HW 7, Numerical Analysis, Lena Kellner, due 27th Oct

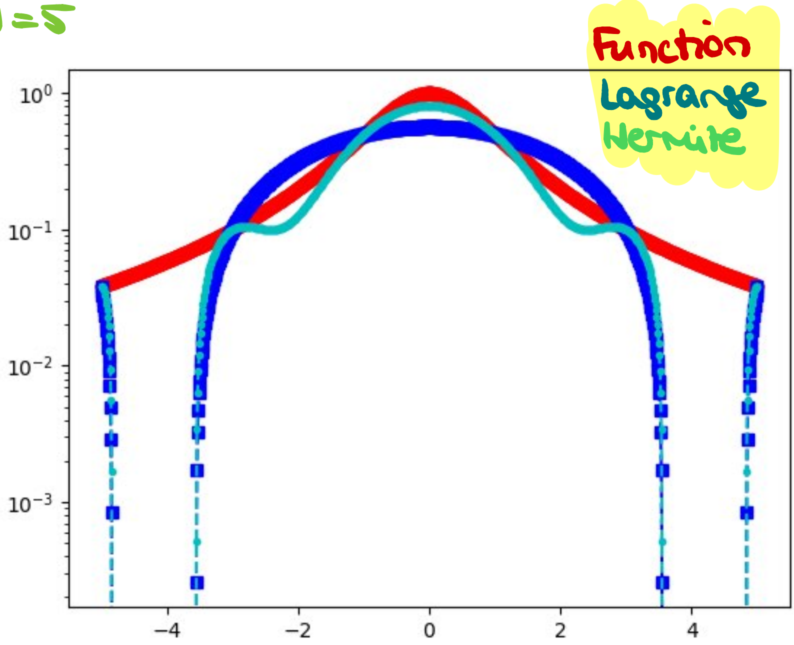
① By comparing the errors, it's clear that the cubic spline methods are better than Lagrange and Hermite. The more intervals one sets the clearer that
of the function in the middle of the interval
is – since the spline methods reachs better to the peak that way.
This is because the spline method looks at each interval as one while the other two methods try to find a general solution for the big interval.

The clamped spline performs slightly better than the natural one.
My guess would be that by adding the more data to matrix and vector b the approximation is more precise because of having more data to use.

Best
↓
Worst

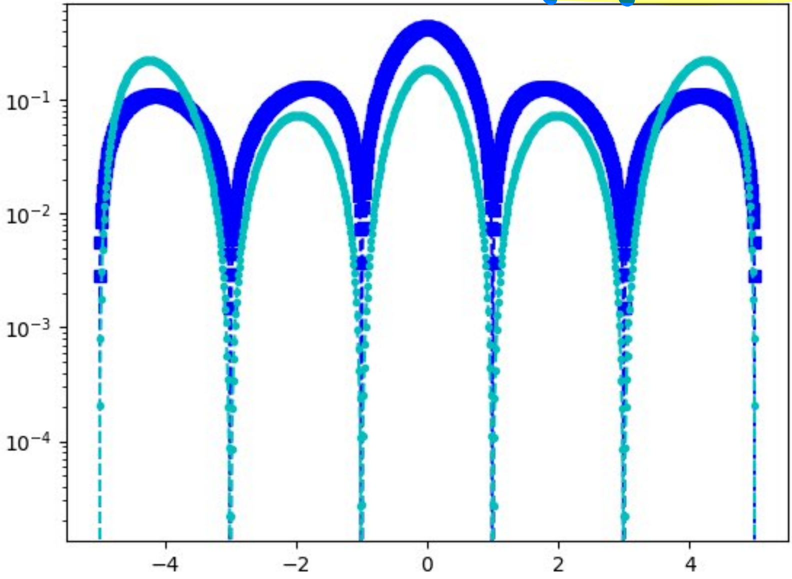
Clamped Spline
Natural Spline
Hermite ← Is more accurate than Lagrange for a bigger part of the interval. However the Runge effect is bad!
Lagrange

