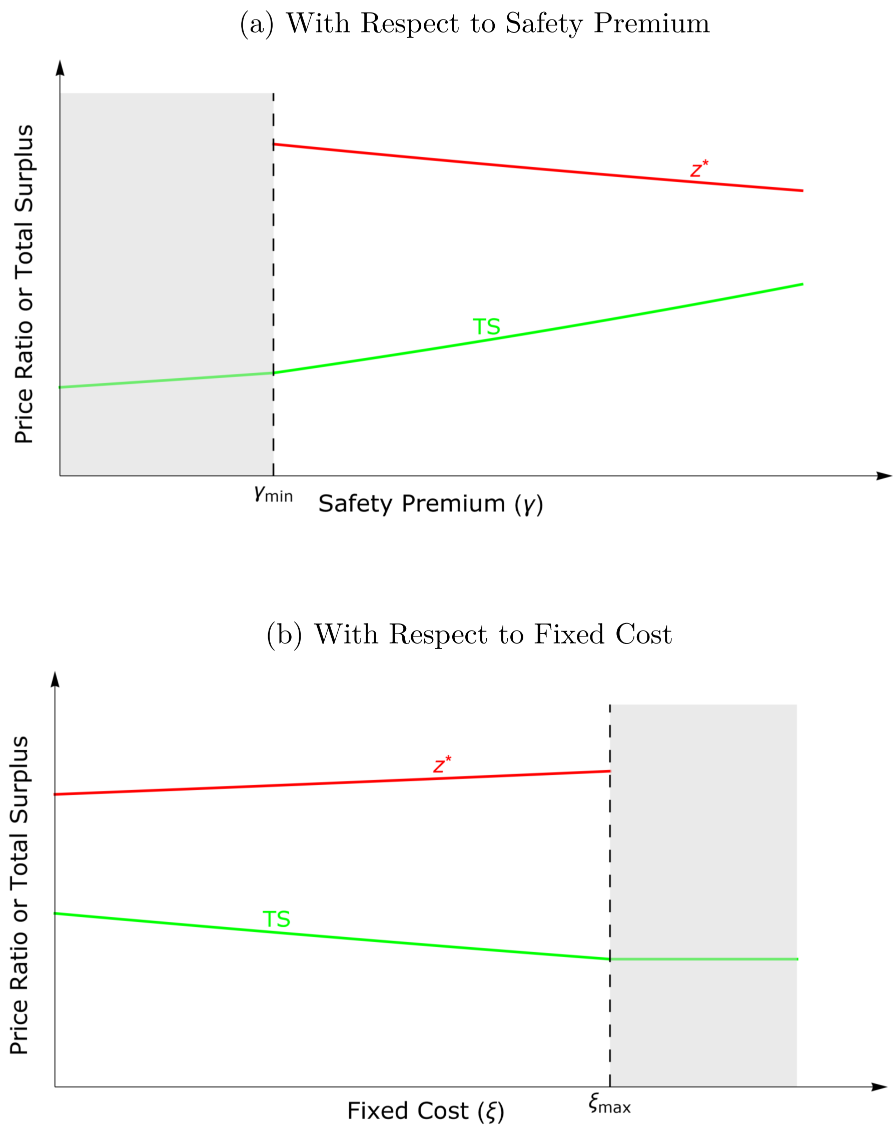


Figure 8: **Loan Price Deviations and the Market's Total Surplus.**
This figure illustrates equilibrium relationship between loan price deviations from fundamentals and the market's total surplus. Panel (a) displays comparative statics of secondary loan price ratio (z^*) and market total surplus (TS) with respect to safety premium (γ). Panel (b) displays comparative statics of these endogenous variables with respect to the fixed cost of commitment (ξ). Shaded gray areas correspond to equilibrium with static portfolios. Parameter values: $p = 0.9$, $R = 1.2$, $\alpha = 0.4$, $\bar{\alpha} = 0.7$, $\gamma = 0.15$, $\xi = 0.05$, and functional form $c(x) = \frac{4}{5}x^{5/4}$.

