HW 12, Numerical Analysis, Lena Kellner 1st Dec

(4) We have a system $\begin{bmatrix} 2 & -6\alpha \\ 3\alpha & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 \\ \frac{3}{2} \end{bmatrix}$.

So, we have 1 solution if $\left[\begin{array}{cc|c} \square & \square & \square \\ 0 & \square & \square \end{array}\right]$ with 0 = zero & $\square$ = non-zero-entry (c)

∞ solutions if $\left[\begin{array}{cc|c} \square & \square & \square \\ 0 & 0 & 0 \end{array}\right]$ (b)

no solution if $\left[\begin{array}{cc|c} \square & \square & \square \\ 0 & 0 & \square \end{array}\right]$ (a)

For (b) and (a) our goal is to have two zero entries in row two by applying Gaussian & choosing α:

$R2 - \mu \cdot R1 = 0 \quad \Leftrightarrow \quad [3\alpha \;\; -1] - \mu \cdot [2 \;\; -6\alpha] = [0 \;\; 0]$

$\Leftrightarrow \quad [3\alpha - 2\mu \;\; -1 + 6\alpha\mu] = [0 \;\; 0]$

$\Leftrightarrow \quad \begin{array}{c|l} I & 3\alpha - 2\mu = 0 \\ II & -1 + 6\alpha\mu = 0 \end{array}$

$\Leftrightarrow \quad \begin{array}{c|l} I' & \alpha = \frac{2}{3}\mu \\ II' & \alpha = \frac{1}{6\mu} \end{array}$

$\Leftrightarrow \quad I' = II' \;\Big|\; \frac{2}{3}\mu = \frac{1}{6\mu}$

$\Leftrightarrow \quad 12\mu^2 = 3$

$\Leftrightarrow \quad \mu^2 = \frac{1}{4}$

$\Leftrightarrow \quad \mu = \pm \frac{1}{2}$

$\Rightarrow \quad \alpha = \pm \frac{1}{3}$

(b) Case $\underline{\underline{\alpha = \frac{1}{3}}}$:
$$\left[\begin{array}{cc|c} 2 & -2 & 3 \\ 1 & -1 & \frac{3}{2} \end{array}\right] \xrightarrow{R2 - R1\cdot(\frac{1}{2})} \left[\begin{array}{cc|c} 2 & -2 & 3 \\ 0 & 0 & 0 \end{array}\right]$$

$$\Leftrightarrow \begin{array}{l|l} \mathrm{I} & 2x_1 - 2x_2 = 3 \\ \mathrm{II} & 0x_1 - 0x_2 = 0 \ \checkmark \end{array} \quad \Leftrightarrow \quad x_1 = \frac{3+2x_2}{2}$$

$\Rightarrow \infty$ solutions are possible

(a) Case $\underline{\underline{\alpha = -\frac{1}{3}}}$:
$$\left[\begin{array}{cc|c} 2 & 2 & 3 \\ -1 & -1 & \frac{3}{2} \end{array}\right] \xrightarrow{R2 - R1\cdot(-\frac{1}{2})} \left[\begin{array}{cc|c} 2 & 2 & 3 \\ 0 & 0 & 3 \end{array}\right]$$

$$\Leftrightarrow \left.\begin{array}{l|l} \mathrm{I} & 2x_1 + 2x_2 = 3 \\ \mathrm{II} & 0 \qquad\quad = 3 \ \lightning \end{array}\right\} \Rightarrow \text{No solution}$$

(c) In all other cases our 2x2 matrix $\begin{bmatrix} 2 & -6\alpha \\ 3\alpha & -1 \end{bmatrix}$ is full rank

since there is no μ s.t. $R2 - \mu \cdot R1 = 0$.

$\Rightarrow \underline{\underline{\forall \alpha \in \mathbb{R} \setminus \{-\frac{1}{3}, \frac{1}{3}\}}}$: There is one unique solution

② Substitution by $t = \frac{1}{x}$, $dt = -\frac{1}{x^2}\,dx$, limits: $\int_1^{\text{"}\frac{1}{\infty}\text{"}} = \int_1^0$, formula: $\int_a^b f(x)\,dx = \int_{\varphi(a)}^{\varphi(b)} f(\varphi(u))\cdot\varphi'(u)\,du \Big|_{x=\varphi(u)}$

Soln.: $\int_1^\infty \frac{\cos(x)}{x^3}\,dx = \int_1^0 t^3 \cos(\frac{1}{t})\cdot(-\frac{1}{t^2})\,dt = -\int_0^1 -\frac{t^3}{t^2}\cos(\frac{1}{t})\,dt = \int_0^1 t\cos(\frac{1}{t})\,dt$

$\Rightarrow$ Simpsons with $a = 0$, $b = 1$, $f(t) = t\cos(\frac{1}{t})$

$\Rightarrow$ Solution: $I \approx 0{,}026739$ with absolute error $1{,}973260$.

```
def driver():

    f = lambda x: x*(np.cos(1/x)) if x != 0 else 0
    a = 0
    b = 1
    n = 5
    # exact integral
    I_ex = 2

    I_simp = CompSimp(a,b,n,f)
    print('I_simp= ', I_simp)
    err = abs(I_ex-I_simp)
    print('absolute error = ', err)


def CompSimp(a,b,n,f):
    h = (b-a)/n
    xnode = a+np.arange(0,n+1)*h
    I_simp = f(xnode[0])

    nhalf = n/2
    for j in range(1,int(nhalf)+1):
        # even part
        I_simp = I_simp+2*f(xnode[2*j])
        # odd part
        I_simp = I_simp +4*f(xnode[2*j-1])
    I_simp= I_simp + f(xnode[n])

    I_simp = h/3*I_simp

    return I_simp
```

PROBLEMS OUTPUT DEBUG CONSOLE TERMINAL PORTS

```
PS C:\Users\kelln> & C:/Users/kelln/AppData/Local/Programs/Python/Pythor
I_simp=  0.026739761759940017
absolute error =  1.97326023824006
PS C:\Users\kelln>
```