N=5

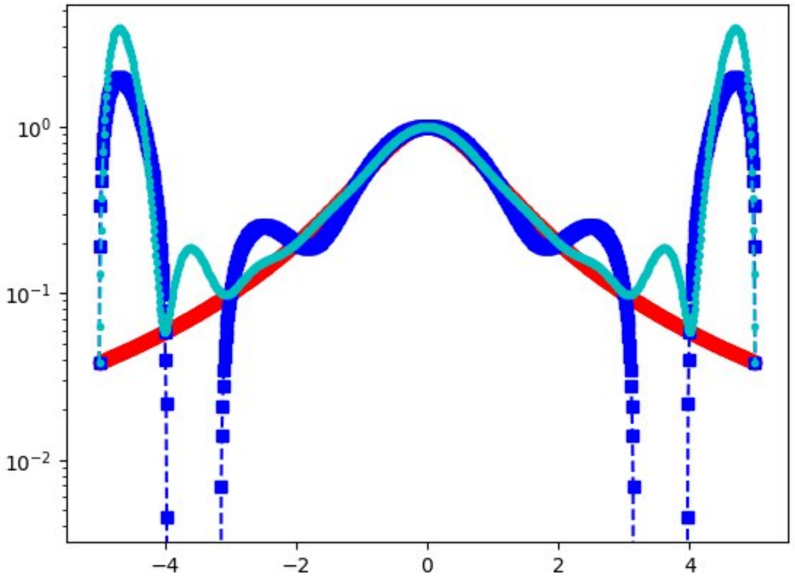


N=5

Error: Lagrange Hermite



N = 10



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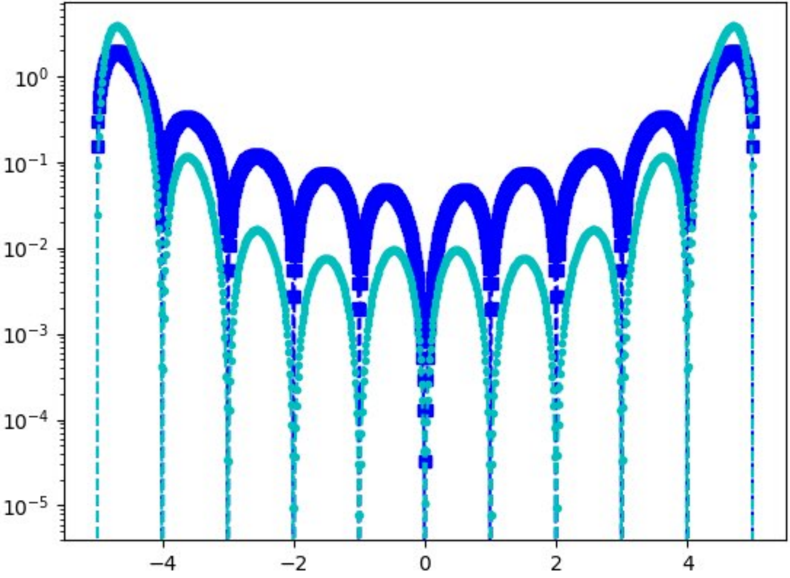
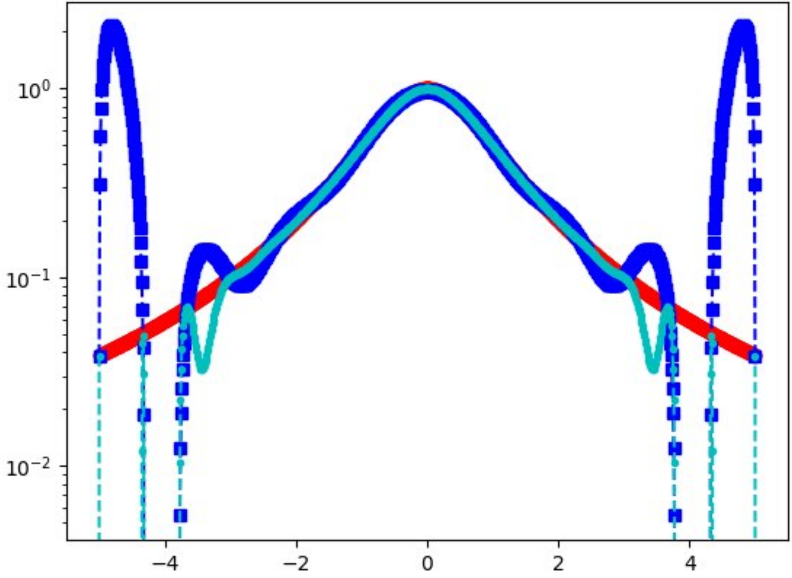
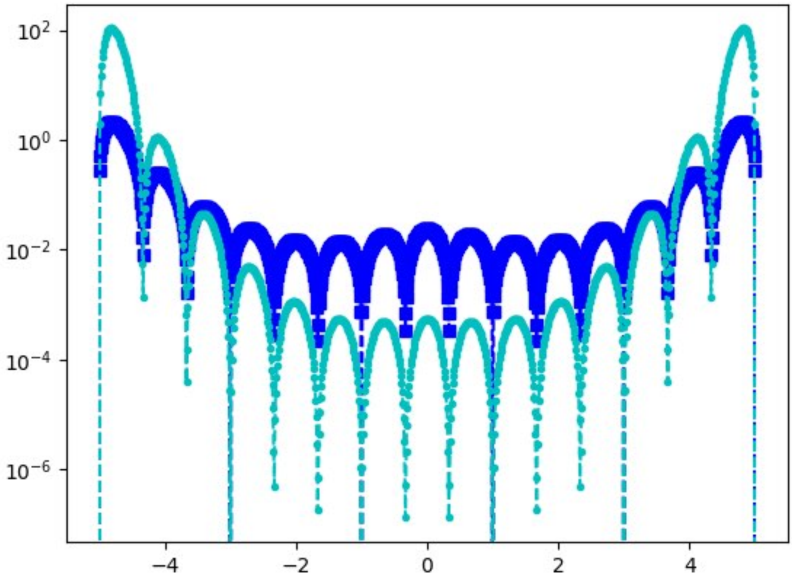