Appendix A Equilibria Without Loan Funds

My model has shown the existence and uniqueness of an equilibrium under Condition 2. In this equilibrium, CLOs and loan funds coexist, and overall CLOs maintain collateral quality by replacing low-quality loans with high-quality loans. This result aligns with empirical facts and thus forms the basis for the positive implications.

This appendix analyzes equilibria when Condition 2 is violated, i.e.,

$$\frac{\alpha}{\bar{\alpha}} \leq \frac{1-p+\gamma}{pR+1-p+\gamma}. \tag{A.1}$$

That is, the average fraction of low-quality loans in institutions' portfolios is relatively small. The following propositions show that in this case, there exist equilibria in which all institutions issue safe debt, with their collateral constraints simultaneously binding at $t = 0$. Such equilibria yield similar implications on the quantities of nonbank lending and safe securities, but the composition of institutions differs: no institution chooses equity financing, so there is no loan fund.

Proposition A.1. *When $\frac{\alpha}{\bar{\alpha}} \in \left(\frac{1-p}{pR+1-p}, \frac{1-p+\gamma}{pR+1-p+\gamma}\right]$ and ξ is small, there exists an equilibrium in which all institutions hold dynamic portfolios and fully use debt capacity, price ratio $z^* = 1 - \frac{\alpha}{\bar{\alpha}} < \frac{\mathbb{E}[R_l]}{\mathbb{E}[R_h]}$, and $d_i = (1-\alpha)x_i$ for all $i \in \mathcal{I}$. In the secondary market, institutions with $\alpha_i > \alpha$ sell low-quality loans and buy high-quality loans, and institutions with $\alpha_i < \alpha$ sell high-quality loans and buy low-quality loans.*

Proof. See Appendix B. $\square$

Proposition A.1 shows that when the average fraction of low-quality loans is in a medium range, there exists an equilibrium in which all institutions operate CLOs. Depending on the realization of idiosyncratic shocks, some of these CLOs provide liquidity to others. In this equilibrium, secondary loan prices also deviate away from fundamental values. But because in aggregate, the quantity of low-quality loans is moderate, liquidity provision by other CLOs is sufficient for keeping all CLOs' debt safe.

Proposition A.2. *When $\frac{\alpha}{\bar{\alpha}} \leq \frac{1-p}{pR+1-p}$ and ξ is small, there exists an equilibrium in which all institutions hold dynamic portfolios and fully use debt capacity, price ratio $z^* = \frac{\mathbb{E}[R_l]}{\mathbb{E}[R_h]}$, and $d_i = (1 - \bar{\alpha} + \bar{\alpha} z^*) x_i$ for all $i \in \mathcal{I}$. In the secondary market, institutions with $\alpha_i > (1 - z^*)\bar{\alpha}$ sell low-quality loans and buy high-quality loans, and some of institutions with $\alpha_i < (1 - z^*)\bar{\alpha}$ sell high-quality loans and buy low-quality loans.*

Proof. See Appendix B. □

Proposition A.2 addresses the case where the average fraction of low-quality loans is relatively low. In this case, there also exists an equilibrium in which all institutions operate CLO, but loan prices exhibit no deviation from fundamentals. As a result, institutions are indifferent about secondary market trades. CLOs with larger realized fractions of low-quality loans are forced to rebalance portfolios, and their demand for liquidity is fully satisfied by other CLOs without exerting any pressure on loan prices.

Appendix B Proofs

Proof of Lemma 1. When $\Delta \mathbb{x}_i = 0$, $v(x_i, d_i, \alpha_i) = x_i(pR + (1-\alpha_i)(1-p)) - d_i$, and the institution's $t = 0$ problem simplifies to

$$V_s = \max_{x_i, d_i \geq 0} \; x_i y_0 + \gamma d_i - c(x_i) \tag{A.2}$$

$$s.t.\; 0 \leq d_i \leq \rho_s x_i \tag{A.3}$$

Since the objective strictly increases in d_i, constraint $d_i \leq \rho_s x_i$ binds. The first-order condition with respect to x_i is $y_0 - c'(x_i) + \eta_s \rho_s = 0$, where $\eta_s = \gamma$ is the shadow price of the binding collateral constraint. This gives $x_s = c'^{-1}(y_0 + \gamma\rho_s)$ and $d_s = \rho_s x_s$.

