Portfolio Dynamics and the Supply of Safe Securities

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Abstract

This paper analyzes the mechanism through which nonbank institutions transform corporate loans into safe securities. The transformation is facilitated by dynamic collateral management whereby Collateralized Loan Obligations (CLOs) rebalance their underlying portfolios as the quality of loans evolves. By maintaining portfolio quality, CLOs create larger safe tranches, thereby enjoying lower funding costs. Yet, replacing deteriorated loans generates pressure on secondary market prices. Such price pressure induces peer institutions, typically investment funds, to coexist and trade loans with CLOs. My framework explains how this mechanism reallocates loans among institutions, distorts loan prices, and increases the quantities of lending and safe securities.

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The transformation of risky assets into safe liabilities is one of the key functions of financial intermediation. This is evident in securitization: Because investors often place special value on highly-rated securities that have low default risk, many financial institutions repackage risky loans into tranches with strictly ordered cash flow priorities.[1] However, the size of safe senior tranches is always constrained by the future cash flows of the underlying loans in bad states of the world. Exceeding this constraint exposes senior tranches to a nontrivial risk of default, as witnessed in the aftermath of the 2007-09 financial crisis.

This paper introduces a new insight: dynamic collateral management helps create larger safe tranches. Relative to static collateral, dynamic portfolios can better prevent extremely bad cash flow realizations, which tend to occur after the quality of loans deteriorates. By selling deteriorated loans and replacing them with less risky loans before their cash flows are realized, an institution can keep its senior tranche away from default. Thus, a larger-sized tranche can remain safe after adverse shocks to the underlying loans.

A prominent example of this idea is Collateralized Loan Obligations (CLOs), which are nonbank institutions that create securities backed by junk-rated corporate loans. Most CLOs actively manage their loan portfolios and trade with other institutions, typically mutual and hedge funds, that also hold these loans (hereafter "loan funds"). Senior CLO tranches, or roughly 65% of the capital structure, are AAA-rated and have never defaulted since the 1990s. Despite the rapid growth of this market and related empirical research, its key mechanism is still not well understood. What drives the size of safe tranches backed by dynamic portfolios? Why do institutions holding similar loans make distinct financing choices? How do portfolio dynamics and loan prices affect lending and the total supply of safe debt?

In this paper, I provide the first study of dynamic collateral management in safe debt creation. To establish the relevance of this mechanism, I begin with direct micro evidence that CLOs replace deteriorated loans, which improves portfolio quality but exerts pressure

[1] Recent literature documents a special demand for highly-rated safe securities that arises from these securities' monetary services and regulatory advantages (e.g., Krishnamurthy and Vissing-Jorgensen, 2012; Gorton, Lewellen, and Metrick, 2012; Nagel, 2016; Van Binsbergen, Diamond, and Grotteria, 2022).

on loan prices. Motivated by new stylized facts, I develop a model of safe debt creation that integrates portfolio dynamics into an equilibrium framework. The model endogenizes institutions' lending and financing choices, from which CLOs and loan funds emerge, coexist, and engage in secondary market trades. Trading affects loan prices, which in turn feedback to safe tranche sizes and lending payoffs. In equilibrium, portfolio dynamics shapes the balance sheets of all institutions, thus affecting total lending and safe debt. My analysis sheds light on how this mechanism distorts loan prices while improving total surplus.

The adoption of dynamic collateral management is essential to both CLOs and loan funds. Like many securitization vehicles, CLOs create long-term securities backed by long-term loans, and the safety of their securities is vulnerable to loan quality deterioration. But unlike other loans, the loans held by CLOs ("leveraged loans") are rated by credit rating agencies and traded in secondary markets. This allows CLO contracts to implement dynamic collateral management. By imposing collateral constraints tied to time-varying loan ratings, these contracts obligate CLO managers to dynamically trade loans and maintain portfolio quality. Nonetheless, it is impossible for all institutions to operate as CLOs and replace deteriorated loans: the secondary market must clear. The market clears because CLOs and loan funds, operated by a common group of managers, coexist and trade as counterparties.

I analyze the equilibrium behind these institutional facts in a simple three-period model. There are two groups of agents: investors, who enjoy a non-pecuniary benefit from safe debt, and financial institutions, who can produce safe debt backed by risky loans. In the first period, institutions issue debt and equity tranches to raise funding from investors and make loans. The loans and tranches pay in the last period. Because of the non-pecuniary benefit, safe debt is issued at a premium and provides cheap funding to its issuers. Ex ante, the size of safe tranches is constrained by the worst-case realization of loan payoffs. This constraint is tight because after issuance, idiosyncratic shocks cause a random fraction of every institution's loan portfolio to deteriorate, leading to potentially very low payoffs.

My model highlights a novel dynamic link between debt safety and loans of different quality. Low-quality loans are riskier and have a lower worst-case payoff, so high-quality loans are better collateral for safe debt. However, loan quality is unknown when tranches are issued and is revealed only after the interim shocks. So the size of safe tranches, if backed by static portfolios, is limited by the uncertainty in collateral quality. Nevertheless, institutions may rebalance portfolios through secondary market trades. By incurring a fixed cost, they can adopt dynamic collateral management, which commits them to sell low-quality loans and buy high-quality loans when quality reveals. This commitment increases the portfolio's worst-case payoff beyond that of a static portfolio, thereby enabling the institution to issue a larger safe tranche ex ante.

While all institutions are identical ex ante, if dynamic collateral management is a viable option, they make distinct financing choices. My model shows that this is a generic equilibrium outcome: Some institutions, which resemble CLOs, specialize in creating safe debt, rebalancing portfolios by trading loans with other institutions, which resemble loan funds. This endogenous mix of institutions is both the cause and result of distorted loan prices. Since CLOs are obligated to trade, the pressure from their trades causes the price of low-quality loans to decrease relative to the price of high-quality loans. As such, the safety premium captured by CLOs will be shared with loan funds through trading interactions. Because dynamic collateral is costly to adopt, in equilibrium, the sharing of surplus will be partial — similar to that equilibrium prices partially reveal costly information in Grossman and Stiglitz (1980) — and CLOs and loan funds will coexist.

By integrating safe debt creation and secondary market trading into an equilibrium framework, my model helps interpret a variety of empirical findings. For example, CLOs typically sell deteriorated loans to mutual and hedge funds (Giannetti and Meisenzahl, 2021), and their loan sales exert downward pressure on secondary market prices (e.g., Elkamhi and Nozawa, 2022; Kundu, 2021; Nicolai, 2020). Conversely, CLOs buy higher-rated loans from mutual funds, supporting the prices of these loans (Emin et al., 2021). Different from these papers, which separately examine CLOs' sales and purchases, my model captures portfolio substitution: empirically, there is a strong, nearly one-to-one relationship between CLOs' loan sales and purchases during market downturns.

My model demonstrates that dynamic collateral management raises the total supply of safe debt through two channels: lending volume and risk sharing. First, since institutions could pledge static portfolios to issue safe debt, they choose dynamic portfolios if and only if doing so improves their lending payoffs. Better payoff induces a larger lending volume and hence more loans to back safe debt. Second, risk sharing increases safe debt backed by each unit of loans because after idiosyncratic shocks, CLOs with deteriorated portfolios can restore quality by trading with other institutions. These two channels are complementary: risk sharing raises lending by improving the collateral value of loans, and greater lending provides more collateral to share across institutions. Overall, while only a subset of institutions operate CLOs, the market produces more safe debt in total.

Microfounding the origin and counterparties of loan trades allows my model to shed new insights on this market structure. I show that a single endogenous variable, the magnitude of price pressure, summarizes the market's total surplus when institutions hold dynamic portfolios. In many markets, price deviation from asset fundamentals is often interpreted as symptoms of market frictions (e.g., Coval and Stafford, 2007; Mitchell, Pedersen, and Pulvino, 2007; Pulvino, 1998; Ellul, Jotikasthira, and Lundblad, 2011). Within the leveraged loan market, however, price pressure is an inherent feature and is more severe precisely when total surplus is greater. This is because, in response to external demand for safe debt, loan funds optimally provide imperfect liquidity, thereby sharing the safety premium with CLOs. As equilibrium prices equalize all institutions' expected payoffs, total surplus is greater when liquidity provision is more profitable, that is, when price pressure is severe.

Finally, my model suggests that aggregate quantities in this market are self-stabilizing: Because of the feedback between institution balance sheets and loan prices, the effects of a shock to the market will have opposite signs at the extensive and intensive margins. For example, consider raising the fixed cost of dynamic collateral management, such as imposing a regulatory burden on CLOs. While this shock reduces the fraction of CLOs, it also mitigates price pressure, which relaxes the remaining CLOs' price-dependent collateral constraints. Hence, the reduction in the fraction of CLOs and the increased lending and debt capacity of the remaining CLOs partially offset each other, stabilizing total loans and safe debt.

This paper is related to the literature on safe debt creation in financial intermediation (Diamond and Dybvig, 1983; Gorton and Pennacchi, 1990). Stein (2012) analyzes banks' short-term debt backed by asset liquidation value, which is determined by exogenous buyers' allocation between liquidity provision and real investments. My paper offers new insights on a distinct setting, which features long-term debt backed by cash flows of dynamic portfolios and the endogenous rise of trading counterparties.[2] More broadly, the literature has studied the specific ways in which intermediaries create safe debt, including risk management (DeAngelo and Stulz, 2015), early liquidation (Stein, 2012; Hanson et al., 2015), deposit insurance (Hanson et al., 2015), asset opacity (Dang et al., 2017), and diversification (Diamond, 2020). My analysis of dynamic collateral management adds a new perspective to these studies.

There is also a fast-growing body of empirical research on leveraged loans and CLOs. Recent papers, including the aforementioned by Giannetti and Meisenzahl (2021), Elkamhi and Nozawa (2022), Kundu (2021), Nicolai (2020), and Emin et al. (2021), study loan trades as a consequence of CLOs' debt covenants. Consistent with a premium for senior tranches, Cordell, Roberts, and Schwert (2021) find that CLO equity earned positive abnormal returns. Foley-Fisher, Gorton, and Verani (2021) and Griffin and Nickerson (2023) use the COVID-19 crisis as a setting to examine CLO tranche bid-ask spreads and credit ratings, respectively.[3] To my knowledge, this paper is the first to explicitly analyze dynamic collateral management

[2]The feature that ex-post trading prices feedback into ex-ante investment and financing choices also exists in theories of fire sales (e.g., Shleifer and Vishny, 1992; Gorton and Huang, 2004; Diamond and Rajan, 2011). In my paper, no liquidation is triggered by short maturity or moral hazard; Instead, long-term contracts obligate CLOs to rebalance portfolios, resulting in a pressure on relative prices.

[3]Consistent with dynamic collateral management, Griffin and Nickerson (2023) document that CLOs' loan trades improved rating agencies' collateral pool risk metrics. However, they do not study the link between loan trades and CLOs' ex-ante financing contracts, the resulting ex-post redistribution of portfolio payoffs across CLO tranches, or the equilibrium implications of these issues.

and its role in shaping institutions' lending and financing choices, their loan trades, and the supply of safe securities. It contributes to this literature with new stylized facts and an equilibrium framework to interpret empirical findings.

The rest of this paper is organized as follows. Section 1 presents empirical facts that motivate my theoretical analysis. Section 2 introduces model setup. Section 3 characterizes the equilibrium. Section 4 discusses the model's implications. Section 5 concludes.

1. Stylized Facts

I begin with institutional background and empirical facts on how CLOs' collateral constraints govern the dynamics of the underlying portfolios. Data details are in Appendix IA.1.

Institutional Background: the Leveraged Loan Market

Leveraged loans are private debt extended to corporations that have a high financial leverage.[4] These loans are originated through syndication deals, where underwriters organize select groups of lenders to privately contract with the borrowers. Following the Federal Reserve Board (2022), I restrict attention to "institutional leveraged loans", which are non-amortizing term loans and mostly held by nonbank institutions.

[Add Figure 1 here]

Collateralized Loan Obligations. CLOs are the largest group of nonbanks that hold leveraged loans. As Figure 1 shows, US leveraged loans grew from $130 billion to $1.2 trillion between 2001 and 2020, and CLOs consistently held about half of these loans. While other types of securitization are mostly backed by static collateral, CLOs' portfolios, consisting of 100–300 loan shares with $300-600 million total par values, are actively managed during a reinvestment period. CLO debt tranches mature in around 10 years, and the reinvestment

[4] S&P Global Market Intelligence defines a loan as leveraged if it is rated below Baa3/BBB-, or if it is secured and has a spread of at least 125 basis points.

period is around 5 years and often extended. After this, the CLO enters its amortization period and repays debt principal gradually.[5] The vast majority of CLOs are "open-market CLOs", whose managers are independent from banks. The manager's compensation consists of size-based fixed fees and incentive fees based on equity tranche performance.