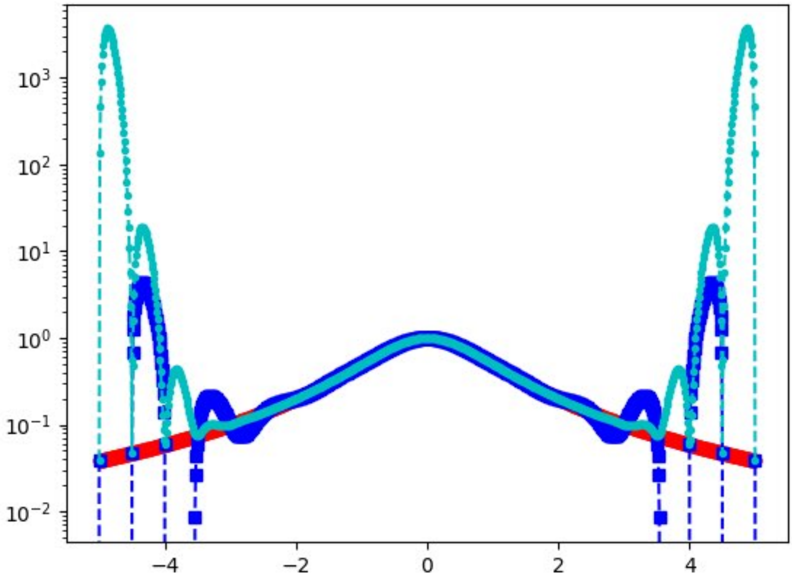
N = 15



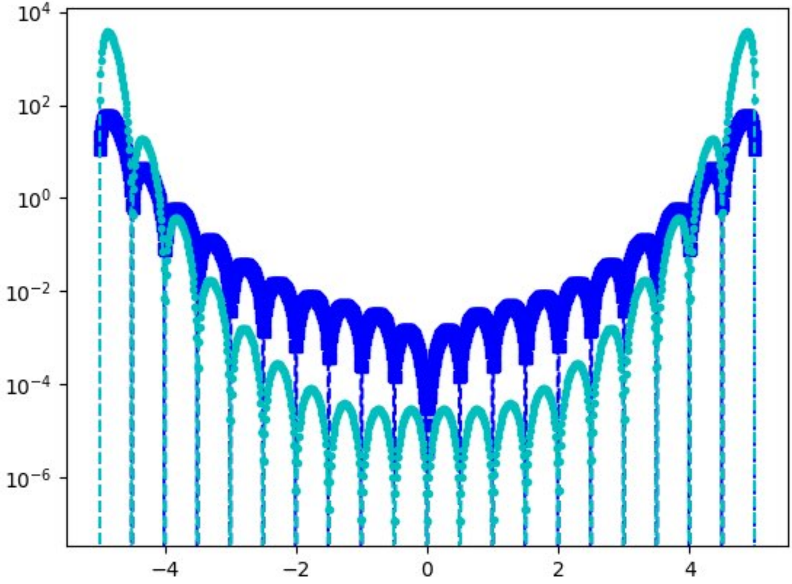
N=15



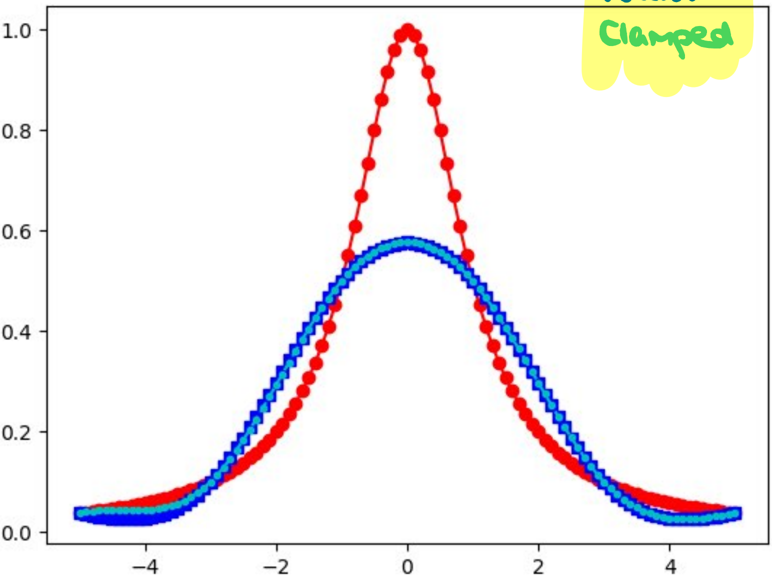
N=20



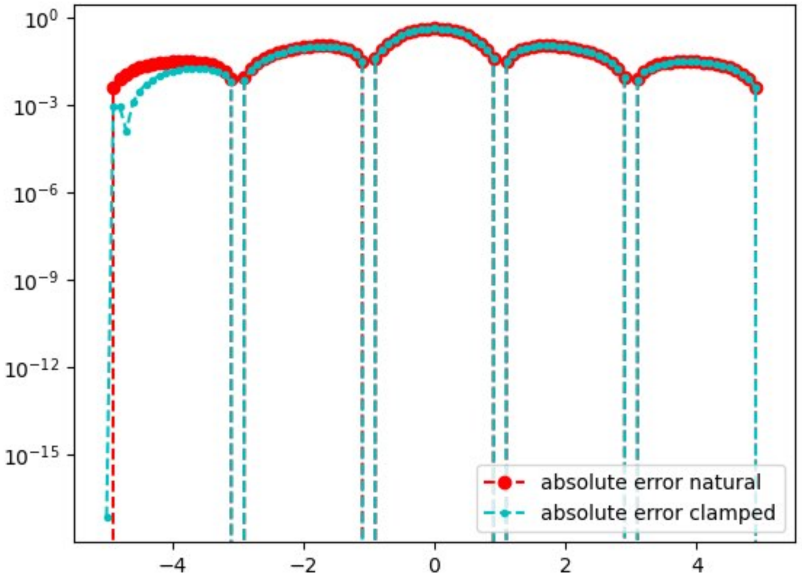
N = 20



N=5

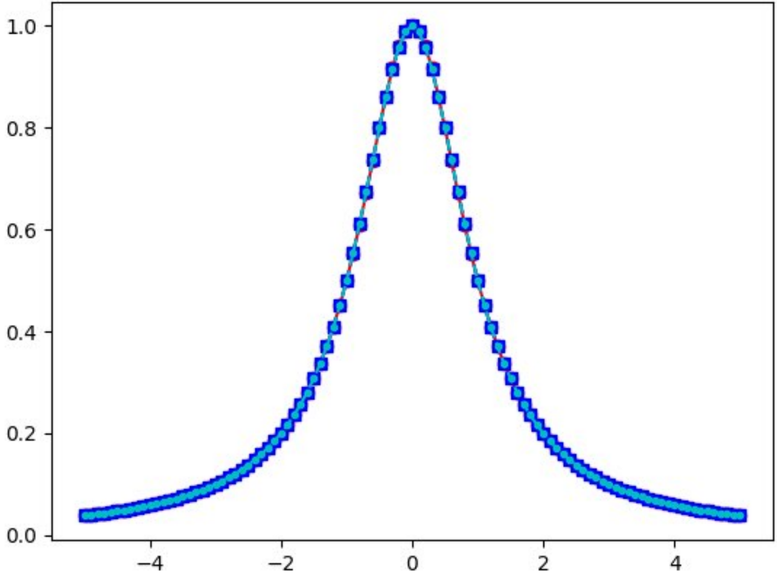


N=5

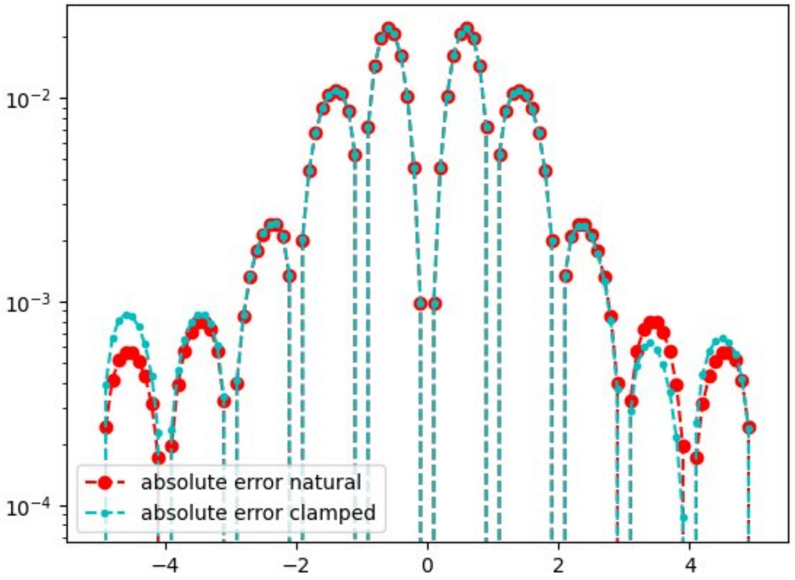


10^{0}
10^{-3}
10^{-6}
10^{-9}
10^{-12}
10^{-15}
−4
−2
0
2
4
absolute error natural
absolute error clamped