It remains to show that given these choices, institutions participate in lending, i.e., $V_s \geq 0$. Substitute x_s, d_s into the objective (A.2) and use function f defined earlier, $V_s = f(y_0 + \gamma\rho_s)$. By construction, f is continuously differentiable, and

$$f'(y) = c'^{-1}(y) + y \cdot \frac{\mathrm{d}}{\mathrm{d}y} c'^{-1}(y) - c'(c'^{-1}(y)) \cdot \frac{\mathrm{d}}{\mathrm{d}y} c'^{-1}(y) = c'^{-1}(y) > 0. \tag{A.4}$$

Hence f is strictly increasing. Given Condition 1, $f(y_0) \geq f(c'(0)) = c'(0) \cdot c'^{-1}(c'(0)) - c(c'^{-1}(c'(0))) = c'(0) \cdot 0 - c(0) = 0$. Therefore, $V_s = f(y_0 + \gamma\rho_s) > f(y_0) \geq 0$.

Proof of Lemma 2. Suppose $\frac{q_l}{q_h} > \frac{\mathbb{E}[R_l]}{\mathbb{E}[R_h]}$, the objective in program (8) would be strictly decreasing in $\Delta x_{i,l}$, and the optimal choice would be $\Delta x_{i,l} = -x_{i,l}$ for all $i \in \mathcal{I}$. This contradicts with market clearing.

Suppose instead, $\frac{q_l}{q_h} = \frac{\mathbb{E}[R_l]}{\mathbb{E}[R_h]}$, then in problem (2), $v(x_i, d_i, \alpha_i) = y_0 - d_i$ would be independent with trading choices, and the objective in problem (5) would be strictly increasing in d_i. Hence, $d_i = \rho x_i = \left(1 - \bar{\alpha} + \bar{\alpha}\frac{q_l}{q_h}\right)x_i$ for all i. Then constraint $\Delta x_{i,l}\frac{q_l}{q_h} + d_i \leq (1-\alpha_i)x_i$ in (8) implies

$$\Delta x_{i,l} \leq \frac{q_h}{q_l}((1-\alpha_i)x_i - d_i)x_i = \frac{q_h}{q_l}\left(\bar{\alpha} - \alpha_i - \bar{\alpha}\frac{q_l}{q_h}\right)x_i, \tag{A.5}$$

and hence,

$$\int_{i\in\mathcal{I}} \Delta x_{i,l}\,\mathrm{d}i \leq \frac{q_h}{q_l}\left(\bar{\alpha} - \alpha - \bar{\alpha}\frac{q_l}{q_h}\right)x_i, \tag{A.6}$$

where

$$\bar{\alpha} - \alpha - \bar{\alpha}\frac{q_l}{q_h} = \bar{\alpha} - \alpha - \bar{\alpha}\frac{pR}{pR+1-p} < \bar{\alpha} - \alpha - \bar{\alpha}\frac{pR}{pR+1-p+\gamma}. \tag{A.7}$$

By Condition 2, $\bar{\alpha} - \alpha < \bar{\alpha}\frac{pR}{pR+1-p+\gamma}$, therefore $\int_{i\in\mathcal{I}} \Delta x_{i,l}\,\mathrm{d}i < 0$, again a contradiction to market clearing.

Proof of Lemma 3. Suppose $\frac{q_l}{q_h} < \underline{z}$, optimal choice in (11) would be $d_i = 0$ for all i such that $s_i = 0$, and their trades would be $\Delta x_{i,h} = -(1-\alpha_i)x_e$. This implies $\int \Delta_{i,h}\,\mathrm{d}i < 0$, which contradicts market clearing.