Demand for Safe Debt. A primary economic force behind the growth of CLOs is the demand for highly-rated securities. Senior tranches, which account for about 65% of the CLO capital structure, are AAA rated. With higher yields than safer assets (e.g., US Treasuries) and fairly low regulatory risk weights, senior CLOs are attractive to, and mostly held by, banks. For example, Fitch (2019) reports that \$94 billion, \$113 billion, and \$35 billion of senior CLOs are held by banks in the US, Japan, and Europe, respectively. Historical data suggest that the AAA rating was not inflated. Since the 1990s, more than two thousand senior tranches have been issued, and none of them ever defaulted.[6] If capital-constrained banks are willing to pay a premium (i.e., accept a lower risk-adjusted return) for AAA tranches, creating these safe tranches becomes a source of cheap funding for CLOs.

Fact 1: Coexistence of Institutions with Distinct Liabilities

The leveraged loan market consists of two types of nonbank institutions. In addition to CLOs, the other type, primarily mutual and hedge funds, hold the majority of the rest of loans. In Appendix IA.2, I summarize the amounts of loans held by these loan funds and decompose public loan funds into open-end, close-end, and exchange-traded funds. A common feature of these funds is that they do not pledge their loans as collateral to back safe securities and hence face few restrictions on portfolio choices.

[Add Figure 2 here]

[5] In the amortization period, CLO managers can buy loans using only prepaid principal of existing loans. See Fitch's report for more details: Reinvestment in Amortization Period of U.S. CLOs.

[6] A subset of senior CLOs were downgraded during 2007–09 but mostly recovered to original ratings (Cordell, Roberts, and Schwert, 2021). No AAA tranche was downgraded in the 2020 COVID-19 crisis.

Notably, CLOs and loan funds are often operated by a common group of asset managers. Figure 2 illustrates that managers choose different type(s) of institutions to operate. For example, CVC Credit Partners only operates CLOs, whereas Fidelity Investments mainly manages leveraged loan mutual funds. Such choices lead to a coexistence of two types of nonbanks that invest in the same asset class but are financed by distinct liabilities.

Fact 2: CLOs' Binding Collateral Constraints

The large size of leveraged loans, typically hundreds of million or even multiple billion dollars, creates economies of scale in information production. Unlike smaller business loans, leveraged loans are traded in secondary markets and have credit ratings reflecting changes in loan quality. Therefore by contracting on individual loan ratings, CLO managers can credibly commit to dynamically replace underperforming loans as their quality deteriorates.

In practice, CLO debt contracts implement this commitment with regular (e.g., monthly) collateral tests that are tied to managerial compensation. The most important test is the over-collateralization (OC) test, which calculates a ratio of quality-adjusted loan holdings to the combined size of a tranche and tranches senior to it. If an OC test fails, the manager stops receiving fees for that tranche until the ratio recovers to a preset threshold. The manager can raise the ratio through either debt acceleration (i.e., divert portfolio cash flows to repaying senior tranches) or portfolio substitution (i.e., replace deteriorated loans with qualified loans).

[Add Figure 3 here]

Collateral constraints imposed by the contracts play a critical role in governing the dynamics of the CLO balance sheet. Figure 3 shows quarterly cross-sectional distribution of the slackness of senior OC constraints between 2010–2019. Among CLOs in reinvestment period, the average senior OC score is slightly (8%) above the threshold and stable over time.[7]

[7] In my data, the senior OC thresholds can be that of AAA and AA tranches, so my calculation may overstate the actual slackness of AAA OC constraints.

In the cross section of CLOs, the slackness is tightly distributed around the average. These persistently binding constraints suggest that managers fully use safe debt capacity provided by their loan portfolios. By contrast, in amortization period, as CLO leverage decreases with principal repayment, the slackness becomes larger and much more dispersed.

Fact 3: Binding Constraints Force CLOs to Replace Loans

Given binding collateral constraints, a shock to the quality of CLOs' underlying loans is likely to trigger secondary market trades. In this paper, I highlight an empirical pattern that is less discussed in the literature: CLOs' loan trades consist of both sales and purchases rather than just one of them. Hence, the portfolio's size remains similar, but its composition changes.

[Add Figure 4 here]

Figure 4 presents CLO balance sheet dynamics before and around the onset of COVID-19 crisis in 2020. Panel (a) shows quarterly average CLO portfolio size for each age cohort. For all cohorts, portfolio size remained stable over time. This implies that overall CLOs did not shrink in size in bad times. Indeed, Panel (b) shows that accelerated repayment of senior debt actually decreased.[8] While the size of portfolios did not change, their composition changed drastically. In Panel (c), the average numbers of loan purchases and sales both nearly doubled upon the arrival of the negative shock.[9] To understand the nature of these trades, Panel (d) examines buys and sells *within* individual CLOs in the first two quarters of 2020. As the bin scatter plot shows, there is a strong positive (and nearly one-to-one) relationship between a CLO's purchases and sales: when a CLO sells loans, it also buys loans to replace them. In other words, CLOs substitute loans in their portfolios.

[8] Earlier cohorts repaid more of their senior tranches when the non-call period ends (typically 2–3 years). Such early repayment discontinued in 2020.

[9] Purchases generally exceed sales because loan holdings generate coupon and principal payments.

Fact 4: Portfolio Substitution Improves Collateral Quality

Using granular data on CLO loan holdings, I examine how such trades affect portfolio quality in 2020. Figure 5 presents the changes from February 15 ("pre") to June 30 ("post"). Panel (a) shows senior OC slackness before and after the shock. As the pandemic caused massive downgrades of leveraged loans, the overall slackness decreased, and the dispersion across CLOs increased. When the crisis settled, however, only 1.2% of CLOs failed senior OC tests.

[Add Figure 5 here]

The reason that test failures are rare, as Fact 3 suggests, could be portfolio substitution. To quantify its causal effect, I track individual loans' quality changes and measure each CLO's counterfactual portfolio quality in the absence of loan trades. Details of this step can be found in Appendix IA.1.3. Panel (b) shows portfolio value-weighted average ratings.[10] Overall, ratings dropped, but managers' trading mitigated deterioration, improving the realized ex-post distribution relative to the counterfactual distribution.

Despite similarly binding constraints ex ante, idiosyncratic exposures to the shock may force CLOs to respond differently. I measure a CLO's exposure using the difference in rating between the pre and counterfactual portfolios. Panel (c) shows that almost all CLOs replaced downgraded loans and that the effect on quality linearly increases in exposure: on average, trading offsets 60% of deterioration caused by COVID-19. Panel (d) replaces the outcome with value-weighted average coupon rate, which measues quality based on loan pricing. In response to a 1-notch downgrading, the manager's trades reduced average coupon by 30 basis points, or roughly one standard deviation. Panels (e) and (f) further show evidence based on the direction of loan trades by comparing ratings and coupons between the loans bought and sold by a CLO. Overall, these facts support the interpretation that binding collateral constraints triggered portfolio substitution that substantially improved portfolio quality.

[10] A larger numeric rating corresponds to a better letter rating. Table IA.2 details the conversion between letter and numeric ratings.

Fact 5: Price Pressure from CLOs

Portfolio substitution triggered by collateral constraints is costly to CLO managers and equity holders because the trades not only reduce portfolio payoff uncertainty, but also exert pressure on loan prices. In Appendix IA.3, I document that during market downturns, the magnitude of transitory price drops is decreasing in loan quality, ranging from nearly 15% for "B-" to only 5% for "BB+". Consistent with price pressure from CLOs, this monotonic pattern is observed among leveraged loans but not high yield bonds. Admittedly, isolating loan price changes caused by CLOs from changes in fundamentals is difficult. While the nature of this evidence is suggestive, given various findings of price pressure in the literature, it is plausible that when a large number of CLOs substitute portfolios in the same direction, they cause the prices of bad loans to decrease relative to the prices of good loans.[11]

2. Model

The empirical facts above suggest a novel tradeoff between the ex-ante benefit of creating more safe debt and the ex-post cost of replacing deteriorated loans. It is still unclear how this tradeoff affects the choices of individual institutions operating as CLOs and loan funds, as well as the equilibrium prices and quantities of risky loans and safe debt. Such an equilibrium view is important for understanding the market and its response to policy interventions. To analyze these issues, I develop a model in which lending institutions can flexibly choose external financing and rebalance portfolios. The economy has three periods, $t \in \{0, 1, 2\}$, and two types of agents: investors and financial institutions.

Investors. There is a unit mass of investors, broadly interpreted as banks, insurance companies, households, and other entities that invest in CLOs and loan funds. Some of these investors face risk-based regulatory requirements and prefer securities with sufficiently low

[11] Griffin and Nickerson (2023) find that relative to CLOs' loan sales, their loan purchases generated negative short-term returns during the COVID crisis. This evidence is consistent with price pressure due to portfolio substitution.

default risks (e.g., AAA rated). To capture this preference, I follow the safe asset literature (e.g., Krishnamurthy and Vissing-Jorgensen, 2012; Stein, 2012; Nagel, 2016) and assume that investors maximize additively separable utility

$$U = \mathbb{E}_0\Big[\sum_{t=0}^{2} C_t\Big] + \gamma D, \tag{1}$$

where C_t is consumption in period t, and D is safe debt held at $t = 0$. Parameter $\gamma \geq 0$ is a non-pecuniary benefit from holding safe debt.[12] Its value is exogenous and determined by forces outside of this model.

At $t = 0$, investors are endowed with an amount e of perishable consumption goods. They cannot directly lend but can buy financial claims backed by loans. Therefore they allocate between consumption and financial claims, taking claim prices as given. I assume e to be sufficiently large, so investors always choose strictly positive consumption.

Financial Institutions. There is a continuum of identical financial institutions ("asset managers"), uniformly populated on $\mathcal{I} = [0, 1]$. Their preference is similar to (1), except for that they do not derive any non-pecuniary benefit from safe debt. Each institution, indexed by $i \in \mathcal{I}$, can lend at $t = 0$ to generate a risky payoff at $t = 2$. Institutions receive zero endowment but can finance their lending by issuing senior and junior financial claims. In particular, a senior claim is referred to as *safe debt* if it is backed by loans whose payoff is enough for repayment with certainty. The key friction in this economy is that markets are incomplete: agents cannot use contracts contingent on future states. For this reason, safe debt can only be supplied by institutions as their senior liabilities.

Investment Technology. Institutions can make loans to form diversified portfolios.[13] Specifically, they can turn x consumption goods into x units of loans at a private cost $c(x) - x$. This private cost captures the effort of participating in syndication deals to put together a

[12] Since senior CLOs are long-term securities, here γ may be interpreted as a shadow price of binding regulatory constraints, instead of monetary services offered by short-term safe assets.

[13] For simplicity, industrial borrowers' output is fully pledgeable, and lenders extract all the rents. Hence, institutions' lending becomes as if they directly control real assets, an approach often used in the literature (e.g., Diamond and Dybvig, 1983).

diversified portfolio. c is twice continuously differentiable and satisfies $c(0) = 0$, $c' > 1$, $c'' > 0$ on $\mathbb{R}_+$. There are two types of loans, denoted by $j \in \{h, l\}$. Every unit of loans generates a risky payoff that depends on state $s \in \{g, b\}$ at $t = 2$. In state g, which realizes with probability $p \in (0, 1)$, both types of loans pay $R_j = R > 1$. In state b, which realizes with probability $1 - p$, high-quality loans (h) pay $R_h = 1$, and low-quality loans (l) pay $R_l = 0$.

Timeline. All institutions simultaneously choose lending and financing in period $t = 0$. Specifically, each institution i raises x_i units of consumption goods from investors by issuing safe debt $d_i \geq 0$ and external equity shares. Meanwhile, the institution makes x_i units of loans without knowing loan types. In this period, the institution can commit to keeping its portfolio static until maturity. This choice is denoted by binary variable $\mathbb{s}_i \in \{0, 1\}$.

In period $t = 1$, an idiosyncratic shock determines loan quality: α_i fraction of an institution's loans deteriorate to low-quality, and $1 - \alpha_i$ fraction are high-quality. Across institutions, α_i is independently drawn from a common distribution with support $[0, \bar{\alpha}] \subseteq [0, 1]$ and mean $\alpha \in (0, \bar{\alpha})$. Loan quality is publicly observable and contractible, and institutions with dynamic portfolios (i.e., $\mathbb{s}_i = 0$) can trade loans in a Walrasian market. In period $t = 2$, payoffs realize. As internal equity holders, institutions repay safe debt and external equity and collect residual portfolio payoffs. All goods are consumed, and the economy ends.