N = 10

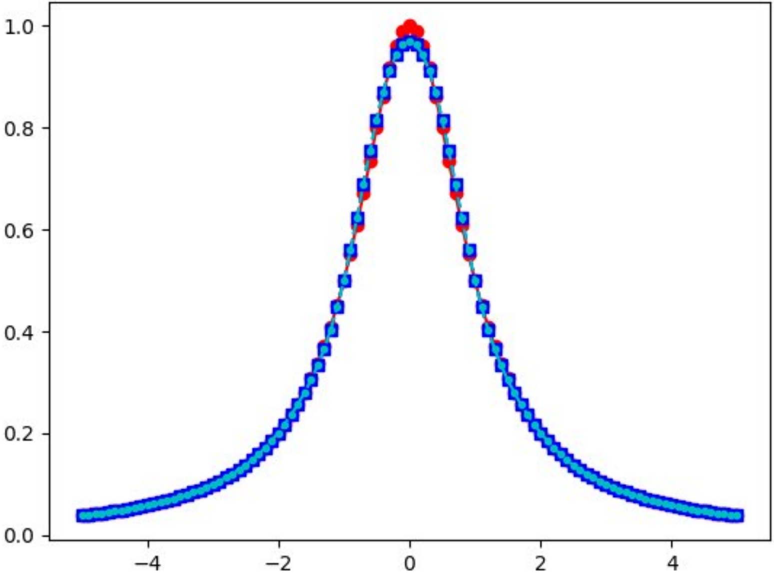


N = 10

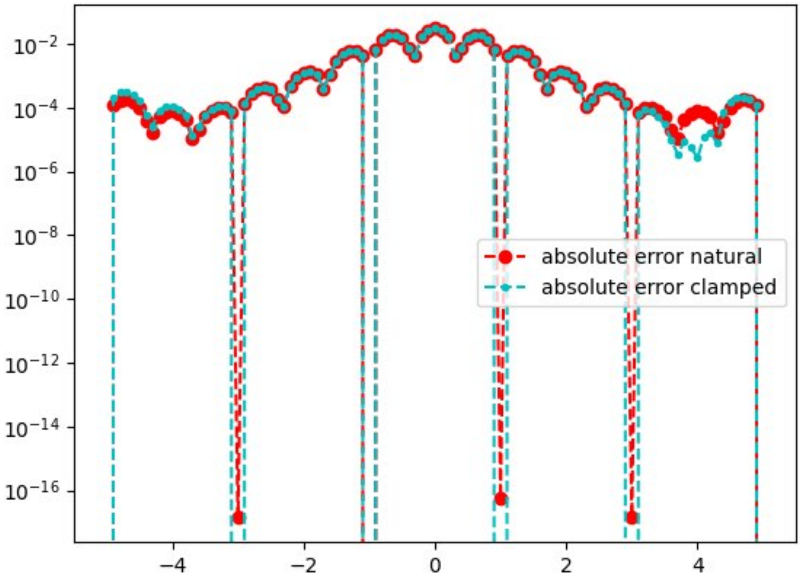


10^{-2}
10^{-3}
10^{-4}
−4
−2
0
2
4
absolute error natural
absolute error clamped

N=15

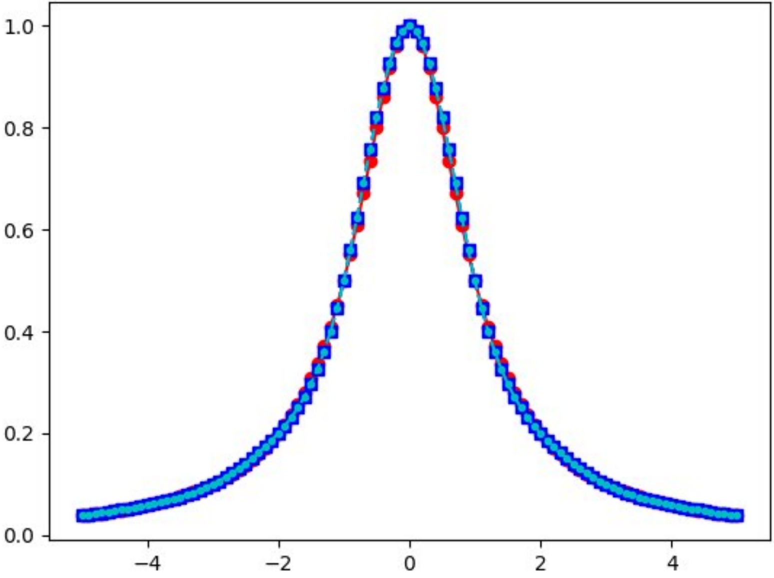


N=15

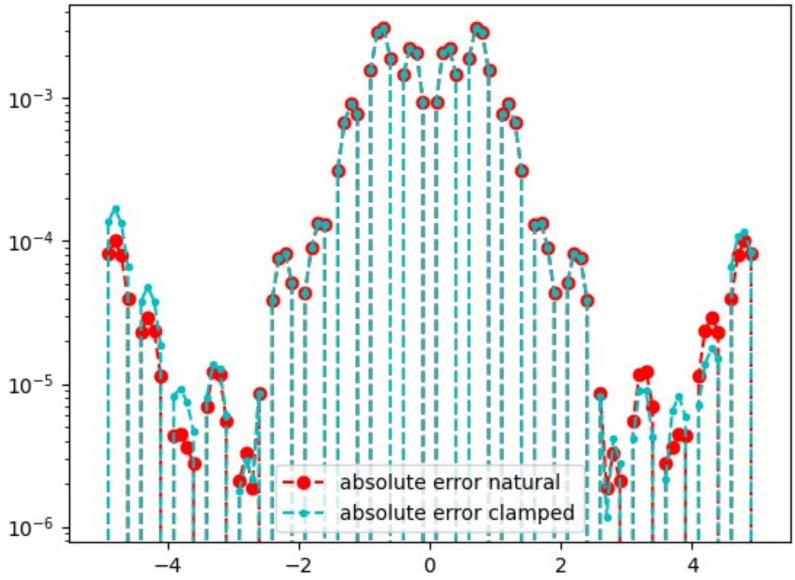


absolute error natural
absolute error clamped
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
10^{-12}
10^{-14}
10^{-16}
−4
−2
0
2
4

N = 20



N=20



absolute error natural
absolute error clamped
10^{-3}
10^{-4}
10^{-5}
10^{-6}
−4
−2
0
2
4

② By using Chebichev nodes the not-spline-methods perform way better. The runge effect is almost non-visible for $N=15$ or more. So, both Lagrange and Hermite perform very stable for $N=20$ and the error for Hermite even never is bigger than 10^{-3}.