Proof of Lemma 4. Let $\mathcal{I}' \subseteq \mathcal{I}$ denote the set of institutions that choose dynamic portfolios. As shown in the proof of Lemma 3, it is impossible for all these institutions to use equity financing. Now suppose they all use debt financing: $d_i = \rho x_d$ for all $i \in \mathcal{I}'$. Then

$$\Delta x_{i,h} = d_i - (1-\alpha_i)x_d = (\alpha_i - \bar{\alpha} + \bar{\alpha}z)x_d, \tag{A.8}$$

and the total demand for high-quality loans would be

$$\int_{i\in\mathcal{I}'} \Delta x_{i,l}\,\mathrm{d}i \propto \alpha - \bar{\alpha} + \bar{\alpha}z \geq \alpha - \bar{\alpha} + \bar{\alpha}\underline{z}, \tag{A.9}$$

where the last inequality follows from Lemma 3. By Condition 2, $\alpha - \bar{\alpha} + \bar{\alpha}\underline{z} > 0$, so $\int_{i\in\mathcal{I}'} \Delta x_{i,l}\,\mathrm{d}i > 0$, a contradiction to market clearing. Therefore, equity financing and debt financing must coexist among institutions in $\mathcal{I}'$.

Proof of Lemma 5. Note that $y \geq y_0 + \gamma\rho_s$ if and only if $z \leq \bar{z}$. Since f is strictly increasing, this implies that $V_s \leq V_e$ if and only if $z \leq \bar{z}$. If $z > \bar{z}$, $V_s > V_e$, and no institution would use equity financing, and by Lemma 4, all institutions choose static portfolios.

Proof of Lemma 6. Given that f is strictly increasing and that $y = pR(\alpha + z^{-1}(1-\alpha))$,

$V_e = f(y)$ is strictly decreasing in z. Also, by Condition 2, $\underline{z} > 1 - \frac{\alpha}{\bar{\alpha}}$, so

$$\frac{\mathrm{d}(y+\eta\rho)}{\mathrm{d}z} = \bar{\alpha}(pR + 1 - p + \gamma) - pR\frac{\bar{\alpha}-\alpha}{z^2} \tag{A.10}$$

$$> \bar{\alpha}\Big(pR + 1 - p + \gamma - pR\frac{\underline{z}}{z^2}\Big) \tag{A.11}$$

$$\geq \bar{\alpha}\Big(pR + 1 - p + \gamma - pR\frac{1}{\underline{z}}\Big) = 0, \tag{A.12}$$

where the last equation follows from the definition of $\underline{z}$. Since $V_d = f(y+\eta\rho) - \xi$, this implies that V_d is strictly increasing in z.

Proof of Proposition 1. Define the utility differential between debt financing and equity financing, $V_d - V_e$, as a function of the price ratio, $\Delta v : [\underline{z}, \bar{z}] \mapsto \mathbb{R}$. By Lemma 6, $\Delta v(z) = V_d - V_e$ is strictly increasing. Also, $\Delta v(\underline{z}) < 0$ for any $\xi > 0$ because by definition of $\underline{z}$ in Lemma 3, when $z = \underline{z}$, η in (14) would be zero, which implies

$$V_d = f(y + 0 \cdot \rho) - \xi = f(y) - \xi = V_e - \xi < V_e. \tag{A.13}$$

Moreover, for any $z > \underline{z}$, $\eta\rho > 0$, and given that f is strictly increasing,

$$V_d + \xi = f(y + \eta\rho) > f(y) = V_e. \tag{A.14}$$

This implies that there always exists some relatively small $\xi > 0$ such that $\Delta v(\bar{z}) > 0$. Since Δv is continuously differentiable by construction, by intermediate value theorem, equation $\Delta v(z) = 0$ has a unique solution $z^* \in (\underline{z}, \bar{z})$. By Lemma 5, at $z = z^*$, $V_d = V_e > V_s$, hence all institutions hold dynamic portfolios.

Given the optimal lending choices (x_d, x_e), optimal financing choices $d_i \in \{0, \rho x_d\}$, and optimal trades in (9), the fraction of institutions choosing $d_i = (1-\rho)x_d$, λ, is determined by market clearing (7):

$$\lambda(\rho - (1-\alpha))x_d = (1-\lambda)(1-\alpha)x_e, \tag{A.15}$$

where λ is the fraction of institutions choosing $d_i = (1-\rho)x_d$. In equilibrium $\rho = 1 - \bar{\alpha} + \bar{\alpha}z^*$, hence

$$\rho - (1-\alpha) = \alpha - \bar{\alpha} + \bar{\alpha} z^* \geq \alpha - \bar{\alpha} + \bar{\alpha}\underline{z} > 0, \tag{A.16}$$

where the first inequality follows from Lemma 3, and the second follows from Condition 2. Therefore

$$\lambda = \frac{(1-\alpha)x_e}{(1-\alpha)x_e + (\rho - (1-\alpha))x_d} \tag{A.17}$$

implied by equation (A.15) is an interior point of $(0,1)$.