Dynamic Collateral Management. The payoff distribution implies that, if the loan portfolio is static, each unit of loans can back at most $\rho_s = 1 - \bar{\alpha}$ safe debt at $t = 0$. To increase its debt capacity beyond ρ_s, an institution may choose a dynamic portfolio. But the ability to trade loans, if not disciplined, prevents an institution from creating any safe debt. The reason is a classic agency problem (Jensen and Meckling, 1976): as equity holders, institutions privately prefer loans with riskier payoffs, which makes their debt default with a positive probability. Given empirical facts in the last section, I assume the existence of a technology that pre-commits institutions to portfolio choices at $t = 1$. This technology can be thought of as third-party services that continuously update loan ratings, perform collateral tests, and seize loans on behalf of debtholders. Adopting the technology and credibly revealing it to investors incur a fixed cost $\xi \geq 0$.

The Institution's Optimization Problem. Institutions with dynamic portfolios make sequential choices to maximize their payoffs. I describe their optimization problem backwardly and consider repayment only in the final period.[14]

Let secondary market prices of the two types of loans be $\mathbb{q} = (q_l, q_h) \in \mathbb{R}^2_+$. When α_i realizes, institution i, with balance sheet (x_i, d_i, α_i), chooses trades $\Delta\mathbb{x}_i = (\Delta x_{i,h}, \Delta x_{i,l})$ to maximize conditional expected payoff to equity

$$v(x_i, d_i, \alpha_i) = \max_{\Delta\mathbb{x}_i} ((1-\alpha_i)x_i + \Delta x_{i,h})\mathbb{E}[R_h] + (\alpha_i x_i + \Delta x_{i,l})\mathbb{E}[R_l] - d_i. \tag{2}$$

These trades are subject to a budget constraint

$$((1-\alpha_i)x_i + \Delta x_{i,h})q_h + (\alpha_i x_i + \Delta x_{i,l})q_l \leq (1-\alpha_i)x_i q_h + \alpha_i x_i q_l, \tag{3}$$

a maintenance collateral constraint

$$d_i \leq (1-\alpha_i)x_i + \Delta x_{i,h}, \tag{4}$$

and short-sale constraints $\Delta x_{i,h} \geq -(1-\alpha_i)x_i, \Delta x_{i,l} \geq -\alpha_i x_i$. The budget constraint (3) requires the trades to be self-financed by the loan portfolio. Constraint (4) reflects the ability to credibly commit to replacing deteriorated loans: After trades, safe debt investors must receive full repayment at $t = 2$ with probability one. This constraint keeps the institution solvent, which is why equity payoff in (2) is linear in portfolio payoff.

All market participants rationally anticipate loan trades at $t = 1$ when institutions choose lending and financing at $t = 0$. Because investors are price-taking, institutions optimally price their safe debt and external equity such that investors break even in expectation. This implies a safety premium: by issuing one unit of safe debt, an institution effectively raises $1 + \gamma$. The rest of funding is raised from external equity, whose expected return will be set to zero. Taking loan prices as given, the institution chooses lending x_i and safe debt d_i to

[14] The option of repaying debt in period $t = 1$ will be discussed in Section 4.

maximize the expected payoff to internal equity:

$$V_i = \max_{x_i, d_i \geq 0} \quad \mathbb{E}_0[v(x_i, d_i, \alpha_i)] - (x_i - (1+\gamma)d_i) - (c(x_i) - x_i) - \mathbb{1}\{d_i > 0\}\xi, \tag{5}$$

$$s.t.\ 0 \leq d_i \leq \rho x_i, \tag{6}$$

where $v(x_i, d_i, \alpha_i)$ is the $t = 1$ maximized equity value in (2). The second term is the expected payoff to external equity, the third term is the institution's private cost of effort, and the last term is the fixed cost of dynamic collateral management: ξ is incurred if and only if safe debt $d_i > 0$ is backed by a dynamic portfolio.

Importantly, the maximization is subject to a price-dependent collateral constraint (6), where $\rho := \rho_s + \bar{\alpha}\frac{q_l}{q_h}$ is the debt capacity provided by each unit of loans. By selling and buying loans, an institution can generate a higher worst-case payoff than that of a static portfolio. This allows for more safe debt than $\rho_s x_i$, and $\bar{\alpha}\frac{q_l}{q_h}$ is the maximum incremental debt capacity that can be obtained from dynamic collateral management.

Different from the above, if an institution commits to a static portfolio (i.e., $\mathbb{s}_i = 1$), it only chooses lending and financing at $t = 0$ and cannot trade loans at $t = 1$.

2.1. Equilibrium Definition

In equilibrium, all institutions take loan prices as given and optimally choose lending, financing, trading, and the adoption of dynamic collateral management. Formally, a market equilibrium in this economy is a collection $\{(\mathbb{s}_i, x_i, d_i, \Delta\mathbb{x}_i)_{i\in\mathcal{I}}, \mathbb{q}\}$ such that given $\mathbb{q}$, choice $\mathbb{s}_i$ maximizes institution i's expected payoff, taking into account that (x_i, d_i) solves the its lending and financing problem at $t = 0$ and that if $\mathbb{s}_i = 0$, $\Delta\mathbb{x}_i$ solves its trading problem at $t = 1$; Moreover, the secondary market clears:

$$\int_i \Delta x_{i,j}\,\mathrm{d}i = 0 \quad \text{for } j \in \{h, l\}. \tag{7}$$

Because of the collateral constraints, institutions' ex-ante lending and financing choices affect their ex-post trades, which in turn affect the lending and financing problem through endogenous loan prices. As such, the equilibrium features a feedback loop between primary and secondary markets.

Before analyzing the model, I introduce two intuitive notations for exposition. First, let $y_0 = pR + (1-p)(1-\alpha)$ be the average hold-to-maturity payoff per unit of lending. Second, I define a function $f(y) := y \cdot c'^{-1}(y) - c(c'^{-1}(y))$, where $c'^{-1}(\cdot)$ is the inverse function of the first-order derivative of c. As will be clear in Section 3, this function maps the marginal payoff of lending to an institution's expected payoff V_i.

I impose three parametric conditions to restrict my analysis to interesting cases. The first condition ensures that lending has a positive NPV for some positive quantity:

Condition 1. $y_0 \geq c'(0)$.

Next, the cost function c will never lead to negative expected payoffs, so institutions always participate in lending:

Condition 2. $f(y_0) \geq 0$.

Last, the mean of portfolio quality deterioration is sufficiently large:

Condition 3. $\frac{\alpha}{\bar{\alpha}} > \frac{1-p+\gamma}{pR+1-p+\gamma}$.

This condition rules out the possibility that all institutions adopt dynamic collateral management while providing secondary market liquidity to each other.

2.2. Discussion of Model Setup

The key feature of my model is that safe debt provides cheap funding, but the underlying loans are overall scarce because of a convex cost of lending. Therefore, committing to replace low-quality loans with high-quality loans allows institutions to create more safe debt from a given quantity of lending. The loan payoff distribution is starker than necessary. What is crucial is the existence of a strictly positive minimum payoff R_h, which makes long-term safe debt possible.[15] Moreover, since only safe debt provides a non-pecuniary benefit, capital structure below the safe tranche is irrelevant, and it is without loss of generality to treat risky junior debt as equity.

The commitment technology is crucial for dynamic collateral management to be viable. In practice, CLOs hold only fairly standardized corporate loans that have credit ratings, so their portfolios can be disciplined by enforceable contracts. With simple collateral, CLOs differ from pre-crisis collateralized debt obligations (CDOs), which held enormous complex derivatives (Cordell, Huang, and Williams, 2011). Consistent with these facts, in my model, every institution's portfolio consists exclusively of risky loans. This occurs because consumption goods are nonstorable, cross-holdings of liabilities are unprofitable, and state-contingent bilateral contracts (e.g., derivatives) are not available.

3. Equilibrium Analysis

3.1. Static Benchmark

To provide a basic benchmark, suppose the technology of dynamic collateral management or the secondary loan market does not exist, and as a result, safe debt must be backed by static portfolios.

Lemma 1. *If portfolios must be static (i.e., $\mathbb{s}_i = 1, \forall i$), all institutions would issue safe debt and fully use debt capacity: $x_i = x_s := c'^{-1}(y_0 + \gamma\rho_s)$ and $d_i = d_s := \rho_s x_s$ for all $i \in \mathcal{I}$.*

Proof. See the Appendix. □

Because safe debt provides cheap funding, every institution pledges loans as collateral and fully uses its debt capacity. Lending x_s increases in γ because loans not only generate

[15]This positive lower bound is consistent with the leveraged loan market, where senior secured first lien term loans' default recovery rate is typically greater than 50%.

monetary payoffs but also can back safe debt, and debt capacity is more valuable when the safety premium is greater. As institutions make identical choices, aggregate lending $X_s = x_s$, and the supply of safe debt $D_s = \rho_s X_s$. This market structure resembles traditional banking, where every bank creates deposits, and loans stay on bank balance sheets.

In the rest of this section, I show how the lending and financing choices differ from this benchmark when institutions can trade loans in a secondary market and credibly commit to future portfolio choices. I first analyze individual institutions' lending, financing, and trading choices for given secondary market prices. I then study balance sheets and loan prices that clear the secondary market. At last, institutions' choices between static and dynamic portfolios will be determined in equilibrium.

3.2. Secondary Market Trades

The lending and financing choices at $t = 0$ depend on continuation value v. To derive v, in this subsection I analyze the institution's secondary market problem (2) in period $t = 1$.

In this period, budget constraint (3) binds, and since $d_i \geq 0$, constraint (4) implies that $\Delta x_{i,h} \geq -(1 - \alpha_i)x_i$ is slack. Omitting predetermined terms, the problem is equivalent to

$$\max_{\Delta x_{i,l}} \quad \left(\mathbb{E}[R_l] - \frac{q_l}{q_h}\mathbb{E}[R_h]\right)\Delta x_{i,l}, \tag{8}$$

subject to constraints $\Delta x_{i,l}\frac{q_l}{q_h} + d_i \leq (1 - \alpha_i)x_i$ and $\Delta x_{i,l} \geq -\alpha_i x_i$.

Essentially, the institution substitutes between the two types of loans. This substitution is constrained by safe debt outstanding d_i and a short-sale constraint on $\Delta x_{i,l}$. I proceed to solve problem (8) based on the following observation.

Lemma 2. *Loan prices are distorted away from fundamental values:* $\frac{q_l}{q_h} < \frac{\mathbb{E}[R_l]}{\mathbb{E}[R_h]}$.

Proof. See the Appendix. □

Lemma 2 shows that secondary market trades triggered by binding collateral constraints exert pressure on the relative price of loans. Intuitively, after idiosyncratic shocks cause the quality of loans to deteriorate, some institutions are obligated to buy high-quality loans and sell low-quality loans, which puts them in demand for liquidity. The natural providers of liquidity are institutions that hold similar loans but face no binding collateral constraints. But for the latter to be willing to provide liquidity, low-quality loans, which are inferior as collateral for safe debt, must offer a higher expected return. Hence, low-quality loans must have a lower price-to-fundamental ratio relative to high-quality loans.

The solution to (8) below indicates that, consistent with Fact 3, the institution's optimal trades lead to portfolio substitution:

$$\Delta x_{i,h} = d_i - (1-\alpha_i)x_i, \;\; \Delta x_{i,l} = \frac{q_h}{q_l}((1-\alpha_i)x_i - d_i) \tag{9}$$

for any i. These trades reallocate loans among institutions. An institution with $d_i > (1-\alpha_i)x_i$ optimally sells just enough low-quality loans to increase its holding of high-quality loans and keep its debt safe. Such portfolio substitution is costly to equity holders because it not only decreases the portfolio's payoff uncertainty, but also moves prices in unfavorable directions. By contrast, an institution with $d_i < (1-\alpha_i)x_i$ sells its high-quality loans and buys low-quality loans to profit from the deviation of loan prices from fundamentals.

3.3. Lending and Financing Choices

Next, I characterize the institution's optimal lending and financing choices at $t = 0$ for given loan prices. Optimal secondary market trades in (9) imply that for given (x_i, d_i, α), equity continuation value v at $t = 1$ is

$$v(x_i, d_i, \alpha_i) = x_i pR\Big(\alpha_i + \frac{q_h}{q_l}(1-\alpha_i)\Big) - d_i p\Big(R\Big(\frac{q_h}{q_l} - 1\Big) + 1\Big). \tag{10}$$

Substitute v into (5) and take expectation over α_i, the institution's lending and financing problem becomes

$$\max_{x_i, d_i} \; x_i pR\Big(\alpha + \frac{q_h}{q_l}(1-\alpha)\Big) - d_i p\Big(R\Big(\frac{q_h}{q_l} - 1\Big) + 1\Big) + (1+\gamma)d_i - c(x_i) - \mathbb{1}\{d_i > 0\}\xi \tag{11}$$

subject to constraint (6). Because the objective is discontinuous at $d_i = 0$, I consider two separate cases.