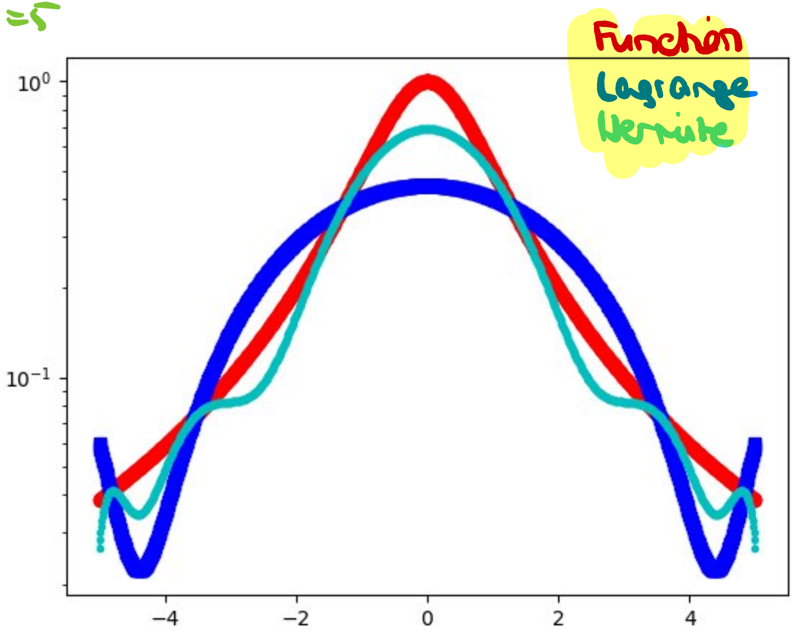
However the spline methods are not performing better with Chebichev nodes. On the ends of the intervals the spline methods still perform better than Hermite – but since in the midst of the interval the error of the spline methods is bigger than 10^{-3}, I would prefer using the Hermite method where the error does not fluctuate that much.

Best

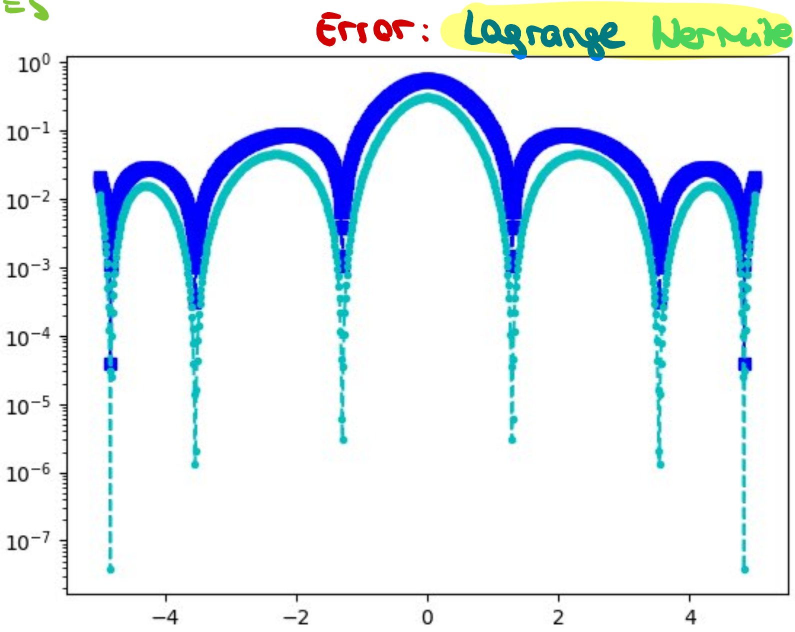
Hermite
Clamped Spline
Natural Spline
Lagrange

