Lab 2 Problems

15 January 2018

1 Bounds

Is there a single comparison-based sorting algorithm that will sort:

a) every sequence of 10 elements in at most 21 comparisons?

b) every sequence of n elements in at most $\frac{1}{2}log_2(n)$ comparisons?

c) every sequence of n elements in at most n^2 comparisons?

(Note: $10! = 3,628,800$ and $2^{21} = 2,097,152$.)

d) What is the minimum number of comparisons needed to sort a single list of n elements?

e) What is the minimum number of comparisons necessary to merge two sorted lists of 10 integers each?

f) Given sorted lists $A = (1, 3, 5, 7, 9)$ and $B = (6, 8)$, what is the minimum number of comparisons needed to merge the lists?

g) Give a sequence of comparisons meeting the bound in f).

2 Posets

a) Given a poset $\{a, b, c, d, e\}$ with $a \leq b, c \leq d, d \leq e$, how many total orders are possible?

b) Given a poset $\{a, b, c, d\}$, $a \leq b, a \leq c, a \leq d$, how many total orders are possible?

c) Given a poset $\{a, b, c, d, e\}$, $a \leq b, b \leq d, a \leq c, c \leq d$, how many total orders are possible?

3 Red-black Trees

a) Draw a red-black tree with black-height 3, a leaf at distance 3 from the root, and a leaf at distance 6 from the root.

b) Is there a red-black tree with a leaf at distance 3 from the root and a leaf at distance 7 from the root?