Equity Financing. In the first case, institution i issues only equity and gives up safe debt: $d_i = 0$. The optimal lending choice, x_e, is given by first-order condition

$$y = c'(x_e), \tag{12}$$

where $y := pR\Big(\alpha + (1-\alpha)\frac{q_h}{q_l}\Big)$ is the payoff per unit of lending when portfolios are dynamic.[16] Hence, x_e is determined by a tradeoff between this payoff and the marginal cost of lending. Different from the hold-to-maturity payoff y_0, here y depends on loan prices at $t = 1$: It is decreasing in price ratio q_l/q_h because of the price pressure, which gives rise to an expected profit that rewards liquidity provision.

Debt Financing. In the second case, institution i adopts dynamic collateral management and issues both safe debt and equity. Let η be the shadow price of constraint $d_i \leq \rho x_i$. The conditions for optimality are

$$y + \eta\rho = c'(x_d), \tag{13}$$

$$\eta = \gamma - \Big(pR\Big(\frac{q_h}{q_l} - 1\Big) - (1-p)\Big), \tag{14}$$

$$\eta \geq 0, \eta(d_i - \rho x_d) = 0. \tag{15}$$

When $\eta > 0$, the collateral constraint binds ($d_i = \rho x_d$): the institution fully uses debt capacity to exploit cheap funding. On the asset side, as characterized by equation (13), optimal lending x_d exceeds x_e. The additional investment is due to $\eta\rho$, the collateral value of loans. Since both debt capacity (ρ) and its per-unit value (η) decrease in price ratio q_l/q_h, price pressure reduces the collateral value that can be extracted from lending.

[16] $x_e > 0$ is guaranteed by Condition 1 and Lemma 5 below: $z \leq \bar{z} < \frac{pR}{pR+1-p}$, which implies $y > y_0$.

3.4. Market Equilibrium of Financial Institutions

Optimal choices in the two cases above determine the balance sheets of institutions that hold dynamic portfolios. Substitute these choices into objective (11), we can write these institutions' payoffs as $V_e = f(y)$ and $V_d = f(y + \eta\rho) - \xi$, respectively, for equity financing and debt financing, where function f was defined in Subsection 2.1. Similarly, using Lemma 1, the payoff of an institution with a static portfolio is $V_s = f(y_0 + \gamma\rho_s)$. In equilibrium, every institution achieves its highest possible expected payoff: it obtains $\max\{V_e, V_d, V_s\}$ from its optimal choices.

Our primary interest of this section is an equilibrium with dynamic loan portfolios. A key endogenous variable in the equilibrium's feedback loop is price ratio $\frac{q_l}{q_h}$, or the rate of portfolio substitution. Evidently, this ratio affects V_e and V_d through the optimal lending and financing choices. Since only the ratio, rather than the level, of loan prices is relevant, I denote $z = q_l/q_h$ for convenience. The lemmas below collectively pin down the properties of the equilibrium.

Lemma 3. *The lower bound of price ratio is $\underline{z} := \frac{pR}{pR+1-p+\gamma}$, at which the collateral value of lending vanishes to zero (i.e., $\eta = 0$).*

Proof. See the Appendix. □

While the safety premium is directly captured by institutions that create safe debt, the price pressure generated by their portfolio substitution transfers part of the surplus to other institutions that provide liquidity. The price ratio determines how the safety premium is shared through secondary market trades. If the price ratio is as low as $\underline{z}$, the sharing would be perfect, which implies $V_d = V_e - \xi < V_e$. Similar to the Grossman and Stiglitz (1980) paradox, if the benefit of creating safe debt is fully shared through prices, then no institution would adopt dynamic collateral management because doing so is privately costly. Therefore in equilibrium, price ratio must be higher that $\underline{z}$ whenever $\xi > 0$.

Lemma 4. *When there exist institutions that choose dynamic portfolios ($\mathbb{s}_i = 0$), secondary market clears only if equity financing and debt financing coexist among these institutions.*

Proof. See the Appendix. $\square$

Obviously, an equilibrium in which all institutions use equity financing cannot exist: Equation (9) suggests that all these institutions trade in the same direction. It is also impossible for all institutions with dynamic portfolios to use debt financing. While some institutions using debt financing will provide liquidity at $t = 1$ as long as the quality of their portfolios does not deteriorate severely (i.e., $\alpha_i < 1 - \rho$), the quantity of high-quality loans they sell is never enough for meeting the demand from institutions that are forced to replace deteriorated loans. Thus, the two financing choices must be both present for dynamic portfolios to show up in equilibrium, which in turn implies $V_e = V_d$ in such an equilibrium.

Lemma 5. *An upper bound of price ratio is $\bar{z} := \frac{pR}{pR+1-p+\gamma\frac{1-\bar{\alpha}}{1-\alpha}}$. $V_e > V_s$ if and only if $z < \bar{z}$. Suppose $z > \bar{z}$, all institutions choose static portfolios.*

Proof. See the Appendix. $\square$

Institutions using equity financing indirectly profit from the safety premium by providing liquidity, and their profit is higher when the price pressure is more severe. Suppose the price ratio is higher than $\bar{z}$, these institutions would not choose dynamic portfolios: they would pledge loans as collateral to issue their own safe debt. This in turn prevents any institution to use dynamic collateral management, so all institutions would end up holding static portfolios. The next lemma proves that institution payoffs are monotone in z over $[\underline{z}, \bar{z}]$, the range of price ratio implied by Lemma 3 and Lemma 5.

Lemma 6. *V_e is strictly decreasing in z, and V_d is strictly increasing in z.*

Proof. See the Appendix. $\square$

If price ratio z is low, liquidity provision is more profitable, and replacing low-quality loans is more costly. This makes equity financing more attractive relative to debt financing. By contrast, if z is high, dynamic collateral management is less costly, and providing liquidity to others is less profitable. In an equilibrium with dynamic portfolios, the price ratio adjusts until $V_e = V_d$, and institutions' lending and financing choices collectively equalize the demand and supply for loans in the secondary market.

Proposition 1. *When $\xi > 0$ is not too large, there exists a unique equilibrium. All institutions hold dynamic portfolios, price ratio $z^* \in (\underline{z}, \bar{z})$, and two distinct financing choices coexist: a fraction $\lambda \in (0, 1)$ of institutions adopt dynamic collateral management and fully use their debt capacity (i.e., operate as CLOs), and $1 - \lambda$ of institutions do not issue any safe debt (i.e., operate as loan funds). In the secondary market, CLOs on average sell low-quality loans and buy high-quality loans, and all loan funds sell high-quality loans and buy low-quality loans.*

Proof. See the Appendix. □

Proposition 1 characterizes the unique equilibrium with dynamic loan portfolios. Notably, distinct financing choices show up on the balance sheets of ex-ante identical institutions. Consistent with Fact 1, two types of financial institutions, resembling CLOs and loan funds, coexist. The CLOs fully use debt capacity, which they maximize by adopting dynamic collateral management at a private cost. Binding collateral constraints generated by this adoption, and the resulting portfolio substitution, are consistent with Fact 2, Fact 3, and Fact 4. Institutions operating as CLOs enjoy cheap funding from larger safe tranches and higher payoffs if realized portfolio deterioration is relatively low.

By contrast, institutions operating as loan funds completely give up issuing safe debt. They do so because market-clearing loan prices deviate from the loans' fundamental values (i.e., $z^* < \mathbb{E}[R_l]/\mathbb{E}[R_h]$), which is consistent with Fact 5. Such price pressure makes providing liquidity in the secondary market as profitable as operating CLOs. After loan quality realizes, loan funds sell their high-quality loans and absorb low-quality loans sold by CLOs. The loans they sell are used by CLOs as collateral to keep senior debt tranches safe.

[Add Figure 6 here]

Figure 6 illustrates the intuition of this equilibrium. As the price ratio increases between $[\underline{z}, \bar{z}]$, V_e decreases because liquidity provision becomes less profitable, whereas V_d increases because replacing deteriorated loans becomes less costly. The unique equilibrium with dynamic portfolios exists if z^*, the single-crossing point between V_e and V_d, is below $\bar{z}$. If instead $z^* \in (\bar{z}, \bar{\bar{z}})$, where $\bar{\bar{z}} := \mathbb{E}[R_l]/\mathbb{E}[R_h]$, then $V_d = V_e < V_s$ at z^*, and all institutions would choose static portfolios.

4. Implications

4.1. The Supply of Safe Debt

How does the presence of dynamic loan portfolios and an interior mix of institutions with distinct financing choices affect the market's total supply of safe debt? Through the lens of the model, I dissect the effects of dynamic collateral management on individual institutions and total quantities.

Lending Volume. Ex ante, every unit of lending generates an uncertain fraction α_i of high-quality loans, which always pay off $R_h \geq 1$ regardless of which state realizes at $t = 2$. Hence, others equal, a larger lending volume always provides greater debt capacity. My model indicates that in the equilibrium with dynamic portfolios, both CLOs and loan funds lend more than what they would had they held static portfolios.

Lemma 7. *When institutions hold dynamic portfolios, they all choose larger lending volumes than with static portfolios:* $\mathbb{s}_i = 0$ *implies* $x_i > x_s$ *for all* $i \in \mathcal{I}$.

Proof. See the Appendix. $\square$

Lending increases because dynamic collateral management helps institutions better exploit the safety premium. This is because trading interactions mitigate the impact of uncertain portfolio deterioration on individual debt capacity (see below), which allows for more cheap funding to be raised from investors for any given quantity of lending. While loan funds do not use their debt capacity, they share part of the safety premium captured by CLOs through equilibrium loan prices. Therefore, when holding dynamic portfolios, all institutions face a higher marginal payoff of lending (i.e., $y + \eta\rho > y > y_0 + \gamma\rho$) and optimally choose larger lending volumes.

Risk Sharing. Ex post, dynamic collateral management facilitates risk sharing across institutions. Sharing risk is valuable because the interim shocks that cause portfolio quality deterioration are unpredictable and idiosyncratic. After these shocks realize, CLOs that experienced severe deterioration can restore portfolio quality by trading with institutions whose collateral constraints are slack. This reallocation of loans achieves a more efficient use of overall collateral.

The value of risk sharing is reflected in total debt capacity. The market-clearing condition (7) and loan trades in (9) jointly imply that with dynamic portfolios, total safe debt

$$D = \int_{i \in \mathcal{I}} d_i \,\mathrm{d}i = (1 - \alpha)X. \tag{16}$$

Hence in aggregate, debt capacity per unit of loans in this market is determined by the mean (α), rather than the minimum ($\bar{\alpha}$), of realized individual portfolio deterioration. So by relaxing collateral constraints, risk sharing increases total debt capacity for any given total quantity of lending.

Overall, despite that only a subset of institutions, namely the CLOs, create safe debt, they produce more safe debt in total because of larger lending volumes and the benefit of risk sharing. The next proposition summarizes these effects of dynamic collateral management on the total supply of safe debt.

Proposition 2. *When dynamic collateral management is adopted, the market produces a* *greater supply of safe debt than the static benchmark: $D > D_s$. The increased supply comes from two complementary channels: (i) Dynamic portfolios increase the payoff, and therefore the quantity, of lending: $X > X_s$. (ii) Risk sharing across institutions allows for greater total debt capacity for any given quantity of loans: $(1-\alpha)X > \rho_s X$ for any X.*

In the presence of dynamic collateral management, the market is more responsive to an increased demand for safe debt. Figure 7 illustrates the comparative statics with respect to γ. When γ is very small, all institutions hold static portfolios. Once γ is above a threshold level γ_{min}, a fraction λ of institutions incur the fixed cost ξ to adopt dynamic collateral management. At this threshold, risk sharing leads to a discrete jump in both debt capacity ρ and CLO lending x_d. Accordingly, there is a discontinuous increase in aggregate lending X and safe debt D. As γ increases further, the lending of both CLOs and loan funds grows. Meanwhile, the equilibrium price ratio z^* drops, thereby keeping the safety premium shared between CLOs and loan funds.[17]

The aggregate quantities with dynamic portfolios can be compared against equilibrium with static portfolios. The equilibrium total lending X exceeds X_s, which translates into greater total debt capacity, even if individual debt capacity per unit of loans remained at ρ_s. The actual total safe debt $D = (1-\alpha)X$ is even greater than $\rho_s X$ because of risk sharing.

4.2. Price Pressure and Total Surplus

A salient feature of dynamic collateral management is that when CLOs rebalance loan portfolios, they generate a price pressure. As a result, the price of low-quality loans will decrease relative to the price of high-quality loans. My model shows that this price pressure is an inherent equilibrium feature, which arises as institutions facing external demand for safe debt optimize their balance sheets without frictions.

