(2) a) The function $f \in C^M[a,b]$ has a root at $\alpha \in [a,b]$ of multiplicity M

if and only if $f(\alpha) = f'(\alpha) = \ldots = f^{(M-1)}(\alpha) = 0$

and $\lim_{x \to \alpha} f^{(M)}(x) \neq 0$.

b) $g(x) = x - \dfrac{f(x)}{f'(x)}$

$$g'(x) = 1 - \frac{f'(x)^2 - f(x) \cdot f''(x)}{f'(x)^2} = 1 - \frac{f'(x)^2}{f'(x)^2} + \frac{f(x) \cdot f''(x)}{f'(x)^2} = \frac{f(x) \cdot f''(x)}{f'(x)^2}$$

$$g'(\alpha) = \frac{f(\alpha) \cdot f''(\alpha)}{f'(\alpha)^2} = \frac{(\alpha - \beta)^M \cdot q(x) \cdot}{}$$

$f(\alpha) = (\alpha - \beta) \cdot q(x)$

$$\begin{aligned}
f(x) &= (x-\alpha)^M \cdot q(x) \\
f'(x) &= M(x-\alpha)^{(M-1)} \cdot q(x) + (x-\alpha)^M \cdot q'(x) = (x-\alpha)^{M-1}\left(M \cdot q(x) + (x-\alpha) \cdot q'(x)\right) \\
f''(x) &= (M-1) \cdot (x-\alpha)^{M-2} \cdot \left(M \cdot q(x) + (x-\alpha) \cdot q'(x)\right) + (x-\alpha)^{M-1}\left(M q'(x) + q''(x)\right) \\
&= (x-\alpha)^{M-2} (M-1)\left(M \cdot q(x) + (x-\alpha) \cdot q'(x)\right) +
\end{aligned}$$

Goal: $0 < g'(x) < 1$

c) $g(x) = x - M \dfrac{f(x)}{f'(x)}$

$\vdots$

Goal: $g'(x) = 0$

$g''(x) \neq 0$

(3)

$\log(|x_{n+1} - a|) = y$

$\log(|x_n - a|) = z$

Def: convergence: $\dfrac{|x_{n+1} - p|}{|x_{n+1} - p|^\alpha} = \lambda$