Worst

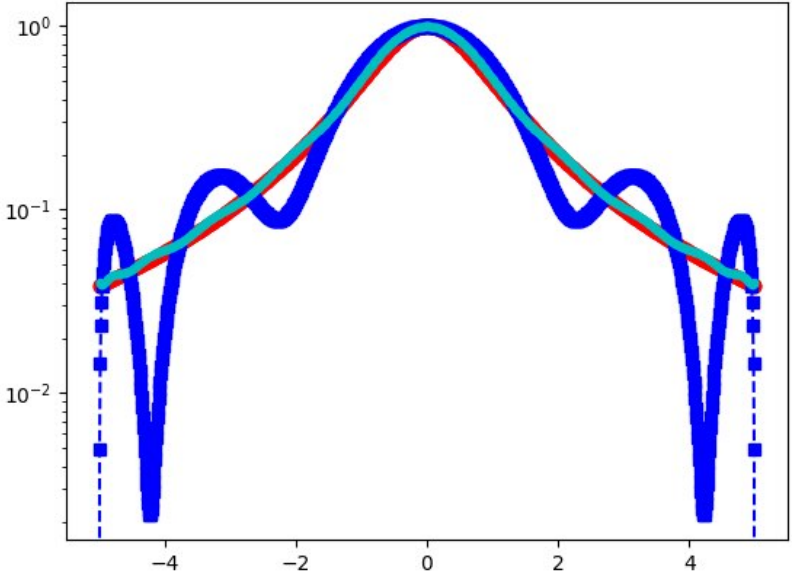
n=5



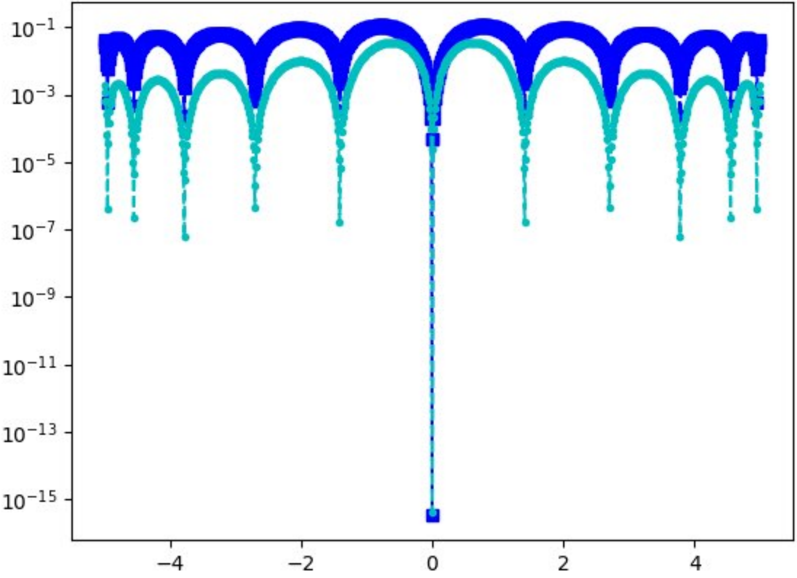
N=5



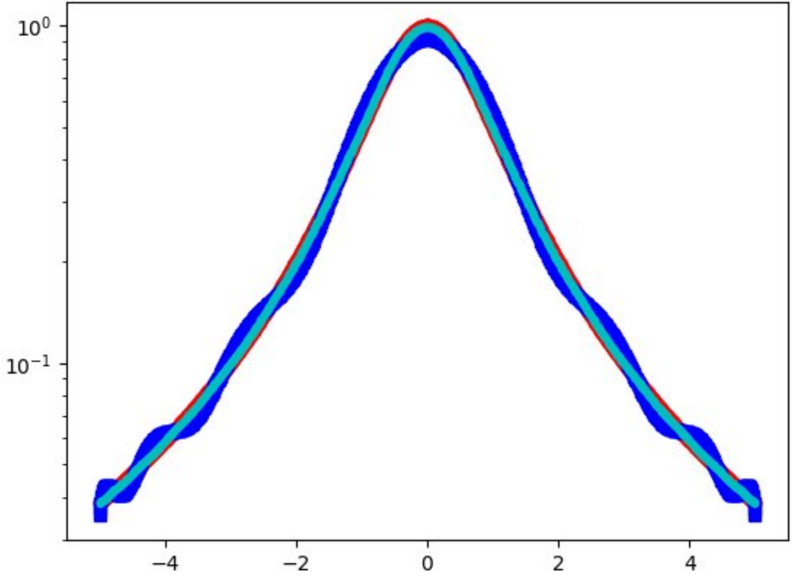
N=10



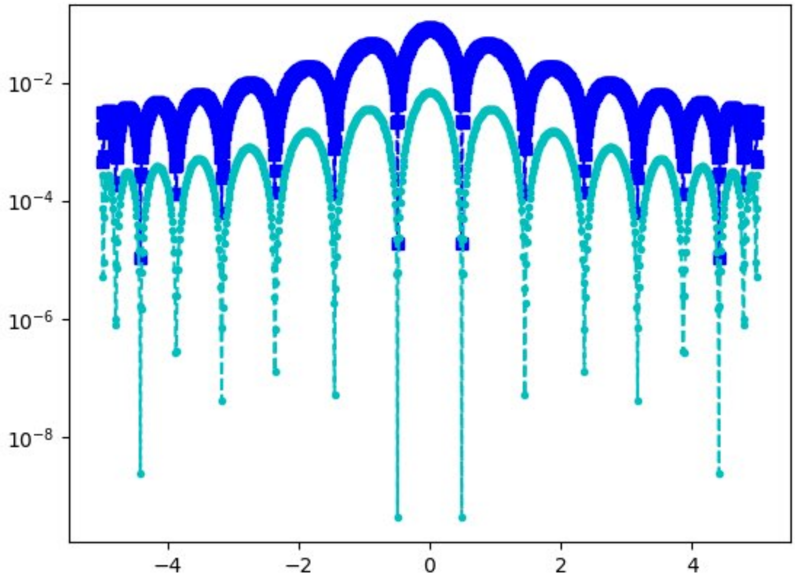
N = 10



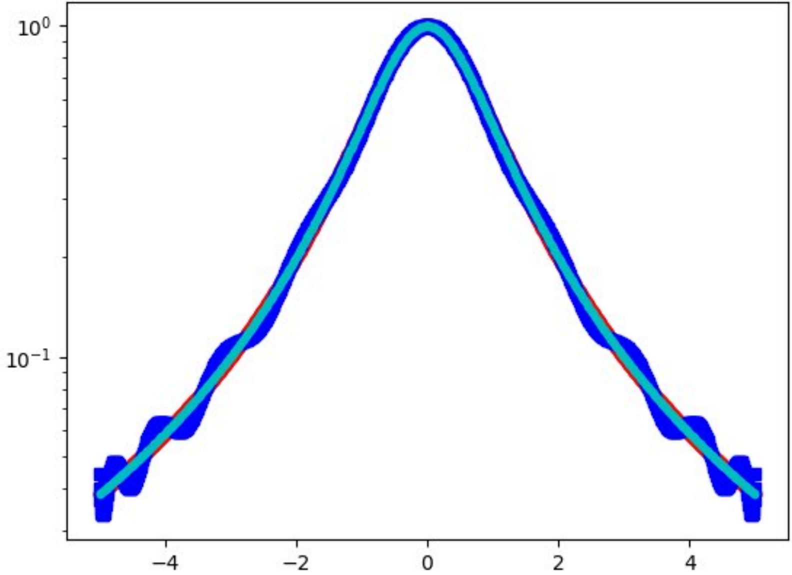
N = 15



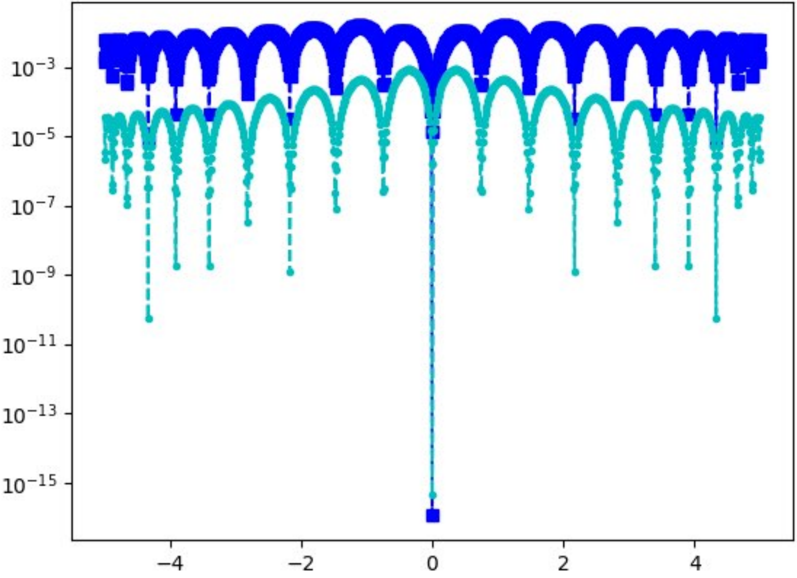