[17] Although a declining price ratio tightens up CLO collateral constraints and lowers ρ, total safe debt increases because the increases in lending volumes and the fraction of institution operating CLOs.

Interestingly, the magnitude of this price pressure is informative about the market-wide total surplus achieved with dynamic collateral management. Given that price-taking investors always break even in expectation, this total surplus is equal to the sum of expected payoffs across all institutions: $TS := \int_{i \in \mathcal{I}} V_i \, \mathrm{d}i$. In equilibrium $V_d = V_e$, so $TS = V_e = f(y)$, where $y = pR(\alpha + \frac{1-\alpha}{z^*})$. Thus, given the exogenous investment technology (e.g., p, R, α), total surplus TS is captured by a single endogenous variable: equilibrium price ratio z^*. Moreover, since f is strictly increasing, TS and z^* are negatively associated with each other.

Proposition 3. *When dynamic collateral management is adopted, equilibrium market-wide total surplus is greater whenever the price pressure is more severe.*

Because individual lending and financing choices adjust together with loan prices, in equilibrium, the price ratio is sufficient to summarize all the gains from using safe debt backed by dynamic portfolios to finance lending. The positive relationship between the magnitude of price pressure and total surplus is intuitive. Given that the equilibrium loan prices always ensure that CLOs and loan funds receive the same expected payoffs, the total surplus is greater whenever institutions operating loan funds are better off. This occurs precisely when the price pressure is more severe and liquidity provision is more profitable. Therefore, my model suggests that in this market, price pressure is a reflection of value creation rather than a symptom of frictions.

4.3. Self-Stabilizing Aggregate Quantities

My model highlights a feedback between institutions' balance sheets and loan prices, which operates through price-dependent collateral constraints. This feedback gives rise to a self-stabilizing effect on aggregate quantities. Specifically, the direct effect of a shock to the market tends to be accompanied by an opposite indirect effect, which attenuates the net effect of the shock on aggregate quantities.

[Add Figure 8 here]

I use two examples to illustrate this effect. Figure 8 presents the equilibrium under different values of ξ, the fixed cost of dynamic collateral management.[18] The direct effect of a higher ξ is that, at the extensive margin, a smaller fraction of institutions operate CLOs, so the total supply of safe debt D may decrease. But meanwhile, an indirect effect exists at the intensive margin. As fewer CLOs demand liquidity in the secondary market, the price pressure eases. As a result, equilibrium price ratio z^* relaxes the remaining CLOs' collateral constraints, which in turn induces the CLOs to lend more than before: both ρ and x_d increase. Overall, the net impact of the increase in ξ on aggregate lending X and safe debt D depends on these two counteracting effects. The impact could be limited, as long as ξ does not exceed the maximum level ξ_{max}, beyond which institutions will hold static portfolios.

[Add Figure 9 here]

In the second example, I consider a change in $\bar{\alpha}$, the worst-case realization of portfolio quality deterioration. This may be thought of as a regime-shifting event that systematically weakens loan covenants, which lowers default recovery rates in bad times. Figure 9 shows that if $\bar{\alpha}$ is very close to α, institutions would hold static portfolios because the benefit of risk sharing is limited. When $\bar{\alpha} > \bar{\alpha}_{min}$, the direct effect of an increase in $\bar{\alpha}$ is at the intensive margin: CLO debt capacity $\rho = 1 - \bar{\alpha} + \bar{\alpha} z^*$ declines as the worst-case portfolio payoff falls. This effect reduces the safe premium captured by CLOs, which in turn causes price ratio z^* to increase and lending by both CLOs (x_d) and loan funds x_e to decrease. However, at the extensive margin, more institutions choose to operate CLOs. As such, the shock's impact on total quantities of lending X and safe debt D is mitigated by an increase in λ.

[18] The increase in ξ can be viewed as a shock to the cost of operating CLOs, such as the introduction of a regulatory burden on CLO managers. In Appendix IA.4.2, I discuss a real-world regulation that increased the cost of operating CLOs.

4.4. Discussions

4.4.1. Why Do CLO Securities Have Long Maturities?

My focus on long-term debt leaves the question open as to why CLOs do not issue short-term debt, which can either rollover in normal times or enforce asset liquidation in bad times. I argue that the observed CLO debt maturity is also an equilibrium outcome: Given that the leveraged loan market is segmented from public markets and that it is difficult for outsiders to buy liquidated loans, issuing long-term safe debt is optimal for CLOs.

This argument can be formalized by introducing a costly storage technology at $t = 0$, which allows investors to purchase loans and institutions to repay debt at $t = 1$. When investors' storage cost is high, so is their required return from liquidated loans. As a result, liquidating loans and repaying debt will be costly to CLO equity holders. Thus, long-term contract, which helps CLOs maximize and maintain cheap leverage, will be preferred. This argument explains the maturity of CLO debt based on the high participation costs faced by potential buyers who are not active lenders in this market.

4.4.2. Will Institutions Internalize Loan Trades?

My model allows institutions to flexibly choose external financing. It is in principle possible that an institution operates two entities with very different liabilities (e.g., a CLO and a loan fund), which seems appealing because the institution can then internalize loan trades after the quality of its portfolio deteriorates. That is, instead of buying and selling loans in the secondary market, the manger could reallocate loans between the entities it operates.

However, doing so is suboptimal for the institution. This is because without trading in the secondary market, the quantity of safe debt an institution can create is constrained by its initial loan portfolio ($d_i \leq \rho_s$), which is dominated by its choices in equilibrium ($V_d = V_e > V_s$). Therefore in equilibrium, institutions tend to specialize, as shown in Fact 1.

4.4.3. Contractual Frictions

Dynamic collateral management relies on long-term contracts to discipline portfolio choices. These contracts are enforced with publicly observable risk metrics, primarily credit ratings, which do not perfectly measure loan quality.[19] To simplify the analysis, my model has assumed that institutions can fully commit to replacing low-quality loans, even if such trades reduce their own payoffs. Nonetheless, as long as imperfectly contractible risk metrics are sufficiently informative about loan quality, the contract should still be calibrated and adjusted to implement desired outcomes (e.g., over-collateralization provisions).[20]

4.4.4. Valuation of CLO Securities and Loan Fund Shares

Because all agents have linear preferences, my model is silent about the pricing of nonbank liabilities when marginal investors are risk-averse. That said, the intuition provided by my analysis suggests that it is important to account for the effects of CLO contracts on portfolio dynamics in the valuation of CLO securities and loan fund shares.

Evaluating the risk of CLO securities solely based on current balance sheets understates both the safety of senior tranches and the risk of junior tranches. Because CLO managers commit to maintain portfolio quality, junior debt and equity tranches tend to lose more value in bad times than they would with static portfolios. Therefore, when portfolios are actively managed, junior tranche investors may require additional compensation for riskier returns. Meanwhile, loan funds' shares are less risky than their current portfolios, because these funds' losses in bad times will be mitigated by their trades with CLOs. These endogenous pro-cyclical and counter-cyclical payoffs should be considered by investors and analysts.

[19] Loumioti and Vasvari (2019) find evidence for CLO managers' strategic trading behaviors that inflate their OC test scores.

[20] In a dynamic environment with repeated interactions, the asset manager's reputation can also provide incentives for maintaining portfolio quality. Analyzing this channel is beyond the scope of the current paper.

4.4.5. Dynamic Collateral Management in Other Markets

While my paper's application focuses on the leveraged loan market, dynamic collateral management has been also adopted in other financial markets. For example, securitization vehicles called CRE CLOs create various tranches of securities backed by actively-managed commercial real estate mortgage portfolios.[21] In the market of securities backed by credit card receivables, the sponsors often purchase high-quality accounts and remove delinquent accounts in the collateral pool under the monitoring of a rating agency. Similarly, to enlarge debt capacity and keep loans solvent, a cryptocurrency-backed lending platform named ALEX developed a Collateral Rebalancing Pool ("CRP") technology. The CRP uses algorithms to dynamically rebalance collateral pools between riskier digital assets (e.g., Bitcoins) and less risky tokens as market conditions evolve. Therefore, the framework in my paper can potentially help interpret empirical findings and inform policy designs in these markets.

5. Conclusion

This paper analyzes a lending market where securitized tranches are backed by dynamic loan portfolios. Before the 2007–09 financial crisis, the securitization industry manufactured large quantities of senior tranches backed by static loan portfolios. Many of these tranches defaulted because their underlying loans deteriorated and failed to generate sufficient cash flows for repayment. By contrast, in the leveraged loan market, CLOs have been creating AAA-rated securities for more than three decades without any default record.

The financial innovation of CLOs is dynamic collateral management, whereby a long-term contract obligates the managers to dynamically maintain the quality of the underlying loan portfolios. This contract generates an intertemporal tradeoff: it helps CLOs create larger safe tranches ex ante but triggers costly ex-post portfolio substitution, which exerts price pressure

[21] According to BofA Global Research, during the COVID pandemic, active portfolio management helped CRE CLOs achieve considerably lower delinquency rates than the conduit and SASB deal.

in the secondary market. This paper develops an equilibrium model to understand how this mechanism drives loan prices and the quantities of lending and safe debt creation. My model provides a framework to interpret the coexistence of CLOs and loan funds and the trades that reallocate loans across institutions. It also sheds light on on how portfolio dynamics raises the supply of safe debt, improves total surplus, and drives the market's responses to shocks.

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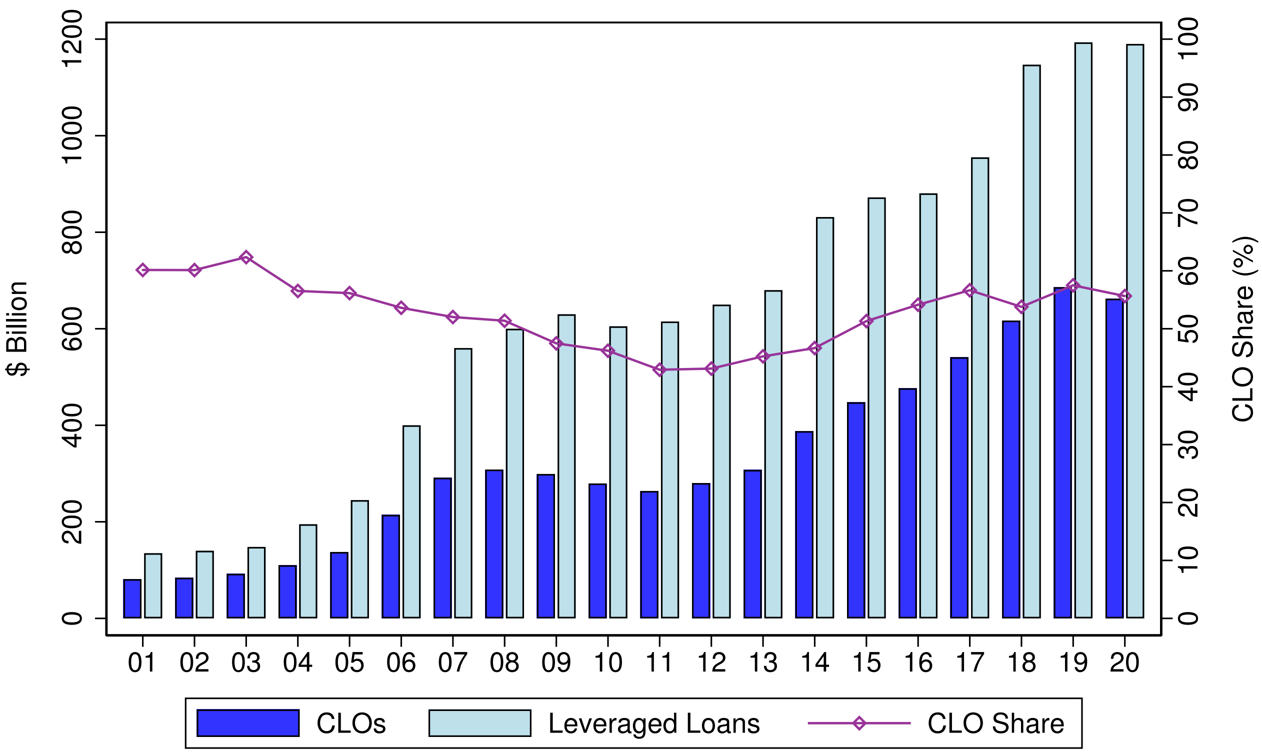


Figure 1: **Leveraged Loans and CLOs Outstanding, 2001–2020.**
This figure plots annual total par values outstanding for leveraged loans (i.e., institutional term loan facilities) and CLOs in the US market. Data source: SIFMA.