Proof of Lemma 7. In equilibrium, institutions choose to hold dynamic portfolios if and only if $V_d = V_e > V_s$. Given the relationship between payoffs (V_s, V_e, V_s) and function f (see Subsection 3.4), $V_d = V_e > V_s$ is equivalent to $f(y + \eta\rho) - \xi = f(y) > f(y_0 + \gamma\rho_s)$. Since f is strictly increasing and $\xi > 0$, this implies $y + \eta\rho > y > y_0 + \gamma\rho_s$. By properties of c, it follows that $c'^{-1}(y + \eta\rho) > c'^{-1}(y) > x_s = c'^{-1}(y_0 + \gamma\rho_s)$. Using equations (12) and (13), $x_d = c'^{-1}(y + \eta\rho)$, $x_e = c'^{-1}(y)$, and $x_s = c'^{-1}(y_0 + \gamma\rho_s)$, therefore $x_d > x_e > x_s$.

Proof of Proposition 4. If there exists a price ratio $\tilde{z}_1 < \frac{\mathbb{E}[R_l]}{\mathbb{E}[R_h]}$ such that all CLOs are able to satisfy maintenance collateral constraint (4) after trading, trades in (9) must be feasible for any realized $\alpha_i \leq \tilde{\alpha}$. Given constraints in problem (8), $\tilde{z}_1$ must satisfy

$$-\tilde{\alpha} x_d \tilde{z}_1 + \rho x_d \leq (1-\tilde{\alpha})x_d, \tag{A.18}$$

or equivalently,

$$\tilde{z}_1 \geq m(\tilde{\alpha}; z^*) := 1 - (1-z^*)\frac{\bar{\alpha}}{\tilde{\alpha}}. \tag{A.19}$$

Note that $m(\tilde{\alpha}; z^*)$ is increasing in z^*. By Lemma 5, $z^* \leq \bar{z}$, hence a sufficient condition for the existence of a $\tilde{z}_1 \in [m(\tilde{\alpha}; z^*), \frac{\mathbb{E}[R_l]}{\mathbb{E}[R_h]})$ is $m(\tilde{\alpha}; \bar{z}) < \frac{\mathbb{E}[R_l]}{\mathbb{E}[R_h]}$. This condition can be simplified to

$$\frac{\tilde{\alpha}}{\bar{\alpha}} < \frac{(pR + 1 - p)(1 - p + \frac{\gamma(1-\bar{\alpha})}{1-\alpha})}{(pR + 1 - p + \frac{\gamma(1-\bar{\alpha})}{1-\alpha})(1-p)} \tag{A.20}$$

where the right-hand side is greater than 1. Therefore, as long as $\frac{\tilde{\alpha}}{\bar{\alpha}}$ is relatively small, there always exists a $\tilde{z}_1$ at which optimal trades in (9) are individually optimal and feasible at

$t = 1$. The total net demand for high-quality loans is

$$\int_{i\in\mathcal{I}} \Delta x_{i,h}\,\mathrm{d}i = (1-\lambda)\int_{i\in\mathcal{I}}(-(1-\alpha_i)x_e)\,\mathrm{d}i + \lambda\int_{i\in\mathcal{I}}(\rho-(1-\alpha_i))x_d\,\mathrm{d}i. \tag{A.21}$$

Because α_i's distribution over $[\underset{\sim}{\alpha}, \tilde{\alpha}]$ preserves the mean, i.e., $\mathbb{E}[\alpha_i] = \alpha$,

$$\int_{i\in\mathcal{I}} \Delta x_{i,h}\,\mathrm{d}i = \lambda(\rho-(1-\alpha))x_d - (1-\lambda)(1-\alpha)x_e \tag{A.22}$$

which equals zero by (A.15). Given trades in (9), the total net demand for low-quality loans is also zero. Hence, the secondary market clears at price ratio $\tilde{\alpha}_1$.