N=15



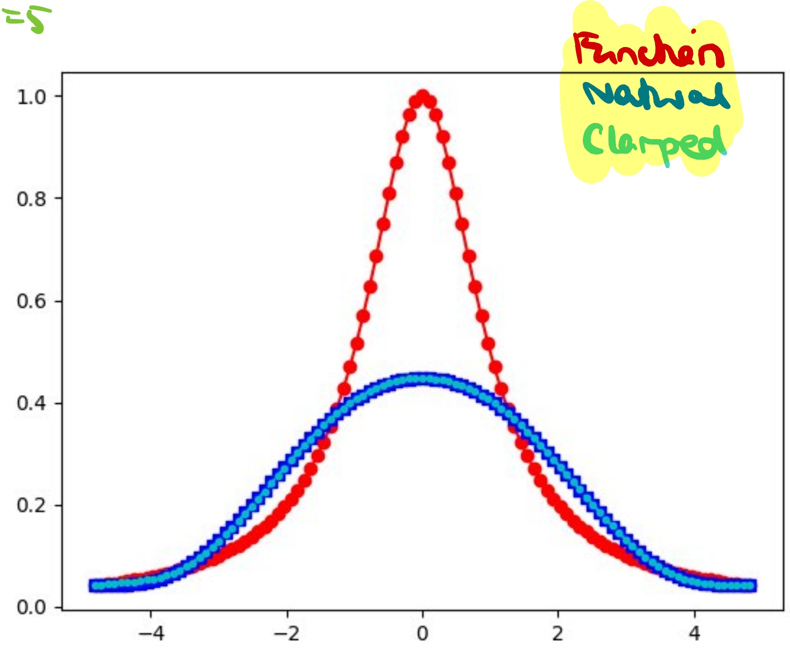
N=20



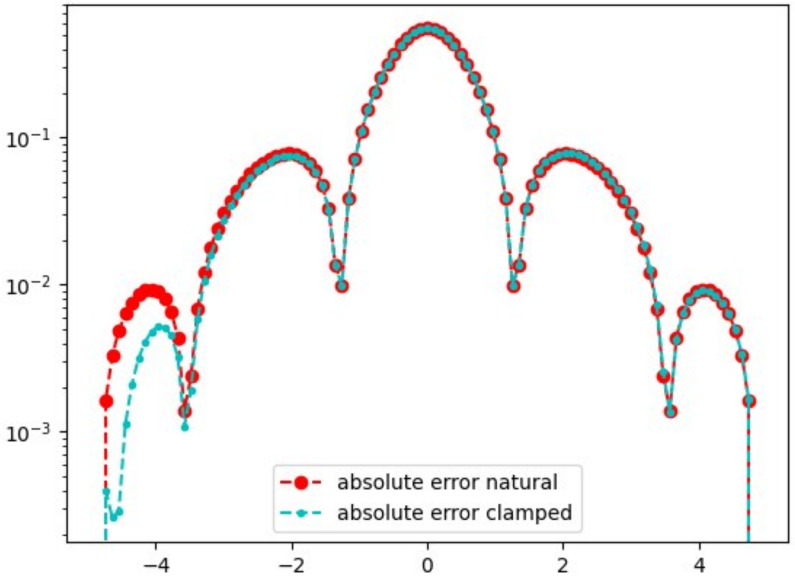
N=20



N=5

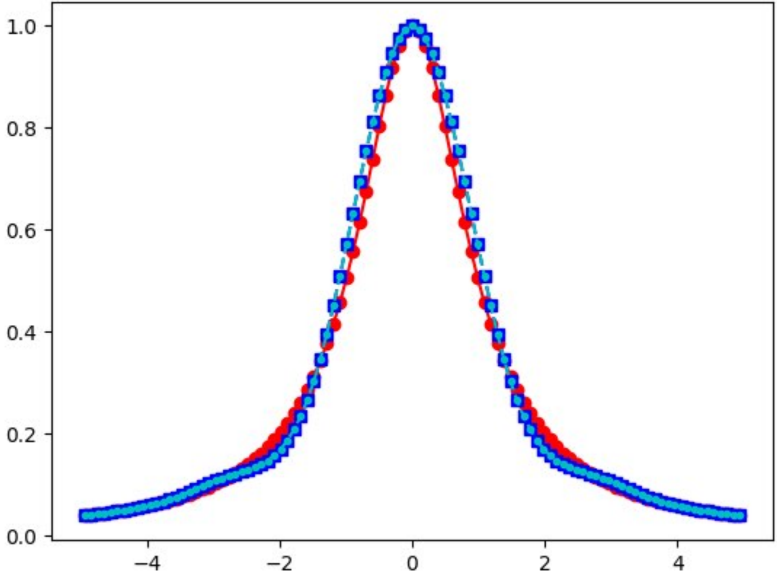


N=5

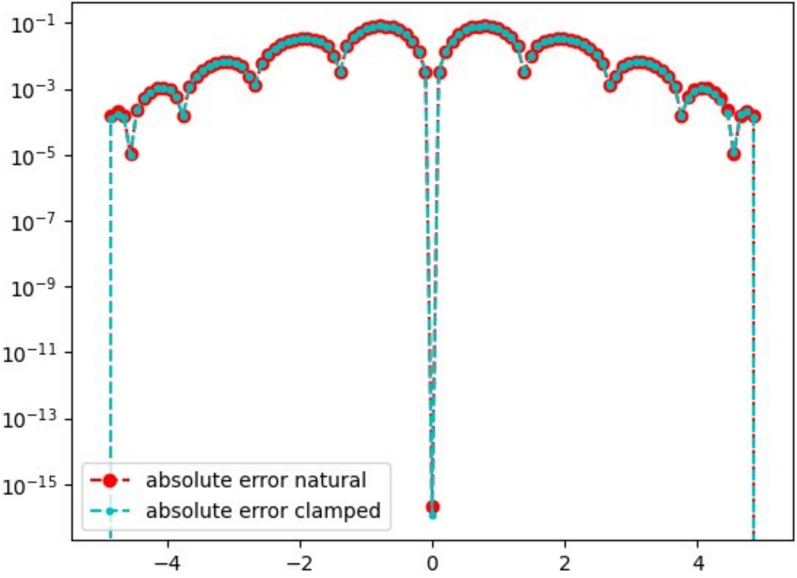


absolute error natural
absolute error clamped
10^{-1}
10^{-2}
10^{-3}
−4
−2
0
2
4