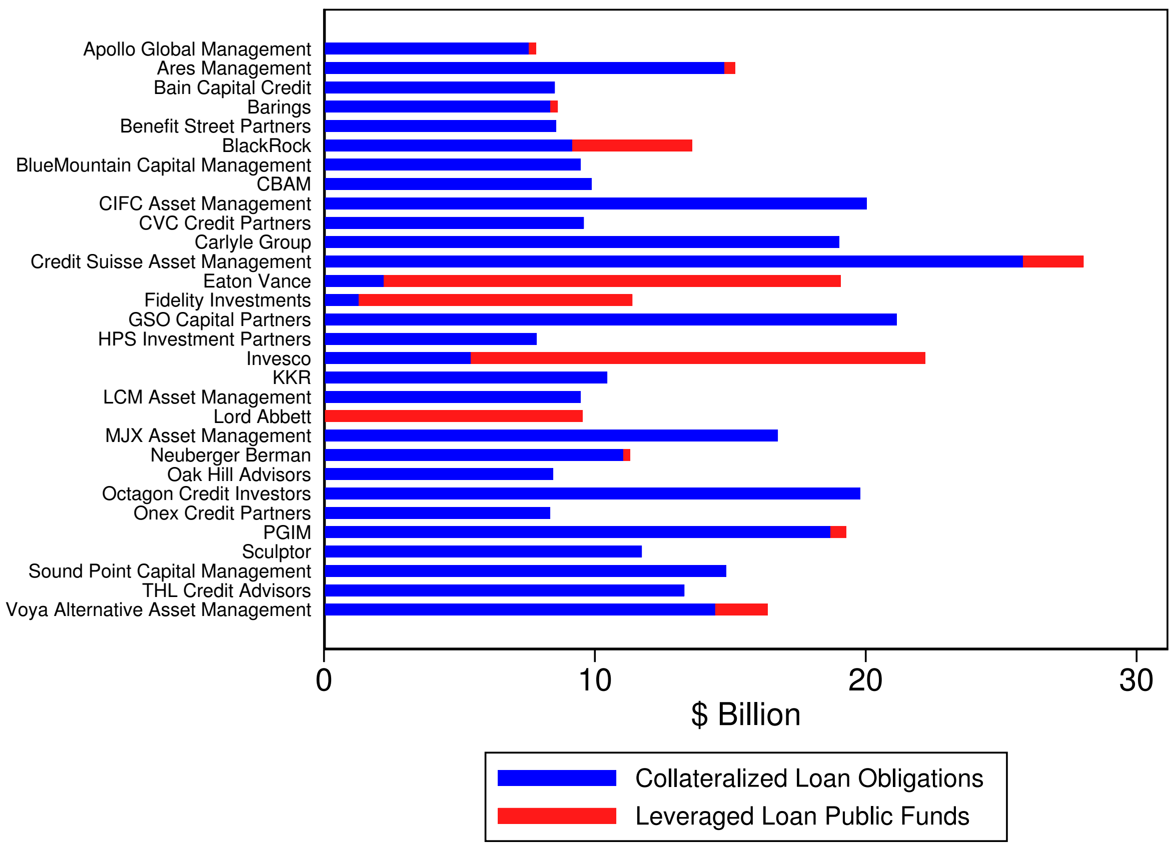
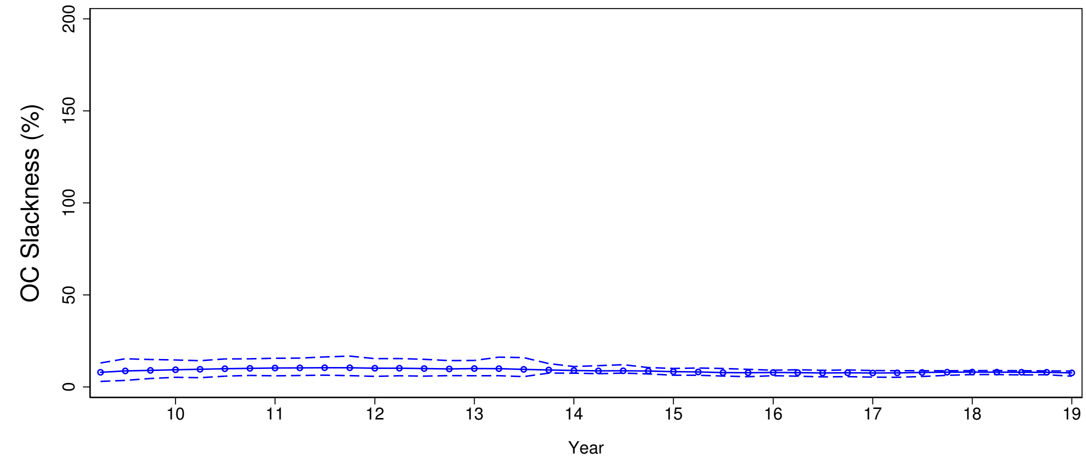
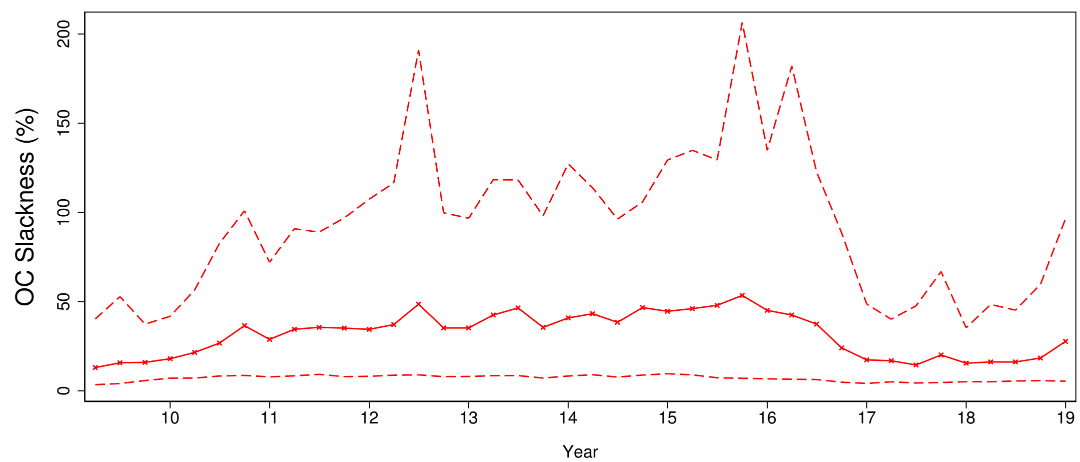


Figure 2: **Asset Managers and Nonbank Institutions.**
This figure presents assets under management for US CLOs and public loan funds (the sum of open-end mutual funds, closed-end mutual funds, and exchange-traded funds) operated by the 30 largest asset managers at the end of 2019. Data come from Creditflux CLO-i, Morningstar, and the SEC's Form ADV databases.

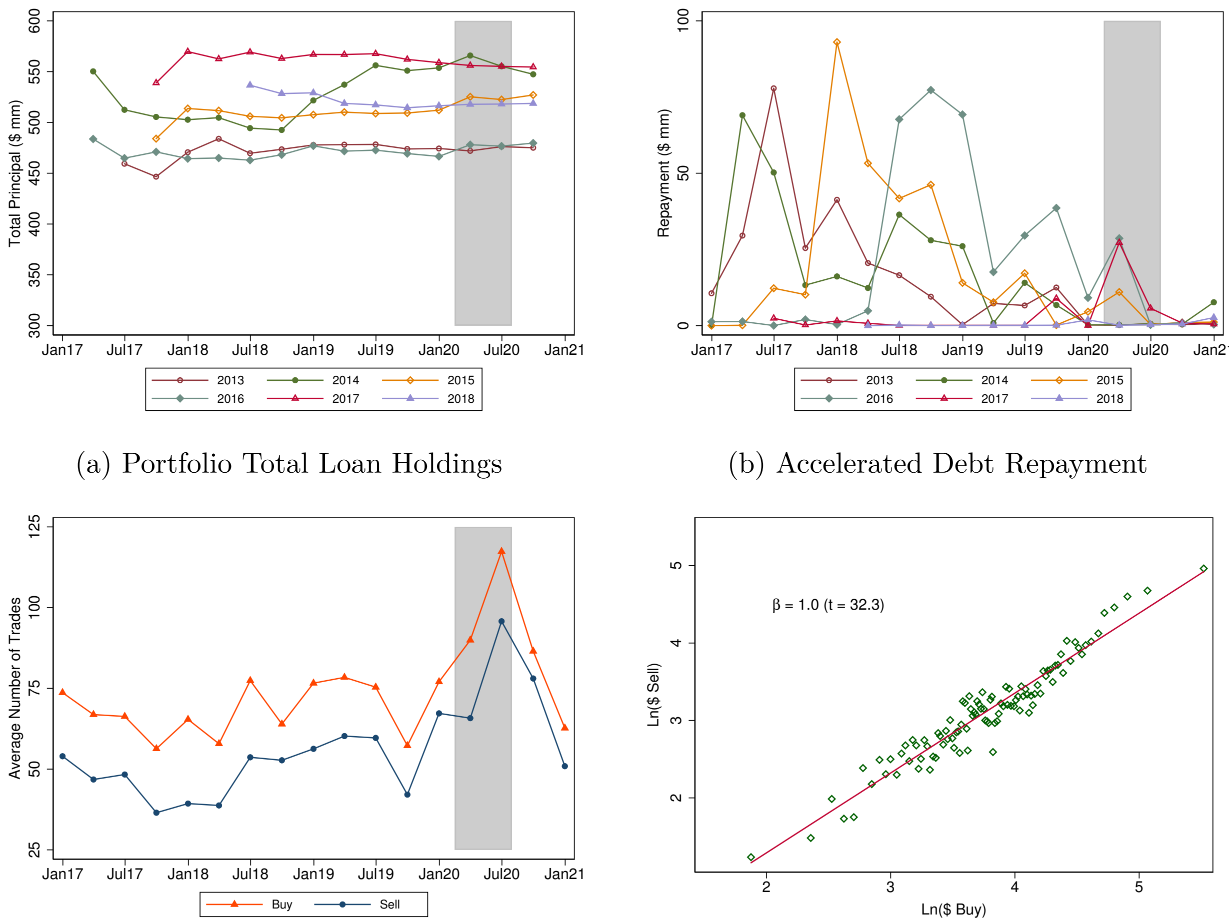


(a) CLOs in Reinvestment Period



(b) CLOs in Amortization Period

Figure 3: **Slackness of Senior Tranche Over-Collateralization Constraint.**
This Figure presents quarterly time series of cross-sectional dispersion in the slackness of CLO senior tranche over-collateralization (OC) constraints between 2010–2019. The slackness is defined as extra OC score scaled by the OC test's predetermined threshold level. Dashed lines indicate 5th and 95th percentiles in each cross section. Panel (a) reports CLOs in reinvestment period, and panel (b) reports CLOs in amortization period.



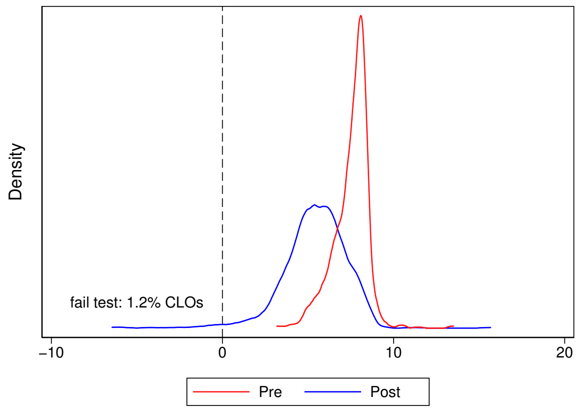
(a) Portfolio Total Loan Holdings

(b) Accelerated Debt Repayment

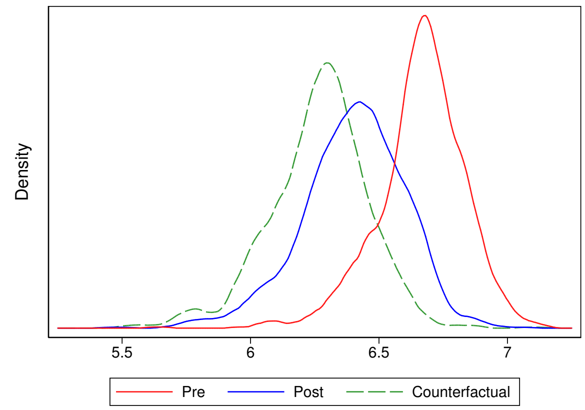
(c) Quarterly Loan Trades

(d) Individual CLOs' Purchases and Sales

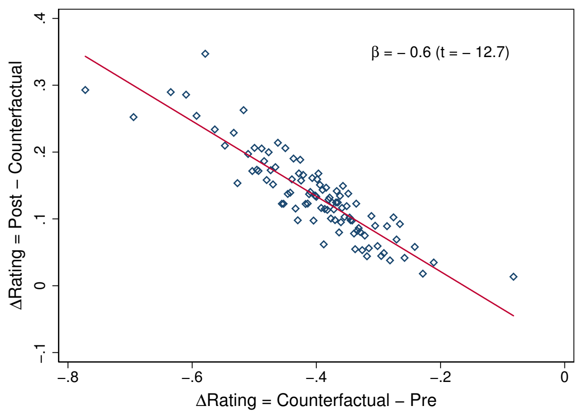
Figure 4: **Balance Sheet Dynamics Around the Onset of COVID-19 Pandemic.** This Figure shows quarterly changes in CLOs' assets and liabilities before and during the COVID-19 shock in 2020. Panel (a) plots average portfolio size by CLO age cohort. Panel (b) plots average accelerated repayment of AAA tranches by CLO age cohort. Panel (c) plots quarterly average numbers of loan purchases and sales. Panel (d) is a scatter plot that groups CLOs into 100 bins based on the dollar volumes of individual CLOs' loan purchases and sales during the first two quarters of 2020. Only CLOs in reinvestment period are included.