Proof of Proposition 5. Conjecture that there exists a cutoff realization $\delta \in (\underset{\sim}{\alpha}, \bar{\alpha})$ such that in the disaster state, there is a market-clearing price ratio $\tilde{z}_2 < \frac{\mathbb{E}[R_l]}{\mathbb{E}[R_h]}$, and a CLO can never satisfy maintenance collateral constraint (4) if and only if $\alpha_i > \delta$. By the definition of δ, both constraints in problem (8) will bind for $\alpha_i = \delta$:

$$-\delta x_d \tilde{z}_2 + \rho x_d = (1-\delta)x_d. \tag{A.23}$$

Under this conjecture, in the disaster state there are three groups of institutions, with different constraints and choices in problem (8):

(i) CLOs with $\alpha_i \le \delta$ will trade are the same as in (9): $\Delta x_{i,h} = \rho x_d - (1-\alpha_i)x_d$.

(ii) CLOs with $\alpha_i > \delta$ will give up the maintenance collateral constraint and shift risk to debt holders: $\Delta x_{i,h} = -(1-\alpha_i)x_d$.

(iii) Loan funds will trade the same as in (9): $\Delta x_{i,h} = -(1-\alpha_i)x_e$.

The total net demand for high-quality loans of these three groups is

$$\begin{aligned}\int_{i\in\mathcal{I}} \Delta x_{i,h}\,\mathrm{d}i =& \lambda\int_0^1 \Big(\mathbb{1}\{\alpha_i \le \delta\}(\rho-(1-\alpha_i))x_d + \mathbb{1}\{\alpha_i > \delta\}(-(1-\alpha_i)x_d)\Big)\,\mathrm{d}i \\ &- (1-\lambda)\int_0^1 (1-\alpha_i)x_e\,\mathrm{d}i.\end{aligned} \tag{A.24}$$

Since α_i is independent and uniformly distributed over $[\underset{\sim}{\alpha}, \bar{\alpha}]$,[31]

$$\int_0^1 \mathbb{1}\{\alpha_i \leq \delta\}\alpha_i \,\mathrm{d}i = \int_{\underset{\sim}{\alpha}}^{\delta} \frac{u}{\bar{\alpha} - \underset{\sim}{\alpha}} \,\mathrm{d}u = \frac{\delta^2 - \underset{\sim}{\alpha}^2}{2(\bar{\alpha} - \underset{\sim}{\alpha})}, \tag{A.25}$$

and

$$\int_0^1 \mathbb{1}\{\alpha_i > \delta\}\alpha_i \,\mathrm{d}i = \int_{\delta}^{\bar{\alpha}} \frac{u}{\bar{\alpha} - \underset{\sim}{\alpha}} \,\mathrm{d}u, = \frac{\bar{\alpha}^2 - \delta^2}{2(\bar{\alpha} - \underset{\sim}{\alpha})}. \tag{A.26}$$

Substitute the above into (A.24), the total net demand can be simplified to

$$\int_{i \in \mathcal{I}} \Delta x_{i,h} \,\mathrm{d}i = \lambda x_d \Big(\frac{\bar{\alpha} + \underset{\sim}{\alpha}}{2} - \frac{\bar{\alpha} - \delta + (\delta - \underset{\sim}{\alpha})(1 - z^*)\bar{\alpha}}{\bar{\alpha} - \underset{\sim}{\alpha}} \Big) - (1 - \lambda) x_e \Big(1 - \frac{\bar{\alpha} + \underset{\sim}{\alpha}}{2} \Big). \tag{A.27}$$

Since λ is determined at $t = 0$ with value in (A.17), market clearing $\int_{i \in \mathcal{I}} \Delta x_{i,h} \,\mathrm{d}i = 0$ implies a unique cutoff:

$$\delta = \underset{\sim}{\alpha} + \frac{(\bar{\alpha} - \underset{\sim}{\alpha})(1 - \frac{\bar{\alpha} + \underset{\sim}{\alpha}}{2})}{1 - \alpha}. \tag{A.28}$$