N = 10

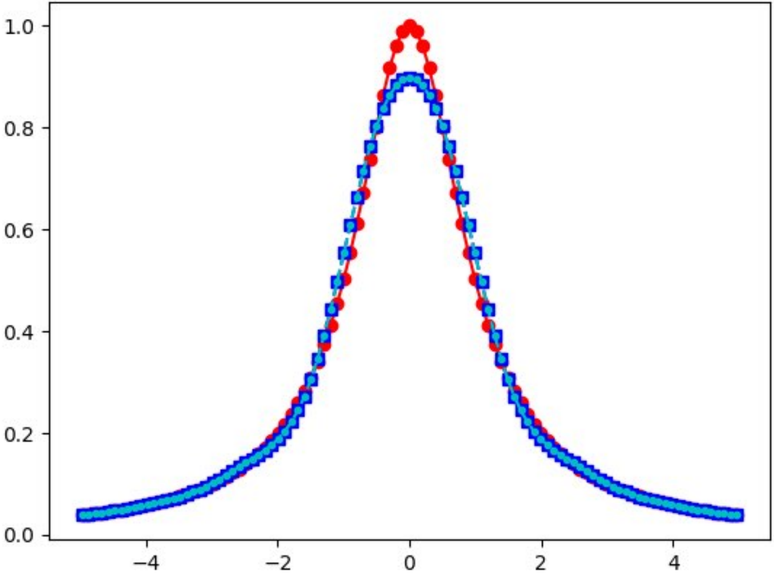


N = 10

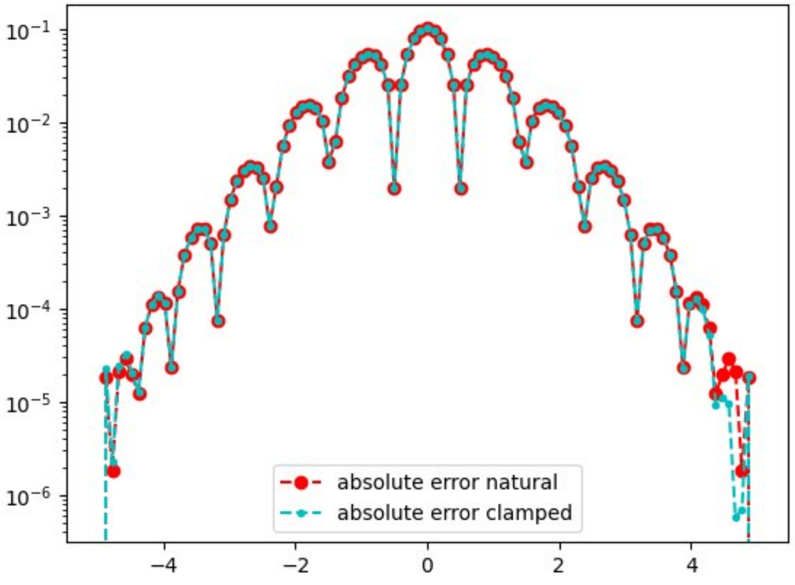


10^{-1}
10^{-3}
10^{-5}
10^{-7}
10^{-9}
10^{-11}
10^{-13}
10^{-15}
absolute error natural
absolute error clamped
−4
−2
0
2
4

N=15

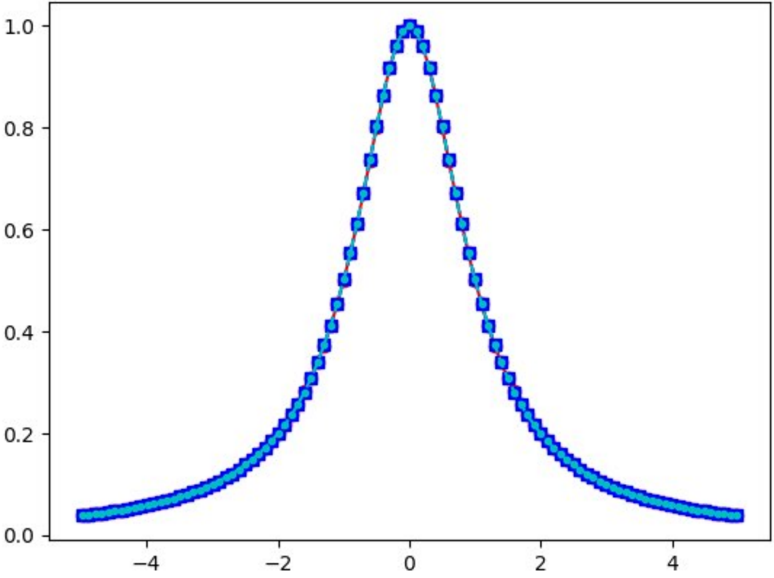


N=15

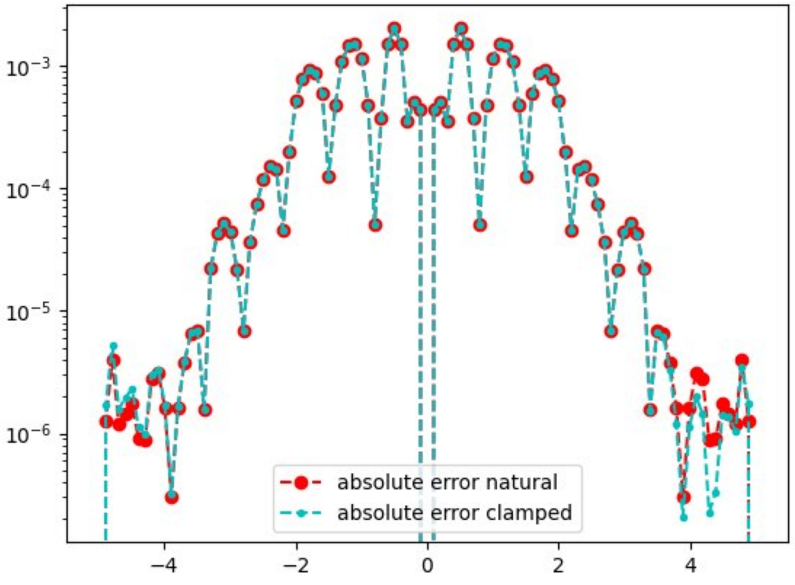


10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
−4
−2
0
2
4
absolute error natural
absolute error clamped

N = 20



N=20



10^{-3}
10^{-4}
10^{-5}
10^{-6}
−4
−2
0
2
4
absolute error natural
absolute error clamped

③ ① Calculate f(x), 1^{st}, 2^{nd} derivative – have to be the same on the end points:

$f(x) = \sin(10x)$

$\rightsquigarrow f(0) = 0$
$\quad\quad \shortparallel$
$\quad f(2\pi) = 0$

$f'(x) = 10\cos(10x)$

$\rightsquigarrow f'(0) = 10 \cdot 1 = 10$
$\quad\quad\quad\quad\quad\quad \shortparallel$
$\quad f'(2\pi) = 10 \cdot 1 = 10$

$f''(x) = -100\sin(10x)$

$\rightsquigarrow f''(0) = 0$
$\quad\quad\quad \shortparallel$ ✓ $\Rightarrow$ natural cubic spline
$\quad f''(2\pi) = 0$