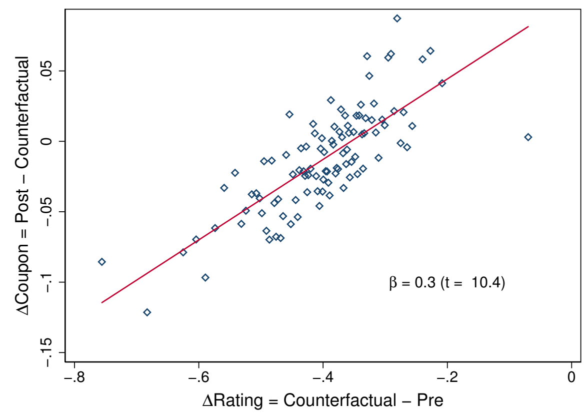
(a) Slackness of OC Constraint (%)



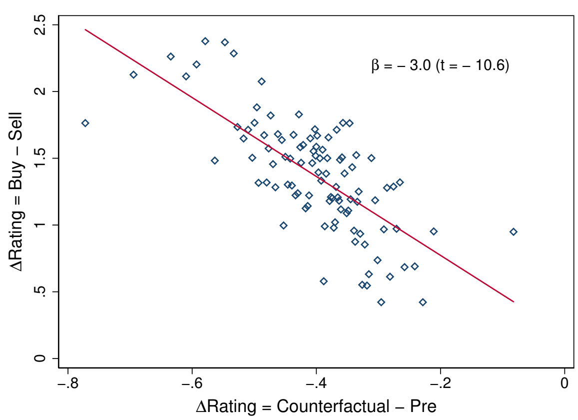
(b) Portfolio Value-Weighted Average Rating



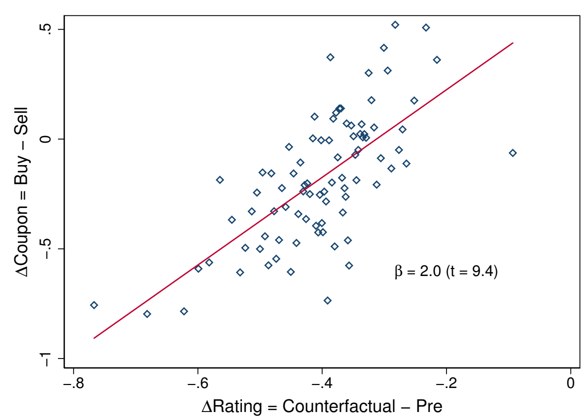
(c) Quality Improvement: Rating



(d) Quality Improvement: Coupon



(e) Trading Direction: Rating



(f) Trading Direction: Coupon

Figure 5: **Portfolio Substitution Improves Collateral Quality.**
This Figure shows the effect of portfolio substitution on CLOs' collateral quality between February 15 and June 30 of 2020. Panel (a) plots kernel density estimates for the distribution of senior OC constraint slackness before and after the onset of COVID-19 pandemic. Panel (b) plots kernel density estimates for the distribution of value-weighted average credit rating for portfolios before and after the shock as well as counterfactual static portfolios. Panels (c)-(f) are scatter plots that group CLOs into 100 bins by counterfactual collateral deterioration and depict the average effect of loan trading within each bin. The fitted lines represent OLS estimates, and t-statistics are based on heteroskedasticity-robust standard errors. Only CLOs in reinvestment period are included.

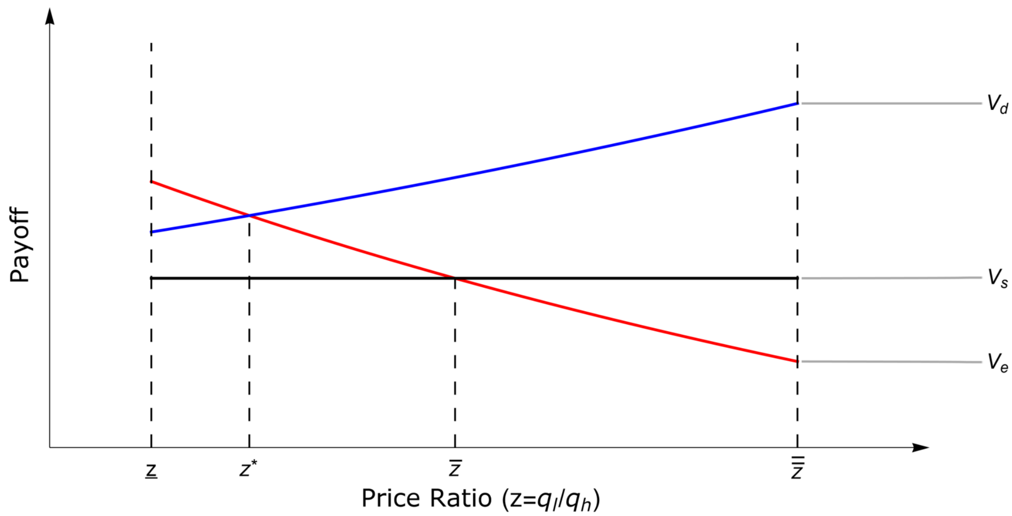


Figure 6: **Determination of Equilibrium.**
This figure illustrates the determination of the equilibrium. Solid lines V_d, V_e, and V_s indicate respectively the payoffs of CLOs, loan funds, and institutions issuing safe debt backed by static portfolios, as functions of secondary market loan price ratio z. Parameter values and functional forms: $p = 0.9$, $R = 1.2$, $\alpha = 0.4$, $\gamma = 0.15$, $\bar{\alpha} = 0.7$, $\xi = 0.05$, and $c(x) = \frac{4}{5}x^{\frac{5}{4}}$.

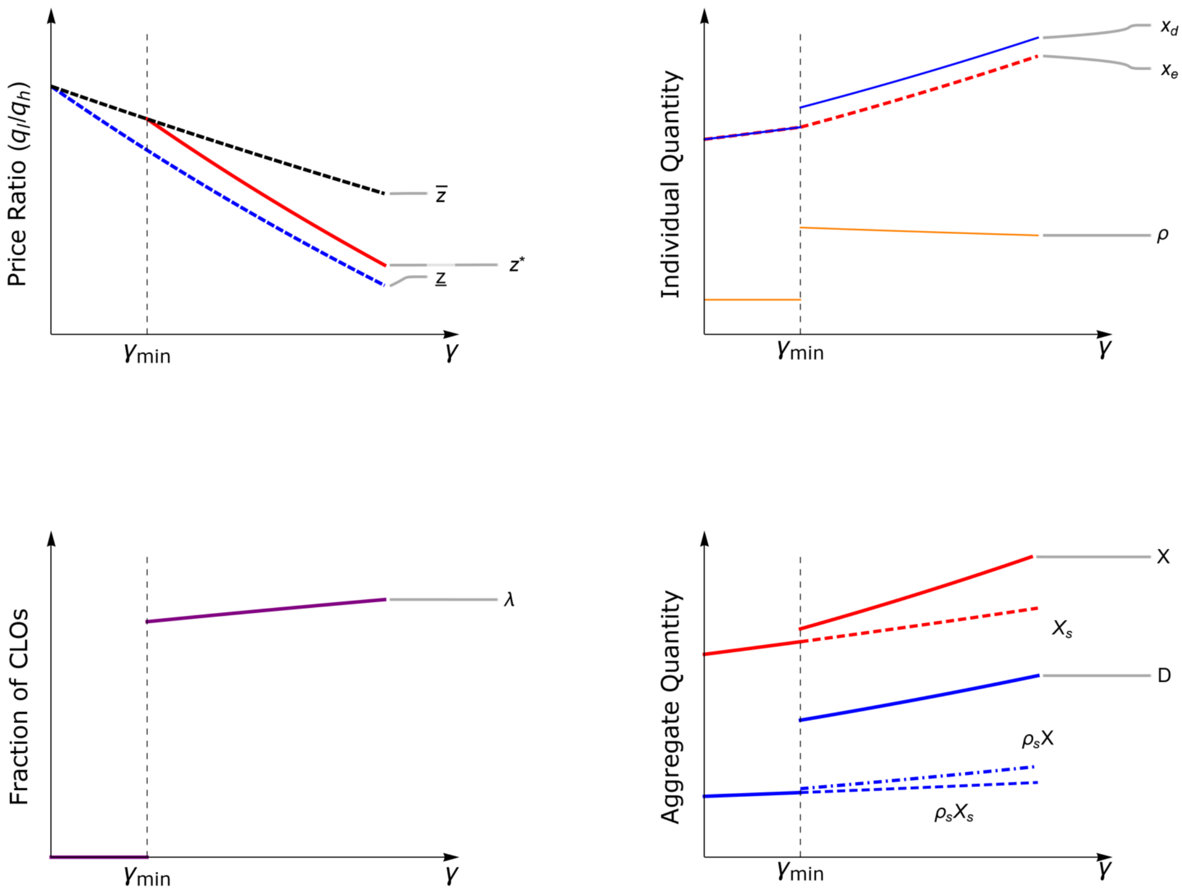


Figure 7: **The Market's Responses to Changes in Safe Premium.**
This figure illustrates equilibrium prices and quantities when the safety premium γ changes. Parameter values: $p = 0.9$, $R = 1.2$, $\alpha = 0.4$, $\bar{\alpha} = 0.7$, $\xi = 0.05$, and functional form $c(x) = \frac{4}{5}x^{5/4}$.

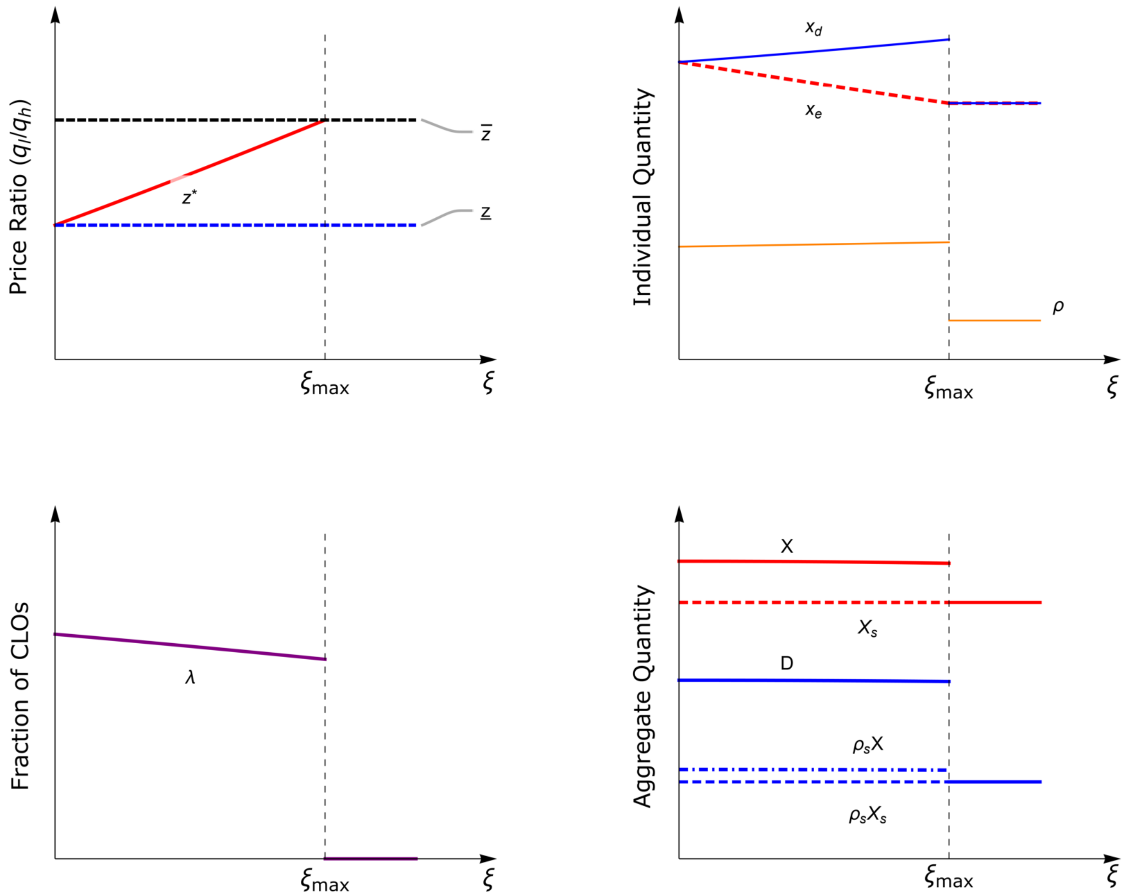


Figure 8: **The Fixed Cost of Dynamic Collateral Management.**
This figure illustrates equilibrium prices and quantities when the fixed cost of dynamic collateral management, $\xi \geq 0$, changes. Parameter values: $p = 0.9$, $R = 1.2$, $\alpha = 0.4$, $\bar{\alpha} = 0.7$, $\gamma = 0.15$, and functional form $c(x) = \frac{4}{5}x^{5/4}$.