It is easy to verify that $\delta \in (\underset{\sim}{\alpha}, \bar{\alpha})$. Moreover, (A.23) implies

$$\tilde{z}_2 = 1 - (1 - z^*) \frac{\bar{\alpha}}{\delta}. \tag{A.29}$$

Since $\delta < \bar{\alpha}$, $\tilde{z}_2 < z^*$, and hence $\tilde{z}_2 < \frac{\mathbb{E}[R_l]}{\mathbb{E}[R_h]}$. This verifies the conjecture that $\tilde{z}_2$ is a market-clearing price ratio.

Proof of Proposition A.1. Similar argument in the proof of Lemma 2 gives $z < \frac{\mathbb{E}[R_l]}{\mathbb{E}[R_h]}$: Note that without Condition 2, $\frac{\alpha}{\bar{\alpha}} > \frac{1-p}{pR+1-p}$ still implies that $\int_{i \in \mathcal{I}} \Delta x_{i,l} \,\mathrm{d}i < 0$ using (A.7). Given optimal trades in (9), the market clears if and only if

$$\int_{i \in \mathcal{I}} (d_i - (1 - \alpha_i) x_i) \,\mathrm{d}i = 0. \tag{A.30}$$

If $d_i = \rho x_i = (1 - \bar{\alpha} + \bar{\alpha} z) x_i$ for all $i \in \mathcal{I}$, equation (A.30) implies $\alpha - \bar{\alpha} + \bar{\alpha} z = 0$, and hence price ratio $z^* = 1 - \frac{\alpha}{\bar{\alpha}}$. Note that $z < \frac{\mathbb{E}[R_l]}{\mathbb{E}[R_h]}$ since $\frac{\alpha}{\bar{\alpha}} > \frac{1-p}{pR+1-p}$. Substitute z^* into debt choice gives $d_i = (1 - \alpha) x_i$. For this debt choice to be indeed optimal, Lemma 3 indicates it is necessary that $z^* \geq \underline{z}$, which is satisfied because $\frac{\alpha}{\bar{\alpha}} \leq \frac{1-p+\gamma}{pR+1-p+\gamma}$. It can also be verified that

[31] By the Exact Law of Large Numbers (e.g., Sun, 2006), integrating a function of α_i over a continuum of iid α_i realizations recovers the function's mean: $\int_0^1 \mathbb{1}\{\alpha_i \leq \delta\}\alpha_i \,\mathrm{d}i = \mathbb{E}[\mathbb{1}\{\alpha_i \leq \delta\}\alpha_i]$ almost surely.

$y+\eta\rho > y_0+\gamma\rho_s$. Therefore when ξ is sufficiently small, $V_d = f(y+\eta\rho)-\xi \geq f(y_0+\gamma\rho_s) = V_s$, and the conjectured d_i and z^* are indeed an equilibrium.

Proof of Proposition A.2. If $z^* = \frac{\mathbb{E}[R_l]}{\mathbb{E}[R_h]}$, using argument in the proof of Lemma 2, the total demand for low-quality loans satisfies

$$\int_{i\in\mathcal{I}} \Delta x_{i,l}\,\mathrm{d}i \leq \frac{1}{z}\Big(\bar{\alpha} - \alpha - \bar{\alpha}z\Big)x_i. \tag{A.31}$$

Since the objective in problem (2) at $t = 1$ is independent with trading choices, $\int_{i\in\mathcal{I}} \Delta x_{i,l}\,\mathrm{d}i = 0$ can be satisfied if and only if $\bar{\alpha} - \alpha - \bar{\alpha}z^* \geq 0$, which holds in this case because $\frac{\alpha}{\bar{\alpha}} \leq \frac{1-p}{pR+1-p}$. Moreover, $y_0 + \gamma\rho > y_0 + \gamma\rho_s$. Therefore when ξ is sufficiently small, $f(y_0 + \gamma\rho) = V_d \geq V_s = f(y_0 + \gamma\rho_s)$, and the conjectured d_i and z^* are indeed an equilibrium.