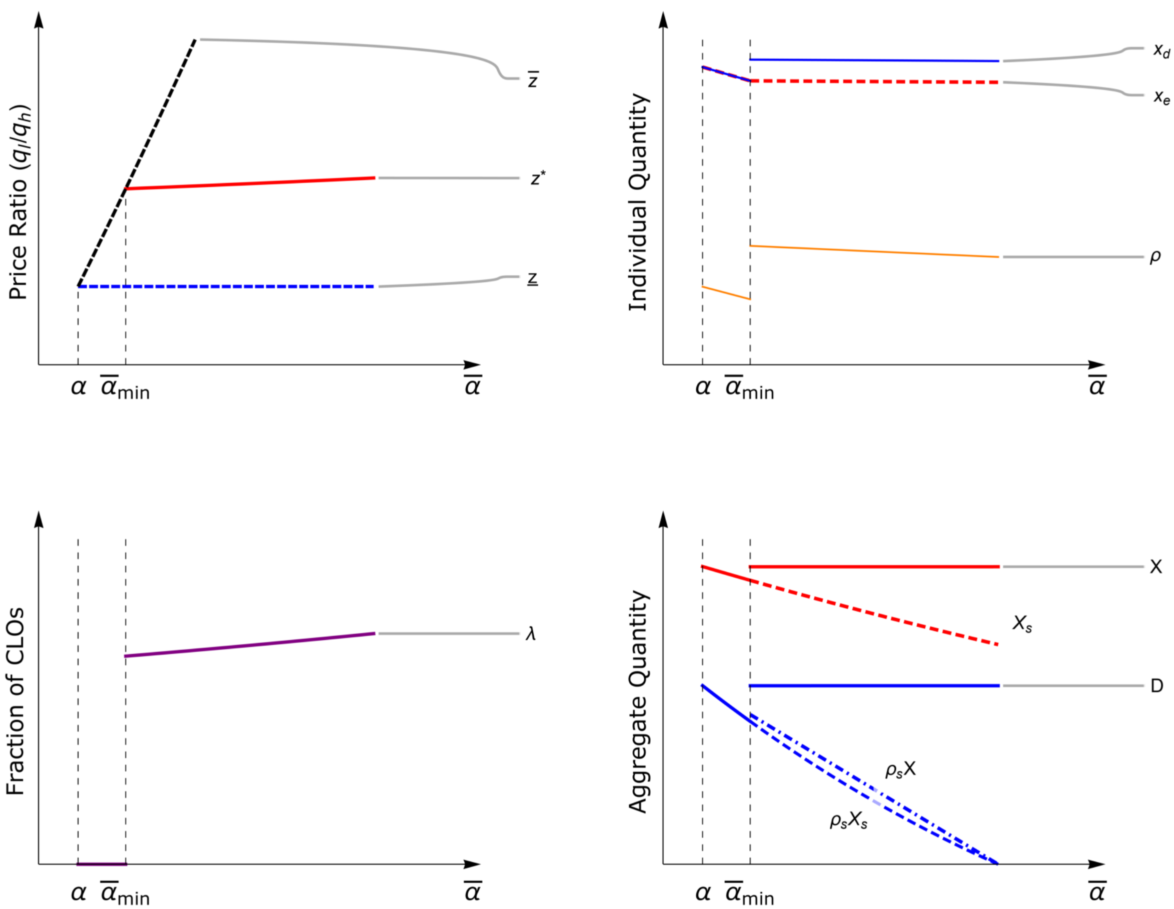


Figure 9: **Worst-Case Realization of Portfolio Quality Deterioration.**
This figure illustrates equilibrium prices and quantities when the worst-case realized fraction of portfolio deterioration, $\bar{\alpha}$, varies over $(\alpha, 1]$. Parameter values: $p = 0.9$, $R = 1.2$, $\alpha = 0.4$, $\gamma = 0.15$, $\xi = 0.05$, and functional form $c(x) = \frac{4}{5}x^{5/4}$.

Appendix: Omitted Proofs

Proof of Lemma 1. If $\Delta \mathbb{x}_i = 0$, $v(x_i, d_i, \alpha_i) = x_i(pR + (1-\alpha_i)(1-p)) - d_i$, so the institution's $t = 0$ problem reduces to

$$V_s = \max_{x_i, d_i \geq 0} \; x_i y_0 + \gamma d_i - c(x_i) \tag{A.1}$$

$$s.t. \; 0 \leq d_i \leq \rho_s x_i \tag{A.2}$$

Since the objective strictly increases in d_i, constraint $d_i \leq \rho_s x_i$ binds. The first-order condition with respect to x_i is then $y_0 - c'(x_i) + \eta_s \rho_s = 0$, where $\eta_s = \gamma$ is the shadow price of the binding collateral constraint. This gives $x_s = c'^{-1}(y_0 + \gamma\rho_s)$ and $d_s = \rho_s x_s$. Note that $c' > 0$ and Condition 1 guarantee that $x_s > 0$, and that Condition 2 ensures all institutions participate.

Proof of Lemma 2. Suppose $\frac{q_l}{q_h} > \frac{\mathbb{E}[R_l]}{\mathbb{E}[R_h]}$, the objective in program (8) would be strictly decreasing in $\Delta x_{i,l}$, and the optimal choice would be $\Delta x_{i,l} = -x_{i,l}$ for all $i \in \mathcal{I}$. This contradicts the market clearing condition (7). It is also impossible that $\frac{q_l}{q_h} = \frac{\mathbb{E}[R_l]}{\mathbb{E}[R_h]}$. This is because otherwise, optimal choice in (5) would be $d_i = \rho x_i$ for all i, where $\rho = 1 - \bar{\alpha} + \bar{\alpha}\frac{\mathbb{E}[R_l]}{\mathbb{E}[R_h]}$. The maintenance collateral constraint in (8) then implies $\int \Delta x_{i,l} \, \mathrm{d}i \leq \frac{q_h}{q_l} x_i(\bar{\alpha} - \alpha - \bar{\alpha}\frac{\mathbb{E}[R_l]}{\mathbb{E}[R_h]})$. Given Condition 3, $\bar{\alpha} - \alpha < \bar{\alpha}\frac{\mathbb{E}[R_l]}{\mathbb{E}[R_h]}$, so $\int \Delta x_{i,l} \, \mathrm{d}i < 0$, a contradiction to market clearing.

Proof of Lemma 3. Substitute $\frac{q_l}{q_h} = \underline{z}$ into equation (14).

Proof of Lemma 4. Let $\mathcal{I}' \subseteq \mathcal{I}$ denote the set of institutions that choose dynamic portfolios. Given (9), it is obvious that if all institutions in $\mathcal{I}'$ choose equity financing ($d_i = 0$), then total demand for high-quality loans is $\int_{i \in \mathcal{I}'} \Delta x_{i,h} \, \mathrm{d}i \propto -(1-\alpha)x_e < 0$, hence the market cannot clear. If instead all institutions choose debt financing ($d_i = \rho x_d$), the total demand is $\int_{i \in \mathcal{I}'} \Delta x_{i,h} \, \mathrm{d}i \propto (\alpha - \bar{\alpha} + \bar{\alpha} z)x_d$. By Lemma 3 and Condition 3, $\alpha - \bar{\alpha} + \bar{\alpha} z \geq \alpha - \bar{\alpha} + \bar{\alpha}\underline{z} > 0$, hence the market cannot clear.

Proof of Lemma 5. Recognize that $y > y_0 + \gamma\rho_s$ if and only if $z < \bar{z}$. Given the monotonicity of f, this implies that $V_e > V_s$ if and only if $z < \bar{z}$. If $z > \bar{z}$, $V_e < V_s$, so no institution would choose equity financing, and hence by Lemma 4, all institutions choose static portfolios.

Proof of Lemma 6. By construction, f is continuously differentiable, and

$$f'(y) = c'^{-1}(y) + y \cdot \frac{\mathrm{d}}{\mathrm{d}y} c'^{-1}(y) - c'(c'^{-1}(y)) \cdot \frac{\mathrm{d}}{\mathrm{d}y} c'^{-1}(y) = c'^{-1}(y) > 0. \tag{A.3}$$

Hence f is strictly increasing. Given that $y = pR(\alpha + z^{-1}(1-\alpha))$, $V_e = f(y)$ is strictly decreasing in z. Also, by Condition 3, $\underline{z} > 1 - \frac{\alpha}{\bar{\alpha}}$, so

$$\begin{aligned} \frac{\mathrm{d}(y + \eta\rho)}{\mathrm{d}z} &= \bar{\alpha}(pR + 1 - p + \gamma) - pR\frac{\bar{\alpha} - \alpha}{z^2} && \text{(A.4)} \\ &> \bar{\alpha}\Big(pR + 1 - p + \gamma - pR\frac{\underline{z}}{z^2}\Big) && \text{(A.5)} \\ &\geq \bar{\alpha}\Big(pR + 1 - p + \gamma - pR\frac{1}{\underline{z}}\Big) = 0, && \text{(A.6)} \end{aligned}$$

where the last equation follows by the definition of $\underline{z}$. Since $V_d = f(y + \eta\rho) - \xi$, this implies that V_d is strictly increasing in z.

Proof of Proposition 1. Define the utility differential between debt financing (V_d) and equity financing (V_e) as a function of the price ratio, $\Delta v : [\underline{z}, \bar{z}] \mapsto \mathbb{R}$. By Lemma 6, $\Delta v(z) = V_d - V_e$ is strictly increasing. At $z = \underline{z}$, $\eta = 0$ and $V_d = V_e - \xi$, so $\Delta v(\underline{z}) < 0$. Moreover, given that f is strictly increasing, for any $z > \underline{z}$, $V_d + \xi = f(y + \eta\rho) > f(y) = V_e$, so $\Delta v(\bar{z}) > 0$ whenever ξ is not too large. Since Δv is continuously differentiable by construction, by intermediate value theorem, equation $\Delta v(z) = 0$ has a unique solution $z^* \in (\underline{z}, \bar{z})$. By Lemma 5, at $z = z^*$, $V_d = V_e > V_s$, hence all institutions hold dynamic portfolios.

Given the optimal lending and financing choices, as well as trades in (9), market clearing requires $\lambda(\rho - (1-\alpha))x_d = (1-\lambda)(1-\alpha)x_e$, where λ is the fraction of institutions choosing $d_i = (1-\rho)x_d$. By Lemma 3 and Condition 3, $\rho - (1-\alpha) = \alpha - \bar{\alpha} + \bar{\alpha}z \geq \alpha - \bar{\alpha} + \bar{\alpha}\underline{z} > 0$, therefore $\lambda = \frac{(1-\alpha)x_e}{(1-\alpha)x_e + (\rho-(1-\alpha))x_d} \in (0, 1)$.

Proof of Lemma 7. In equilibrium $V_d = V_e > V_s$, or equivalently, $f(y+\eta\rho) > f(y+\eta\rho) - \xi = f(y) > f(y_0 + \gamma\rho_s)$. Since f is strictly increasing, this implies $y + \eta\rho > y > y_0 + \gamma\rho_s$. Therefore, $x_d = c'^{-1}(y + \eta\rho) > x_e = c'^{-1}(y) > x_s = c'^{-1}(y_0 + \gamma\rho_s)$.